THE ALMOST IDEAL SUPPLY SYSTEM AND AGRICULTURAL PRODUCTION IN THE UNITED STATES

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Abstract:
This paper estimates an Almost Ideal Supply System using aggregate U.S. agricultural data. Share equations derived from an indirect production function yield elasticities that are consistent with production theory. A nested test comparing the Almost Ideal Supply System to the Translog Production Function finds little difference between the two models.

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1. Introduction

An important concern in applied economics is the appropriate way to model consumer demand and firm supply. Several methods have been proposed for modeling consumer demand. Prominent among these are the Rotterdam model of Theil (1965), the Linear Translog model of Christensen, Jorgenson, and Lau (1975) and the Almost Ideal Demand System (AIDS) of Deaton and Muellbauer (1980). Among these, the AIDS has gained widespread support in both direct applications and extensions to more complex applications, such as the inverse demand framework (for recent examples see Holt and Goodwin [1997], Ryan and Wales [1996], Eales and Unnevehr [1994]). The AIDS model is appealing among the potential demand systems due to its second order flexibility, compatibility with demand theory, and relative ease of estimation.

Given its clear intuitive appeal, it is somewhat surprising that only one attempt has been made to apply the almost ideal framework to the firm's supply decision (Chambers and Pope, 1994). Previous work in production economics has focused on alternative functional forms, such as the Translog or the Generalized Leontief. The properties that make the framework appealing for demand should also make it appealing for supply. Namely, the Almost Ideal Supply System (AISS) we propose should provide an easily estimable and sufficiently general functional form that can be used to test the restrictions imposed in producer theory.

In this paper, we are the first to estimate an indirect production function using the AISS model with time-series data on U.S. aggregate agricultural output. We find estimated elasticities that are consistent with production theory. To assess the relative goodness of fit of the AISS to the Translog Production Function, we use the nested test of Lewbel (1989) and find no significant difference between the two models. Hence our results suggest that the AISS is a potentially appealing method for estimating modern supply theory.

2. The Almost Ideal Supply System

The classic almost ideal demand system of Deaton and Muellbauer (1980) is derived from the indirect utility function

(1) $$h(P,m) = \prod_{k=1}^{L} p_k^{-B_k} (\ln m - \ln g(P))$$

where h(P,m) represents indirect utility, P is a vector of consumer prices, m is income, and g(p) is a price index defined as

(2) $$\ln g(P) = \alpha_0 + \sum_{k=1}^{L} \alpha_k \ln p_k + \frac{1}{2}\sum_{k=1}^{L}\sum_{j=1}^{L} B_{kj} \ln p_k \ln p_j .$$

To convert this almost ideal framework to supply, assume output, y, is produced by combining capital, land, chemicals, other intermediate inputs, and labor. Let W represent a vector of input prices, E represent aggregate expenditure, and t represent a time trend measuring technological change. The almost ideal indirect production function is defined by substituting y, W, and E for the indirect utility, price and income measures in (1) and adding t to capture technological change. We then have

(3) $$y(W,E,t) = \prod_{k=1}^{5} w_k^{-B_k} (\ln E - \ln g(W,E,t))$$

where ln g(W,E,t) is defined as

(4) $$\ln g(W,E,t) = \alpha_0 + \sum_{k=1}^{5} \alpha_k \ln w_k + \frac{1}{2}\sum_{k=1}^{5}\sum_{j=1}^{5} B_{kj} \ln w_k \ln w_j + V_t t + t\sum_{k} V_k \ln w_k + B_T t \ln E + \frac{1}{2} V_{tt} t^2$$

Note that the additional terms in (4) relative to (2) are the result of adding technological change to the production process.

Economic theory implies that production functions should be homogeneous of degree one and symmetric in the second order parameters. Additionaly, because we will be estimating share equations, adding-up restrictions are needed. These desirable properties are satisfied by imposing the following restrictions: $\sum \alpha_k = 1, \sum B_k = 0, \sum B_{jk} = 0, \sum V_k = B_t$, and $B_{kj} = B_{jk}$ for all k and j. Finally, to guarantee convergence we make the additional identifying restriction that $\alpha_0 = 0$.

Imposing the above restrictions and applying Roy's identity to equation (3) yields a system of share equations:

$$(5) \qquad S_i = \frac{\alpha_i + \sum_k B_{ik} \ln w_k + tV_i + B_i(\ln E - \ln g(W,E,t))}{1 - B_T t}$$

where S_i is the cost share of the ith input and $\sum S_i = 1$.[2]

Next, we want to examine the relevant elasticities implicit in the AISS model. The elasticities are computed by converting the share equations in (5) to ordinary input demand functions, X_i, by $X_i = S_i E / W_i$. The subsequent uncompensated own- and cross-price elasticities of demand are:

$$(6) \qquad \varepsilon_{ii}^{u} = \frac{\partial \ln X_i}{\partial \ln w_i} = -1 + \frac{B_{ii}}{S_i(1\text{-}B_T t)} - \frac{B_i(\alpha_i + \sum_k B_{ik} \ln w_k + tV_i)}{S_i(1\text{-}B_T t)}$$

$$(7) \qquad \varepsilon_{ij}^{u} = \frac{\partial \ln X_i}{\partial \ln w_j} = \frac{B_{ij}}{S_i(1\text{-}B_T t)} - \frac{B_i(\alpha_j + \sum_k B_{jk} \ln w_k + tV_j)}{S_i(1\text{-}B_T t)}$$

and the expenditure elasticities of input demand are:

$$(8) \qquad \varepsilon_{iE} = \frac{\partial \ln X_i}{\partial \ln E} = 1 - \frac{B_i}{S_i}$$

Combining (6), (7), and (8) according to the Slutsky decomposition yields the compensated own- and cross-price elasticities:

$$(9) \qquad \varepsilon_{ij}^{c} = \varepsilon_{ij}^{u} + S_j \varepsilon_{iE}.$$

[2] Note that $S_i = W_i X_i / E = -(\partial \ln y(W,E,t)/\partial \ln w_i)/(\partial \ln y(W,E,t)/\partial \ln E)$

Finally, following Chambers (1988) the Allen elasticities of substitution are

(10) $\sigma_{ij} = \varepsilon_{ij}^{c} / S_j$.

Turning to the effect of technology on production, the rate of technical change for the AISS indirect production function is measured by

(11) $$\varepsilon_{YT} = \frac{\partial \ln y}{\partial \ln t} = \prod_k w_k^{-B_k} (-V_t - \sum_k V_k \ln w_k - B_t \ln E - V_{tt} t) \frac{t}{y}$$

and input bias in technical change is measured by

(12) $$b_i = \frac{\partial \ln w_i}{\partial \ln t} = \frac{t(V_i - B_i (V_t + \sum_k V_k \ln w_k + B_t \ln E + V_{tt} t)}{S_i (1 - B_t t)} + \frac{B_t t}{(1 - B_t t)} .$$

According to Kim (1988), if technical change is occurring ε_{YT} will be positive. Further, if $b_i>0$ input i is technology intensive, if $b_i=0$ input i is technology neutral, and if $b_i<0$ input i is technology saving.

3. Data Description and Empirical Results

The Almost Ideal Supply System consists of the indirect production function in (3) and the five share equations in (5). To explore the potential usefulness of the AISS, we estimate the above model using the familiar data on U.S. agricultural production found in Ball et. al. (1997). These data consist of time series observation on U.S. aggregate agricultural output (Y), capital (K), land (L), chemicals (C), other inputs (I), and labor (LB) for the period 1948-1994.[3] Note that other inputs consist of fuels, electricity, feed, seed, and livestock purchases. Data are aggregated using the Divisia indexing method and the base year is defined as 1987.

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[3] See Ball et. al. (1997) for an excellent description of the original data sources as well as the methods used to construct the subcategory indices.

The AISS model is estimated using a maximum likelihood routine in TSP subject to the parameter restrictions imposed by symmetry and homogeneity. The labor equation is dropped to avoid the singular variance-covariance matrix that occurs because the share equations add to one. Hence, five equations are jointly estimated, specifically, the indirect production function and four share equations. The resulting R^2 values are above .80 in each of the equations indicating a fairly good fit. However, the Durbin-Watson statistics are all below .25 suggesting the presence of autocorrelation. Accordingly, the model was re-estimated using the Berndt-Savin, Moschini-Moro, and Symmetric-R autocorrelation correction methods. The results presented below are from the Symmetric-R specification, which had the highest log likelihood value, and provided R^2 values above .95 and Durbin-Watson statistics above two.

Compensated own- and cross-price elasticities calculated using (9) and the AISS parameter estimates are presented in Table 1.[4] The estimated cross-price elasticities are nearly all positive suggesting that the five inputs are substitutes. Statistically significant exceptions include the demand for chemicals with respect to the price of capital, the demand for land with respect to the price of other inputs, and the demand for land with respect to the price of labor. Estimated own price elasticities are all negative and significant, suggesting that the inputs follow the law of factor demand. The fact that these values are greater than negative one, suggests that all five inputs are inelastic, with chemicals having the most elastic factor demand and land having the most inelastic factor demand. The compensated elasticities can be used to test whether the indirect production function is quasi-convex. The leading eigenvalue in the substitution matrix (calculated at the data means) is 0.9243. To have proper curvature,

[4] Allen elasticities of substitution were also estimated. However, these values are not reported as they contain the same information as the compensated elasticities.

eigenvalues of should all be negative. Therefore, we reject the quasi-convexity of the indirect production function.

A well known aspect of compensated elasticities is that they isolate input price effects without considering expenditure effects that accompany changes in input prices. To consider both effects, we calculate the uncompensated elasticities in Table 2. As expected, the estimated own-price elasticities are all negative. Turning to cross-price elasticities, the estimated uncompensated values in Table 2 differ markedly from the estimated compensated values in Table 1. Most of the uncompensated elasticities are negative, suggesting that when the expenditure effect is accounted for, the majority of inputs are complements. The only statistically significant exception is land, for which increases in the price of capital and chemicals increase the demand for land. The difference in sign between compensated and uncompensated elasticities is consistent with previous findings by Kim (1988) and Gajanan and Ramaiah (1996).[5]

Table 3 presents the estimated expenditure elasticities in (8). Economic theory predicts that these elasticities should be positive, as firms should increase the amount of all inputs used in response to increases in expenditure. All inputs have the correct sign, except for land. The negative sign for land may result because land is a relatively fixed input. The expenditure elasticity for capital is less than one, suggesting that capital is expenditure inelastic. Conversely, the expenditure elasticities for chemicals, other inputs and labor are greater than one, suggesting that those inputs are expenditure elastic. These results differ from those of Chambers and Lee (1986) who find all expenditure elasticities are positive and close to one.

[5] Based on the sign differences, Gajanan and Ramaiah (1996) caution policy makers to consider uncompensated rather than compensated elasticities.

Table 4 investigates the effect of technological innovation by computing the rate of technical change in (11) and the input bias terms in (12). The estimated rate of technical change is positive and significant, implying that technology increased during the period from 1948 to 1994. The bottom panel of Table 4 suggests that other inputs and labor were technology intensive during that time period, capital and chemicals were technology saving, and land was technology neutral.

As the empirical results above are generally consistent with production theory, the AISS would seem a strong candidate for future work in production economics. We would therefore like some idea how the AISS compares to the popular Translog model. Lewbel (1989) proposes a model that nests both the AIDS and the Translog indirect utility models and allows for a simple test between the two. By substituting supply for demand measures and adding technology we develop an analogous nested model for testing between the AISS and the Translog indirect production functions.[6] A test between the alternative models does not reject the null hypothesis of equality between the AISS and the Translog models. Specifically, the log likelihood value for the AISS is 913.435 while the log likelihood value for the Translog model is 909.894. In other words, it appears that the performance of these two models is not significantly different as a means of estimating indirect production functions.

4. Conclusion

The almost ideal demand system of Deaton and Muellbauer (1980) has become one of the most popular functional forms in empirical studies of consumer demand theory. In fact, a recent literature search found in excess of one hundred applications since its initial development. In this study we are the first to estimate an almost ideal indirect production system. Using time

[6] Details are available upon request.

series data on U.S. agricultural production, we find that the AISS is a potentially beneficial method of estimating firm supply theory. The functional form is sufficiently general, the estimation is computational simple, the estimated coefficients are highly significant, and the majority of the results conform to conventional supply theory. These findings suggest that the AISS may be preferable to alternative methods, such as the Translog, as a means of studying firm supply theory.

In the future we plan to extend the AISS model estimated here to examine several important issues. We would like to estimate a short run model that treats capital and land as fixed. Within the context of this short run model, we plan to take a closer look at the curvature issue and, if needed, impose local concavity. Current research by Pitarakis and Tridimas (1999) allows expenditure to be endogenous in a system of demand equations. We plan to adopt this methodology to the supply side and treat expenditure as endogenous within the indirect production framework.

Table 1. Estimated Compensated Price Elasticities

	Elasticity with respect to the price of				
	Capital	Land	Chemicals	Other Inputs	Labor
Capital	-0.4213*	0.1581*	-0.0063	0.0810*	0.1885*
	(0.061)	(0.035)	(0.030)	(0.039)	(0.036)
Land	0.4891*	-0.1330*	0.1370*	-0.1486*	-0.3446*
	(0.054)	(0.036)	(0.039)	(0.047)	(0.059)
Chemicals	-0.3768*	-0.0596	-0.7046*	0.6692*	0.4718*
	(0.073)	(0.049)	(0.072)	(0.076)	(0.075)
Other Inputs	-0.0376	0.0087	0.0409*	-0.1550*	0.1430*
	(0.012)	(0.013)	(0.016)	(0.029)	(0.021)
Labor	0.0143	0.0163	-0.0009	0.1852*	-0.2149*
	(0.034)	(0.018)	(0.023)	(0.041)	(0.035)

Notes: Standard errors in parentheses. * denotes significance at the 5% level.

Table 2. Estimated Uncompensated Price Elasticities

	Elasticity with respect to the price of				
	Capital	Land	Chemicals	Other Inputs	Labor
Capital	-0.4931* (0.065)	0.0505 (0.041)	-0.0431 (0.033)	-0.2069* (0.047)	0.0257 (0.041)
Land	0.5391* (0.064)	-0.0581 (0.051)	0.1626* (0.044)	0.0518 (0.066)	-0.2313* (0.074)
Chemicals	-0.6056* (0.084)	-0.4023* (0.057)	-0.8219* (0.074)	-0.2481* (0.086)	-0.0469 (0.092)
Other Inputs	-0.1731* (0.026)	-0.1942* (0.014)	-0.0285 (0.018)	-0.6980* (0.020)	-0.1641* (0.027)
Labor	-0.1369* (0.045)	-0.2102* (0.020)	-0.0784* (0.026)	-0.4209* (0.033)	-0.5577* (0.046)

Notes: Standard errors in parentheses. * denotes significance at the 5% level.

Table 3. Estimated Expenditure Elasticities

Elasticity	Estimate
Capital	0.6668* (0.116)
Land	-0.4641* (0.200)
Chemicals	2.1247* (0.203)
Other Inputs	1.2579* (0.068)
Labor	1.4040* (0.116)

Notes: Standard errors in parentheses. * denotes significance at the 5% level.

Table 4. Estimated Input Biases and the Rate of Technical Change

	Estimate
Rate of Technical Change	0.6875* (0.213)
Bias	
Capital	-1.5283* (0.253)
Land	0.1277 (0.160)
Chemicals	-0.7589* (0.334)
Other Inputs	0.1075* (0.056)
Labor	1.5358* (0.193)

Notes: Standard errors in parentheses. * denotes significance at the 5% level.

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