Finite Sample Properties of Nonstationary Binary Response Models: A Monte Carlo and Response Surface Analysis

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1. Introduction

A host of studies, beginning with Granger and Newbold (1974), have investigated the properties of linear model parameter estimators when some or all of the model variables are generated by nonstationary processes. This article builds on that work by extending the current research to cover latent variables generated by nonstationary stochastic processes. We find that, as with standard linear models involving nonstationary series, the commonly used tests almost always reject the null hypothesis of no relationship between x_t and y_t, even when they are, in fact, generated by independent random walks. Analogous to the outcome for fully observed nonstationary processes, spurious correlation may be avoided if at least one of the regressors is correlated with the independent variable. To our knowledge, it is the first empirical work to show that the problem of spurious regression extends to the realm of latent variable analysis.

This paper looks at the finite sample distributions of statistics commonly associated with binary models when regressors are generated by nonstationarity processes. Three cases are discussed. First, I examine the case where the latent variable and regressor are cointegrated processes.[1] Small sample parameter estimates and test statistics exhibit significant bias. Using response surface analysis, the probit estimates are shown to converge at n, reminiscent of the properties of cointegrating vectors associated with standard cointegrated models. Next, I study the case where the regressor and the latent dependent variable are generated by independent integrated processes. Probit parameter estimates and related test-statistics are shown to be unbiased, but diverge at rate n, again suggestive of the standard observed variable case. Finally, I examine the statistical properties of a three-variable model containing one latent variable. Of the two independent variables, only one is actually cointegrated with the latent variable while the other is independent. This case reveals the dual rates of convergence found by Park and Phillips (1999b) in their asymptotic analysis of nonstationary binary choice models. The results show that when one of the variables is cointegrated with the latent variable, spurious inference may be avoided.

2. The Basic Model

Consider the following data generating process (DGP):

$$\begin{aligned} x_t &= x_{t-1} + u_t \\ y_t^* &= \beta x_t + v_t \end{aligned} \tag{2.1}$$

where u_t is a stationary random vector and v_t is a stationary random variable, with covariance matrices Σ_u and Σ_v, respectively. For non-zero values of β, x_t and y^*_t are cointegrated and a large body of research suggests methods for estimating β and its associated t-statistics; see Hamilton (1994) for a thorough discussion. In contrast, we

[1] Two I(1) processes are said to be cointegrated if some linear combination of the variables is a stationary I(0) process (Hamilton 1994).

examine the latent dependent variable case where the y_t is observed only when y_t*

exceeds some threshold value α so that:

$$y_t = \begin{cases} 1 \ for \ y^*_t > \alpha \\ 0 \ for \ y^*_t \leq \alpha \end{cases} \tag{2.2}$$

This DGP could be used to characterize a time-based index function model with index function $\beta x_t - \alpha$. Assuming a normal distribution for v_t:

$$P(y^*_t > \alpha) = P(\beta x_t + v_t > \alpha) = P(v_t < \beta x_t - \alpha) = \Phi(\beta x_t - \alpha) \tag{2.3}$$

The conditional log likelihood is for the standard (independent errors) probit model (Amemiya 1985):

$$\log L = \sum_{i=1}^{n} \{ y_i \log \Phi(\beta' x_i - \alpha) + (1 - y_i) \log[1 - \Phi(\beta' x_i - \alpha)] \} \tag{2.4}$$

Parameter estimates are typically estimated using Newton's method. For the probit model the matrix of second derivatives is calculated as:

$$H = \frac{\partial^2 \log L}{\partial \beta \partial \beta'} = -\sum_i \frac{q_i \phi(q_i \beta' x_i)}{\Phi(q_i \beta' x_i)} \left[\frac{q_i \phi(q_i \beta' x_i)}{\Phi(q_i \beta' x_i)} + \beta' x_i \right] x_i x_i \tag{2.5}$$

where $q_i = 2 y_i - 1$.

Previous authors have estimated the parameters of this model using logit or probit, assuming v_t is independent and identically distributed as a normal or logistic random variable, respectively (Alm and Whittington 1995, Alm, McKee and Skidmore 1993). Other articles have proposed estimators for cases where error term, v_t, is serially correlated (White 1994). Park and Phillips'(1999b) paper investigated the asymptotic properties of the model parameters and test statistics when the model is correctly specified.

Amemiya (1985) shows that for the cross-section data that is typically studied in the literature $\sqrt{n}(\hat{\beta}-\beta_0) \rightarrow N(0, H^{-1})$. The Hessian is negative semi-definite and the function is globally concave. These results rest upon the assumption of the independence amongst the observations. For time series applications, independence assumptions are largely invalid. In the extreme case of dependence, latent nonstationarity, Park and Phillips (1999b) show that the H converges to a random limit matrix, and is therefore not constant. The matrix is, however, almost surely negative definite and the limit function is globally concave leaving open the possibility of convergent ML models. They also find that the though probit-based model parameters are consistent, a different rate of convergence applies to nonstationary regressors than when regressors are stationary. Interestingly, dual convergence rates are observed, with $n^{3/4}$ rate convergence in the direction of the null hypothesis, and $n^{1/4}$ rate convergence applying in all other directions.

Park and Phillip's (1999b) findings support the conjecture that ML estimators and resulting test statistics for nonstationary models may have very different properties from those found in stationary models. Although insightful, their investigation addressed non-cointegrated and correctly specified models in an asymptotic framework. In practice, some examination of spurious regression and cointegration could be very useful for econometricians. If spurious regression is a weakness of nonstationary ML models, then a theory of cointegration would be exceedingly useful. Therefore, in the next sections, we investigate finite sample properties of ML estimators and the rate of convergence for parameter distributions under first, the cointegrated case, the independence case, then finally a case where the model is partially cointegrated.[2]

3. The cointegrated case

To investigate parameter test statistics when the latent variable is cointegrated with the independent variable, 24,000 replications were generated for the model:

$$\begin{aligned} x_t &= x_{t-1} + u_t \\ y_t^* &= \beta x_t + v_t \\ y_t &= \begin{cases} 1 \ for \ y^*_t > \alpha \\ 0 \ for \ y^*_t \le \alpha \end{cases} \end{aligned} \tag{3.1}$$

Where x_t, u_t, v_t, y_t and y^*_t are nx1 vectors indicating the two variable model with one observed variable and one latent variable. The series v_t and u_t are distributed as N(0,1), making x_t and y^*_t I(1) cointegrated processes for nonzero values of β. We allow α to range from

0.2 to 0.9 and β values from -0.1 to -0.9. The sample size, n, ranges from 25 to 245 in increments of 20. There are 10 experiments total, with 12 values of n each replicated 2,000 times, giving 24,000 observations for the cointegrated case. The parameters of the model are estimated using probit and the BHHH maximization algorithm in GAUSS. The covariance matrix of the parameters is computed from the quasi-maximum

[2] Independence in the nonstationary model means independence between the latent variable and the regressor rather than independence amongst the observations.

likelihood covariance matrix of the parameters[3]: $\hat{A}^{-1}\hat{B}\hat{A}^{-1}$ where $\hat{A} = \frac{1}{n}\sum_{i=1}^{n}\frac{\partial^2 L_i}{\partial\theta\partial\theta'}$ and

$$\hat{B} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\partial L_i}{\partial\theta}\right)'\left(\frac{\partial L_i}{\partial\theta}\right).$$

Figure 1 charts the kernel density of the ML probit estimate of β for β= -0.15 and β=-0.9 at sample sizes of 24 and 245. The distribution is clearly right skewed for β= -0.9, although the skewness falls as the sample size rises. For β= -0.15, the asymmetry is very mild at n equal to 25, and imperceptible at the larger sample size.

The standard error of β and the empirical means of the distribution of β are shown in Table 1 for different values of and β and n. The ML probit estimate of β is biased in small samples. The bias tends to diminish as the sample size grows. A rather unique feature of the distribution is its clear dependency on the value of β. As the absolute value of β increases, reflecting more certainty in the cointegrating relationship between x_t and the latent variable y^*_t, the bias becomes stronger. From this result, we infer that sample sizes must be very large (greater than 250 observations) for the asymptotic consistency to apply when the model is strongly cointegrated.

The standard errors of β are positively correlated with β. The response surface regression below indicates that the distribution of β converges at rate n-- a rapid rate relative to the convergence rate expected in the stationary case of $n^{1/2}$.

$$\begin{aligned} se(\beta) &= -0.224 + 0.475\beta \;\; + 0.472n^{-1/4} + 4.865n^{-1} \\ &\quad\;\; (0.039)\;\; (0.010) \quad (0.156) \qquad (0.877) \qquad\qquad (3.4) \\ R^2 &= 0.960 \end{aligned}$$

[3] Amemiya (1985) shows that $\hat{\theta} \to N(\theta, A^{-1}BA^{-1})$. When the equation is correctly specified plim(A)=plim(B) and $\hat{\theta} \to N(\theta, \hat{A}^{-1})$ (Gauss Maximum likelihood Applications Module). Because we

Unlike the distribution of β arising from probit estimation of related stationary series, the standard error falls as β nears 0 and the strength of cointegration wanes. In the extreme case, the regression suggests that asymptotic standard errors become negative for the independence case. This is of course, not possible. Rather it implies that the relationship exhibited in 3.4 breaks down as β approaches 0, the independence case.

Skewness in the distribution of β implies skewness in the associated z-statistic, as well. To investigate the properties of the z-test for ML estimation of nonstationary models, we calculate z statistic $(\hat{\beta}-\beta_0)/\hat{\sigma}_\beta$ for each replication,. The percentiles of the z-statistic for the two-variable cointegrated case are reported in Table 2. The critical values indicate that strong deviations from the normal distribution are not found in the cointegrated case. Nevertheless, differences do exist. When plotted against the cumulative probability function of the normal distribution using a quantile-quantile plot, the distribution is platykurtic, having thinner tails than the standard normal distribution. The departure is not extreme, however. More importantly, there is evidence of right skewness in the distribution. For β close to zero (β=0.1 or .2), the empirical test statistic for the 50% percentile is slightly negative in 4 of the 6 cases. However, at higher absolute values of β, a strong negative bias is observed with virtually all of the test-statistics. The situation is reversed when moving toward the left tail. Here, for β close to zero, the z-statistic is very close to the value expected from the normal distribution. As β increases

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are interested in finite sample results, we utilize the quasi-maximum likelihood covariance matrix.

in absolute value, upward bias is observed in the lower tail quantiles. The skewness becomes more pronounced at larger sample sizes[4].

To evaluate the tails of the distribution, we ran response surface regressions of the form:

$$C_i(p) = \gamma_0 + \gamma_1 \alpha_i + \gamma_2 \beta_i + \gamma_{3j} f_{ij}(n) + \varepsilon_i \tag{3.1}$$

where $C_i(p)$ is the pth percentile of the ith experiment and $f_{ij}(n)$ are inverse polynomial functions of n such as n^{-1}, $n^{-1/2}$, $n^{-3/4}$ and so on. These regressions did not exhibit any explanatory power for the right tail percentiles of the distribution. Percentiles of the left tail, for probabilities close to 0, are correlated with the cointegrating coefficient β so that:

$$\underset{}{\hat{C}(.01)} = \underset{(0.037)}{-2.85} - \underset{(0.029)}{0.628\beta_i} + \underset{(0.107)}{1.83n^{-1/4}} \tag{3.2}$$

Equation 3.2 suggests that, for values of approaching zero, that the statistic converges to an asympotic value of –2.85, somewhat larger than the statistic associated with the normal distribution. However, as the strength of cointegration increases, the left tail moves closer to the mean value, and the distribution begins to take on the skewed characteristic evidenced in Table 2.

The apparent right skewness of the z distribution under the cointegrated case and small samples will impact inference. Normal based inference will be conservative because we will be more likely to accept the null hypotheses than if we had the empirical distribution from Table 2. This will be particularly true for test statistics falling below the mean and for two-tail tests. For instance, when using two-tail tests for independence,

[4] This is supported by regressions of the left tail empirical critical values using powers of the sample size as independent variables.

we will be more likely to fail to reject the null of no correlation between the latent variable and the regressor than if we knew the true distribution.

Table 3 presents the empirical Wald statistic for the null hypothesis that $\alpha=\beta=0$ under the cointegrated case.

Like the z-statistics, the empirical distribution of the Wald statistic does not deviate in a dramatic fashion from its asymptotic distribution, the chi-square. Nevertheless, like the z-statistic, the tail percentiles vary with the degree of cointegration between the independent and latent variable. Equation 3.3 below reveals that the 0.99 percentile of the Wald statistic, W(.99), rises with the level of cointegration. For values of β close to -1, the asymptotic Wald statistic approaches 8.28, somewhat less than the chi-square value. The distribution converges to the asymptotic distribution at rate of n. The impact of sample size is dampened, particularly in small samples, by the $n^{-1/4}$ term in the regression.

$$\underset{}{\hat{W}(.99)} = \underset{(0.907)}{6.707} - \underset{(0.262)}{1.573\beta_i} + \underset{(3.633)}{7.461n^{-1/4}} - \underset{(20.187)}{67.44n^{-1}} \tag{3.3}$$

In review, several pertinent observations arise from the Monte Carlo experiment performed on the cointegrated case. First, small sample parameter distributions are right skewed, making normal-based inference conservative for two-tail tests. The skewness diminishes with sample size. Perhaps more importantly, the bias and asymmetry diminish as β approaches zero. This fact, together with the well-known result that OLS potentially understates parameter variances when faced with nonstationarity, suggests that the results for the standard errors and z-statistics may be entirely different for the independence case than for the cointegrated case. Therefore, we turn to an investigation of the independence case

4. Independence Case

In case two, we investigate the independence model where β=0, specifically:

$$x_t = x_{t-1} + u_t$$
$$y_t* = y*_{t-1} + v_t \tag{4.1}$$
$$y_t = \begin{cases} 1 \text{ for } y^*_t > \alpha \\ 0 \text{ for } y^*_t \leq \alpha \end{cases}$$

Table 4 reports the standard errors of the cointegrated case. The table also lists the empirical sizes of a z-test of the null hypothesis that the parameter in question equals zero with nominal size 0.05. The results are quite striking. For both the intercept and slope term, the probability we reject the null hypothesis when it is, in fact, true actually increases with the sample size.

For the β coefficient, at sample sizes over 200, we reject the true null hypothesis over 65% of the time. In other words, over 65% of the time, we are likely to infer correlation that is entirely spurious. Further, at large samples, we are likely to estimate non-zero threshold term, α, even when the actual threshold is zero.

The response surface regression in 4.2 reveals that the fast n rate of convergence associated with OLS estimation of nonstationary series applies in the latent variable case as well. This result, together with the severe size distortions that are observed for the z – statistic imply that extreme caution must be taken when drawing inference from probit-based regressions of latent variable models subject to nonstationarity. More often than not, using standard t-statistics will lead to erroneous inference, and one will accept apparent relationships between entirely unrelated series.

$$se(\beta) = 0.016 + 8.168n^{-1}$$
$$\quad (0.002)\ (0.162) \tag{4.2}$$
$$R^2 = 0.996$$

The empirical z-statistics are reported in Table 5 for increasing values of n. As expected, the empirical values are greatly exaggerated. Even for small sample sizes, the likelihood of rejecting a true null hypothesis is disturbing large if the critical values are mistakenly taken from the normal distribution. For modest sample sizes in excess of 100, the empirical z-statistic is 2-3 times that taken from the corresponding normal. At samples greater than 100 the distortion causes the empirical statistic to exceed the normal by a factor of 3 to 4.

We recall the seminal works by Granger and Newbold (1974) and Phillips (1986) that showed that for the model $y_t = \alpha + \beta x_t + u_t$ unless some cointegrating value of β exists so that u_t is I(0), OLS is likely to produce spurious results. In the observed variable case, the existence of a cointegrating value suggests a cure for spurious correlation: evaluate the stationarity properties of the error term. If the error term is stationary, OLS parameters estimates are super-consistent converging at rate n rather than the $\sqrt{n}$ rate of expected when the model variables are stationary (Hamilton 1994).

Unfortunately, simple error-based tests for cointegration are not available for the latent variable case, because the error term is unobservable. Further, standard cures for spurious regressions such as first differencing are also inappropriate due to the binary nature of the independent variable. This poses a serious dilemma for practitioners using latent variable models based on nonstationary series. When regressions are spurious, the standard errors and the test-statistics diverge as the sample size increases. As such, until methods are developed that are capable of correcting for spurious regressions in latent variable models, all parameter estimates based on nonstationary series are highly suspect.

Fortunately, the result obtained in case 1, the cointegrating regression, suggests a situation where the distribution of the probit parameter is better behaved than when the model is spurious. If at least one of the model regressors is cointegrated with the latent variable, Park and Phillips (1999b) show that the parameter estimates are consistent and asymptotically normal. In the next section, we investigate the small sample properties for partially correctly specified models. We show that spurious results are much less likely if some cointegration exists in the model.

5. Three Variable Mixed Independent and Cointegrated Case

In our final case, of the two nonstationary independent variables, only one is cointegrated with the latent variable. Here, we hope to observe the dual rates of convergence found by Park and Phillips (1999b). The DGP is:

$$x_{1t} = x_{1,t-1} + u_{1,t}$$
$$x_{2t} = x_{2,t-1} + u_{2t}$$
$$y_t^* = \beta_1 x_{1t} + \beta_2 x_{2t} + v_t$$
$$y_t = \begin{cases} 1 \ for \ y^*_t > \alpha \\ 0 \ for \ y^*_t \le \alpha \end{cases}$$

We let β_1 vary between -0 .1 and -1 while holding β_2 constant at 0.

Table 6 reports the empirical means and standard errors of β_1 and β_2 for different values of the parameters and different sample sizes. The results in the three variable case are closer to the cointegrated case than the independence case. The estimate of β_1 is strongly biased in small samples. For instance, the mean estimate for β_1 is –0.2614 for an actual parameter value of –0.20, a difference of approximately 30%. The bias diminishes as the sample size grows. Nevertheless, even at relatively large sample sizes, the ML estimate is still 10 percent off the true parameter value, on average. The estimate of β_2, equal to zero for all of the replications in the three variable case, is apparently unbiased.

According to Park and Phillips (1999) dual rates of convergence should be found in the model. Convergence to the null distribution (in our case β_2) proceeds at a rate of $n^{3/4}$ and convergence in all other directions, including the true parameter vector, at a rate of $n^{1/4}$. The dual convergence rates are evident in Figure 2. Here, the parameter vector, β, is (1,0)', $\beta^{\perp}$ is (0,1)'and $x_t'\beta_t = x_{1t}\beta_1$. For β_1=-1, the parameter converges at a relatively slow rate compared to the orthogonal direction, β_2=0. This particular case is charted in Park and Phillips (1999) paper for sample sizes 100 to 500. In their results, the right skewness of the distribution is not apparent. At the smaller sample sizes depicted here, the asymmetry dominates the β_1 distribution, just as the β_2 distribution retains its symmetrical shape for all sample sizes.

The asymmetry of the distribution of β_1 is reflected in the critical values for the z-statistic $(\beta_1 - \beta_0)/\sigma_{\beta_1}$ reported in Table 7. Like the cointegrated case, the z-statistic is negatively correlated with β_1 and positively correlated with the sample size.

The response surface regression for the left tail of the distribution is reported in equation 5.1.

$$\underset{}{\hat{C}(.01)} = \underset{(0.035)}{-2.73} - \underset{(0.018)}{0.398\beta_1} + \underset{(0.105)}{1.61n^{-1/4}} \tag{5.1}$$

The regression is virtually indistinguishable from the results found in the two variable cointegrated case reported above. The statistic converges to an asympotitic value of –2.73, larger than the statistic associated with the normal distribution. As before, as the strength of cointegration increases, the left tail moves closer to the mean value, and the distribution begins to take on the skewed form found in the simple cointegrated case.

The distribution of the z-statistic for the second coefficient, β_2, is perhaps more interesting because we have shown in case II, the independence case, that the z-statistic performs poorly when the null hypothesis of independence is in fact true. Table 7 reports the empirical size of the test statistic for a nominal size of 0.05. As the results show, when at least one of the model variables is, in fact, correlated with the latent variable, spurious inference is avoided. The empirical size is perhaps slightly smaller than the nominal size, suggesting that the z-test is slightly conservative.

Examination of Table 8 reveals some important differences between the z-statistic for the non-correlated variable, β_2, and the correlated variable, β_1. Most importantly, the statistic is centered about zero for β_2, and the distribution does not exhibit the skewness found in distribution associated with the β_1 coefficient. The distribution has a smaller variance than the normal distribution, exhibited by the lower absolute critical values for the distribution as compared to the normal. This support the finding that the z-test is somewhat conservative, more likely to fail to reject independence than under the true underlying distribution.

Mild correlation is found between β_1 and the empirical z-statistic so that the strength of the cointegrated relationship actually influences the distribution of the unrelated independent variable. Given that x_{2t} is in fact completely unrelated to either x_{1t} or y_t*, this is a rather unexpected, yet fortunate result. In contrast to the independence case, the empirical z-distribution is tighter than the normal. Thus, the presence of one truly cointegrating relationship forces the z-distribution of the parameter of the other, unrelated, variable, into a distribution rather close to the normal. Therefore, the z-test in the three variable case is far less likely to find spurious correlation than in the independence case, where no cointegrating relationships exist between any of the variables.

Although z-tests are often used in econometric modeling, joint tests are equally important for evaluating model validity. Table 9 reports the results of the Wald test for the null hypothesis that both coefficients, β_1 and β_2, are equal to zero. The results are reminiscent of the Wald test in the cointegrated case. Strong cointegration between x_1 and y*, represented by relatively large absolute values of β_1, imparts higher empirical Wald statistics than found in the true chi-square distribution with two degrees of freedom. As the strength of cointegration weakens, and β_1 moves closer to 0, the empirical distribution of the test statistic more closely matches that of the chi-square.

The results from the empirical Wald distribution and the empirical z-distributions from the three variable case indicate the if some level of cointegration exists in the model, spurious inference concerning extraneous model variables may be avoided. The result is quite striking and is the core finding of the paper. This means that in a model at least one cointegrating relationship exists, the z-test will work reasonably well to eliminate other irrelevant variables. In contrast, if all of the model regressors are independent of the latent variable, ML estimates will we excessively prone to accepting spurious results. Thus, although one cannot be certain of a model's validity, conditioning on the relationship between two variables will lead to tests for other, more suspect, variables. For instance, assuming a non-zero relationship between quantity demanded and price would allow us to test for income effects on quantity demanded. Nevertheless, assuming economic relationships is not standard econometric practice, and will leave some practitioners uneasy. A clear need for a cointegration test is apparent, but, because of the latent error term, does not immediately suggest itself.

6. Summary

The basic finding of this study is that many of the familiar findings of OLS models containing nonstationary variables extend to the binary choice model. When probit is used to estimate the relationship between independent integrated series, spurious correlation is a problem. Because parameter estimates diverge at rate n, spurious correlation becomes more likely at large sample sizes, a disturbing result.

Fortunately, like standard OLS models of nonstationarity, cointegration and corresponding n rate parameter convergence is also a feature of the nonstationary binary choice model. Thus, if any of the model variables are cointegrated with the latent variable, normal-based tests are consistent. However, caution should be used in cointegrated models because the parameter distribution may exhibit significant skewness in small samples.

As a final note, although the potential for cointegration and the resulting convergent test-statistics is comforting, a test for cointegration does not exist for nonstationary binary choice models. Until a test is developed, a high degree of uncertainty surrounds the validity of nonstationary binary choice models.

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Table 1. Standard errors for β coefficient, se(β) and empirical mean, E($\hat{\beta}$), for different values of n and b: cointegrated two-variable case.

n	β	se($\hat{\beta}$)	E($\hat{\beta}$)	β	se($\hat{\beta}$)	E($\hat{\beta}$)	β	se($\hat{\beta}$)	E($\hat{\beta}$)
25	-0.1	0.173	-0.128	-0.5	0.435	-0.699	-0.8	0.644	-1.044
45	-0.1	0.093	-0.106	-0.5	0.271	-0.599	-0.8	0.494	-1.018
65	-0.1	0.065	-0.103	-0.5	0.241	-0.599	-0.8	0.428	-0.991
85	-0.1	0.050	-0.106	-0.5	0.222	-0.599	-0.8	0.403	-0.967
105	-0.1	0.041	-0.105	-0.5	0.207	-0.606	-0.8	0.361	-0.953
125	-0.1	0.036	-0.105	-0.5	0.183	-0.573	-0.8	0.356	-0.949
145	-0.1	0.032	-0.105	-0.5	0.183	-0.580	-0.8	0.321	-0.941
165	-0.1	0.023	-0.102	-0.5	0.150	-0.557	-0.8	0.283	-0.919
185	-0.1	0.025	-0.101	-0.5	0.146	-0.552	-0.8	0.310	-0.935
205	-0.1	0.023	-0.102	-0.5	0.149	-0.558	-0.8	0.295	-0.927
225	-0.1	0.022	-0.102	-0.5	0.153	-0.564	-0.8	0.293	-0.929
245	-0.1	0.021	-0.102	-0.5	0.146	-0.556	-0.8	0.287	-0.920

Table 2. Percentiles of the z statistic for different values of n, and β and the cointegrated case.

n	β	0.010	0.025	0.05	0.1	0.5	0.9	0.95	0.975	0.99
25	-0.1	-2.0299	-1.862	-1.6196	-1.3735	-0.1565	1.2518	1.6236	1.9120	2.1953
25	-0.2	-1.9683	-1.8041	-1.5891	-1.2147	-0.0947	1.1933	1.5846	1.8715	2.1492
25	-0.4	-1.7681	-1.5783	-1.4187	-1.2300	-0.1994	1.1334	1.4877	1.7654	2.0053
25	-0.5	-1.6865	-1.4772	-1.3256	-1.1492	-0.1853	1.2426	1.6528	2.0406	2.4025
25	-0.6	-1.5436	-1.423	-1.2791	-1.1034	-0.1556	1.1935	1.5864	1.9756	2.3303
25	-0.7	-1.5841	-1.3787	-1.2303	-1.0720	-0.1506	1.1697	1.6810	2.0524	2.4338
25	-0.8	-1.4328	-1.2942	-1.1652	-0.9851	-0.1204	1.2359	1.6700	2.1519	2.4606
125	-0.1	-2.4323	-1.9639	-1.7508	-1.4103	-0.0769	1.2721	1.6022	1.8776	2.2008
125	-0.2	-2.3262	-1.9375	-1.6618	-1.3133	-0.1511	1.1833	1.5264	1.7841	2.2220
125	-0.4	-2.056	-1.7545	-1.5391	-1.3057	-0.1659	1.1900	1.5771	1.9722	2.4468
125	-0.5	-1.9261	-1.6981	-1.4443	-1.2139	-0.1509	1.1149	1.5786	1.9290	2.3551
125	-0.6	-1.941	-1.6711	-1.4862	-1.2631	-0.1582	1.1769	1.4987	1.8468	2.2364
125	-0.7	-1.9296	-1.6891	-1.5043	-1.2560	-0.1658	1.1583	1.5800	1.9875	2.3083
125	-0.8	-1.8355	-1.5926	-1.4124	-1.1870	-0.2213	1.1883	1.7313	2.0925	2.6959
245	-0.1	-2.3118	-2.0089	-1.6729	-1.3548	-0.0287	1.3009	1.6400	1.9824	2.3662
245	-0.2	-2.1756	-1.8757	-1.6187	-1.3053	-0.0749	1.1887	1.5666	1.9615	2.2292
245	-0.4	-2.0911	-1.8461	-1.5563	-1.2985	-0.1815	1.1771	1.5852	1.8101	2.2352
245	-0.5	-2.0201	-1.7418	-1.5474	-1.2827	-0.1292	1.1767	1.5666	2.0071	2.4656
245	-0.6	-2.027	-1.8404	-1.5224	-1.2546	-0.1799	1.1782	1.5695	1.8577	2.3700
245	-0.7	-1.8868	-1.6691	-1.4683	-1.2618	-0.1171	1.2383	1.7080	2.1295	2.5550
245	-0.8	-1.915	-1.6722	-1.4628	-1.2139	-0.1688	1.1870	1.5998	2.0224	2.3620
z-statistic		-2.3263	-1.9600	-1.6449	-1.2816	0.0000	1.2816	1.6449	1.9600	2.3263

Table 3. Percentiles of the Wald statistic for different values of n and β: Cointegrated case.

n	β	0.9	0.95	0.975	0.99
25	0.1	4.0178	4.907	5.7714	6.8027
45	0.1	4.3273	5.4741	6.4734	8.0932
65	0.1	4.5448	5.8872	6.8416	8.1126
85	0.1	4.4262	5.7136	6.9198	8.5712
105	0.1	4.4902	5.6289	6.6655	8.3152
125	0.1	4.8708	6.0273	7.4325	9.2405
145	0.1	4.6682	5.843	7.1641	8.6889
165	0.1	4.5977	5.8569	7.5786	8.9715
185	0.1	4.5024	5.7106	6.9929	8.9993
205	0.1	4.3142	5.6783	7.1149	9.0737
225	0.1	4.4169	5.6915	6.701	8.6643
245	0.1	4.6518	6.1288	7.4056	8.685
25	0.5	3.8816	4.9167	6.3408	7.9737
45	0.5	4.1393	5.2322	6.4779	7.9778
65	0.5	4.1418	5.2608	6.7826	8.9535
85	0.5	4.4309	5.7603	7.2814	8.8778
105	0.5	4.0781	5.285	7.0675	8.3196
125	0.5	4.1031	5.1561	6.6691	8.9615
145	0.5	4.1342	5.3784	6.7063	9.1204
165	0.5	4.2024	5.548	6.8347	8.6479
185	0.5	4.2007	5.6258	7.2722	9.8328
205	0.5	4.2836	5.6918	7.0929	9.3505
225	0.5	4.2348	5.6831	7.1186	9.3082
245	0.5	4.0586	5.2447	6.8313	8.9129
25	0.8	3.8743	5.3422	7.1964	9.8342
45	0.8	3.9291	5.1136	6.6112	10.2564
65	0.8	4.0618	5.6062	7.1968	10.1526
85	0.8	4.1324	5.2549	6.6131	8.3532
105	0.8	3.9735	5.9501	7.9961	9.9244
125	0.8	4.1925	5.786	7.9161	10.7701
145	0.8	4.1784	5.7287	7.5398	9.2966
165	0.8	4.137	5.1789	6.8367	9.6573
185	0.8	4.3625	5.5633	7.2475	10.1438
205	0.8	4.0928	5.5419	7.3361	9.2924
225	0.8	4.3263	5.8637	7.4545	9.3082
245	0.8	4.0775	5.4816	6.6303	8.5938
χ^2,d.f=2		4.605	5.991	7.378	9.21

Table 4. Standard errors, empirical sizes, and nominal sizes for α and β at different sample sizes: independence case

n	empirical size α	empirical size β	nominal size	se($\hat{\alpha}$)	se($\hat{\beta}$)
25	0.3113	0.1753	0.05	0.7019	0.3415
45	0.4591	0.3194	0.05	0.511	0.2007
65	0.576	0.4251	0.05	0.4292	0.1398
85	0.6331	0.4862	0.05	0.3766	0.1155
105	0.6714	0.4916	0.05	0.3386	0.098
125	0.6761	0.5224	0.05	0.3315	0.0873
145	0.6839	0.5536	0.05	0.2928	0.0723
165	0.7099	0.5739	0.05	0.2484	0.0525
185	0.7295	0.6102	0.05	0.252	0.0577
205	0.7449	0.6275	0.05	0.2484	0.0525
225	0.7471	0.6547	0.05	0.2349	0.0566
245	0.7613	0.6487	0.05	0.2276	0.0549

Table 5. Percentiles of the z-statistic, independence case.

n	0.01	0.025	0.05	0.1	0.5	0.9	0.95	0.975	0.99
25	-2.876	-2.666	-2.423	-2.027	-0.100	1.987	2.401	2.569	2.831
45	-3.756	-3.430	-3.155	-2.605	-0.056	2.791	3.370	3.683	3.886
65	-4.599	-4.277	-3.834	-3.252	0.045	3.295	3.917	4.331	4.689
85	-5.326	-4.812	-4.404	-3.827	-0.042	3.670	4.468	4.964	5.437
105	-5.827	-5.319	-4.861	-4.060	-0.112	3.913	4.735	5.297	5.705
125	-6.288	-5.791	-5.342	-4.363	-0.089	4.432	5.457	5.987	6.576
145	-7.041	-6.519	-5.828	-4.931	0.053	4.963	5.918	6.441	6.830
165	-8.284	-7.558	-6.764	-5.689	-0.298	5.631	6.829	7.681	8.359
185	-7.972	-7.235	-6.487	-5.544	0.213	5.387	6.508	7.054	7.565
205	-8.284	-7.558	-6.764	-5.689	-0.298	5.631	6.829	7.681	8.359
225	-8.652	-7.965	-7.117	-5.780	-0.237	5.706	7.136	7.906	8.533
245	-8.987	-8.110	-7.238	-5.982	-0.116	6.184	7.465	8.412	9.242
Z-stat	-2.326	-1.960	-1.645	-1.282	0.000	1.282	1.645	1.960	2.326

Table 6. Standard errors, empirical means, power and size for β_1 and β_2: Three variable case.

n	β_1	β_2	se($\hat{\beta}_1$)	se($\hat{\beta}_2$)	E($\hat{\beta}_1$)	E($\hat{\beta}_2$)	Power β_1	Size β_2
25	-0.2	0	0.2535	0.2470	-0.2614	0.0082	0.1583	0.0389
45	-0.2	0	0.1411	0.1387	-0.2396	-0.0062	0.4478	0.0431
65	-0.2	0	0.1054	0.1027	-0.2302	-0.0021	0.6653	0.0477
85	-0.2	0	0.0896	0.0864	-0.2272	0.0005	0.7801	0.0421
105	-0.2	0	0.0738	0.0725	-0.2206	-0.0020	0.8884	0.0450
125	-0.2	0	0.0697	0.0653	-0.2228	0.0007	0.9257	0.0537
145	-0.2	0	0.0632	0.0609	-0.2201	-0.0014	0.9480	0.0535
165	-0.2	0	0.0491	0.0448	-0.2127	0.0008	0.9584	0.0512
185	-0.2	0	0.0548	0.0492	-0.2197	0.0016	0.9635	0.0465
205	-0.2	0	0.0491	0.0448	-0.2127	0.0008	0.9730	0.0440
225	-0.2	0	0.0493	0.0436	-0.2186	-0.0023	0.9730	0.0495
245	-0.2	0	0.0462	0.0394	-0.2136	0.0000	0.9734	0.0376
25	-0.5	0	0.4723	0.3868	-0.7050	0.0094	0.3305	0.0215
45	-0.5	0	0.3329	0.2618	-0.6632	-0.0114	0.6438	0.0294
65	-0.5	0	0.2870	0.2076	-0.6479	-0.0086	0.7416	0.0319
85	-0.5	0	0.2508	0.1665	-0.6273	0.0089	0.8327	0.0384
105	-0.5	0	0.2314	0.1553	-0.6194	0.0030	0.8460	0.0300
125	-0.5	0	0.2226	0.1312	-0.6226	0.0003	0.8544	0.0355
145	-0.5	0	0.2029	0.1275	-0.5982	0.0036	0.8682	0.0410
165	-0.5	0	0.1851	0.1093	-0.5959	0.0043	0.8723	0.0426
185	-0.5	0	0.1795	0.1056	-0.5864	0.0013	0.8895	0.0403
205	-0.5	0	0.1851	0.1093	-0.5959	0.0043	0.8907	0.0382
225	-0.5	0	0.1774	0.1000	-0.5943	0.0058	0.9016	0.0412
245	-0.5	0	0.1638	0.0937	-0.5810	-0.0006	0.9048	0.0430
25	-1	0	0.8786	0.5158	-1.3413	-0.0315	0.3474	0.0054
45	-1	0	0.6909	0.3406	-1.3184	0.0205	0.6015	0.0115
65	-1	0	0.5731	0.2649	-1.2487	0.0190	0.6958	0.0199
85	-1	0	0.5288	0.2494	-1.2365	-0.0182	0.7468	0.0305
105	-1	0	0.4907	0.2197	-1.2155	0.0108	0.7703	0.0288
125	-1	0	0.4798	0.1903	-1.2259	-0.0054	0.7879	0.0305
145	-1	0	0.4695	0.1855	-1.2254	0.0068	0.8001	0.0268
165	-1	0	0.4177	0.1601	-1.2119	-0.0088	0.8254	0.0367
185	-1	0	0.4233	0.1525	-1.2056	-0.0091	0.8228	0.0274
205	-1	0	0.4177	0.1601	-1.2119	-0.0088	0.8393	0.0375
225	-1	0	0.3737	0.1357	-1.1590	-0.0023	0.8631	0.0350
245	-1	0	0.3600	0.1257	-1.1573	0.0033	0.8674	0.0322

Table 7. Percentiles of the z-statistic for β_1, three variable case.

n	β_1	0.01	0.025	0.05	0.1	0.5	0.9	0.95	0.975	0.99
25	-0.2	-1.995	-1.817	-1.630	-1.333	-0.096	1.307	1.610	1.854	2.062
25	-0.4	-1.783	-1.602	-1.431	-1.226	-0.266	1.179	1.539	1.800	2.216
25	-0.4	-1.790	-1.603	-1.413	-1.211	-0.200	1.172	1.538	1.857	2.222
25	-0.5	-1.753	-1.581	-1.444	-1.209	-0.224	1.193	1.541	1.852	2.206
25	-0.7	-1.543	-1.391	-1.300	-1.111	-0.221	1.159	1.509	1.924	2.410
25	-0.8	-1.574	-1.426	-1.273	-1.066	-0.191	1.221	1.747	2.096	2.473
25	-0.9	-1.423	-1.288	-1.141	-1.005	-0.157	1.302	1.728	2.135	2.643
25	-1.0	-1.391	-1.287	-1.136	-0.965	-0.174	1.237	1.642	2.053	2.524
125	-0.2	-2.117	-1.888	-1.681	-1.352	-0.161	1.184	1.632	1.969	2.416
125	-0.4	-2.048	-1.834	-1.637	-1.363	-0.240	1.195	1.533	1.950	2.404
125	-0.5	-1.982	-1.754	-1.586	-1.369	-0.297	0.993	1.437	1.914	2.318
125	-0.7	-1.897	-1.661	-1.519	-1.286	-0.203	1.151	1.601	1.936	2.444
125	-0.8	-1.878	-1.678	-1.465	-1.251	-0.246	1.105	1.628	2.050	2.666
125	-0.9	-1.833	-1.624	-1.419	-1.225	-0.267	1.159	1.574	1.875	2.397
125	-1.0	-1.824	-1.621	-1.439	-1.189	-0.206	1.169	1.635	2.013	2.646
245	-0.2	-2.233	-1.904	-1.567	-1.286	-0.128	1.155	1.526	1.869	2.192
245	-0.4	-2.130	-1.859	-1.638	-1.342	-0.187	1.154	1.530	1.863	2.413
245	-0.5	-2.145	-1.899	-1.643	-1.354	-0.276	1.055	1.498	1.937	2.430
245	-0.7	-2.084	-1.788	-1.601	-1.312	-0.274	1.128	1.583	1.968	2.427
245	-0.8	-1.936	-1.694	-1.508	-1.247	-0.219	1.156	1.552	1.986	2.450
245	-0.9	-1.907	-1.672	-1.489	-1.284	-0.286	1.133	1.562	2.036	2.550
245	-1.0	-1.830	-1.615	-1.468	-1.236	-0.215	1.222	1.661	2.126	2.718
Z-statistic		-2.326	-1.960	-1.645	-1.282	0.000	1.282	1.645	1.960	2.326

Table 8. Percentiles of the z-statistic for β_2, three variable case.

n	β_1	β_2	0.01	0.025	0.05	0.1	0.5	0.9	0.95	0.975	0.99
25	-0.2	0	-2.132	-1.862	-1.616	-1.351	0.027	1.317	1.666	1.928	2.106
25	-0.4	0	-2.002	-1.792	-1.547	-1.284	-0.015	1.265	1.496	1.742	2.026
25	-0.4	0	-2.123	-1.788	-1.549	-1.254	0.006	1.286	1.632	1.833	2.070
25	-0.5	0	-1.956	-1.738	-1.490	-1.228	-0.011	1.248	1.542	1.804	2.050
25	-0.7	0	-1.832	-1.582	-1.412	-1.184	-0.011	1.211	1.471	1.653	1.849
25	-0.8	0	-1.857	-1.638	-1.458	-1.207	-0.047	1.123	1.370	1.621	1.847
25	-0.9	0	-1.840	-1.667	-1.470	-1.188	-0.003	1.177	1.423	1.544	1.712
25	-1.0	0	-1.790	-1.608	-1.417	-1.194	-0.055	1.161	1.427	1.644	1.851
125	-0.2	0	-2.137	-1.921	-1.558	-1.256	0.016	1.326	1.732	2.042	2.305
125	-0.4	0	-2.062	-1.826	-1.590	-1.275	-0.025	1.285	1.563	1.873	2.192
125	-0.4	0	-2.060	-1.830	-1.590	-1.256	0.023	1.266	1.591	1.851	2.041
125	-0.5	0	-2.281	-1.841	-1.629	-1.293	0.025	1.230	1.566	1.829	2.147
125	-0.7	0	-2.167	-1.859	-1.631	-1.259	0.014	1.257	1.528	1.777	2.016
125	-0.8	0	-2.179	-1.912	-1.624	-1.298	0.015	1.295	1.565	1.805	2.111
125	-0.9	0	-2.078	-1.809	-1.529	-1.209	0.026	1.271	1.590	1.851	2.030
125	-1.0	0	-2.182	-1.823	-1.583	-1.280	0.004	1.242	1.584	1.798	2.080
245	-0.2	0	-2.359	-1.908	-1.601	-1.284	0.027	1.248	1.530	1.858	2.068
245	-0.4	0	-2.362	-2.050	-1.689	-1.321	0.019	1.308	1.654	2.009	2.284
245	-0.4	0	-2.201	-1.768	-1.531	-1.195	-0.043	1.231	1.533	1.797	2.169
245	-0.5	0	-2.252	-1.891	-1.633	-1.266	0.033	1.256	1.624	1.927	2.270
245	-0.7	0	-2.244	-1.874	-1.604	-1.282	0.022	1.260	1.538	1.809	2.163
245	-0.8	0	-2.178	-1.906	-1.613	-1.295	0.010	1.255	1.526	1.762	1.972
245	-0.9	0	-2.208	-1.875	-1.628	-1.250	0.004	1.296	1.606	1.831	2.164
245	-1.0	0	-2.200	-1.862	-1.598	-1.269	0.015	1.234	1.533	1.784	2.061
Z-statistic			-2.326	-1.960	-1.645	-1.282	0.000	1.282	1.645	1.960	2.326

Table 9. Percentiles of the Wald statistic for different values of n β_1, β_2, and n: three variable case.

n	β_1	β_2	0.9	0.95	0.975	0.99
25	-1	0	3.711	4.778	6.204	9.202
45	-1	0	3.511	4.824	6.604	8.779
65	-1	0	4.040	5.815	7.995	10.779
85	-1	0	4.115	5.974	8.115	11.345
105	-1	0	4.111	5.903	7.444	10.379
125	-1	0	4.084	5.948	7.832	11.081
145	-1	0	4.160	5.388	7.271	10.668
165	-1	0	4.289	5.980	7.352	9.622
185	-1	0	4.244	5.774	7.365	9.046
205	-1	0	4.059	5.353	6.814	10.058
225	-1	0	4.293	5.770	7.154	10.304
245	-1	0	4.323	5.829	8.024	12.100
25	-0.5	0	3.788	4.852	5.629	7.250
45	-0.5	0	3.704	4.910	6.332	8.171
65	-0.5	0	4.002	5.385	6.639	8.775
85	-0.5	0	3.970	5.058	6.765	9.315
105	-0.5	0	3.999	5.219	6.600	7.657
125	-0.5	0	4.157	5.437	6.876	9.320
145	-0.5	0	4.134	5.670	6.934	9.558
165	-0.5	0	4.113	5.457	6.943	8.999
185	-0.5	0	3.994	5.144	6.549	8.530
205	-0.5	0	4.245	5.367	6.905	9.765
225	-0.5	0	3.975	5.500	6.950	8.689
25	-0.2	0	3.954	4.741	5.639	6.663
45	-0.2	0	4.214	5.269	6.223	7.358
65	-0.2	0	4.375	5.483	6.937	8.243
85	-0.2	0	4.515	5.560	6.575	8.139
105	-0.2	0	4.219	5.360	6.425	8.103
125	-0.2	0	4.460	5.502	6.796	8.382
145	-0.2	0	4.276	5.750	7.057	8.389
165	-0.2	0	4.617	5.849	6.653	7.810
185	-0.2	0	4.489	5.853	6.927	8.470
205	-0.2	0	4.259	5.189	6.594	8.528
225	-0.2	0	4.536	5.638	6.857	8.828
245	-0.2	0	4.152	5.382	6.731	8.288
χ^2 d.f=2			4.605	5.991	7.378	9.21

Figure 1. Kernel densities for β=-0.90 and β=-0.15 and n=25 and n=245: cointegrated model

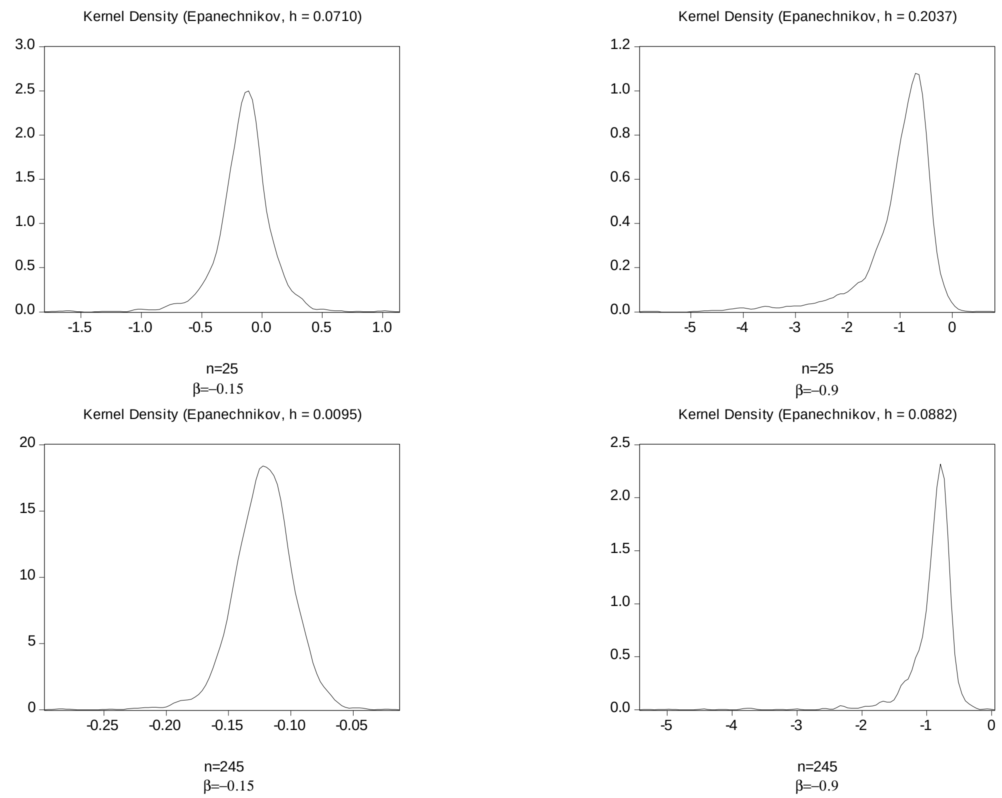

Figure 2. Kernel densities for the three variable case