Spatial Break-Even Variability for Custom Hired Variable Rate Technology Adoption

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Abstract

A theoretical model identified ranges of spatial variability required within multiple-land-class fields for economically viable variable rate technology (VRT) and the spatial variability required for maximum return to VRT. An example illustrated that return to VRT and the viable range of spatial variability increased for higher corn and nitrogen prices.

Key Words

Precision farming, site-specific farming, nitrogen, corn, spatial variability, yield variability, profit function

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Introduction

Agricultural fields consist of numerous areas that differ from one another with respect to the factors that condition crop growth (Carr et al.; Hannah, Harlan, and Lewis; Hibbard et al.; Malzer et al.; Sawyer; Spratt and McIver). The concept of precision farming recognizes that farm fields are rather heterogeneous units. Precision farming refers to treating within-field variability with spatially variable input application rates using a set of technologies to identify the variability and its causes, and prescribe and apply inputs to match spatially variable crop and soil needs. Two important benefits claimed of precision farming include increased profits to farmers and reduced environmental harm as a result of more precise placement of inputs (Kitchen et al.; Koo and Williams; Sawyer; Watkins, Lu, and Huang). The key, however, to the acceptance of site-specific farming is the profitability of the technology (Daberkow; Reetz and Fixen; Sawyer).

The presence of variability in soil and field characteristics is key to the economic viability of precision farming (English, Roberts, and Mahajanashetti; Forcella; Hayes, Overton, and Price; Snyder). From an economic standpoint, the factors that drive the adoption of precision farming technology are spatial variability, or the distribution across a field of land types with different yield responses, and the magnitudes of the differences in yield response (English, Roberts, and Mahajanashetti; Forcella).

The objectives of this study were 1) to determine the range of spatial variability over which the return to variable rate technology (VRT) covers the cost of VRT in fields with two or more land types, 2) to identify the amount of spatial variability that would maximize the return to VRT, and 3) to evaluate the impacts of changes in crop and input prices on the profitable range of spatial variability and on the spatial variability required to maximize the return to VRT.

Theoretical Model

Suppose fields suited to corn production in a particular area can be classified into m land types, each having a different yield response to applied nitrogen. Suppose further that corn fields in this area can be characterized by any of these land types in any proportion. Assume farmers are profit maximizers and have knowledge of the following land-specific yield response functions.

$$Y_i = Y_i\,(N_i) \qquad i = 1,2,\ldots,m \qquad (1)$$

Where Y_i is corn yield (bu/ac) and N_i is nitrogen applied (lb/ac) on the i^{th} land type. A farmer using VRT on a particular field determines the optimum application rates for the m land types by equating the marginal physical products of the respective response functions with the nitrogen-to-corn price ratio. Optimum return above nitrogen cost per acre for the field under VRT (R^*_{VRT}) is then calculated from the following profit function (Nicholson),

$$R^*_{VRT} = \sum_{i=1}^{m-1} \lambda_i\,[\,P_C\,Y_i\,(N_i^*) - P_N\,N_i^*] + [1 - \sum_{i=1}^{m-1} \lambda_i\,][\,P_C\,Y_m\,(N_m^*) - P_N\,N_m^*]\text{ , or}$$

$$R^*_{VRT} = R^*_{VRT}\,(\,l_1,\,l_2,\ldots,\,l_{m-1},\,P_C\,,\,P_N\,) \qquad (2)$$

Where P_C is the corn price; P_N is the nitrogen price; N_i^* is the optimum nitrogen application rate for the i^th land type; N_m^* is the optimum nitrogen application rate for the m^th land type; λ_i is the proportion of the field in the i^th land type such that $\sum_{i=1}^{m} \lambda_i = 1$.

When applying nitrogen using uniform rate technology (URT), the farmer determines the optimum uniform nitrogen rate based on the field average response function. The field average response function can be expressed as:

$$Y_u (N_u) = \sum_{i=1}^{m-1} \lambda_i \, Y_i (N_u) + [1 - \sum_{i=1}^{m-1} \lambda_i] Y_m (N_u) \tag{3}$$

Where $Y_u (N_u)$ is the weighted average corn yield response function and N_u is the uniform nitrogen application rate. The optimum return above nitrogen cost per acre for URT (R^*_{URT}) is calculated from the following profit function:

$$R^*_{URT} = P_C Y_u (N_u^*) - P_N N_u^* \text{, or}$$

$$R^*_{URT} = R^*_{URT} (l_1, l_2, ..., l_{m-1}, P_C, P_N) \tag{4}$$

Where N_u^* is the optimum uniform nitrogen rate obtained by equating the marginal physical product of the average yield response function in equation (3) with the nitrogen-to-corn price ratio.

The difference between R^*_{VRT} and R^*_{URT}, which is the optimum return to VRT ($RVRT^*$), can be specified as a profit function:

$$RVRT^* = RVRT^*(\lambda_1, \lambda_2, ..., \lambda_{m-2}, \lambda_{m-1}, P_C, P_N) \tag{5}$$

Where all variables are defined earlier.

Equation (5) is assumed to be concave in λ_i. The assumption of concavity can easily be understood by considering fields with only two land types, types 1 and 2. For fields that are uniformly land type 1 (λ_1 = 1 and λ_2 = 0), RVRT* = 0. Fields with a positive λ_2 **(**λ_1 < 1) have both land types and farmers can consider using VRT. Since optimization of input use with VRT is more suited to the site-specific yield response functions than with URT, RVRT* now becomes positive and continues to increase to a maximum as λ_2 increases (λ_1 decreases) over some range. Eventually, RVRT* begins to decline until it reaches zero for fields with only land type 2 (λ_1 = 0 and λ_2 = 1). The above discussion can be generalized to m land types.

Spatial Break-Even Variability Proportions (SBVPs) (English, Roberts, and Mahajanashetti; Mahajanashetti) for l_{m-1} are defined as the lower and upper limits of l_{m-1} for given levels of $\lambda_1, \lambda_2, ..., \lambda_{m-2}$, P_C, P_N and VC such that RVRT* = VC, where VC equals the additional charges for custom hiring VRT compared to URT. In this case, l_m varies inversely with l_{m-1} when the other land proportions are fixed. Mathematically, equation (5) can be modified and used to locate the SBVPs of λ_{m-1} as follows:

$$RVRT^* = RVRT^*(\lambda_{m-1} \mid \bar{\lambda}_1, \bar{\lambda}_2, ..., \bar{\lambda}_{m-2}, \bar{P}_C, \bar{P}_N) = V\bar{C} \quad (6)$$

Where $\bar{\lambda}_1, \bar{\lambda}_2, ..., \bar{\lambda}_{m-2}$, $\bar{P}_C$, $\bar{P}_N$, and $V\bar{C}$ are given levels of the respective variables.

Solving equation (6) for λ_{m-1} provides the SBVPs of λ_{m-1} and l_m that bound the range over which $RVRT^* \geq V\bar{C}$. However, for certain $\bar{\lambda}_1, \bar{\lambda}_2, ..., \bar{\lambda}_{m-2}$, $\bar{P}_C$ and $\bar{P}_N$, RVRT* may be less than $V\bar{C}$ for all possible levels of λ_{m-1}, implying that SBVPs do not exist and that economic losses from using VRT would occur at all levels of λ_{m-1}. In some cases, RVRT* may be greater than $V\overline{C}$ for all possible levels of λ_{m-1}, implying that SBVPs do not exist and that economic gains would occur from using VRT regardless of the level of λ_{m-1}. Finally, in the remaining cases, only an upper or a lower SBVP may exist, but not both. If RVRT* > $V\overline{C}$ for λ_{m-1} = 0, and RVRT* = $V\overline{C}$ for 0 < λ_{m-1} ≤ (1 – $\sum_{1=1}^{m-2}\overline{\lambda}_i$), only an upper SBVP exists. In this case, the maximum this upper SBVP can be is 1 – $\sum_{1=1}^{m-2}\overline{\lambda}_i$ when λ_m = 0. However, if RVRT* > $V\overline{C}$ for λ_{m-1} = 1 – $\sum_{1=1}^{m-2}\overline{\lambda}_i$ and RVRT* = $V\overline{C}$ for 0 ≤ λ_{m-1} < (1 – $\sum_{1=1}^{m-2}\overline{\lambda}_i$), only a lower SBVP exists. In this case, the minimum this SBVP can be is 0 when λ_m = 1 – $\sum_{1=1}^{m-2}\overline{\lambda}_i$.

Three land types suited to corn production in a hypothetical geographic area were considered to illustrate the concepts presented above. These land types were assumed to have high, medium, and low yield responses to applied N. Corn fields in this area were assumed to consist of any of these three land types in any proportion. The following quadratic corn yield response functions were assumed for high, medium, and low response lands, respectively.

$$Y_1 = 120 + 1.11N_1 - 0.0023N_1^2 \quad (7)$$

$$Y_2 = 100 + 1.05N_2 - 0.0026N_2^2 \quad (8)$$

$$Y_3 = 75 + 0.5N_3 - 0.0014N_3^2 \quad (9)$$

Where Y_1, Y_2, and Y_3 are corn yields (bu/ac) and N_1, N_2, and N_3 are N application rates (lb/ac) for high, medium, and low response lands, respectively.

After defining l_1, l_2, and l_3 as the proportions of the field in high, medium, and low yield response lands, respectively, the functional forms of equations (2), (4), (5), and (6) were determined, the SBVPs were identified, and the RVRT* maximizing land proportions were found. The RVRT* maximizing l_i were found from equation (6) by taking the partial derivative with respect to l_i, setting it equal to zero, and solving the resulting equation for l_i.

Spatial break-even and RVRT* maximization analyses were conducted using the 1993-1997 mean corn and N prices of $\overline{P}_C$ = \$2.79/bu and $\overline{P}_N$ = \$0.26/lb (Tennessee Department of Agriculture). Sensitivity analysis examined 10 percent increases and decreases in $\overline{P}_C$ and $\overline{P}_N$.

The additional custom charge for variable rate N application was assumed to be $V\overline{C}$ = \$3.00/ac. This additional charge was obtained from personal communication with two local farmers' cooperatives in West Tennessee (Names of providers are not given to prevent disclosure.). Each cooperative indicated variable rate application of one plant nutrient would cost the farmer \$2.00/ac more than if it were applied uniformly across a field. In addition, they would charge \$1.00/ac to create the N application map based on soil survey maps, a visit to the field, and an interview with the farmer.

Results

Table 1 presents the land proportions that maximize RVRT* and the SBVPs for fields with two or three land types. When calculating these proportions, the proportion of one land type was given at 0, 20, 40, 60, or 80 percent of the field area.

The maximum RVRT* for fields with no high yield response land ($\bar{l}_1$ = 0 percent) was $1.95. This maximum RVRT* occurred in fields with 58 percent low yield response land (42 percent medium response land). Thus, fields with only low and medium response land types would not be able to cover the additional custom charge of $3.00/ac, implying that the adoption of VRT would lead to economic losses. Similarly, the maximum RVRT* ($2.33/ac) for fields having only medium and high response lands ($\bar{l}_3$ = 0 percent) was less than the additional custom charge, suggesting that adoption of VRT would not be profitable. For fields with only low and high yield response lands ($\bar{l}_2$ = 0 percent), SBVPs were clearly identified at 15 percent and 90 percent low response land, with maximum RVRT* ($7.07) occurring at 56 percent low response land. Thus, farmers would have an economic incentive to adopt VRT on fields with only high and low yield response land types if the fields were between 15 and 90 percent low response land (85 and 10 percent high response land).

When $\bar{l}_1$ was specified at 20, 40, 60, or 80 percent of a field, economically viable ranges of spatial variability in l_3 were identified. These ranges, however, had only a minimum SBVPs. For example, on fields with 20 percent high yield response land ($\bar{l}_1$ = 20 percent), a minimum of 22 percent low response land (a maximum of 58 percent medium response land) was required for VRT adoption to be economically viable. No maximum SBVP existed for l_3 because RVRT* was still greater than \$3.00/ac when l_3 reached its maximum at 80 percent low response land (100 percent minus 20 percent high response land) and l_2 reached 0 percent medium response land. Therefore, with $\bar{l}_1$ = 20 percent, RVRT* was greater than or equal to $V\bar{C}$ between 22 and 80 percent low response land, or equivalently, between 58 and 0 percent medium response land. Within this range of l_3 (and l_2) farmers would have an economic incentive to adopt VRT, given $\bar{l}_1$ = 20 percent.

As another example, when fields were assumed to be 40 percent low yield response land ($\bar{l}_3$ = 40 percent), VRT would only be viable for farmers of fields with at most 50 percent medium response land (at least 10 percent high response land). No lower SBVP existed for l_2 because RVRT* was still greater than \$3.00/ac when l_2 reached its minimum of 0 percent. With $\bar{l}_3$ = 40 percent, farmers would consider VRT for fields between 0 and 50 percent medium yield response land (60 and 10 percent high response land).

When the share of medium response land was specified at 60 or 80 percent ($\bar{l}_2$ = 60 or 80 percent), no economically viable mix of l_1 and l_3 could be found. However, given $\bar{l}_2$ = 20 or 40 percent, VRT could be employed profitably on fields provided they had all three land types. For example, for $\bar{l}_2$ = 40 percent, fields with between 7 and 53 percent low yield response land (53 and 7 percent high response land) would be considered for VRT instead of URT.

Sensitivity Analysis

Sensitivity analysis was conducted to examine how changes in P_N and P_C influenced the SBVPs and the $RVRT^*$ maximizing proportion through their impact on the $RVRT^*$ function. For simplicity, the sensitivity analysis was conducted only for fields with 20 or 40 percent medium yield response land.

The results in Table 2 show that given 20 or 40 percent medium yield response land, the $RVRT^*$ maximizing proportion of low response land varied directly with P_N and inversely with P_C. However, maximum $RVRT^*$ varied directly with each price variable. Irrespective of whether the field was 20 or 40 percent medium response land, a 10 percent change in P_C had a larger impact on the SBVPs and the maximum $RVRT^*$ compared to an equivalent change in P_N.

Sensitivity analysis also revealed that an increase in P_N or P_C expanded the range of spatial variability of low response land over which positive net returns to VRT were possible. In other words, higher prices caused the lower SBVP for l_3 to decrease and the upper SBVP to increase. A fall in P_N or P_C, on the other hand, reduced the economically viable range of spatial variability.

Conclusions

Adoption of VRT depends to a large extent on the expected net economic benefits received by adopting farmers. Fields generally exhibit yield variability; however, not all fields warrant VRT from an economic standpoint. The results of this study emphasize the importance of both spatial and yield response variability. In this analysis, the economic benefit from using VRT instead of URT did not cover the difference in custom charges for fields consisting of only high and medium yield response lands, or medium and low response lands. In these fields, yield response variability was not sufficient to warrant VRT regardless of spatial variability. Nevertheless, fields with high and low yield response lands had positive net returns to VRT for certain ranges of spatial variability. These results highlight the importance of yield response variability, while illustrating the importance of spatial variability in precision farming.

Sensitivity analysis showed that lower nitrogen and/or corn prices decreased the optimal return to variable rate technology and reduced the range of spatial variability providing positive net returns to VRT. Thus, the lower crop and input prices of recent times likely will reduce the economic incentive for farmers to adopt VRT.

Table 1. Return-to-Variable-Rate-Technology-Maximizing Land Proportions and Spatial Break-Even Variability Proportions in Hypothetical Corn Fields with Three Land Types.

Given $\bar{l}_i$ [a] (percent)	$RVRT^*$ Maximizing l_i [b] (percent)	Maximum $RVRT^*$ for l_i ($/ac)	SBVPs for l_i [b] Lower (percent)	SBVPs for l_i [b] Upper (percent)
$\bar{l}_1$	……………………………………	l_3	…………………………………	
0	58	1.95	[c]	[c]
20	79	5.22	22	[d]
40	60	7.03	8	[d]
60	40	6.38	7	[d]
80	20	3.89	12	[d]
$\bar{l}_2$	…………………………………...	l_3	………………………………...	
0	56	7.07	15	90
20	43	5.68	9	73
40	31	4.28	7	53
60	18	2.89	[c]	[c]
80	5	1.50	[c]	[c]
$\bar{l}_3$	…………………………………...	l_2	………………………………...	
0	48	2.33	[c]	[c]
20	21	4.38	[d]	58
40	0	6.37	[d]	50
60	0	7.03	[d]	33
80	0	5.22	[d]	12

[a] l_1, l_2, and l_3 are the proportions of the field in high, medium, and low yield response land, respectively.

[b] When $\bar{l}_1$ or $\bar{l}_2$ is given in column 1, the $RVRT^*$ maximizing l_i and the SBVPs are calculated in terms l_3. When $\bar{l}_3$ is given, they are calculated in terms of l_2.

[c] Because the maximum $RVRT^*$ attainable by varying l_3 or l_2 is less than the additional custom charge of $3.00/ac, a break-even l_3 or l_2 does not exist.

[d] An upper or lower SBVP does not exist because $RVRT^*$ is greater than the additional custom charge for VRT ($3.00/ac) when l_i is at its maximum or minimum.

Table 2. Impact of Changes in Nitrogen and Corn Prices on Return-to-Variable-Rate-Technology-Maximizing Land Proportions and Spatial Break-Even Variability Proportions in Hypothetical Corn Fields with Three Land Types.

Given $\overline{l}_2$ [a] (percent)	Changes in P_N and P_C	$RVRT^{*b}$ maximizing l_3 (percent)	Maximum $RVRT^*$ ($/ac)	SBVPs for l_3 Lower (percent)	Upper
20	Mean prices	43.4	5.68	9.1	73.2
20	P_N 10% higher	43.8	5.86	8.8	74.0
20	P_N 10% lower	43.1	5.50	9.3	72.4
20	P_C 10% higher	43.2	6.07	7.4	73.9
20	P_C 10% lower	43.8	5.30	11.9	72.4
40	Mean prices	30.8	4.28	6.5	52.8
40	P_N 10% higher	31.4	4.41	6.4	54.1
40	P_N 10% lower	30.1	4.16	6.6	51.5
40	P_C 10% higher	30.1	4.59	3.9	53.8
40	P_C 10% lower	31.5	3.99	9.6	51.6

[a] l_2 and l_3 are the proportions of the field in medium and low yield response land.
[b] $RVRT^*$ is the optimum return to variable rate technology for given prices and land proportions.

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