Delivery Option in Futures Contracts and Basis Behavior at Contract Maturity

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In designing a futures contract, the exchange has to make sure that the contract attracts both hedgers and speculators to the market, at the same time preventing manipulation of prices. These interests are sometimes in direct contradiction and a balance has to be found. One of the necessary conditions for attracting hedgers to the futures market is to design contract provisions that minimize basis risk as well as minimize the possibility of market manipulation. Delivery options are introduced that expand deliverable supply, thus reducing the probability of a market squeeze, but on the other hand are a source of basis risk. The exchange has to monitor the degree to which the delivery options reduce futures and spot price correlations.

In particular, net benefits of using a futures market as a risk management tool depend on hedging effectiveness. A perfect hedge can be completed if the futures price converges exactly to the spot price at maturity; i.e., the basis converges to zero. Complexities of delivery specifications of futures contracts as well as arbitrage costs cause imperfect convergence. Delivery options embedded in contract specifications introduce uncertainty about the relevant spot price to which the futures contract will converge at maturity, resulting in basis risk. Variability of the basis at maturity is a cost to hedgers and negatively influences hedging demand. Thus, from the point of view of an exchange as well as that of risk management demand, it is important to understand price and basis behavior at contract maturity.

This paper analyzes the timing and location options jointly. The quality option is not included in the analysis due to unavailability of data on grades delivered. The majority of the literature has evaluated each option separately (Hemler, 1988, Silk, 1988, Pirrong et al., 1994).

However, the estimates from these studies are likely to be biased as the interaction effects of the different options are ignored. Boyle (1989) incorporates such effects in the evaluation of the timing and quality option in Treasury bonds.

This paper evaluates the joint timing and location option for CBOT corn futures contract during the years 1989-97. Similar to Boyle's approach, the effects of the two delivery options are interacted. In addition, the importance of considering the institutional background of delivery is highlighted. Two institutional setups are considered, one that imposes a one-day waiting period between entering the futures market and delivering and one that allows delivery on the same day as a futures contract is entered. We find that the possibility of immediate delivery lowers the value of delivering early.

The estimates of the joint option values under the two assumptions are used to examine the impact of the delivery options on basis behavior at contract month. Delivery options appear not to have a significant impact on basis non-convergence in Chicago, but increase the basis significantly in the Toledo location. The direction of the effect is robust to the changes in the institutional assumptions about delivery.

The paper is organized as follows. Section I presents models for calculating futures prices with the embedded timing and location options under two institutional assumptions about delivery. Section II describes the methodology and data used in the estimation of the joint option values for CBOT corn futures contract. Section III presents the empirical estimates of the joint option values obtained under the two institutional assumptions. Section IV uses the estimated values to examine the role of delivery options on the basis behavior at contract month. Section V concludes.

I. Model

The timing option is defined here as the option that allows the short to deliver the underlying commodity any time during the first three weeks of the expiration month. These are the days when trading in the currently deliverable contract persists (12-16 business days), and the seller has the option to offset, deliver, or defer the choice. The other type of timing options, like end-of-the-month option and wild-card options, are not included in the current definition. Thus, for the CBOT corn futures contract, it is assumed that all short positions in the expiring contract have to be closed by delivery or offset by the last trading day of the expiration month.[1]

The location option expands the deliverable supply by extending the set of deliverable locations. The short can deliver at alternative locations and receive a price adjusted according to a given discount/premium schedule. This option is designed to prevent manipulation due to shortage of supply in a single market place and high transportation costs from distant markets.

The location option can be viewed as a type of quality option, where the state variables are the spot prices in the deliverable locations, adjusted for any discounts/premia. Thus, the economy is assumed to have n risky assets, each asset being represented by the adjusted spot price in each of the n deliverable locations, and one riskless bond. Since the short will choose the location where delivery is the cheapest, the location option is an option on the minimum of n assets. For the CBOT corn futures contract, only two deliverable locations are active: Chicago and Toledo and n = 2.

The value of the joint timing and location option V_{JO} is determined as the difference between the price of a currently deliverable futures contract without the joint option, $F_{w/oJO,}$ and

[1] Actual physical delivery can occur up to the last business day of the month.

the futures price of the same contract with the option, $F_{w/JO,}$

$$V_{JO}(0) = F_{w/oJO}(0) - F_{w/JO}(0),$$

where the values and prices are estimated on the first delivery day.

Using risk neutral valuation with martingale equivalent probabilities of up, middle, and down movements p_1, p_2, and $1\text{-}p_1\text{-}p_{2,}$ respectively, the no-arbitrage futures price without options is a martingale

$$\begin{aligned} &F_{w/oJO}(t;s_t) \\ &\quad = \tilde{E}_t(F_{w/oJO}(t+1;s_{t+1})) = p_1 F_{w/oJO}(t+1;s_t u) + p_2 F_{w/oJO}(t+1,m) + (1-p_1-p_2)F_{w/oJO}(t+1;s_t d), \end{aligned}$$

where $\tilde{E}$ denotes expectation under martingale equivalent probabilities and $F(t;s_t)$ is the futures price at time period t in state s_t. Using the law of iterative expectations a well-known result is obtained

$$F_{w/oJO}(t) = \tilde{E}_t(S(T)).$$

Assumption 1

Under Assumption 1, a trader cannot sell and deliver on the same day. A short trader has to have a pre-existing position in the futures contract to be able to close it by delivery. The futures price is determined by backward induction, recognizing that the value of the contract is reset to zero every day through marking to market. Each state of the world in every time period is checked for optimal exercise, where the cheapest-to-deliver asset is delivered if delivery is optimal.

For simplicity of exposition, the subscript denoting the presence of the joint option is omitted, as well as the adjustment for location discounts/premia. In the subsequent analysis, $S_k(.)$ represents the adjusted spot price in location k and F(.) is the price of the currently deliverable futures contract with the joint option.

In an economy with two deliverable assets (spot commodity in two deliverable locations), the futures price on the last delivery day converges to the price of the asset with the lowest price, $min(S_1(T;s_T), S_2(T;s_T))$ for all s_T. On this last delivery day, no option to delay exists, and the short position must be closed by delivery (or offset), yielding a zero cash flow. Thus, the only possible non-zero payoff comes from the marking-to-market cash flow.

$$V_{T-1} = 0 = \tilde{E}_{T-1}\left(\frac{F(T-1;s_{T-1}) - F(T;s_T)}{R}\right),$$

yielding

$$F(T-1;s_{T-1}) = \tilde{E}_{T-1}(\min(S_1(T;s_T), S_2(T;s_T)).$$

At T-1, the short has an option to deliver, offset or delay. The short will choose the strategy that maximizes her payoff. The value of the futures contract at T-2 is then the expected payoff (under martingale equivalent probabilities) from following the optimal strategy in period T-1, plus the expected cash flow from marking to market at T-1,

$$\begin{aligned} V_{T-2} = 0 = {} & \tilde{E}_{T-2}\{F(T-2;s_{T-1}) - F(T-1;s_{T-2})\} \\ & + \tilde{E}_{T-2}\{\max[F(T-1;s_{T-1}) - \min(S_1(T-1;s_{T-1}), S_2(T-1;s_{T-1}), 0]\}. \end{aligned}$$

The value of the futures contract consists of two components – the value due to the futures price (the term in the first curly brackets) and the value due to the timing option interacted with the location option (the term in the second curly brackets). The zero value of the futures contract uniquely determines the arbitrage-free futures price at T-2,

$$F(T-2;s_{T-2}) = \tilde{E}_{T-2}\{F(T-1;s_{T-1})\} - \tilde{E}_{T-2}\{\max[F(T-1;s_{T-1}) - \min(S_1(T-1;s_{T-1}), S_2(T-1;s_{T-1})), 0]\}.$$

Continuing the same argument inductively backward in time generates the result for the futures price at $0 \le t < T-1$,

$$F(t;s_t) = \tilde{E}_t\{F(t+1;s_{t+1})\} - \tilde{E}_t\{\max[F(t+1;s_{t+1}) - \min(S_1(t+1;s_{t+1}), S_2(t+1;s_{t+1})), 0]\}.$$

At every time period $0 \le t < T-1$, the price is lower by the expected value of the joint option in the following period.

Assumption 2

Assumption 2 allows delivery on the same day as a futures position is established. The futures price is now bounded from above, such that at any time t and any state of the world s_t $F(t, s_t) \le \min(S_1(t; s_t), S_2(t; s_t))$. Suppose $F(t, s_t) > \min(S_1(t; s_t), S_2(t; s_t))$. Then, selling an expiring futures contract and delivering the asset in the cheapest to deliver location immediately would yield positive arbitrage profits.

The value of the timing option V_{TO} is again determined as the difference between the price of a currently deliverable futures contract without the timing option, $F_{w/oTO,}$ and the futures price of the same contract with the option, $F_{w/TO,}$

$$V_{TO}(0) = F_{w/oTO}(0) - F_{w/TO}(0),$$

where the values and prices are estimated on the first delivery day.

Under the assumption that delivery is possible on the same day as the futures contract is entered, the short position does not have to be pre-existing from a previous period for the short to take advantage of the timing and location option. The short decides between delivering at time t in either of the two deliverable locations and holding on to the futures contract, whose value at time t is zero.

In both cases, he/she receives cash flows from marking to market. The futures price is again determined by backward induction, recognizing that the value of the position is reset to zero through marking to market. Each state of the world in every time period needs to be checked for optimal exercise. For all $0 \le t < T$, the futures price is determined such that the no arbitrage boundary condition holds

$$\max\,[F_{w/TO}(t;s_t) - \min(S_s(t;s_t), S_2(t;s_t)),\ \tilde{E}_t(\frac{F_{w/TO}(t) - F_{w/TO}(t+1)}{R}) + V_t] = 0\,,$$

where $V_t = 0$ and $\tilde{E}_t$ denotes expectation under martingale equivalent probabilities.

The first term in the square brackets represents the payoff to the short if delivery occurs at time t, and the second term is the value of the futures contract if delay is the optimal strategy at time t. For simplicity of exposition, the subscript denoting the presence of the timing option is omitted in the subsequent analysis and F(.) represents the price of a futures contract with the joint option.

It is again assumed that the futures price on the last day of delivery converges to the spot price on that day, $F(T, s_T) = \min\,(S_1(T;s_T), S_2(T;s_T))$ for all s_T. On this last delivery day, no option to delay exists, and the short position must be closed by delivery (or offset), yielding a zero cash flow. Thus, the only possibly non-zero payoff comes from the marking to market cash flow, $F(T-1) - \min\,(S_1(T), S_2(T))$.

At T-1, the short has an option to deliver at T-1, yielding the payoff from delivery $F(T-1) - \min\,(S_1(T-1), S_2(T-1))$, or delay delivery and obtain the marking to market cash flow at time T. He/she will follow the strategy that maximizes her payoff. The futures price at T-1 is determined simultaneously with the optimal decision to exercise, by setting the value of the futures contract with the embedded option to zero

$$V_{T-1} \equiv \max\,[F(T-1) - \min\,(S_1(T-1), S_2(T-1)), \tilde{\mathrm{E}}_{\mathrm{T\text{-}1}}\frac{(F(T-1) - \min\,(S_1(T-1), S_2(T-1))}{R}] = 0,$$

where R is the risk neutral discount factor.

At T-2, the short again has an option to deliver at T-2 or delay delivery until T-1 and obtain the next period's marking to market cash flows and the value of the futures contract at T-1. The condition to be checked for optimal exercise is

$$V_{T-2} \equiv \max\,[F(T-2) - \min(S_1(t-2), S_2(T-2), \tilde{\mathrm{E}}_{\mathrm{T\text{-}2}}(\frac{(F(T-2) - F(T-1)) + V_{T-1}}{R})] = 0\,.$$

Since the value of the futures contract at any time is reset to zero through marking to market, the futures price at any time $0 \le t < T$ is determined by a condition

$$V_t \equiv \max\,[F(t) - \min\,(S_1(t), S_2(t)), \tilde{\mathrm{E}}_t(\frac{F(t) - F(t+1)}{R})] = 0\,.$$

Due to the presence of the location option, early exercise is not always optimal[2]. By allowing immediate delivery under Assumption 2, the futures price has an upper boundary, $F(t) \le \min\,(S_1(t), S_2(t))$. This is because if $F(t) > \min\,(S_1(t), S_2(t))$, the strategy of selling an expiring futures contract and delivering immediately the cheapest to deliver asset would yield an arbitrage profit of $F(t) - \min\,(S_1(t), S_2(t)) > 0$.

II. Methodology and Data

Methodology

Models used in the analysis assume perfectly competitive markets, no transaction costs, and no taxes. As we are interested in the joint value of the delivery options on the first delivery date, the no transaction cost assumption is perhaps a reasonable abstraction. Most small hedgers offset their positions prior to the expiration month. Thus, traders who potentially might make delivery are likely those with low transaction costs. The other two assumptions are widely used in the theoretical as well as empirical literature on option pricing, but it is true that futures markets become more concentrated as the last day of trading approaches.

The two underlying state variables, i.e., the spot prices in the two deliverable locations, are assumed to follow two correlated lognormal processes

[2] In the absence of the location option, the first payoff is always greater under martingale equivalent probabilities and early exercise is always optimal.

$$dS_1(t) = \mu_1 S_1(t)dt + \sigma_1 S_1(t)dW_1(t)$$

$$dS_2(t) = \mu_2 S_2(t)dt + \sigma_2 \rho S_2(t)dW_1(t) + \sigma_2 \sqrt{1-\rho^2} S_2(t)dW_2(t),$$

where $dS_k(t)$ denotes a change in the spot price of asset k during a (infinitesimally small) time increment dt, and $dW_k(t)$ is a Wiener process with zero mean and variance dt. μ_k and σ_k represent the drift and volatility of asset k's instantaneous return and ρ is the correlation coefficient between instantaneous returns on the two assets. This is a model with independent shocks, reflecting the fact that the two Wiener processes are independent.

A number of authors have derived closed-form solutions to partial differential equations for valuing European options on several assets. Margrabe (1978), Johnson (1987), and Hemler (1988) provide such solutions for valuing European options on maximum and/or minimum of n assets. To accommodate the early exercise feature of American options, numerical procedures must be used to solve the partial differential equations. However, when more than one underlying asset is involved, these procedures become computationally expensive. A discrete approximation approach offers a potentially more efficient procedure for valuing American options.

A generalization of the binomial lattice framework can be used as a discrete approximation of a multivariate diffusion process with more than one state variable. Discrete-time models assume that the price of an underlying asset moves in discrete jumps. In order to describe the process, jump sizes and jump probabilities have to be specified. There are basically two ways of doing this. The approach taken by Cox, Ross and Rubinstein (1979) and extended for n state variables by Boyle, Evnine, and Gibbs (BEG, 1989) is to fix the jump sizes and calculate the probabilities such that convergence to the continuous diffusion process is ensured.

An alternative approach fixes the jump probabilities and determines the ensuing jump sizes (He, 1990; Amin, 1995).

BEG assume that each of the assets follows a binomial distribution, moving up or down in every time period. The combinations of the values thus generate 2^n possible states for the n assets after a time increment h. For given up and down factors of movement, probabilities are determined for the states. However, this approach does not guarantee positive probabilities. For a large absolute value of the assets' covariance, negative probabilities can arise. Further, although the BEG discrete approximation converges under a risk neutral probability measure, it does not satisfy the property of complete markets. For an economy with n risky assets and one riskless bond, 2^n payoffs cannot be dynamically replicated and options cannot be priced by arbitrage. This makes the discrete-time model inconsistent with its continuous counterpart, in which markets are dynamically completed by continuous trading in the n assets and the bond.

Cheyette (1988) and Madan, Milne, and Shefrin (MMS, 1989) also approximate an n-dimensional lognormal process and establish the convergence of the discrete approximation to its continuous-time counterpart. However, although the MMS model does satisfy the complete market property of the continuous model, it does not guarantee convergence of multivariate contingent claims, such as options on the minimum of two assets. Cheyette's discrete multinomial approximation does not rely on risk neutral probabilities.

The analysis in this paper uses the approach that fixes the probabilities and determines the up and down factors such that the discrete-time process converges to the corresponding continuous process. A sequence of n-variate, (n+1)-nomial processes is used to approximate the n-dimensional diffusion process for the spot prices. The model consists of n risky assets and one bond, which form a dynamically complete securities market. Hua He (1990) shows convergence

of these discrete-time state price processes to their continuous-time counterparts. That is, individual spot price processes, contingent claims prices and replicating portfolio strategies converge in the limit.

To satisfy the market completeness property, each random variable $\tilde{X}_1$ and $\tilde{X}_2$ in the discrete analog to the continuous solution, is allowed to take three values, (x_{1u}, x_{1m}, x_{1d}) and (x_{2u}, x_{2m}, x_{2d}) respectively, where x_{kj} is the realization of random variable in location k in state j = up, middle and down. The realizations must satisfy the properties that $E(\tilde{X}_1) = E(\tilde{X}_2) = 0$, $Var(\tilde{X}_1) = Var(\tilde{X}_2) = 1$ and $Cov(\tilde{X}_1, \tilde{X}_2) = 0$. The resulting system of equations has infinitely many solution pairs. One such pair is $(\sqrt{\frac{3}{2}}, 0, -\sqrt{\frac{3}{2}})$ and $(\sqrt{\frac{1}{2}}, \sqrt{2}, -\sqrt{\frac{1}{2}})$. Thus, a two-variate trinomial model approximates the diffusion process with two state variables (asset prices in the two delivery locations).

Another advantage of this type of discrete approximation is its computational efficiency. As the two correlated spot price processes are path independent, the trinomial trees recombine. The trees do not grow exponentially and at any time t, the number of nodes is equal to *(t+1)(t+2)/2*.

Convenience yield is ignored in the estimation of the option value. Assuming a constant riskless rate and dt =1, the up, middle, and down factors for the two assets are

$$U_1 = 1 + \alpha_1 + \sigma_1\sqrt{\frac{3}{2}} \qquad U_2 = 1 + \alpha_2 + \sigma_2\rho\sqrt{\frac{3}{2}} + \sigma_2\sqrt{1-\rho^2}\,\frac{1}{\sqrt{2}}$$

$$M_1 = 1 + \alpha_1 \qquad M_2 = 1 + \alpha_2 - \sigma_2\sqrt{1-\rho^2}\,\frac{2}{\sqrt{2}}$$

$$D_1 = 1 + \alpha_1 - \sigma_1\sqrt{\frac{3}{2}} \qquad D_2 = 1 + \alpha_2 - \sigma_2\rho\sqrt{\frac{3}{2}} + \sigma_2\sqrt{1-\rho^2}\,\frac{1}{\sqrt{2}}$$

where $\alpha_k = r - \frac{1}{2}\sigma_k^2$ for asset k = 1, 2 .

Estimation proceeds as follows. First, a sequence of two trinomial trees for the Chicago and Toledo spot prices is generated. Then the futures price on the first delivery day, t=0, is calculated by backward induction, incorporating the interacted timing and location options as specified in the model section. It is assumed that the futures price on the last delivery day T converges to $\min(S_1(T), S_2(T))$. Then, the futures price tree without options is generated. The value of the joint option on the first delivery day is obtained as the difference between the two futures prices at the initial date 0.

Data

The value of the joint timing and location option is estimated for the corn futures contract traded at the CBOT. Daily data are used for each expiration month (March, May, July, September and December) in 1989 to 1997. The years before 1989 are influenced heavily by price support programs and substantial government stocks and are excluded from the analysis. Futures prices are the daily settlement prices. Cash prices are those reported by USDA for Chicago and Toledo terminal markets.[3] The prices are reported in ranges and the midpoint is used as a representative price. The 90-day T-bill rates obtained from CRSP database are used as risk-free rates. The number of trading days in individual delivery months ranges from 12 to16.

Volatilities are estimated as standard deviations of the log of spot price returns during the expiration month, and the correlation coefficient represents the correlation between the logs of spot price returns in locations 1 and 2 during the same time period. If location 1 stands for

[3] Chicago spot market has for the last decade been practically inactive and the quoted spot prices may not be representative transaction prices. The central Illinois average prices may be considered more representative, but represent a wrong location. The problem of low quality spot prices for agricultural commodities is well-known, and no 'good' spot prices are available.

Chicago and 2 for Toledo, d_1=0 and d_2=3 cents, since Chicago is the par location for corn delivery.

III. Empirical Results

Assumption 1

The value of the joint option estimated under Assumption 1 averaged 2.2 cents during years 1989-97, ranging from 0.3 to 3.7 cents (Table 1). This value represents 0.8% of the average futures price on the first delivery day and 105% of the absolute value of the Chicago basis and 122% of the Toledo basis (calculated as percent of the absolute value of the basis F-S).

Table 1: Joint option values estimated under Assumption 1 (in cents)[a]

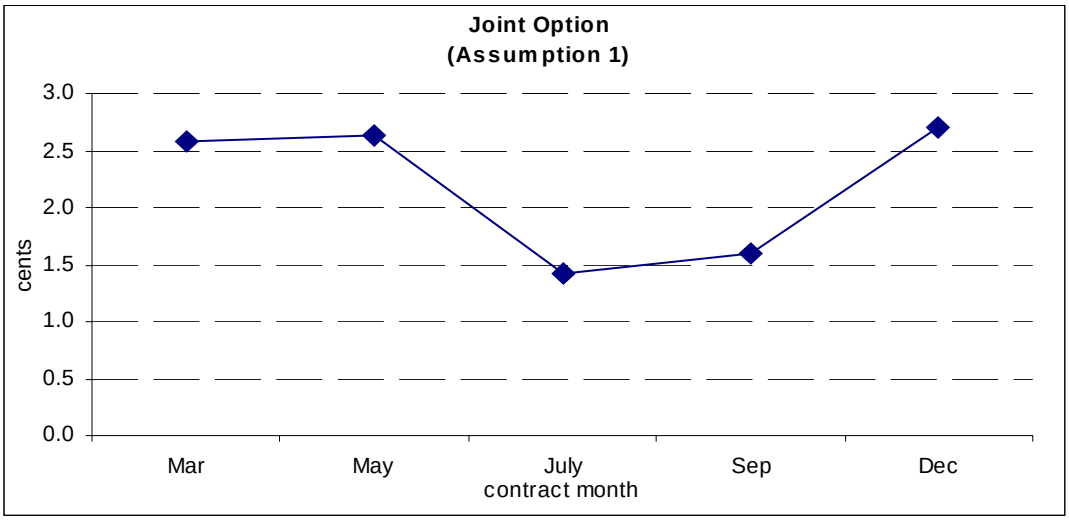
Month \ Year	89	90	91	92	93	94	95	96	97
March	3.674	1.902	3.299	2.770	3.096	2.796	3.312	1.319	1.027
May	2.557	3.230	3.367	2.841	2.857	3.071	2.917	0.697	2.086
July	0.535	0.507	2.339	1.488	2.449	1.762	2.305	0.817	0.550
September	0.347	0.621	2.740	1.889	2.707	3.115	0.987	0.419	1.516
December	1.827	3.291	2.569	2.904	3.141	2.872	2.947	1.496	3.261

a. Values are estimated for the first delivery day.

Figure 1 illustrates the seasonal movement of the joint timing and location options. The value is on average the highest for the post-harvest month December and the lowest for pre-harvest month July.

The negative delta effect jointly with the correlation effect dominate the vega effect, as July is the month when spot prices and the volatility are the highest. At harvest time and immediately thereafter, crop size is known, and prices are least volatile. As vega ($\partial P / \partial \sigma$) for an option is positive, where P is the value of a put option, values of the option should decrease during the months with low price volatility.

Figure 1



However, corn prices are typically at their lowest levels in December. The effect of the low prices through the negative delta of a put option ($\partial P / \partial S$) has an opposite seasonal effect on the option values. For the CBOT corn futures contract, the negative delta effect offsets the positive vega, resulting in high option values in December. Perhaps surprisingly, these effects appear to persist through May.

Assumption 2

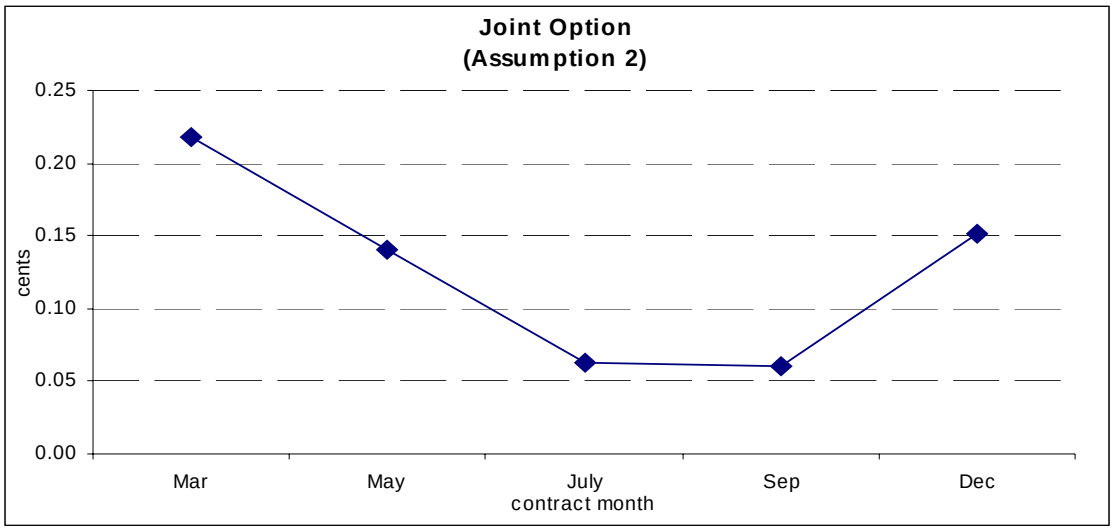
Estimating the joint option under Assumption 2 results in lower values of the joint timing and location options. This is because the possibility of immediate delivery in two different locations makes the option to deliver early less valuable. The value estimates during years 1989-97 range from 0 to 0.7 cents, averaging 0.1 cents (Table 2). This value represents 0.04% of the average futures price on the first delivery day and 4.8 % and 5.5% of the Chicago and Toledo bases (in absolute value).

Table 2: Joint option values estimated under Assumption 2 (in cents)[a]

Month \ Year	89	90	91	92	93	94	95	96	97
March	0.737	0.133	0.327	0.116	0.111	0.084	0.346	0.096	0.012
May	0.205	0.318	0.396	0.000	0.012	0.077	0.158	0.046	0.056
July	0	0.487	0	0	0	0.002	0.074	0	0
September	0.222	0.004	0.124	0.005	0.010	0.140	0.009	0	0.028
December	0.127	0.327	0.056	0.075	0.156	0.085	0.179	0.070	0.286

a. Values are estimated for the first delivery day.

Figure 2



The seasonal fluctuation in the joint option values is similar to the estimates under Assumption 1 (Figure 1), although the option value declines from March to July. The negative delta effect due to high spot prices in pre-harvest months dominates the positive vega effect. The values are the lowest for the pre-harvest month of July continuing through the transition month of September.

IV Role of Delivery Options in Basis Convergence

The above estimates of the joint option are used to explain basis behavior at contract maturity. The option value captures the importance of the option for that particular month and may be a factor in explaining basis non-convergence. That is, the option value may help predict

basis behavior and thereby help understand basis risk. In general, the larger the basis risk, the lower the hedging demand, ceteris paribus. Heifner (1966) and Peck and Williams (1991) analyze hedging effectiveness in terms of the predictability of the change in the basis. Heifner evaluates gains from basing storage decisions on predicted basis changes over various storage intervals. Peck and Williams focus on the effect of delivery timing, during the expiration month, on basis convergence. They find that convergence is better predicted on the last day of trading than on the first day of the expiration month.

The presence of the location option affects the price dynamics of the contract's bases. Garbade and Silber (1983) demonstrate that the addition of a new location increases hedging effectiveness at that location, but reduces the hedging effectiveness in the original delivery point. Pirrong, Kormendi and Meguire (1994) examine the effect of the location option embedded in the CBOT corn and soybean contracts on hedging effectiveness in different locations. They show that adding/expanding a location option may actually improve hedging effectiveness, with an asymmetric impact in different locations.

All the above studies treated delivery options separately and additively, omitting the interaction effect between the options. As a result, evaluations of the impact of delivery options on basis behavior are likely to be biased. Our analysis should achieve a more accurate assessment of the impact of the options on basis behavior, as it uses the estimates of the *joint* value of the delivery options.

Basis Model

Theoretical models of futures prices assume a single expiration day T and perfect convergence of the futures and spot prices on this date. For any date $t < T$, the futures price in perfect and frictionless markets equals

$$F(t,T) = S(t) \cdot e^{(r-y+c)(T-t)},$$

where c is storage cost y is convenience yield, , and F(t,T), S(t), and r are as defined above. This is a result of a cash-and-carry no arbitrage argument. Delivery options add value to the short and result in a lower futures price

$$F(t,T) = S(t) \cdot e^{(r-y+c)(T-t)} + JO(t),$$

where JO(t) is the value at time t of the timing option. Thus, basis at time t is a function of the interest rate, convenience yield, storage cost, time to maturity and the timing option. Separate estimates are obtained for the Chicago and Toledo bases according to the model

$$\frac{F(t,T)}{S_k(t)} = e^{(r-y+c)(T-t)} + \frac{JO(t)}{S_k(t)}$$

for k = 1,2, where JO represents the value of the joint option. The following loglinear model is estimated for each location basis using OLS,

$$LnB_k = \beta_0 + \sum_{i=1}^{4} \alpha_i D_i + \beta_1 Ln(\frac{JO}{S_k}) + \beta_2 FS + \beta_3 IntRate + e,$$

where B_k is the basis in location k, D's represent contract month dummies and JO is in turn defined using the values of the joint option estimated under Assumption 1 and 2, respectively. FS is the futures spread defined as the difference between the price of the deliverable contract and the next nearby and stands as a proxy for convenience yield. IntRate is the riskless rate. The models are also fitted linearly, with observed basis as a function of JO/S_k.

Empirical Results

The estimated values of the joint option are used in regressions of Chicago and Toledo bases behavior. The models help explain how the degree of convergence in different delivery locations vary across delivery months and years and what role the delivery options play in bases non-convergence.

Coefficient estimates from the regressions using the estimates of the joint option value are presented in Tables 3 and 4. The results suggest that the delivery options have a significant effect on the Toledo basis, but not on the Chicago basis. A 1% increase in the joint option value increases the basis in Toledo by 2.6%. The impact on the hedging effectiveness in the Toledo location is thus negative. Although the R^2 for Chicago basis is relatively modest, it compares favorably with other attempts to model basis behavior at or near contract maturity (e.g., Leuthold). The model explains better the variation in the Toledo basis, with R^2 of 64% for the full model and 62% for the reduced model.

Seasonal dummies are in general insignificant. The seasonal component of the basis seems to be picked up by the seasonality of the option values or other variables. Also, the signs on the interest rate and futures spread in the Toledo basis model are not consistent with theory.

Table 3: Chicago basis behavior using joint option estimated under Assumption 1

	Log model		Linear model	
	Full model[a]	Reduced model[a]	Full model[a]	Reduced model[a]
Constant	-0.025(-1.5)	-0.027 (-1.8)	0.996 (107.7)	0.991 (127.1)
D1 (Mar)	-0.007 (-1.4)		-0.007 (-1.3)	
D2 (May)	-0.007 (-1.4)		-0.007 (-1.4)	
D3 (Jul)	-0.005 (-0.9)		-0.005 (-0.9)	
D4 (Sep)	-0.007 (-1.2)		-0.006 (-1.1)	
Futures Spread	-0.113 (-2.0)	-0.111 (-2.1)	-0.134 (-2.5)	-0.128 (-2.6)
Interest Rate	0.056 (0.5)	0.050 (0.5)	0.072 (0.7)	0.062 (0.6)
Joint Option	-0.004 (-1.1)	-0.003 (-1.0)	-0.363 (-0.6)	-0.323 (–0.6)
R^2	50%	32%	36%	36%

a. Numbers represent regression coefficients and t-statistics (in the brackets)

Table 4: Toledo basis behavior using joint option estimated under Assumption 1

	Log model		Linear model	
	Full model[a]	Reduced model[a]	Full model[a]	Reduced model[a]
Constant	0.158 (6.8)	0.159 (7.2)	0.991 (78.8)	0.988 (87.0)
D1 (Mar)	-0.010 (-1.3)		-0.010 (-1.4)	
D2 (May)	-0.008 (-1.1)		-0.008 (-1.2)	
D3 (Jul)	-0.004 (-0.5)		-0.002 (-0.2)	
D4 (Sep)	-0.013 (-1.6)		-0.013 (-1.7)	-0.008 (-1.5)
Futures Spread	-0.153 (-1.8)	-0.170 (-2.2)	-0.117 (-1.6)	-0.126 (-1.8)
Interest Rate	-0.253 (-1.6)	-0.248 (-1.6)	-0.278 (-1.9)	-0.306 (-2.2)
Joint Option	0.026 (5.3)	0.028 (6.1)	4.421 (6.0)	4.353 (6.1)
R^2	64%	60%	68%	65%

a. Numbers represent regression coefficients and t-statistics (in the brackets)

Results listed in Tables 5 and 6 show that both the significance and the direction of the delivery options effect are robust to using estimates under Assumption 2. The options are insignificant in explaining the basis behavior in Chicago and significantly influence the convergence of the basis in Toledo.

Table 5: Chicago basis behavior using joint option estimated under Assumption 2

	Log model		Linear model	
	Full model[a]	Reduced model[a]	Full model[a]	Reduced model[a]
Constant	-0.006 (-0.4)	-0.009 (-0.7)	0.991 (146.9)	0.987 (168.0)
D1 (Mar)	-0.007 (-1.3)		-0.006 (-1.2)	
D2 (May)	-0.007 (-1.3)		-0.007 (-1.4)	
D3 (Jul)	-0.004 (-0.6)		-0.005 (-0.8)	
D4 (Sep)	-0.005 (-0.8)		-0.005 (-1.0)	
Futures Spread	-0.158 (-3.2)	-0.157 (-3.3)	-0.149 (-3.3)	-0.141 (-3.5)
Interest Rate	0.079 (0.7)	0.064 (0.6)	0.118 (0.9)	0.107 (0.9)
Joint Option	0.000 (0.2)	0.000 (0.3)	-1.483 (-0.4)	-1.516 (-0.4)
R^2	35%	31%	35%	31%

a. Numbers represent regression coefficients and t-statistics (in the brackets)

Table 6: Toledo basis behavior using joint option estimated under Assumption 2

	Log model		Linear model	
	Full model[a]	Reduced model[a]	Full model[a]	Reduced model[a]
Constant	0.097 (3.7)	0.086 (3.4)	1.053 (87.7)	1.045 (95.5)
D1 (Mar)	-0.011 (-1.2)		-0.015 (-1.6)	
D2 (May)	-0.003 (-0.3)		-0.005 (-0.6)	
D3 (Jul)	0.005 (0.4)		-0.006 (-0.5)	
D4 (Sep)	-0.017 (-1.7)		-0.020 (-2.1)	
Futures Spread	0.052 (0.6)	0.038 (0.4)	0.080 (1.0)	0.080 (1.0)
Interest Rate	-0.749 (-3.5)	-0.734 (-3.5)	-0.830 (-3.7)	-0.858 (-3.7)
Joint Option	0.005 (2.2)	0.004 (2.3)	16.121 (2.4)	16.960 (2.5)
R^2	43%	33%	44%	35%

a. Numbers represent regression coefficients and t-statistics (in the brackets)

The magnitude of the effect in Toledo is now decreased and a 1% increase in the joint option value under Assumption 2 increases the Toledo basis by 0.5%. This is the result of the overall lower values for the joint option under Assumption 2.

As noted earlier, the estimated option values have no explanatory power for the variability of the Chicago basis. Moreover, Toledo spot prices are typically lower than Chicago prices, even after adjusting for the discount for delivery in Toledo. Thus, the data imply that most deliveries should occur in Toledo, which is inconsistent with the actual delivery patterns. On average, 71% of deliveries took place in Chicago. Alternatively, in 78 % of the expiration months during the years 1989-97 the percentage of the total deliveries delivered in Chicago was larger than that in Toledo.

These seemingly inconsistent results are probably related to the quality of the Chicago prices and 'unobservability' of full costs of delivery. With respect to the data, Chicago is a relatively inactive spot market, and the reported prices are perhaps not representative of transactions' prices. When Central Illinois spot prices are substituted for the Chicago series, the resulting option values do have a significant explanatory power in the basis equation. However, these option values are not for the Chicago location. In other words, the variability of the option

value, based on Central Illinois prices, is helpful in explaining variability of the associated Illinois basis, but these estimates cannot be interpreted as representing the true joint location-timing option values for Chicago and Toledo delivery.

The option model, using the Chicago and Toledo spot prices, captures the fact that Toledo appears optimal for delivery, i.e., the estimated option values are internally consistent with the model and the available data. The data, however, do not reflect some factors (costs) that influence options values and delivery decisions. As Peck and Williams point out, the demand for various commercial uses of corn (livestock feed, processing, and exporting) likely is larger in Toledo than in Chicago; i.e., the opportunity costs of delivering in Chicago are likely smaller than in Toledo. An important function of corn stored in Chicago is delivery in futures contracts; corn located in Toledo has more alternative uses. If so, this is not captured by the data and the model we are using.

Also, the assumption of competitive markets is probably not fully met.[4] Those firms short futures positions in the delivery month likely own corn in certified-for-delivery locations that they own; thus, it is relatively inexpensive for them to make delivery. For one major firm, Chicago was probably the cheapest-to-deliver location. If, for instance, a firm owned corn in Chicago, not Toledo, but nonetheless wanted to make delivery in Toledo, they would be obligated to buy corn in Toledo, possibly pay to have it inloaded to a certified warehouse, and then pay 0.15 cent per bushel per day storage cost (CBOT rule 1056.1). The 0.15 cent cost is equivalent to 4.5 cents per month, while typical commercial storage charges for the 1989-97 period were about 2.7 cents (in elevators not certified for delivery on the contract). Clearly, the

[4] As of July 1, 1996, only two locations were certified for delivery in Chicago, each owned by a major grain merchant. Three firms owned facilities in Toledo. One firm, Cargill, owned certified delivery facilities in both Chicago and Toledo. Thus, they presumably could have made delivery in either location. This suggests that the

cost of delivery in Toledo would have to be sufficient to cover the storage costs in certified warehouses in Toledo, a cost that exceeds common commercial rates.

Further, if a firm short futures does not own corn in a certified warehouse in either deliverable location, then they face the additional costs of transporting and inloading the grain into the approved-for-delivery location. These costs are unobservable and not accounted for in the model. In this case, however, it seems likely that it would be less costly to transport corn to Toledo than Chicago. Also, as mentioned above, the Chicago price series may not capture the transactions' prices for firms to purchase corn for delivery in Chicago.

V Conclusions

The value of the timing option jointly with the location option is estimated for the CBOT corn futures contract for all expiration months during years 1989-97. Two institutional assumptions are used to obtain the value of the option. Assumption 1 allows delivery only on the next day after entering a futures contract and in this sense aims to simulate the discreteness of the actual delivery process. Assumption 2 is consistent with the one frequently used in the literature. It allows delivery on the same day as a futures position is established. The institutional assumptions lead to different estimates of the joint option values, the values being smaller under Assumption 2. This is because the possibility of immediate delivery in combination with the location option lowers the value of delivering early. This result highlights the importance of taking institutional arrangements into account.

The value of the joint option under Assumption 1 averaged 2.2 cents over the years 1989-97, constituting 0.8% of the average futures price on the first delivery day. On average, the joint option value is lowest during the months of July and September. July is characterized by the

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notion of a lower opportunity cost of delivering corn in Chicago may be an important, but difficult to estimate, factor in determining location of deliveries.

highest price levels as well as by relatively high price volatility. The resulting low option values demonstrate that the delta effect dominates in the delivery options implicit in the CBOT corn futures contract. The dominating delta effect persists through September in spite of decreasing price levels and decreasing volatility.

Allowing immediate delivery under Assumption 2 in combination with the location option reduces the value of delivering early and results in lower estimated option values. The joint option values estimated under Assumption 2 averaged 0.1 cent over the years 1989-97.

The joint option has low explanatory power for basis variability in Chicago. Delivery options significantly explain basis non-convergence in Toledo. Results indicate that the option increases the Toledo basis by 2.2% on average and thus decreases hedging effectiveness in this location. However, the results have to be interpreted with caution. Low quality of spot price data (mainly for the Chicago terminal) as well as inadequate data on 'true' costs of delivery make interpretation of the results difficult. Using the narrow concept of costs allocates a disproportionate percent of deliveries to the Toledo location. The option values capture this by reflecting the variability in Toledo basis to a higher degree than the one in Chicago. If there exist transportation and other costs of delivery that eliminate the advantage of delivering in Toledo versus Chicago, the option values and consequently basis regression results may be biased.

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