Optimal Spatial Scale for Evaluating Economic and Environmental Tradeoffs

By

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Introduction

This paper develops a conceptual framework that can provide a scientific foundation for formulating policies that consider environmental and economic tradeoffs, such as the policy issues related to carbon sequestration in the Kyoto protocol. It addresses a critical problem recognized in the environmental sciences, namely, choosing the appropriate spatial scale for measurement and analysis of spatially variable economic and bio-physical processes. The importance of selecting an appropriate scale for analysis has been acknowledged in many different sciences (for example, Sivapalan and Kalma; Turner *et al*.; Wu and Segerson; Aspinall) and has been identified as "one of the major impediments, both conceptually and methodologically, to advancing all sciences that use geographic information" (NCGIA).

It is estimated that U.S. agriculture has the potential to sequester between 75 to 208 million tons of carbon (C) annually in the soil through modified management practices (Lal *et al*.). Recent experience with existing emissions trading programs suggests that for agriculture to participate in meeting U.S. commitments to reduce greenhouse gas emissions, it will be necessary to provide estimates of soil C under baseline conditions and under alternative C-sequestration policies.

Farmer decision making and soil C are spatially variable and their measurement is scale dependent (Cao and Lam; Bian and Walsh). This means that estimates of base soil C levels and soil C under proposed policies will vary depending on the scale[1] chosen for policy analysis

[1] There are many meanings of the term scale (Lam and Quattrochi; Cao and Lam). In this paper, the term scale refers to the measurement scale or resolution of the data. For example, common scales (areal units) for economic data are census block and county. Within each of these areas, variables take on a single value.

(Davies *et al.*; Wong and Amrhein). The scale of analysis can affect the value of information produced as well as the cost of that information.

The appropriate scale for analysis is an important consideration in policies designed to encourage soil C-sequestration for two reasons. Firstly, estimates of economic and environmental tradeoffs must meet accepted scientific standards and be verifiable to support contracts between farmers and the government or private entities. Secondly, the costs of collecting disaggregate data over large spatial regions can be high; therefore, the question of whether the benefits justify the costs becomes relevant to agencies responsible for conducting these analyses.[2]

Formulation of the scale problem within an economic framework will provide a theoretically consistent analysis of the tradeoffs encountered in choosing a scale for empirical analysis. The framework and techniques can be used by any discipline to determine the economically efficient scale at which to conduct research.

Previous Research on the Scale Problem

In the late 1970's Openshaw and Taylor formally defined the scale problem as the Modifiable Areal Unit Problem (MAUP). They demonstrated that by changing the scale at which data are gathered, the level of correlation between variables could span values that range from +1 to –1. This effectively illustrates that the scale of data aggregation can significantly affect the outcomes of models and the decision making process (Bian). MAUP encompasses two dimensions. Firstly, the scale problem that refers to the variation in results when areal units are

[2] Antle and Just; and Crissman, Antle and Capalbo suggest that it may be necessary to collect highly disaggregate economic data to accurately measure the spatial variability in management behavior that is relevant to environmental processes.

progressively aggregated into fewer and larger units; and secondly, the aggregation problem that refers to the variation in results due to the use of alternative aggregation schemes at equal or similar scales.

The existence of the scale problem has led to a number of studies that seek to determine the “appropriate” scale for analysis. Cao and Lam; Meyers; and Fotheringham suggest that the appropriate scale is that at which the data exhibit the maximum inter-zonal variability and minimum intra-zonal variability. These studies do not consider the trade-offs between the cost and benefits from analyses at different scales.

The costs of data collection can be estimated in a reasonably straightforward fashion. However, the benefits generated by analyses at different spatial scales are harder to estimate. Two measures of the benefit of information are entropy (Theil) and the comparative performance of decisions made under alternative states of information (Chavas and Pope). In this paper, the benefits of additional information are calculated as the change in producer profit (or other economic measure of well being) as a result of making decisions based on information generated by analyses conducted at different spatial scales. This is consistent with several studies that have considered the value of information in the context of improved decision making (Baquet, Halter and Conklin; Mjelde *et al*.; Wu and Segersen; Babcock, Carriquiry and Stern; Swinton and Jones; Adams *et al*.; Costello, Adams and Polasky).

Theory of Economically Optimal Spatial Scale

There is a benefit and a cost to more accurate measurement or from data collection at various scales, hence, the economically optimal scale is a function of the factors affecting these benefits and costs. The framework is illustrated using the following stylized presentation.

Define the following variables:

$y(x,\alpha)$	= outcome of interest, with parameter α
x	= spatially referenced variable affecting $y(\cdot)$ where, $x \sim \psi(x\|\text{h})$
h	= measure of the heterogeneity of x in the population (e.g., the population variance of x)
S	= spatial scale index, S = 1,...,N
v(S)	= measure of the variability of x measured at scale S
N	= spatial scale at which v(S) achieves its maximum value
$x_i(\text{S})$	= value of x for the i th cell for scale S < N
$x_{ij}(\text{N})$	= value of x for the j th cell in the neighborhood of x_i
w	= data cost per cell
e_{ij}	= measurement error, $y(x_i(\text{S}),\alpha) - y(x_i(\text{N}),\alpha)$
$L(\text{S},\text{N},\text{h})$	= loss function resulting from inaccurate measurement of $y(\cdot)$

Consider a region for which we wish to measure values of y. In the simplest case, a single value of x and hence of y can be measured for this region. If the value of x is uniform over the region, then a single measurement of x is sufficient to measure y accurately for every part of the region. If the value[3] of x varies over the region, a single measurement will produce inaccurate estimates of y for places in the region. The problem is to determine the economically optimal spatial scale for measuring x, i.e., the scale to measure x that maximizes the net value of the data collected.

Spatial scale is defined using the index S. For any scale S the region is divided into $4^{\text{S-1}}$ uniform cells. As S increases, the number of cells increase and their area decreases. The variable x takes on a value $x_i(\text{S})$ for the i th cell, where the cells are numbered in rows from

[3] The value of x does not need to be determined from a single sample point, it can be determined as the average

northwest to southeast (Figure 1), and for spatial representation, these values are attributed to the center of each cell.

At some scale, N, the variability of the measured x across all cells, $v(\mathrm{S})$, achieves its maximum, i.e., $v(\mathrm{S}) < v(\mathrm{N})$ for all S < N. This denotes the scale at which the maximum inter-cell variation of x has been captured. For a value of N, define a neighborhood around $x_i(\mathrm{S})$ as the set of $4^{\mathrm{N-S}}$ cells nearest to $x_i(\mathrm{S})$ (see Figure 1). The measurement error e_{ij} is calculated as the difference between the outcome measured at each cell in the neighborhood at scale N, $y(x_{ij}(\mathrm{N}),\alpha)$, and the outcome measured at the center of the neighborhood measured at scale S, $y(x_i(\mathrm{S}),\alpha)$. Inaccurate measurement results in a loss of valuable information.

To illustrate consider the quadratic loss function

$$L(\mathrm{S},\mathrm{N},\mathrm{h},\alpha,\theta) = \int \theta \sum_{i=1}^{4^{\mathrm{S}-1}} \sum_{j=1}^{4^{\mathrm{N}-\mathrm{S}}} \left[y(x_i(\mathrm{S}),\alpha) - y(x_{ij}(N),\alpha) \right]^2 \psi(x|\mathrm{h})\,\mathrm{d}x \qquad (1)$$

where θ is a value parameter with units \$/y, used to translate the loss into dollar terms. Loss decreases in S for given N and h, i.e., as the number of data points increases the accuracy of measurement increases. This function also embodies the notion that loss is a convex function of S.

Comparing the loss functions at each scale can assess the benefits from measuring x at a variety of scales. For example, the greatest loss occurs when S=1,[4] hence, we can define the benefits associated with measurement at scale S > 1 as the reduction in loss compared to S=1.

$$B(\mathrm{S},\mathrm{N},\mathrm{h},\alpha,\theta) = L(1,\mathrm{N},\mathrm{h},\alpha,\theta) - L(\mathrm{S},\mathrm{N},\mathrm{h},\alpha,\theta) \qquad (2)$$

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value of several sample points (or similar). However, once determined, the value of x is taken to represent the entire cell.

[4] Assuming that the data exhibit heterogeneity. If the data are spatially homogeneous, N=S=1 and there is no loss when S=1.

$B(\mathrm{S}, \mathrm{N}, \mathrm{h}, \alpha, \theta)$ is not differentiable with respect to the integers S and N. In (1), $L(\mathrm{S}, \mathrm{N}, \mathrm{h}, \alpha, \theta)$ is hypothesized to be decreasing in S, therefore benefits are increasing in S. In addition, if loss is convex in S, it follows that (2) is a concave function of S.

Assuming that the data cost per cell, *w*, is independent of S, the cost of measurement at scale S > 1 can be defined as the increase in cost relative to cost at S = 1.

$$C(\mathrm{S}, w) = w(4^{S-1} - 1) \tag{3}$$

$C(\mathrm{S}, w)$ is not differentiable with respect to S but is increasing and convex in S. Combining (2) and (3), the net benefit of data collection and analysis at S > 1 compared to S=1 is

$$NB(\mathrm{S}, \mathrm{N}, \mathrm{h}, \alpha, \theta) = B(\mathrm{S}, \mathrm{N}, \mathrm{h}, \alpha, \theta) - C(\mathrm{S}, w) \tag{4}$$

Under the assumption that benefits are increasing and concave in S and costs are increasing and convex in S, it follows that the optimal value of S will occur where marginal benefit equals marginal cost . Thus the economically optimal scale is hypothesized to be a function $\mathrm{S}(\mathrm{N}, \mathrm{h}, \alpha, \theta, w)$.

To explore its properties in more detail, we treat S as a continuous variable rather than an integer. Then the economically optimal spatial scale satisfies the first-order condition $NB_S = -L_S(\mathrm{S}, \mathrm{N}, \mathrm{h}, \alpha, \theta) - C_S(\mathrm{S}, w)$. Differentiating with respect to the variables N, h, α, θ and *w* shows that the comparative static properties of the optimal S satisfy

$$d\mathrm{S}/d\mathrm{N} = -L_{\mathrm{SN}}/(L_{\mathrm{SS}} + C_{\mathrm{SS}})$$

$$d\mathrm{S}/d\mathrm{h} = -L_{\mathrm{Sh}}/(L_{\mathrm{SS}} + C_{\mathrm{SS}})$$

$$d\mathrm{S}/d\alpha = -L_{\mathrm{S}\alpha}/(L_{\mathrm{SS}} + C_{\mathrm{SS}})$$

$$d\mathrm{S}/d\theta = -L_{\mathrm{S}\theta}/(L_{\mathrm{SS}} + C_{\mathrm{SS}})$$

$$dS/dw = -C_{Sw}/(L_{SS} + C_{SS})$$

Maintaining the assumption that both L and C are convex in S, the denominator $(L_{SS} + C_{SS}) > 0$. The comparative static properties depend on the signs of the cross-partial derivatives L_{Si}, i = N, h, α, θ and C_{Sw}. From equation (3) we know that C_{Sw} > 0, hence, it follows unambiguously that $dS(N, h, \alpha, \theta, w)/dw$ < 0, i.e., that the optimal number of data points is decreasing with the unit cost of data collection. From (1) we know L_θ > 0, L_S < 0, and $L_{S\theta}$ < 0, showing that as the per unit value of the loss (θ) increases, the optimal value of S increases. Intuitively, we expect that the optimal S should be increasing in N and h, implying the hypothesis that L_{SN} < 0 and L_{Sh} < 0. The parameter α is not specified, but under the assumption that the value of the outcome *y* is increasing in α, our intuition suggests that S should be increasing in α and hence $L_{S\alpha}$ < 0.

Optimal Spatial Scale for C-Sequestration Policy

An application of the above theory, is presented in this section in the context of developing a policy for agricultural soil carbon sequestration. Consider a policy that pays farmers to adopt management practices that increase soil carbon. The US government determines that the social value of a ton of atmospheric carbon sequestered is *v* dollars. The government also estimates that the marginal cost of sequestering carbon on the *i* th land unit is $mc_i(B)$ for B tons sequestered in excess of the *status quo* without policy intervention.

The government enters into a contract to obtain the amount B_{ig} that satisfies $v = mc_i(B_{ig})$. However, at this quantity of carbon sequestered the true (or actual) marginal cost is $\text{amc}_i(\text{B}_{ig}) > \text{mc}_i(\text{B}_{ig})$. In the case in which the actual marginal cost is linear (Figure 2), $\text{amc}_i = \text{am}_{0i} + \text{am}_{1i}\text{B}$, the cost (loss) of the government over-allocating resources to carbon sequestration on this land unit is

$$L_i = \tfrac{1}{2}\text{am}_{1i}(\text{B}_{ig} - \text{B}_{ia})^2 .$$

In this case loss is a quadratic function of the error in estimating the efficient quantity of carbon that should be sequestered on each unit of land.

Let the farmer's profit from managing a land unit be $\pi_i = p_i q_i(x_i, \phi_i) - w_i x_i$ where p_i and w_i represent output and input prices respectively, q_i and x_i represents output and input quantities respectively and ϕ_i is a parameter. The profit-maximizing level of input use, $x_{i\pi}$, solves the first-order condition $p_i \, {\delta q_i}/{\delta x_i} - w_i = 0$. Also let the amount of carbon sequestered by the farmer beyond the amount in the absence of any policy be a linear, increasing function of *x*, i.e., $\text{B}_i = \text{B}(x_i - x_{i\pi}, \gamma_i) = \gamma_i(x_i - x_{i\pi})$, where γ_i is a parameter. The government believes that the marginal cost of increasing x beyond the profit maximizing level is equal to the farmer's loss in profits, which assuming a linear marginal product function is

$$\text{mc}_i = w_i - p_i \, {\delta q_i(x_i, \phi_{ig})}/{\delta x_i} = w_i - p_i(\phi_{0ig} - \phi_{1ig} x_i) \text{ for } x_i > x_{i\pi} .$$

The government's estimate of the carbon sequestration function also leads to the estimate γ_{ig} of γ_i. Substituting the government's carbon sequestration function into the above expression gives the marginal cost function:

$$\begin{aligned} \text{mc}_i(\text{B}) &= w_i - p_i \, \delta q_i(x_i, \phi_{ig}) / \delta x_i \\ &= w_i - p_i(\phi_{0ig} - \phi_{1ig} x_i) \\ &= w_i - p_i(\phi_{0ig} - \phi_{1ig}(\text{B}/\gamma_{ig} + x_{i\pi})) \\ &= w_i - p_i\phi_{0ig} - p_i\phi_{1ig} x_{i\pi} + (p_i\phi_{1ig}/\gamma_{ig})\text{B} \end{aligned} \qquad (5)$$

The above equation shows that the marginal cost function depends on the parameters of the farmer's production function and the parameters of the processes linking management decisions to soil carbon. The marginal cost is linear as in Figure 2 when the carbon sequestration function and the marginal product functions are linear.

To induce the farmer to undertake management that increases carbon sequestration, the government enters into a contract with the farmer that specifies that the farmer will employ $x_{ig} > x_{i\pi}$ units of *x*. The amount of payment required to induce the farmer to produce with x_{ig} will depend on the farmer's knowledge of the production function. Assuming that the farmer knows the true marginal product function $(\phi_{0ia} - \phi_{1ia} x_i)$, the actual marginal cost (which is the marginal cost perceived by the farmer) is:

$$\begin{aligned}\mathrm{amc}_i(\mathrm{B}) &= w_i - p_i(\phi_{0ig} - \phi_{1ig} x_i) \\ &= w_i - p_i(\phi_{0ig} - \phi_{1ig}(\mathrm{B}/\gamma_{ig} + x_{i\pi})) \\ &= w_i - p_i\phi_{0ig} - p_i\phi_{1ig} x_{i\pi} + (p_i\phi_{1ig}/\gamma_{ig})\mathrm{B} \\ &= \mathrm{am}_{0\mathrm{i}} + \mathrm{am}_{1i}\mathrm{B}\end{aligned} \qquad (6)$$

where $\mathrm{amc}_i(\mathrm{B}) > \mathrm{mc}_i(\mathrm{B})$ for all B > 0. The farmer receives a price per ton of carbon of $v' = \mathrm{amc}_i(\mathrm{B}_{ig}) = w_i - p_i(\phi_{0ia} - \phi_{1ia} x_{ig})$ (see equation 6), and a total payment $v'(\mathrm{B}_{ig})$, of which the portion equal to $L_i = \frac{1}{2}\mathrm{am}_{1i}(\mathrm{B}_{ig} - \mathrm{B}_{ia})^2$ is deadweight loss. This example could be carried through with the assumption that the government underestimates the optimal amount of C, giving a deadweight loss in the amount $L = \frac{1}{2}\mathrm{am}_{1i}(\mathrm{B}_{ig} - \mathrm{B}_{ia})^2$.

The economically optimal spatial scale for analysis of this C sequestration policy introduced above can be determined from measuring model parameters ϕ, γ, and the management variable x at different scales. For example, if management and C sequestration processes operate at the field level, we can interpret the field as the scale that yields the most information about these processes.[5] Measurements of the model's parameters may be made at coarser scales, however, such as 10 km^2 or 100 km^2 grids, or at the level of a political unit such as a county.[6] Thus, to operationalize the assessment, we select scales S_j where j = 1,…..,N for comparison, where S_N provides the maximum variability from the sampled x (the field scale in this discussion). At each scale, the parameters are estimated and a value $B_{ig}(S_j)$ is estimated, giving a loss at each scale S_j of

$$L(S_j) = \frac{1}{2}\sum_{i=1}^{4^{N-1}} am_{1i}\left(B_{ia}(S_N) - B_{ig}(S_j)\right)^2$$

The cost of data collection at scale S_j is, as discussed above, $C(S_j, w) = w(4_j^{S-1} - 1)$. Net benefits at each scale are calculated as $NB(S_j) = L(S_1) - L(S_j) - C(S_j, w)$. The optimal scale is then defined as S_O where $NB(S_0) \geq NB(S_j)$ for all j. These net benefits should be a function of w and the other variables discussed above that affect benefits, including the population heterogeneity.

Conclusions

In this paper, a theoretical framework is presented for characterizing the economically optimal spatial scale for conducting analysis of spatially variable economic and bio-physical processes. The economically optimal spatial scale is defined as the one that maximizes net benefits of information produced. The framework and illustrative C-sequestration policy application presented in this paper highlight the importance of selecting the appropriate scale for data collection and analysis. The economically optimal scale for analysis was found to be an increasing function of the scale at which the observed data exhibit maximum variability (N) and

[5] Field level data accounts for heterogeneity across farms.
[6] Note that not all of the scales listed are represented by uniform areal units.

the heterogeneity of the data. In addition the optimal number of data points is decreasing with the unit cost of data collection, and increasing as the per unit value of the outcome increases.

While the linearity assumptions in the C-sequestration example are not valid for all situations, the framework illustrated that there are trade-offs between the benefits and costs of analyses conducted at different scales and several results emerge. First, the marginal cost function for C-sequestration is a function of the parameters of the farmer's production technology and the parameters defining the processes that relate management to soil C. Second, errors in estimation of these parameters will lead to errors in the government's soil C-sequestration targets for each land unit, and the cost of these errors increase approximately with the squared error.

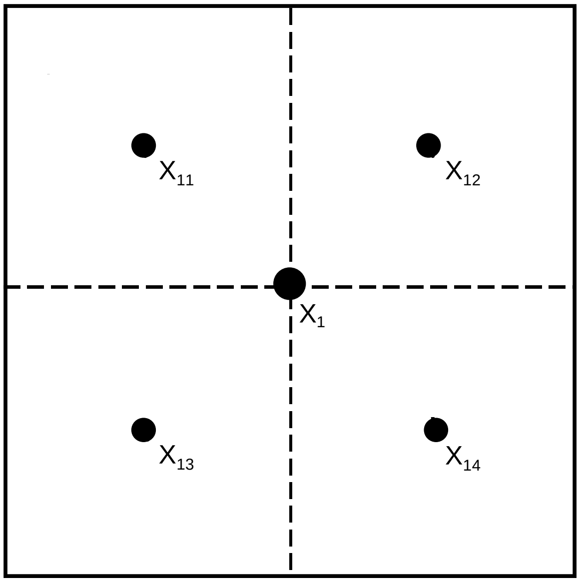

Figure 1. The neighborhood around point X_1 for S = 1 and N = 2.

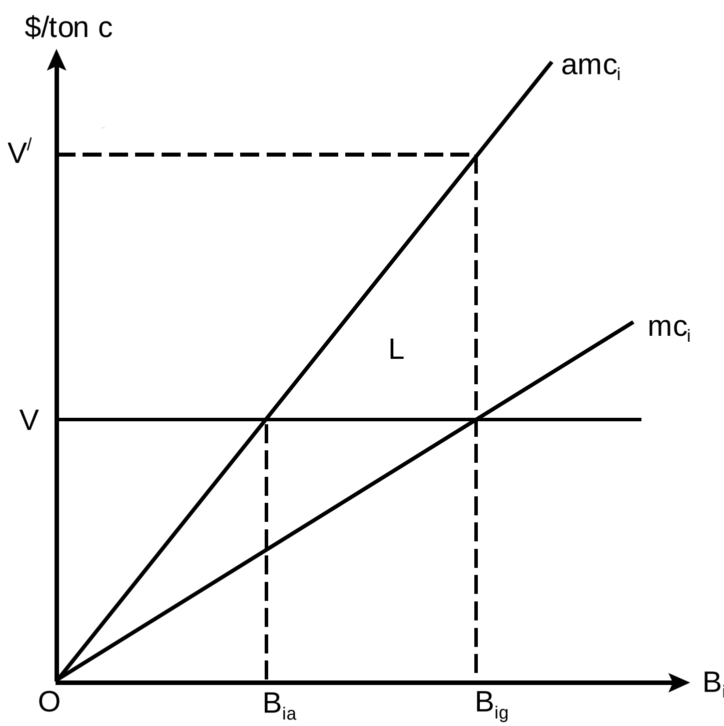

Figure 2. Marginal cost of carbon sequestration and loss L.

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