Designing Optimal Crop Revenue Insurance*

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Abstract

The optimal crop revenue insurance contract is designed from recent developments in the theory of insurance economics under incomplete markets. The message is twofold. Firstly, when the indemnity schedule is contingent on individual price and individual yield, the optimal contract depends only on the individual gross revenue. Secondly, this policy is shown to fail if the indemnity function is based on aggregate price and/or aggregate yield. A closed-form solution, in which basis risks are ignored, is proposed. It differs from actual revenue insurance programs proposed to the U.S. farmers. When insurance and capital markets are unbiased, it can be replicated with existing crop yield and revenue insurance policies and hedging contracts if the decision variables are not constrained. The impact of yield and price basis risks on the form of the optimal crop revenue insurance contract is examined and a closed-form solution is derived.

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Introduction

Crop farmers as well as many other primary commodity producers face joint price and output (yield) risk. Alternatives for managing the sources of risk have recently expanded with innovations in revenue insurance which aims at providing a protection against price declines and/or low yield. The U.S. Risk Management Agency developed a pilot revenue insurance program known as Income Protection in 1996. Two private sector programs were also approved: Crop Revenue Coverage and Revenue Assurance. Revenue insurance choices continue to expand with a new product called Group Risk Income Protection launched in 1999. In Europe, the first revenue insurance contract is proposed to English farmers since 1999 and other policies should be offered to European producers in the near future.

This paper is a first attempt to investigate the design of an optimal crop revenue insurance contract through recent theoretical developments on optimum insurance in incomplete markets. When the indemnity schedule is contingent on the individual price and the individual yield, the optimal indemnity schedule turns out to depend only on the individual gross revenue, defined as the individual yield times the price at which the producer sells his output. However, real-world markets often are not complete because the indemnity function is based on imperfect estimators of the individual yield and/or price. For instance, under Income Protection and Crop Revenue Coverage programs, indemnity payments are based on national output price rather than individual price. The optimal revenue insurance is designed in this context of incomplete markets. Indemnity payments are contingent not only on the aggregate gross revenue, equal to the aggregate yield multiplied by the aggregate price, but also on the aggregate price and/or on the aggregate yield. A first closed-form solution of the optimal revenue insurance contract is proposed when basis risks are ignored. A second one is derived in the presence of price and yield basis risks when the producer's preferences exhibit constant absolute risk aversion. The latter stresses the role of the producer's prudent behavior and of the variability of basis risks on the optimal coverage.

The second purpose of this paper is to examine how the optimal revenue insurance policy could be replicated with crop and revenue insurance policies and with hedging instruments offered by real-world insurance and financial markets. The results are used to comment on empirical findings about the risk-reducing performance of insurance and hedging contracts recently presented in the agricultural economic literature.

Optimum Insurance Design against Individual Yield and Price Risk

In the expected utility framework, the producer's preferences are represented by a monotone increasing and strictly concave von Neumann-Morgenstern utility function u. His gross revenue is exposed to multiple risks, such as random yield and random price, that affect each crop. The stochastic gross revenue is defined as a deterministic function R of n nonnegative random variables $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_i, ..., \tilde{x}_n)$ with a joint density function $f(x_1, ..., x_n)$ defined over the support $X \equiv [0, \bar{x}_1] \times ... \times [0, \bar{x}_n]$, where $\bar{x}_i > 0$ for $i = 1, ..., n$. Hence, the producer's gross revenue is $R(x)$ when $x = (x_1, ..., x_n) \in X$ is realized. The function R is assumed to increase with respect to each argument:

(1) $R_i(x) \equiv \partial R(x) / \partial x_i \geq 0$ for all $x \in X$, for $i = 1, ..., n$.

To protect against the occasional occurrence of low revenue levels, the producer has the opportunity to purchase an insurance contract. It is described by a couple $[I(.), P]$ where $I(x)$ is the payment transferred from the insurance company to the insured producer when x is realized, and P is the insurance premium. A central assumption is that the indemnity function depends upon individual random parameters, such as individual yields and prices at which the farmer sells his production. A feasible indemnity function must be nonnegative:

(2) $I(x) \geq 0$ for all $x \in X$

and the premium is assumed to depend only on the actuarial value of the policy:

(3) $P = c[EI(\tilde{x})]$

with $c(0)=0$, $c'(I) \geq 1$ for all $I \geq 0$ and E is the expectation operator.

An optimal insurance contract is obtained by finding the insurance premium and the indemnity function that maximize the insured producer's expected utility function of gross revenue under the above-mentioned constraints:

(4) $\max_{I(.),P} Eu[R(\tilde{x})+I(\tilde{x})-P] = \int_{x\in X} u[R(x_1,x_2,...,x_n)+I(x_1,x_2,...,x_n)-P] f(x_1,x_2,...,x_n)dx_1dx_2...dx_n$

subject to conditions (2) and (3).

Since the unknown function $I(x)$ depends on n variables, the extension of the simple Euler equation can be used to include n dimensions in order to derive the optimal insurance policy. The objective function and the constraints do not contain the partial derivatives of the optimal indemnity function. This entails that the Euler equation is a succession of pointwise first-order conditions for $I(.)$ and, consequently, problem (4) can be solved by using Kuhn-Tucker conditions for $I(x)$ for all $x \in X$. The first-order condition with respect to $I(x)$ is

(5) $u'[R(x)+I(x)-P]+l(x)-mc'[EI(\tilde{x})]=0$

for all $x \in X$, where m and $l(x)$ are the Lagrangian multipliers associated to constraints (3) and (2), respectively, with

(6) $l(x)\begin{cases} =0 & \text{if } I(x)>0 \\ \geq 0 & \text{otherwise.} \end{cases}$

Equation (5) can be rewritten as

(7) $u'[R(x)+I(x)-P]=mc'[EI(\tilde{x})]$ for all $x \in X$ such that $I(x)>0$.

In every state of the world in which indemnity payments are made, the marginal utility function of the insured producer must be constant. Therefore, the optimal indemnity function depends only on the realized gross revenue, i.e. there exists a function $J(.)$ such that

(8) $I^*(x) = J[R(x)]$ for all $x \in X$.

Since the producer's utility function is concave and the gross revenue increases with respect to each argument, an optimal insurance contract has the following form.

PROPOSITION 1. *The optimal insurance contract, solution to program (4) subject to constraints (2) and (3) when P is fixed, provides full coverage below a trigger gross revenue $\hat{R} \geq 0$: $I^*(x) = \max(\hat{R} - R(x), 0)$.*

When the producer faces multiple uncertainty affecting his gross revenue, Proposition 1 states that it is optimal to purchase a unique insurance contract covering all sources of risks at the same time, with full insurance below a trigger gross revenue. In other words, the payoff function I^* is the least expensive risk-sharing tool to reach a predetermined insurance coverage. Such a result holds whatever the degree of correlation between the sources of risk.

The first-order condition of the maximization problem (4) with respect to P is

(9) $Eu'[R(\tilde{x}) + I(\tilde{x}) - P] - m = 0$.

Introducing equation (9) in equation (5) and taking the expectation with respect to $\tilde{x}$ yields

(10) $E\,l(\tilde{x}) = \{c'[EI(\tilde{x})] - 1\} Eu'[R(\tilde{x}) + I(\tilde{x}) - P]$.

When insurance is sold at an actuarially fair price, i.e. $c'(I) = 1$ for all $I \geq 0$, $E\,l(\tilde{x}) = 0$ and therefore the optimal trigger revenue $\hat{R}$ is equal to the maximum gross revenue $\overline{R} = R(\overline{x}_1, \overline{x}_2, ..., \overline{x}_n)$. The insured producer chooses to be fully insured: his gross revenue net of the indemnity and the insurance premium is equal to its expectation with certainty. When insurance is costly, i.e. $c'(I) > 1$ for some $I \geq 0$, $E\,l(\tilde{x}) > 0$ and consequently $\hat{R}$ is lower than $\overline{R}$.

Several results can be derived from Proposition 1, depending on the form of the gross revenue function R. They also contribute to re-examining results recently provided by Hennessy, Babcock and Hayes.

First, let the gross revenue be $R(\tilde{x}_1,\tilde{x}_2)=\tilde{x}_1\tilde{x}_2$ where $\tilde{x}_1$ and $\tilde{x}_2$ are the individual random price and the individual random yield of a single crop, respectively. Applying Proposition 1 yields that the optimal insurance against joint yield and price risk displays full coverage whenever the realized individual gross revenue x_1x_2 falls below a trigger level $\hat{R}\geq 0$, for a fixed premium:

(11) $I^*(x_1,x_2)=\max(\hat{R}-x_1x_2,0)$.

This revenue insurance contract is thus the cheapest way of providing a predetermined coverage level. Consequently, revenue insurance displaying full coverage under $\hat{R}=\hat{x}_1\hat{x}_2$ is less costly than price insurance providing full coverage under $\hat{x}_1$ and crop insurance providing full coverage under $\hat{x}_2$ (Hennessy, Babcock and Hayes, Result 1).

Second, consider that the gross revenue satisfies $R(\tilde{x}_1,\tilde{x}_2)=\tilde{x}_1+\tilde{x}_2$ where $\tilde{x}_1$ and $\tilde{x}_2$ represent the random gross revenues of two agricultural activities. From Proposition 1, the optimal insurance policy in the presence of $\tilde{x}_1$ and $\tilde{x}_2$ provides full coverage whenever the sum of these realized gross revenues (x_1+x_2) falls below a trigger level $\hat{R}$, for a fixed premium:

(12) $I^*(x_1,x_2)=\max(\hat{R}-(x_1+x_2),0)$.

For a given insurance premium, the revenue insurance policy based on the whole-farm revenue, often called portfolio revenue insurance, provides a better coverage than the sum of insurance contracts based on commodity-specific revenue, often called commodity-specific revenue insurance. Consequently, for a predetermined coverage level, the portfolio revenue insurance is less costly than commodity-specific revenue insurance (Hennessy, Babcock and

Hayes, Result 2). It should be noticed that this finding is just a reinterpretation of Raviv's result which claims that full insurance above a deductible on the aggregate loss is optimal when there are multiple additive losses.

The above two cases can be combined to obtain a third one in which the stochastic gross revenue is $R(\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_n) = \sum_{j=1}^{n/2} A_j \tilde{R}_j$, where n is an even number, with $\tilde{R}_j = \tilde{x}_{2j-1}\tilde{x}_{2j}$ for $j = 1,...n/2$, $\tilde{x}_{2j-1}$ and $\tilde{x}_{2j}$ are the individual random price and the individual random yield of crop j, respectively, and A_j is the area devoted to crop j. For a multi-product farm, the optimal revenue insurance thus displays full coverage whenever the sum of the realized gross revenues $\sum_{j=1}^{n/2} A_j R_j$ falls below a trigger level $\hat{R}$, for a fixed premium:

$$(13) \qquad I^*(x_1, x_2, ..., x_n) = \max\left(\hat{R} - \sum_{j=1}^{n/2} A_j R_j, 0\right)$$

Therefore, the portfolio insurance on individual gross revenue below $\hat{R}$ is less costly than the sum of price and yield insurance policies (Hennessy, Babcock and Hayes, Result 3).

The superiority of revenue insurance on separate price insurance and yield insurance has been shown when the indemnity payments are contingent on individual revenue losses. However, real-world markets often are not complete in that indemnity schedule is based on an imperfect signal of the producer's gross revenue. We examine in the next section the design of an optimal insurance against joint yield and price uncertainty in this context of incomplete markets.

Optimum Insurance Design against Aggregate Yield and Price Risk

The indemnity schedule is now assumed to be based on imperfect estimators of individual yield and price. These indexes can represent aggregate yield estimated in a surrounding geographic area and futures on board of trade. To model the imperfect mechanism provided by the insurance markets, individual yield $\tilde{y}_i$ and individual price $\tilde{p}_i$ are written as a linear function of the yield index $\tilde{y}$ and price index $\tilde{p}$, respectively:

(14) $\tilde{p}_i = a_1 + b_1 \tilde{p} + \tilde{e}_1$

(15) $\tilde{y}_i = a_2 + b_2 \tilde{y} + \tilde{e}_2$

where $p \in [0, p_{\max}]$, $y \in [0, y_{\max}]$, $\tilde{e}_1$ and $\tilde{e}_2$ are zero-mean random variables. Such a relationship is obtained when the stochastic individual yield (price) is projected orthogonally onto the stochastic yield (price) index.[1] In addition, we assume that $\tilde{e}_i$ is independent of $(\tilde{p}, \tilde{y})$ for $i = 1,2$, and $\tilde{e}_1$ is independent of $\tilde{e}_2$. No specific assumptions need to be made about the stochastic dependence between the price index and the yield index. An optimal insurance policy $[I(.,.), P]$ is solution of the following maximization program:

(16) $\underset{I(.,.),P}{Max}\, Eu[(a_1 + b_1\tilde{p} + \tilde{e}_1)(a_2 + b_2\tilde{y} + \tilde{e}_2) + I(\tilde{p}, \tilde{y}) - P]$

subject to

(17) $I(p, y) \geq 0$ for all (p, y)

(18) $P = c[EI(\tilde{p}, \tilde{y})]$

where the administrative cost function $c(.)$ is defined as previously. The resolution of this maximization problem leads to the following proposition.

[1] We thus have $b_1 = \mathrm{cov}(\tilde{p}_i, \tilde{p})/\mathrm{var}(\tilde{p})$, $a_1 = E\tilde{p}_i - b_1 E\tilde{p}$, $b_2 = \mathrm{cov}(\tilde{y}_i, \tilde{y})/\mathrm{var}(\tilde{y})$ and $a_2 = E\tilde{y}_i - b_1 E\tilde{y}$.

PROPOSITION 2. *The insurance contract, solution to program (16) subject to constraints (17) and (18) when P is fixed, takes one of the following four forms:*

(i) *If $b_1 > 0$ and $b_2 > 0$, a decreasing trigger function $\hat{p}(.)$ exists such that:*

$$I^*(p,y)\begin{cases} >0 & \text{if } p < \hat{p}(y) \\ =0 & \text{otherwise.} \end{cases}$$

(ii) *If $b_1 > 0$ and $b_2 < 0$, an increasing trigger function $\hat{p}(.)$ exists such that:*

$$I^*(p,y)\begin{cases} >0 & \text{if } p < \hat{p}(y) \\ =0 & \text{otherwise.} \end{cases}$$

(iii) *If $b_1 < 0$ and $b_2 > 0$, an increasing trigger function $\hat{p}(.)$ exists such that:*

$$I^*(p,y)\begin{cases} >0 & \text{if } p > \hat{p}(y) \\ =0 & \text{otherwise.} \end{cases}$$

(iv) *If $b_1 < 0$ and $b_2 < 0$, a decreasing trigger function $\hat{p}(.)$ exists such that:*

$$I^*(p,y)\begin{cases} >0 & \text{if } p > \hat{p}(y) \\ =0 & \text{otherwise.} \end{cases}$$

In the four cases, for all $(p,y): I^*(p,y) > 0$, the marginal indemnity functions satisfy:

(19) $$\frac{\partial I^*(p,y)}{\partial p} = -b_1 \frac{E_2\left[(a_2 + b_2 y + \tilde{e}_2) E_1 u''(\tilde{p})\right]}{E_1 E_2 u''(\tilde{p})}$$

(20) $$\frac{\partial I^*(p,y)}{\partial y} = -b_2 \frac{E_1\left[(a_1 + b_1 p + \tilde{e}_1) E_2 u''(\tilde{p})\right]}{E_1 E_2 u''(\tilde{p})}$$

where $p = (a_1 + b_1 p + e_1)(a_2 + b_2 y + e_2) + I(p,y) - P$ and E_i is the expectation operator with respect to $\tilde{e}_i$, for $i = 1,2$.

The proof of Proposition 2 is found in the appendix. The form of the optimal crop revenue insurance contract depends on the sign of the regression coefficients b_1 and b_2. If b_1 is positive (negative), indemnity payments are made whenever the realized price index is below (above) the trigger level. This trigger price is a function of the realized yield index. It decreases (increases) as the yield index increases if $b_1 b_2$ is positive (negative). The indemnity function increases (decreases) with the realized price index if b_1 is positive (negative) and it increases (decreases) with the realized yield index if b_2 is positive (negative).

The impact of actuarially fair insurance, i.e. $c'(I)=1$ for all $I \geq 0$, and costly insurance, i.e. $c'(I)>1$ for some $I \geq 0$, on the trigger function is presented in the following proposition.

PROPOSITION 3. *Under actuarially fair insurance,* $\hat{p}(y)= p_{\max}$ *for all* y *if* $b_1 > 0$ *and* $\hat{p}(y)=0$ *for all* y *if* $b_1 < 0$. *Under costly insurance,* $\hat{p}(y)< p_{\max}$ *for some* y *if* $b_1 > 0$ *and* $\hat{p}(y)>0$ *for some* y *if* $b_1 < 0$.

The proof of Proposition 3 is found in the appendix. The producer is thus fully covered against the revenue variability if the insurance policy is sold at a fair price. He is partially covered otherwise. The remaining part of this section focuses on the most realistic case where the regression coefficients b_1 and b_2 are positive: the individual price and the price index are positively correlated, and the individual yield is positively correlated with the yield index. It is first worthwhile to note that if the insurance indemnity is based on individual price and individual yield, i.e. $\tilde{p} \equiv \tilde{p}_i$ and $\tilde{y} \equiv \tilde{y}_i$, then the partial derivatives (19) and (20) are equal to $-y_i$ and $-p_i$ for all $(p_i, y_i): I^*(p_i, y_i) > 0$, respectively. The optimal insurance contract is thus $I^*(p_i, y_i) = \max\left[\hat{R} - p_i y_i, 0\right]$, where $\hat{R} \geq 0$. This result has been obtained in the previous section as a consequence of Proposition 1.

If the indemnity schedule is based on yield index and price index, equations (19) and (20) can be rewritten as:

(21) $$\frac{\partial I^*(p,y)}{\partial p} = b_1\left\{- E[\tilde{y}_i / \tilde{y} = y] + \text{cov}_2\left[\tilde{e}_2, \frac{E_1 u''(\tilde{p})}{-E_1 E_2 u''(\tilde{p})}\right]\right\}$$

(22) $$\frac{\partial I^*(p,y)}{\partial y} = b_2\left\{- E[\tilde{p}_i / \tilde{p} = p] + \text{cov}_1\left[\tilde{e}_1, \frac{E_2 u''(\tilde{p})}{-E_1 E_2 u''(\tilde{p})}\right]\right\}$$

for all $(p, y): I^*(p,y) > 0$, where cov_j is the covariance operator with respect to the $\tilde{e}_j$ basis risk, for $j = 1,2$. The optimal marginal indemnity function with respect to the price index, expressed in equation (21), is affected by the regression coefficient between the individual price and the price index, by the bias between the individual yield and the yield index through the expectation of the individual yield conditional on the yield index, and by the $\tilde{e}_2$ yield basis risk through the covariance term. This slope is higher or lower than $-b_1 E[\tilde{y}_i / \tilde{y} = y]$ depending on whether the covariance term is positive or negative. Since the profit function increases with $\tilde{e}_2$, this covariance is positive, null or negative as the marginal utility function, is convex, linear or concave in wealth. The notion of prudence, which is linked to the convexity of u', is recognized as a realistic behavioral assumption (Kimball). It is a necessary condition for decreasing absolute risk aversion. The role of prudence in the design of an optimal insurance contract has recently been emphasized by Mahul (2000a) in the presence of an insurable risk and an uninsurable and independent risk. The marginal indemnity function with respect to the aggregate price is thus higher than $-b_1 E[\tilde{y}_i / \tilde{y} = y]$ if the producer is prudent. From a similar analysis, the marginal indemnity function with respect to the aggregate yield in (22) is higher than $-b_2 E[\tilde{p}_i / \tilde{p} = p]$ if the producer exhibits prudence.

Mahul (1999) shows that the form of an optimal area yield crop insurance contract under nonrandom price depends only on the regression coefficient between the individual yield and the area yield. Consequently, the producer's preferences does not affect the form of the contract. When the price is stochastic and the indemnity schedule is contingent on an approximation of this price, equation (21) shows that the optimal marginal coverage with respect to the yield index depends on the producer's attitude towards risk, and especially his prudent behavior.

The optimal coverage function characterized by equations (21) and (22) is analyzed in two steps. Firstly, the covariance terms are ignored in order to focus on the impact of the bias between individual and aggregate yield and between individual and aggregate price. Secondly, we assume there is no bias and the effect of yield and price basis risks on the form of the optimal indemnity schedule is highlighted.

As a first approximation, the covariance terms in (21) and (22) are ignored.[2] The partial derivatives become

(23) $$\frac{\partial I^*(p,y)}{\partial p} \approx -b_1 E[\tilde{y}_i / \tilde{y} = y]$$

(24) $$\frac{\partial I^*(p,y)}{\partial y} \approx -b_2 E[\tilde{p}_i / \tilde{p} = p]$$

for all $(p,y): I^*(p,y) > 0$. The closed-form solution of the optimal crop revenue insurance contract thus takes the following form:

(25) $$I^*(p,y) \approx I^{A1}(p,y) = \max\left[\hat{S} - E(\tilde{y}_i / \tilde{y} = y)E(\tilde{p}_i / \tilde{p} = p), 0\right].$$

The approximate indemnity function is contingent on the product of the conditional expectation of the individual yield and of the individual price $S(p,y) = E(\tilde{y}_i / \tilde{y} = y)E(\tilde{p}_i / \tilde{p} = p)$. Payoffs are made whenever $S(p,y)$ is lower than the trigger level $\hat{S}$. From equations (14) and (15), this can be rewritten as

(26) $$I^{A1}(p,y) = \max\left[\hat{T} - (b_1 a_2 p + b_2 a_1 y + b_1 b_2 py), 0\right]$$

[2] The covariance term in (21) (in (22)) equals zero if the producer's utility function is quadratic, an unrealistic assumption, and/or if the individual price (the individual yield) is a deterministic linear function of the price index (the yield index), i.e. there is no price basis risk (yield basis risk).

where $\hat{T} = \hat{S} - a_1 a_2$. The approximate indemnity schedule depends not only on the gross revenue index py but also on the price index and on the yield index taken separately. It is worthwhile to note that, contrary to the optimal insurance policy, the closed-form solution does not depend on the producer's attitude towards risk. This approximate form can also be viewed as a consequence of Proposition 1 where the gross revenue is equal to the function S.

Two particular cases are derived from this result. Firstly, if the insurance indemnity is based on the price index and on the individual yield, i.e. $\tilde{y} \equiv \tilde{y}_i$, then the closed-form solution (25) becomes

(27) $$I^{A2}(p, y_i) = \max\left[\hat{S} - y_i E(\tilde{p}_i / \tilde{p} = p), 0\right] = \max\left[\hat{S} - (a_1 y_i + b_1 p y_i), 0\right].$$

The approximate indemnity function is contingent on the insurable gross revenue py_i and on the individual yield. Secondly, if the insurance indemnity is based on the individual price, i.e. $\tilde{p} \equiv \tilde{p}_i$, and on the yield index then the closed-form solution of the optimal insurance contract can be rewritten as

(28) $$I^{A3}(p_i, y) = \max\left[\hat{S} - E(\tilde{y}_i / \tilde{y} = y) p_i, 0\right] = \max\left[\hat{S} - (a_2 p_i + b_2 p_i y), 0\right].$$

The approximate indemnity schedule is contingent on the insurable gross revenue $p_i y$ and on the individual price.

The impact of the price and yield basis risks on the design of an optimal crop revenue insurance contract is now examined. For the sake of simplicity, we assume that $a_1 = a_2 = 0$ and $b_1 = b_2 = 1$. This entails that $\tilde{p}_i = \tilde{p} + \tilde{e}_1$ and $\tilde{y}_i = \tilde{y} + \tilde{e}_2$, respectively. From equations (21) and (22), the first derivatives of the optimal coverage become

(29) $$\frac{\partial I^*(p, y)}{\partial p} = -y + \text{cov}_2\left[\tilde{e}_2, \frac{E_1 u''(\tilde{p})}{-E_1 E_2 u''(\tilde{p})}\right]$$

(30) $$\frac{\partial I^*(p, y)}{\partial y} = -p + \text{cov}_1\left[\tilde{e}_1, \frac{E_2 u''(\tilde{p})}{-E_1 E_2 u''(\tilde{p})}\right]$$

for all $(p, y): I^*(p, y) > 0$, where $\pi = (p + e_1)(y + e_2) + I^*(p, y) - P$. An approximation of the covariance term in (29) is first provided. Taking a Taylor series expansion of $u''(\pi)$ around $\pi_1 = p(y + e_2) + I^*(p, y) - P$ and ignoring the terms associated with the fourth or higher derivatives of the utility function yield

(31) $$u''(\pi) \approx u''(\pi_1) + (y + e_2)e_1 u'''(\pi_1).$$

The expectation of $u''(\pi)$ with respect to $\tilde{e}_1$ is approximated by $E_1 u''(\tilde{\pi}) \approx u''(\pi_1)$ because $E_1 \tilde{e}_1 = 0$. From a Taylor series approximation of $u''(\pi_1)$ around $\bar{\pi} = E_2 \tilde{\pi}_1 = py + I^*(p, y) - P$, we have

(32) $$u''(\pi_1) \approx u''(\bar{\pi}) + pe_2 u'''(\bar{\pi}).$$

Approximating the expectation of $u''(\pi_1)$ with respect to $\tilde{e}_2$ yields $E_2 u''(\tilde{\pi}_1) \approx E_1 E_2 u''(\tilde{\pi}) \approx u''(\bar{\pi})$. From the above approximations, we deduce that

(33) $$\operatorname{cov}_2\left[\tilde{e}_2, \frac{E_1 u''(\tilde{\pi})}{-E_1 E_2 u''(\tilde{\pi})}\right] \approx \operatorname{cov}_2\left[\tilde{e}_2, \frac{u''(\bar{\pi}) + pu'''(\bar{\pi})\tilde{e}_2}{-u''(\bar{\pi})}\right] = pR(\bar{\pi})\operatorname{var}(\tilde{e}_2)$$

where $R(\pi) \equiv -u'''(\pi)/u''(\pi)$ is the index of absolute prudence. A similar method is used to approximate the covariance term in (30). Therefore, closed-form solutions of the first derivatives of the optimal coverage (29) and (30) are

(34) $$\frac{\partial I^*(p, y)}{\partial p} \approx -y + pR(\bar{\pi})\operatorname{var}(\tilde{e}_2)$$

(35) $$\frac{\partial I^*(p, y)}{\partial y} \approx -p + yR(\bar{\pi})\operatorname{var}(\tilde{e}_1)$$

for all $(p, y): I^*(p, y) > 0$. Contrary to the previous approximation where basis risks were ignored, the above approximation highlights the impact of the policyholder's prudent behavior and of the basis risk variability on the optimal crop revenue insurance design. As mentioned in the general case, the presence of price and yield basis risks will induce the prudent producer to select an indemnity function such that its slope with respect to price (yield) is higher than $-y$ ($-p$). Comparative statics results can be derived from equations (34) and (35). The more prudent the insured producer, the higher the first derivatives of the optimal coverage. Suppose that the insured producer's index of absolute prudence R is constant with wealth. This is equivalent to assume that his preferences exhibit constant absolute risk aversion (CARA). Then the derivative of the optimal coverage with respect to price (yield) increases with the variance of the yield (price) basis risk. Under CARA, the closed-form solution of the optimal crop revenue insurance is obtained by integrating equations (34) and (35) with respect to yield and price:

(36) $$I^*(p,y) \approx I^B(p,y) = \max\left[\hat{D} - \left\{py - \frac{1}{2}R\left[p^2 \operatorname{var}(\tilde{e}_2) + y^2 \operatorname{var}(\tilde{e}_1)\right]\right\}, 0\right].$$

Payoffs are thus made whenever $D(p,y) = py - \frac{1}{2}R\left[p^2 \operatorname{var}(\tilde{e}_2) + y^2 \operatorname{var}(\tilde{e}_1)\right]$ is lower than a trigger value $\hat{D} \geq 0$. The presence of price (yield) basis risk implies that D is concave in yield (price).

Optimal Insurance and Hedging Decisions

One of the first insurance product proposed to the U.S. farmers was the multiple peril crop insurance (MPCI) contract. Under this government-subsidized program, producers receive indemnities when the realized individual yield falls below a yield guarantee $\hat{y}_i$:

(37) $$I^{MPCI}(y_i) = p_s \max[\hat{y}_i - y_i, 0]$$

where p_s is the nonrandom price selection at which the insurer compensates the farmer for a unit loss of the commodity. The farmer selects his yield guarantee $\hat{y}_i$ between 50% and 75% of the individual average historical yield m_i, refereed to as the actual production history

(APH), in 5% increments, and any price selection between 30% and 100% of the FCIC estimated market price (GAO, Harwood et al.).

Since 1996, three government-subsidized revenue insurance plans are offered to the U.S. farmers. For both Income Protection (IP) and Revenue Assurance (RA), the farmer's gross revenue guarantee is defined when crops are planted. It equals the product of the yield guarantee $\hat{y}_i$, which is between 50% and 75% of the farmer's APH, and the projected price. The realized gross revenue is established by multiplying the realized yield and a price at harvest. The producer receives an indemnity when the realized gross revenue falls below the revenue guarantee:

(38) $I^j(p, y_i) = q_i \max\left[d^j \hat{p}\hat{y}_i - d^j p y_i, 0\right]$ for $j = RA, IP$

where p is the Chicago Board of Trade's November price for the December contract, $\hat{p}$ is the Chicago Board of Trade's February price for the December contract, $q_i \in \{0,1\}$ is the coverage level chosen by the producer, and d^j is a fixed coefficient of adjustment. This coefficient equals one under IP and a county factor under RA. Therefore, indemnity under IP is based on individual yield and national price, whereas indemnity under RA is based on individual yield and local price. Under the third revenue insurance plan, called Crop Revenue Coverage (CRC), indemnities are triggered if the farmer's realized gross revenue falls below a revenue guarantee measured by the product of the realized individual yield and the higher of the price at planting and the price at harvest:

(39) $I^{CRC}(p, y_i) = q_i \max\left[\max(p, \hat{p})\hat{y}_i - p y_i, 0\right]$.

where the yield guarantee $\hat{y}_i$ is selected between 50% and 85%, in 5% increments, of the farmer's APH, and $q_i \in \{0, 0.95, 1\}$ is the coverage level chosen by the producer. This indemnity schedule can be rewritten as

(40) $I^{CRC}(p, y_i) = q_i \max\left[\left(\hat{p} + \max(p - \hat{p}, 0)\right)\hat{y}_i - p y_i, 0\right]$.

This revenue insurance thus provides a replacement-cost protection to producer characterized by the call option $\max(p-\hat{p},0)$.[3] The producer's revenue guarantee may increase over the season, allowing the producer to purchase "replacement" bushels if yields are low and prices increase during the season. An innovative insurance product has been proposed to English producers in 1999. Its indemnity schedule looks like the CRC program, except that it is contingent on an area yield index rather than the individual yield. Other revenue insurance contracts are under study and they may be launched in Europe in the near future.

U.S. revenue insurance programs continue to expand, with a new product introduced in 1999 and called Group Risk Income Protection (GRIP). Under this program, coverage is based on county-level gross revenue, calculated as the product of the county yield and the harvest-time futures market price. The GRIP program is available as a pilot program in selected counties for corn and soybean (Dismukes). Formally, its indemnity schedule is

(41) $$I^{GRIP}(p,y)=q_i \max\left[\hat{p}\hat{y}-py,0\right]$$

where y is the realized area yield, and $\hat{y}$ is the area yield guarantee selected between 70% and 90%, in 5% increments, of the expected area yield, and q_i is the coverage level. This insurance policy adds a revenue component to the area yield insurance program.

Farmers have also the opportunity to hedge against yield and price variations on financial markets. Beside price futures contracts and options on futures to manage price risk, they can use innovative instruments to hedge against crop yield risk, called Crop Yield Insurance (CYI) futures and options. They were launched by the Chicago Board of Trade in 1995. The underlying instruments are the official state-based yield estimates released during the growing and harvesting season by the U.S. Department of Agriculture (Vukina, Li and Holthausen).

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[3] The price at harvest is subject to a maximum upward price movement: $p \le \hat{p}+m$ where m is equal to $1.50 per bushel for corn and $3.00 for soybeans.

Our purpose is to examine how these revenue insurance contracts can be combined with the crop insurance policy and the hedging instruments in order to replicate the closed-form solutions of the optimal hedging strategy against joint yield and price risk. We assume that the crop and revenue insurance programs do not exhibit constraints on yield and price guarantees and on the coverage level, and that insurance and hedging instruments are provided at fair prices. These assumptions will be discussed hereafter.

The yield and price basis risks are first ignored. When the indemnity schedule of the revenue insurance contract based on individual yield and individual price is available, we know from Proposition 1 that the first-best optimal insurance policy depends only on the individual gross revenue and this policy displays full insurance under a critical level. Consequently, crop insurance and hedging tools turn out to be redundant. Nevertheless, to our knowledge, such revenue insurance policy is not provided by real-world insurance markets.

When the indemnity schedule of the revenue insurance policy based on individual yield and aggregate price is available at a fair price, we deduce from Proposition 3 and equation (27) that the closed-form solution of the optimal indemnity function net of its premium is

(42) $$J^{A2}(p, y_i) = I^{A2}(p, y_i) - EI^{A2}(\tilde{p}, \tilde{y}_i) = [a_1 E\tilde{y}_i + b_1 E(\tilde{p}\tilde{y}_i)] - [a_1 y_i + b_1 p y_i].$$

It can be rewritten as

(43) $$J^{A2}(p, y_i) = b_1[E(\tilde{p}\tilde{y}_i) - p y_i] + a_1[E\tilde{y}_i - y_i].$$

The approximate net indemnity function can be replicated by purchasing the IP policy at a coverage level $q_i = b_1$, or the RA policy at $q_i = b_i / d^{RA}$, with a revenue guarantee $\hat{p}\hat{y}_i = p_{\max} y_{i\max}$, where $p_{\max}$ and $y_{i\max}$ are the maximum national price and the maximum individual yield respectively. It thus provides full coverage against the random variable $b_1 \tilde{p}\tilde{y}_i$. The farmer also purchases the MPCI contract with $p_s = a_1$, if positive, and $\hat{y}_i = y_{i\max}$. It provides full coverage against individual yield variations in volume. Therefore the producer is fully covered against the price risk through the revenue insurance contract. He is partially insured against the yield risk and thus he is induced to purchase the MPCI contract, if a_1 is positive. Revenue insurance and individual crop insurance turn out to be complementary, whereas futures contracts and options on futures are redundant. The final wealth of the insured producer becomes

(44) $$w^{ins} = p_i y_i + J^{A2}(p, y_i) = a_1 E\tilde{y}_i + b_1 E(\tilde{p}\tilde{y}_i) + e_1 y_i \, .$$

The only source of uncertainty borne by the insured producer stems from the zero-mean random variable $\tilde{e}_1 \tilde{y}_i$. When the indemnity schedule of the revenue insurance contract depending on the yield index and the price index is available at a fair price, Proposition 3 and equation (26) yield that the approximate indemnity function net of its premium is

(46) $$J^{A1}(p,y) = I^{A1}(p,y) - EI^{A1}(\tilde{p},\tilde{y}) = [b_1 a_2 E\tilde{p} + b_2 a_1 E\tilde{y} + b_1 b_2 E(\tilde{p}\tilde{y})] - [b_1 a_2 p + b_2 a_1 y + b_1 b_2 py]$$

which can be rewritten as

(44) $$J^{A1}(p, y) = b_1 b_2 [E(\tilde{p}\tilde{y}) - py] + b_1 a_2 [E\tilde{p} - p] + b_2 a_1 [E\tilde{y} - y].$$

This closed-from solution can be replicated as follows: the producer purchases the GRIP policy at a coverage level $q_i = b_1 b_2$ and at a revenue guarantee $\hat{p}\hat{y} = p_{\max} y_{\max}$, where $y_{\max}$ is the maximum yield index. It thus provides full insurance against $b_1 b_2 \tilde{p}\tilde{y}$. In addition, he selects on financial markets a short (long) CYI futures position against the $\tilde{y}$ yield risk with a futures yield at planting equal to the expected aggregate yield and an optimal hedge ratio equal to $|b_2 a_1|$ if a_1 is positive (negative), and a short (long) price futures positions with a futures price at planting equal to the expected aggregate yield and an optimal hedge ratio equal to $|b_1 a_2|$ if a_2 is positive (negative).[4] Therefore, revenue insurance, crop yield insurance futures contracts and price futures contracts are complementary. The final wealth of the insured producer is

[4] The CYI hedging ratio is defined up to a multiplicative factor which depends on the unit of trading of the financial contracts. For example, the unit of trading for Iowa Corn Yield Insurance Futures is the Iowa yield estimate in bushels per acre times $100.

(47) $\quad w^{ins} = p_i y_i - J^{A1}(p,y) = a_1 a_2 + b_1 a_2 E\tilde{p} + b_2 a_1 E\tilde{y} + b_1 b_2 E(\tilde{p}\tilde{y}) + p_i e_2 + E[\tilde{y}_i / \tilde{y} = y] e_1 .$

The insured producer bears two sources of risk caused by the uninsurable and unhedgeable price and yield basis risks.

The insurance and hedging strategy is now examined in the presence of yield and price basis risks. Under fair insurance and financial markets, Proposition 3 and equation (36) yield that the approximate indemnity function net of its premium is

(48) $\quad J^B(p,y) = I^B(p,y) - EI^B(\tilde{p},\tilde{y}) = [E(\tilde{p}\tilde{y}) - py] + \frac{R}{2}\mathrm{var}(\tilde{e}_2)[p^2 - E(\tilde{p}^2)] + \frac{R}{2}\mathrm{var}(\tilde{e}_1)[y^2 - E(\tilde{y}^2)]$

The first RHS term in brackets can be replicated by purchasing the IP policy at a coverage level $q_i = 1$, with a revenue guarantee $\hat{p}\hat{y} = p_{\max} y_{\max}$. The second RHS term in brackets depends only on the price index. In order to replicate as close as possible the increasing and convex form with respect to p, the optimal hedging strategy should require a long futures position and long straddles positions against the price risk (Mahul 2000b).[5] Likewise, the third RHS term can be partially replicated by buying yield futures and yield straddles. Therefore, insurance policy based on aggregate gross revenue, yield futures and options contracts and price futures and options contracts turn out to be complementary. It should be noticed that options were not useful in order to replicate the closed-form solution (26) with no basis risk, but they become a useful hedging instrument in the presence of basis risks because of the non-linearity of the closed-form solution (38) with respect to yield and price.

When the revenue insurance program is based on individual yield and national price, the insurance and hedging strategy is replicated with the IP or RA contract rather than the CRC policy. However, this insurance product turns out to have the highest enrollment among the U.S. farmers, accounting for about one-third of the total crop insurance sales in the areas where they are offered (GAO). This observation is *a priori* in contradiction with the above

[5] A long (short) straddle is the market position established by buying (selling) a put and a call with the same strike price.

theoretical results. This may be the consequence of existing constraints on yield guarantee and coverage level in the actual IP and CRC programs. Since these insurance policies are sold at a price which is less than the actuarial premium thanks to the premium subsidies, producers will be induced to select the policy which provides us the largest coverage. By construction, the CRC product generates a higher coverage than the IP and RA products. Therefore, we can conjecture that, for an identical level of coverage, the IP or RA product should be preferred to the CRC product. This seems to be confirmed by recent empirical results obtained by Heifner and Coble.

These authors also show that revenue insurance based on individual yield does not substitute completely for forward pricing. Our previous theoretical results, when basis risk is ignored, seem to contradict their empirical findings. Once again, this may due to the existence of an upper bound on yield guarantee. This seems to be confirmed by their simulations about the effect of yield guarantee on the optimal hedge ratio (see Heifner and Coble, Figures 19 to 22). When a revenue insurance contract like the IP program is available, the hedge ratio tends to zero as the yield guarantee is higher than 125% of the historic individual yield in most of the counties.[6] The presence of price basis risk could also rationalize the use of hedging instruments in addition to revenue insurance contract, even if there is no constraint on the yield guarantee.

Under the IP and RA insurance programs, the coverage level is equal to one, i.e. the producer insures all his crop, or zero, i.e. the producer does not purchase the insurance policy. This constraint may induce him to prefer the RA or IP plan depending on whether b_1 is close to coefficient of adjustment or to one. He may refuse to purchase both of them if b_1 is close to zero or the GRIP plan if $b_1 b_2$ is close to zero.

Conclusions

This paper has examined the design of an optimal crop revenue insurance contract when the producer faces joint yield and price risk. This is a first step in a long-term research project on the rational insurance purchasing decisions against multiple risks in the context of incomplete markets. The message of this paper is twofold. Firstly, if the indemnity schedule is contingent on individual yield and individual price, the optimal insurance contract has been shown to depend only on the individual gross revenue. It displays full insurance under a trigger revenue. In this context of complete markets, crop yield insurance contracts and hedging instruments turn out to be redundant. Secondly, we have demonstrated that this result fails when the indemnity schedule is contingent on yield index and/or price index that are imperfectly correlated with their associated individual parameter. In this context of incomplete markets, the optimal revenue insurance contract does not depend only on the gross revenue index. A first closed-form solution of the optimal insurance contract against yield and price indexes has been derived when yield and price basis risks are ignored. It can be replicated with unconstrained yield and revenue insurance policies and hedging instruments sold at a fair price. The IP and RA programs are complementary with the MPCI program, and hedging contract against price risk are redundant. The GRIP plan has been shown to be complementary with the yield and price futures contracts. We have also studied how the presence of yield and price basis risks affects the optimal crop revenue insurance contract. The impact of the producer's prudent behavior and of the variability of basis risks has been highlighted. A closed-form solution has been derived. The convexity of its indemnity function with respect to both yield and price has provided a rationale for the use of options.

[6] These figures also show that, under the CRC program, the optimal hedge ratio increases with the yield guarantee.

The design of these two closed-form solutions could provide useful information to policy makers in order to design future revenue insurance programs. In addition, it questions the efficiency provided by the Crop Revenue Coverage.

If constraints on the yield guarantee and on the coverage level exist or if insurance and hedging contracts are sold at a price higher than the fair one, then the closed-form solution of the optimal revenue insurance policy, where basis risk is ignored, cannot be replicated any longer. In particular, the revenue insurance contract cannot provide a full coverage against the revenue uncertainty and, therefore, separate contracts for yield and price risk will play a key role, even if the indemnity schedule of the revenue insurance policy is contingent on individual yield and price. The correlation between yield and price will play a central role in the optimal insurance and hedging decisions. These restrictions on the insurance contracts and hedging instruments create a new source of incompleteness that will be investigated in further research.

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Appendix

Proof of Proposition 2

The optimal indemnity schedule, solution to the maximization problem (16) subject to constraints (17) and (18) is derived by using Kuhn-Tucker conditions for $I(p,y)$ for all (p,y). The first-order condition with respect to $I(p,y)$ is

(A1) $Eu'[(a_1+b_1p+\tilde{e}_1)(a_2+b_2y+\tilde{e}_2)+I(p,y)-P]+l(p,y)-mc'[EI(\tilde{p},\tilde{y})]=0 \quad \forall\,(p,y)$

where m and $l(x)$ are the Lagrangian multipliers associated to constraint (18) and constraint (17) respectively with:

(A2) $l(p,y)\begin{cases}=0 & \text{if } I(p,y)>0\\ \geq 0 & \text{otherwise.}\end{cases}$

For all couple (p,y) such that $I(p,y)=0$, equation (A1) can be rewritten as

(A3) $K(p,y)\equiv Eu'[(a_1+b_1p+\tilde{e}_1)(a_2+b_2y+\tilde{e}_2)-P]-mc'[EI(\tilde{p},\tilde{y})]\leq 0$

and its partial derivatives are

(A4) $\partial K(p,y)/\partial p\equiv b_1E\{(a_2+b_2y+\tilde{e}_2)u''[(a_1+b_1p+\tilde{e}_1)(a_2+b_2y+\tilde{e}_2)-P]\}$

(A5) $\partial K(p,y)/\partial y\equiv b_2E\{(a_1+b_1p+\tilde{e}_1)u''[(a_1+b_1p+\tilde{e}_1)(a_2+b_2y+\tilde{e}_2)-P]\}$

From the concavity of u, $K(.,.)$ decreases (increases) with p if b_1 is positive (negative); it decreases (increases) with y if b_2 is positive (negative). For all y, let $\hat{p}(y)$ be such that $K(\hat{p}(y),y)=0$. Under realized price index p and realized yield index y, indemnity payments thus are made whenever p is lower (higher) than $\hat{p}(y)$ if b_1 is positive (negative). From the theorem of implicit functions, the trigger function $\hat{p}(.)$ decreases (increases) with y if the product b_1b_2 is positive (negative).

For couple (p,y) such that $I(p,y)>0$, equation (A1) becomes

(A6) $Eu'[(a_1+b_1p+\tilde{e}_1)(a_2+b_2y+\tilde{e}_2)+I(p,y)-P]-mc'[EI(\tilde{p},\tilde{y})]=0$

Differentiating equation (A6) with respect to p and y and rearranging the terms lead to the first derivatives of optimal coverage expressed in equations (19) and (20).

Proof of Proposition 3

Optimizing problem (16) with respect to the insurance premium P yields the following first-order condition:

(A7) $Eu'[\tilde{p}_i\tilde{y}_i+I(\tilde{p},\tilde{y})-P]=m$.

Combining the above equation with equation (A1) yields

(A8) $E\{u'[\tilde{p}_i\tilde{y}_i + I(\tilde{p},\tilde{y}) - P]/(\tilde{p},\tilde{y}) = (p,y)\} + l(p,y) - Eu'[\tilde{p}_i\tilde{y}_i + I(\tilde{p},\tilde{y}) - P]c'[EI(\tilde{p},\tilde{y})] = 0$

for all (p, y). Taking the expectation of the above equality with respect to $(\tilde{p},\tilde{y})$ gives

(A9) $El(\tilde{p},\tilde{y}) = \{c'[EI(\tilde{p},\tilde{y})] - 1\}Eu'[\tilde{p}_i\tilde{y}_i + I(\tilde{p},\tilde{y}) - P]$.

If insurance is sold at a fair price, i.e. $c'(I) = 1$ for all $I \geq 0$, then $El(\tilde{p},\tilde{y}) = 0$. Since $l(.,.)$ is a nonnegative by definition, this implies that $l(p,y) = 0$ for all (p,y). Therefore, indemnity payments are made is all states of nature. This proves the first part of the proposition.

If insurance is costly, $c'(I) > 1$ for some $I \geq 0$, the non-negativity constraint (17) must be binding for some (p, y) with a positive probability. Indemnity payments are thus made is the less favorable states of nature. This leads to the second part of the proposition.