Evaluating Risk Management Strategies Using Stochastic Dominance with a Risk Free Asset

ABSTRACT: The stochastic dominance with a risk free asset (SDRA) criteria are evaluated. Results show that the inclusion of the risk free asset (combined with the traditional assumptions of stochastic dominance) produce risk efficiency criteria that are very powerful at narrowing the risk management alternatives relevant for farm manager's further consideration.

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INTRODUCTION

Uncertainty is an important aspect of many production and investment activities. Two key sources of uncertainty in agricultural production and investment activities are output quantity variability and output price variability (Harwood, et.al., 1999). Together, these uncertainties create revenue uncertainty. Risk management is the process of identifying, evaluating, and implementing strategies that affect revenue uncertainty.

Many research and extension programs have been dedicated to risk management (Baker and Patrick, 1997; Iowa State, 1997; Schnitkey, Miranda, and Irwin, 1996). Although agricultural economists have provided producers assistance in identifying risk management strategies and the associated gross revenue distributions, they have provided little assistance in choosing among these strategies.

The objective of this research is to identify and evaluate methods that can significantly reduce the set of strategies that the risk manager must consider. Several risk management strategies will be simulated to produce a set of revenue distributions. Then, methods that can systematically reduce the set by making assumptions about economic behavior will be applied to the set of revenue distributions. Specifically, the methods to be considered are first and second degree stochastic dominance with and without a risk free asset. The size of the efficient sets will be compared across the methods, and conclusions about the usefulness of these criteria will be developed.

Making Choices Under Uncertainty

Risk management choices must made ex ante, or before uncertainty is resolved. The dominant framework that academics have used to describe choice under these conditions is the expected utility hypothesis (EUH) of Von-Neumann and Morganstern (1947). The EUH is an extremely powerful framework because, given the specification of the utility function and return distributions, one can exactly describe a decision maker's optimal choices by maximizing expected utility. However, this framework loses much of its usefulness as the exact specification of an individual decision maker's utility function is extremely difficult and impractical.

Because individual risk aversion levels are idiosyncratic, several more general statements of the choice under uncertainty problem have been developed. These more general statements of the EUH are often called risk efficiency criteria. The goal of these criteria is to reduce the set of all risky choices to a subset that must contain the expected utility maximizing solution for a general class of risk preferences, i.e., the risk averse case. The inoffensive assumptions that agents prefer more return to less and that agents are risk averse produce the two most widely known risk efficiency criteria, first and second degree stochastic dominance.

Stochastic Dominance

Several authors developed and tested the efficiency of the stochastic dominance criteria (Fishburn, 1964; Hadar and Russell 1969; Hanoch and Levy, 1969; Rothschild and Stiglitz, 1970; Levy and Sarnat, 1972). In general, the efficient sets implied by FSD (first order stochastic dominance) tend to be quite large because FSD often does not apply (i.e., the distributions cross). Even with the added assumption of risk aversion, SSD (second order stochastic dominance) efficient sets tend to be large. This is due to the naïve assumptions that support the SD rules. While the generality of these assumptions is desirable, the criteria are weak and not typically useful for ranking investments that present the decision maker with a risk-return tradeoff, such as risk management strategies.

The Common Failings of Stochastic Dominance

Two common failings of the SD rules make them undesirable efficiency criteria for comparing risk management strategies. The common failings are the difference of means and the lower tail crossing problems. The difference of means problem manifests itself by requiring that the mean of the dominating distribution be at least as large as the mean of the dominated distribution. The lower tail crossing problem rules out dominance of the alternative with the largest cumulative probability at the worst possible outcome. Unfortunately, risk management strategies often suffer from the equality of means problem as they present the decision maker with a trade-off of expected return for risk reduction. On the other hand, base strategies often suffer from the lower tail crossing problem. This means that the traditional SD rules are not usually empirically efficient tools for evaluating risk management strategies because the efficient sets tend to be large.

The common failings of the SD rules are a result of the their focus on only business risk. The SD criteria are only capable of identifying strategies with less business risk for each given level of expected return. Tobin saw this failing in the mean-variance model and allowed the possibility that agents could borrow or lend a risk free asset to adjust the financial leverage of the portfolio. Similarly, Levy and Kroll (1976) realized that including the possibility that agents could borrow or lend a risk free asset would allow agents to adjust financial leverage to compensate for mean differences or lower tail crossings. This realization has the potential to significantly reduce the efficient set. The alternative criteria developed by Levy and Kroll (1978) are called stochastic dominance with a risk free asset (SDRA) and will be discussed in the next section.

Stochastic Dominance with a Risk Free Asset (SDRA)

The SDRA criteria further refine the SD rules by adding the assumption that agents can borrow or lend a risk free asset. This assumption allows agents to use leverage to transform the distributions of risky alternatives. Borrowing the risk free asset pivots the CDF clockwise around the risk free return, while lending the risk free asset pivots the CDF counterclockwise around the risk free return. These transformed distributions are then compared using the traditional stochastic dominance rules.

To understand why these pivots might be useful, consider the a typical risk management strategy. Risk management strategies generally move the CDF to the left (mean reduction) and increase its slope (variability reduction). An agent might compensate for a mean reduction by borrowing and investing in more of the risky activity. If financial leverage can be used to increase the mean enough to alleviate the differences of means problem without increasing the risk enough to produce a lower tail crossing, SD of the risk management strategy over the base strategy may emerge. (Similar logic holds that by lending the risk free asset it is possible to reduce the cumulative probability in the lower tail of the base strategy enough to produce dominance of the base strategy over the risk management strategy.)

SDRA is contingent upon the existence of an asset with a risk free return, i.e., no variance in return. There are several assets or investments in agriculture that can be viewed as having no variance in return. The reason that the returns are not variable is that they are fixed over the time frame that the agent is making his/her production investments. For example, the farmer who cash leases farmland must pay his/her cash rent in advance of the season. Default on the cash lease obligation is not permitted if the business expects to continue operation. Likewise, borrowing money from a financial institution generates an obligation that must be repaid with probability one, unless the agent defaults on the debt and exits farming. The agent is then bound by these obligations, thus producing an asset whose return is fixed or risk free.

The SDRA rules are derived by constructing combinations of the return to risky actions and the return to the risk free action. Consider the case where there are two risky outcomes *X* and *Y*, and a risk free outcome *r*. The risky outcomes can be combined with the risk free outcome as shown in (1).

$$\begin{aligned} X_\alpha &= (1-\alpha)r + \alpha X \qquad && \alpha \in [0,\infty) \\ Y_\beta &= (1-\beta)r + \beta Y \qquad && \beta \in [0,\infty) \end{aligned} \tag{1}$$

Where X_α and Y_β are the sets of all combinations of the risky outcomes and the risk free return, α and β are weights of the original risky outcomes *X* and *Y* in these combinations, and *r* is the risk free return. Each outcome in the set X_α or Y_β has an associated CDF. The sets of CDF's can be denoted $F_{X\alpha}$ and $G_{Y\beta}$. Where any particular element in $F_{X\alpha}$ or $G_{Y\beta}$ has the form shown in (2).

$$\begin{aligned} F_{X\alpha}(z) &= \Pr(X_\alpha \le z) \\ G_{Y\beta}(z) &= \Pr(Y_\beta \le z) \end{aligned} \tag{2}$$

Where *z* is some monetary outcome and *Pr* returns the probability that a particular X_α is less than or equal to *z*.

$G_{Y\beta}$ will dominate $F_{X\alpha}$ by SDRA if and only if for each combination of *X* and *r* there is at least one combination of *Y* and *r* that dominates it by first or second degree stochastic dominance (Levy and Kroll, 1978). Because there are an infinite number of distributions in the sets $F_{X\alpha}$ and $G_{Y\beta}$, it is necessary to determine when dominance is possible, i.e., a range of β that produces CDF's that dominate all the potential CDF's in $F_{X\alpha}$. These conditions are derived by considering the potential movement of the CDF when the risk free asset is included.

Before discussing the movement of the CDF it is useful to introduce quantile notation.

$$Q_F(p) = F(x)^{-1} \qquad (3)$$

The quantile function, $Q_F(p)$, is the inverse of the cumulative distribution function of X. The cumulative distribution function, $F(x)$ produces the probability that an outcome of X is below a given outcome x. The quantile function returns the monetary outcome x that is associated with a given cumulative probability level, p.

Levy and Kroll (1978) first develop a necessary condition for G(x) to dominate F(x) by first order stochastic dominance with a risk free asset (FSDRA). It states that the cumulative probability under G(x) must be lower than the cumulative probability under F(x) at the risk free rate, and G(x) must lie below F(x) to one side of r. The condition says nothing about the number of times the CDF's may cross to one side of r. This implies that multiple crossings to one side of r do not rule out FSDRA. It also implies that FSDRA is a special case of FSD (cumulative probability lower at all points).

More importantly Levy and Kroll derive and prove that (4) is a necessary and sufficient condition for G(x) to dominate F(x) by FSDRA.

$$\inf_{0 \le p < G(r)} \frac{a}{b} = \frac{Q_F(p) - r}{Q_G(p) - r} \ge \sup_{G(r) < p \le 1} \frac{c}{d} = \frac{Q_F(p) - r}{Q_G(p) - r} \qquad (4)$$

Where *inf* represents the infimum, or greatest lower bound, *p* is probability, *G(r)* is the cumulative distribution function of activity *Y* evaluated at the risk free rate, $Q_F(p)$ and $Q_G(p)$ are the quantile functions for activities *X* and *Y*, *r* is the risk free return, and *sup* is the supremum, or least upper bound.

The condition can be interpreted as a method to find the amount of pivoting needed to induce dominance on one side of *r* relative to the amount of pivoting that is allowed without removing dominance to the other side of r. If (4) holds, it is possible to find β that generates a CDF ($G_{Y\beta}$) that lies below F(x) for every value of *p*.

A Graphical Interpretation of FSDRA

Figure 1 shows one possible case that might be considered. Here the solid CDF, F(x), might represent the returns to a base strategy such as using the natural hedge, and the dashed CDF, G(x), could represent the returns to a risk management strategy such as buying crop insurance. G(x) has less probability in the lower tail, but suffers from the equality of means problem as the agent must give up expected return for the risk reduction. The difference of means problem then rules out any degree of SD of G(x) over F(x). On the other hand F(x) suffers from the lower tail crossing problem and SD of F(x) over G(x) is also ruled out.

Figure 1 shows that the necessary condition is met. At the risk free rate, *r*, G(r) is less than F(r), and G(x) lies below F(x) everywhere to the left of *r*. To the left of *r*, the solution to the infimum is always greater than one as *b* is less than *a*. The smallest value of *a/b* will occur where *b* is the greatest proportion of *a* or where the distance $Q_G(p)$ – $Q_F(p)$ is minimized. Because the infimum is greater than one, if FSDRA exists in this case, positive amounts of leverage must be used to induce dominance below the risk free return. Recalling that adding leverage ($\beta > 1$) to the activity associated with G(x) pivots the CDF clockwise, one can interpret the solution to the infimum as the most leverage that can be added to the risk management strategy without producing a lower tail crossing.

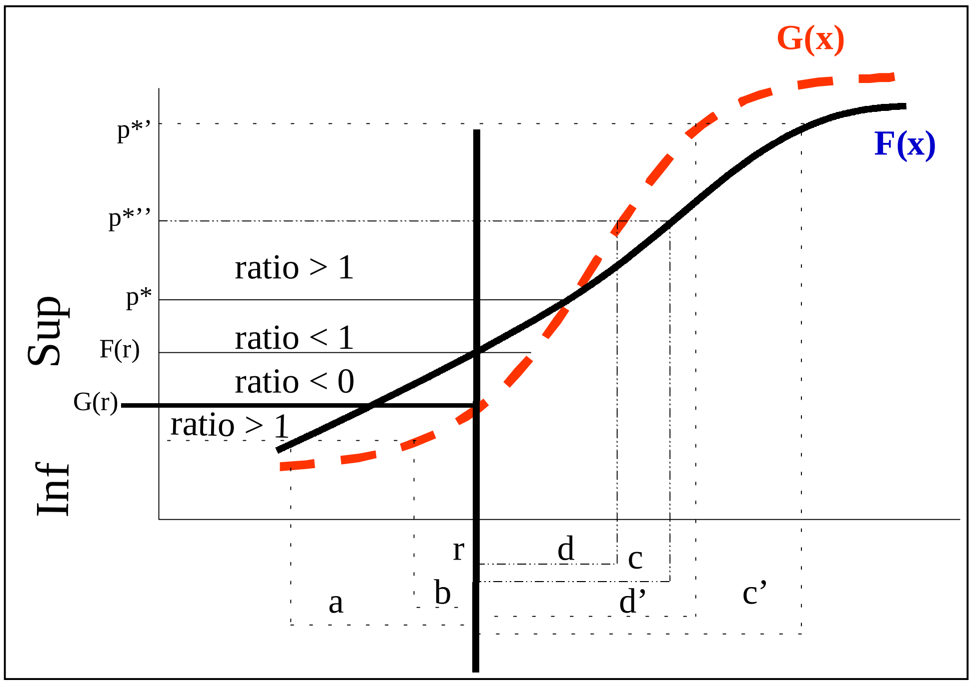

Figure 1. A graphical interpretation of the necessary and sufficient conditions for FSDRA.

The supremum problem contains three ranges to discuss. In the probability interval above G(r) up to F(r), the ratio, c/d, is negative, as $Q_G(p) - r$ is always positive, and $Q_F(p) - r$ is always negative. Above F(r), but below the intersection of the CDFs denoted on the probability axis as p*, the ratio is less than one as *d* is greater than *c*. If the distributions cross, as they do in the example, the supremum will be greater than one as *d* is less than *c* above p*. The solution to the supremum problem will occur when *d* is the smallest proportion of *c*, or when $Q_F(p) - Q_G(p)$ is maximized at probabilities greater than p*. For instance, at p*' the value of *c'/d'* will be greater than the value of *c/d* at p*''.

In the example depicted in Figure 1, the supremum is finding the largest distance that the CDF must be pivoted to induce FSD above *r*. Because the solution is greater than one, the leverage used must be positive ($\beta > 1$). The requirement that the infimum be greater than the supremum means that the allowed negative impact of leverage in the lower tail must be greater than the required positive impact of leverage in the upper tail. In other words, the amount of room available to pivot G(x) must be greater than the amount of pivoting needed.

The above case demonstrates that SDRA allows one to remove the difference of means failing of ordinary stochastic dominance. If the necessary condition holds so that F(r) is less than G(r), it is also possible to remove a lower tail crossing and induce FSD of the base case over the risk management strategy by lending enough of the risk free asset, *r*, to pull the lower tail of the CDF of the base strategy below the lower tail of the CDF of the risk management strategy. Although the FSDRA criterion alone demonstrates the potential value of SDRA, more efficiency is likely to be gained with the addition of higher order SDRA.

Second Order Stochastic Dominance with a Risk Free Asset (SSDRA)

Strictly increasing utility, risk aversion, and the existence of a risk free return produce the SSDRA criterion. Like the FSDRA criterion, the necessary and sufficient condition is stated in quantiles. Formally, the condition for G(x) to dominate F(x) by SSDRA is shown as (5) and (6).

$$\inf_{0 \le p < p_0} \frac{\int_0^p [Q_F(t) - r]dt}{\int_0^p [Q_G(t) - r]dt} \ge \sup_{p_0 < p \le 1} \frac{\int_0^p [Q_F(t) - r]dt}{\int_0^p [Q_G(t) - r]dt} \qquad (5)$$

Where p_0 solves the following :

$$rp_0 = \int_0^{p_0} Q_G(t)dt \qquad (6)$$

Proofs of its necessity and sufficiency can be found in Levy and Kroll, 1978. This condition can again be interpreted as finding the amount of pivoting required to produce SSD to one side of the pivot point relative to the amount of pivoting allowed to the other side of the pivot point.

The SDRA criteria refine the SD rules by acknowledging that agents can manage risk by adjusting business risk and financial risk. This acknowledgement has the potential to significantly reduce the size of the FSD and SSD efficient sets. The goal of the analysis section will be to determine the reduction in the efficient set that can be expected with this acknowledgement.

Analysis

To examine the ability of SDRA to reduce the efficient set, two simulation models were used to generate return distributions. AgRISK, was used to simulate thirteen pre-harvest marketing strategies for a 300 acre corn/soybean farm. The strategies included the natural hedge (no marketing strategies), hedging various amounts of expected production, buying options on varying amounts of expected production, and buying put options and selling call options on varying amounts of expected production. The risk free return was calculated by considering the cash renal rate and variable operating costs. The ordinary stochastic dominance and SDRA efficient sets from the AgRISK simulation are shown in Table 1.

Table 1. Number of AgRISK strategies in the efficient set.

	FSD	SSD	FSDRA	SSDRA
Strategies in Efficient Set	12	7	5	3
Percent of Total	92%	54%	38%	23%

The results show that the SDRA rules reduced the efficient set considerably. For instance, the manager using the SSDRA rule would only need to choose between three of the thirteen potential risk management strategies. In this case, FSDRA proved to be a stronger criteria than SSD. This implies that acknowledging the role of leverage greatly reduces the ordinary SD efficient set. None of the SDRA sets contained the strict cash sale option (natural hedge) while the ordinary SD efficient sets did. This implies that by leveraging the risk management strategies, one can produce distributions that dominate the natural hedge strategy.

A simulation model was also used to generate return distributions for 23 risk management strategies for a 1,000 acre corn/soybean farm with a 190 sow farrow-to-finish operation (Nydene, 1999). The strategies included a natural hedge, hedging inputs, hedging outputs, purchasing options, buying crop insurance, and combinations of the methods. The risk free return was calculated based on a 8 percent interest rate. Table 2 shows the size of the efficient sets for the ordinary SD rules and the SDRA rules.

Table 2. Number of Hog Risk strategies in the efficient set.

	FSD	SSD	FSDRA	SSDRA
Strategies in Efficient Set	23	7	8	3
Percent of Total	100%	33%	26%	13%

Again the results show that the SDRA criteria reduce the efficient set considerably. The FSDRA set was nearly as small as the SSD set. In this case, the SSDRA efficient set was less than half of the SSD efficient set. The SSDRA set contains three strategies, all of which involve the use risk management tools. This suggests that risk management decisions should not be made without considering the potential impact of leverage. In fact, when increased risk is viewed as a cost, leverage can prove to be a more efficient way to increase expected return than reverting to base strategies such as the natural hedge.

Summary

The SDRA criteria consider all possible combinations of the strategies and financial leverage. This consideration allows the possibility that strategies with less business risk, less expected return, and greater leverage may dominate strategies with greater business risk and greater expected return. This is important when one compares investments that present the decision maker with a risk-return trade-off such as risk management strategies. Results show that the inclusion of the risk free asset (combined with the traditional assumptions of stochastic dominance) produce risk efficiency criteria that are very powerful at narrowing the risk management alternatives relevant for a manager's further consideration.

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