Estimation of Risk Aversion Coefficients for Dryland Wheat and Irrigated Corn Enterprises in Kansas

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Abstract

The risk attitudes of dryland wheat and irrigated corn enterprises in Kansas have been examined in this paper using the nonlinear mean-standard deviation approach. Our results showed that dryland wheat enterprises are characterized by increasing absolute and relative risk aversion while irrigated corn enterprises are characterized by decreasing absolute and increasing relative risk aversion. The level of wealth is noted to have an impact on risk attitudes.

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Introduction

Risk constitutes an important factor in the production and marketing decisions of farmers. Therefore, knowledge of how farmers react to risk is important in farm decision analyses. Since the pioneering works of Pratt and Arrow, the Arrow-Pratt measures of risk aversion have been widely used in describing a decision maker's response to risk. The Arrow-Pratt measures of absolute and relative risk aversion rely heavily on the expected utility maximization theorem. This theorem describes the benefits (utility) derived from a stream of uncertain wealth by a utility function and proposes that an individual's objective is to maximize his or her utility. Thus, individuals act so as to derive the maximum benefit from their actions.

In agriculture, uncertainty abounds and farmers are faced with making decisions in an uncertain environment. At best, farmers form expectations about the uncertain events and take decisions based on these expectations. Individual farmers react differently to changes in the expectation about uncertain events and to changes in their asset position or wealth level. In general, risk aversion is widely accepted as a stylized fact in agriculture and aversion to risk is expected to decrease as wealth level increases. This risk attitude is known as decreasing absolute risk aversion (DARA) and is seen as a rational behavioral pattern for risk averse agents. While it has been difficult to verify other risk preference structures using the Arrow-Pratt measures of risk aversion in applied work, the existence of other risk attitudes can not be discounted (Anderson, Dillon and Hardaker) and research findings have supported the existence of other forms of risk preferences (Saha).

Saha observed that the Arrow-Pratt measures impose implicit restrictions on the risk aversion ratios based on the functional form chosen. As an alternative, a nonlinear mean-standard deviation framework of risk aversion measurement was proposed. This new method is capable of describing all possible risk preference structures depending on parameter values. Although this new method is theoretically plausible, its application has been limited to a small sample set. Applications of this method to richer samples could offer an opportunity for validating Saha's findings.

Our aim in this paper is to estimate the risk preference structures for different farm enterprises in Kansas using the nonlinear mean-standard deviation approach. Dryland wheat and irrigated corn enterprises are considered for analysis. The effects of wealth level on farmers' risk attitudes are also evaluated. In the rest of the paper, we have discussed the Arrow-Pratt measures of risk aversion and the implied restrictions on preferences. The nonlinear mean-standard deviation approach is then presented and the results from the empirical models are discussed. The paper concludes with a summary of our findings.

The Arrow-Pratt Measures of Risk Aversion

In the expected utility maximization framework, utility (U) is defined as a function of uncertain wealth (w) and utility increases with wealth but at a decreasing rate. The Arrow-Pratt measures of absolute risk aversion and relative risk aversion describe how aversion to risk change as wealth level changes and how aversion to risk change as the risky prospect and wealth level are changed by the same proportion, respectively (Arrow). If we specify a utility function of the form:

(1) $$U = U(w),$$

where $U'(w) > 0$, $U''(w) < 0$, and the primes denote order of derivatives. The Arrow-Pratt measures of absolute risk aversion ($R_A(w)$) and relative risk aversion ($R_R(w)$) are defined as:

(2) $$R_A = -\frac{U''(w)}{U'(w)},$$

(3) $$R_R = -\frac{U''(w)}{U'(w)} w = R_A w .$$

Decreasing, constant and increasing absolute (relative) risk aversion is represented by $R'_A(w)[R'_R(w)] > 0, = 0, \text{and } < 0$, respectively. Saha showed that the particular form of U dictates what the values of R'_A and R'_R will be. In particular, regardless of its flexibility, when a utility function exhibits constant absolute risk aversion (CARA) or increasing absolute risk aversion (IARA), decreasing relative risk aversion (DRRA) and constant relative risk aversion (CRRA) are both impossible. The complete relationship between preference structures is presented in Table 1. While some risk preferences may not be verified with the implicit restrictions placed on preferences by the Arrow-Pratt measures, they can not be ruled out in practical settings (Wolf and Pohlam; Cohn et al.; Szpiro). This suggests the need for an alternative framework for describing risk attitudes.

The Nonlinear Mean-Standard Deviation Framework

Under the nonlinear mean-standard deviation approach proposed by Saha, utility is defined over the first two moments (mean and standard deviation) of uncertain wealth and risk attitude is described in terms of the negative of the ratio of partial derivatives of the utility function with respect to the standard deviation and mean of wealth, respectively. The risk attitude measure implies risk aversion, risk neutrality, or risk affinity depending on the sign of the ratio. In the special case of risk aversion, the magnitude of the risk attitude measure also reflects the degree of risk aversion. Other absolute and relative properties of risk aversion are defined by the values of the utility function parameters. The use of a particular flexible form of utility function makes it possible to verify all types and degrees of risk attitudes.

Consider an individual decision maker with utility defined by the mean (M) and standard deviation (S) of his or her uncertain wealth as follows:

(4) $$U = U(M, S).$$

The agent's risk attitude (A) is defined by:

(5) $$A(M, S) = -[U_S / U_M],$$

where U_S and U_M represent the partial derivative of the utility function with respect to the standard deviation of wealth and the mean of wealth, respectively. Risk aversion, neutrality, and affinity correspond to $A(M, S) > 0, = 0$, and < 0, respectively. In the case of risk aversion, the magnitude of A represents the degree of risk aversion. The sign of the derivative of the risk attitude measure with respect to the mean of wealth (A_M) denotes the absolute risk aversion measure. DARA, CARA, and IARA are represented by $A_M < 0, = 0$ and > 0, respectively. DRRA, CRRA, and IRRA are also denoted by $A_t(tM, tS) < 0, = 0$ and > 0, where $t > 0$. Using this framework, Saha proposed a particular flexible utility function of the form:

(6) $$U(M, S) = M^q - S^g,$$

where q and g are parameters and $q > 0$. Using this mean-standard deviation utility (MSU) function, the risk attitude measure is defined by

(7) $$A(M, S) = (g / q)M^{1-q} S^{g-1}.$$

Risk aversion, neutrality, and affinity thus correspond to $g > 0, = 0,$ and < 0, respectively. DARA, CARA, and IARA are represented by $q > 1, = 1,$ and < 1, respectively. Also, DRRA, CRRA, and IRRA are represented by $q > g, q = g$, and $q < g$, respectively. The complete risk preference relationships are presented in Table 2 and show that all preference structures are possible under different parameter values.

Empirical Model

In our present study, farmers are assumed to operative in a perfectly competitive market and random wealth $(\tilde{W})$ is a function of random output price $(\tilde{P})$ as follows:

(8) $$\tilde{W} = \tilde{P}Q - C(\mathbf{v}, Q) + w,$$

where $C(\mathbf{v}, Q)$ is the cost function, Q is output quantity, $\mathbf{v}$ is a vector of input prices, and w is off-farm income. If it is assumed that expectations about current price is formed from past realized prices, price moments can be constructed from historical data. The moments of uncertain wealth are thus of the form:

(9) $$M = \overline{P}Q - C(\mathbf{v}, Q) + w\,,$$

(10) $$S = s_{P}Q\,,$$

where $\overline{P}$ and s_{P} represent the mean and standard deviation of the price distribution, respectively. The decision problem then becomes that of maximizing utility by choosing the optimal quantity to produce at each period. The first-order condition of the utility maximization when utility is of the MSU form is given by :

(11) $$\overline{P} - C_{Q}(\mathbf{v}, Q) = (g\,/q)M^{1-q}Q^{g-1}(s_{P})^{g}\,,$$

where $C_{Q}(\mathbf{v}, Q)$ is the marginal cost. The expression in (11) can also be expressed in implicit form as follows:

(12) $$[\overline{P} - C_{Q}(\mathbf{v}, Q)](q\,/g)M^{q-1}Q^{1-g}(s_{P})^{-g} = 1\,,$$

and once the price moments are estimated, (12) can also be estimated. In estimating the price moments, the annual average price for each enterprise (in Kansas) was assumed to follow an autoregressive (AR) process with an autoregressive conditional heteroskedastic (ARCH) time-varying variance (Engle). The exponential ARCH (EARCH) was used because it has the desirable property of making s_{P} positive (Nelson). The mean price is forecasted based on the predicted value of the last significant AR order in equation (13) below. Once the AR order is known, the variance of output price is then calculated using equation (14):

(13) $$P_{t} = a_{0} + \sum_{l=1}^{L} a_{l}P_{t-l} + e_{t}$$

(14) $$s^{2}{}_{P} = \exp(d_{0} + d_{1}e_{t-1}^{2})\,,$$

where L represents the last significant lag in the AR process. By substituting for e_{t-1} from (13) in (14), s_P^2 can be expressed in terms of parameters and lagged prices only as follows:

$$s_P^2 = \exp[d_0 + d_1(P_{t-1} - a_0 - \sum_{l=2}^{L+1} a_l P_{t-l})^2] \tag{15}$$

Data and Estimation

Firm-level data from the Farm Management Data bank maintained by the Department of Agricultural Economics, Kansas State University are employed is this study. Observations in the data set covered the years 1993 to 1997 for each enterprise. Variables of interest included farm expenditures, crop acreage, output, farm revenue, and other incomes. The state-wide market year average output price for each enterprise from 1950 to 1997 as well as the index of price paid on farm inputs were also collected from the records of the Kansas State Department of Agriculture and the United States Department of Agriculture. Farms with incomplete records for the five years were removed from our observations. Those farms that recorded no output for any particular year or that gave a negative mean wealth were also deleted from our records. This gave us a total of 1125 observations (225 farms) for the dryland wheat enterprise and 200 observations (40 farms) for the irrigated corn enterprise. These represent substantially large data sets compared to 60 observations used by Saha.

Estimation of the parameters in equation (15) was done by formulating the log likelihood function for each enterprise. Maximization of this function provides the maximum likelihood (ML) estimates of the parameters. The log likelihood function was solved using MINOS5.3 solver in GAMS. The mean and standard deviation of price distribution for each enterprise were then calculated using (13) and (15). In order to determine the technology structure in each enterprise, quadratic and Cobb-Douglas cost functions were specified and estimated. The log likelihood functions, after the Jacobian correction has been performed, were compared for each enterprise and the Cobb-Douglas was observed to dominate the quadratic form for both enterprises. Hence, a Cobb-Douglas cost function was specified and the marginal cost corresponding to this functional form derived. The cost function and the first-order condition (12) with an added term for errors in optimization were then jointly estimated as a system. The estimated system is given as follows:

(16) $$\ln C_{it} = b_0 + b_1 \ln v_{1it} + b_2 v_{2it} + b_3 v_{3t} + b_4 Q_{it} + e_{1it}$$

(17) $$[\overline{P}_t - (\frac{b_4 C_{it}}{Q_{it}})](q/g) M_{it}^{q-1} Q_{it}^{1-g} (s_{Pt})^{-g} + e_{2it} = 1,$$

where v_1, v_2, and v_3 represent the price for material inputs, price for capital inputs and wage rate, respectively. The indexes i and t represent individual farmer and time period, respectively, and the e_{it}'s are error terms. In our study, material inputs include seed, fertilizer, agro-chemical, repair, machinery hire and other services. Capital inputs include rent, interest charge, tax, insurance and depreciation. Composite input prices were calculated using expenditure shares and the USDA index of prices paid for farm inputs. The marginal cost structure from the Cobb-Douglas technology has been substituted for $C_Q(\mathbf{v}, Q)$ in the estimable system. Joint estimation of equations (16) and

(17) for each enterprise was done by nonlinear three-stage least squares (3SLS) and tests were performed on the utility parameters to determine the risk structure for each enterprise. All system estimation were done using SHAZAM.

Results and Discussion

The output price autoregressive process indicated that wheat price followed an AR(3) process while corn price followed an AR(1) process. The ML estimates of the parameters of price distribution for wheat and corn are presented in Table 3. Using these estimates, the price moments (mean and standard deviation) for 1993-1997 were generated for the two enterprises. These values are reported in Table 4. The results of the joint estimation of the cost function and the first-order condition for dryland wheat and irrigated corn enterprises are presented in Table 5. The parameters of the MSU function are significant at 1% level for both enterprises. The cost function parameters are also significant at the 1% level, except for the parameter of price of capital inputs that was insignificant for dryland wheat and the prices of material and capital inputs that were insignificant for irrigated corn. The estimate of q and g are positive for both enterprises and $g > q$ for both enterprises. In addition $q > 1$ for irrigated corn.

The findings from our results indicate that both dryland wheat and irrigated corn enterprises in Kansas are characterized by risk aversion. The degree of risk aversion is greater for dryland wheat than for irrigated corn, as depicted by the risk aversion measure. Our results also support preferences described by IARA and IRRA for dryland wheat and DARA and IRRA for irrigated corn. The null hypotheses for linear mean-standard deviation model, CARA and CRRA were all rejected at 1% level for both enterprises. The risk preference structure for irrigated corn is similar to those found by Saha. However, the structure for dryland wheat is intuitively unappealing. While IRRA has been found in previous studies (Wolf and Pohlman; Saha), DARA not IARA is normally assumed. Further investigation of our sampled farmers indicated that the mean wealth level, as well as the minimum and maximum levels, are much lower for dryland wheat than for irrigated corn. Thus, wheat farmers may be expected to be highly averse to risk due to low income and any marginal increase in wealth may push farmers farther away from the risky prospects.

Conclusions

In this study, we have applied the nonlinear mean-standard deviation framework to analyze risk attitudes for dryland wheat and irrigated corn enterprises in Kansas. This approach offers an alternative to the Arrow-Pratt measures of risk aversion and does not impose any restrictions on the risk preference structures a priori. Our results indicated that dryland wheat enterprises are characterized by IARA and IRRA while preferences in irrigated corn enterprises are of the form of DARA and IRRA. The level of wealth is also observed to have significant influence on risk structure. For both enterprises, preferences are adequately represented by a non-linear utility function in mean, standard deviation of wealth space.

Table 1. Alternative Risk Preference Configurations Using Arrow-Pratt Measures

	DRRA	CRRA	IRRA
DARA	Feasible	Feasible	Feasible
CARA	Not feasible	Not feasible	Feasible
IARA	Not feasible	Not feasible	Feasible

Source: Saha.

Table 2. Alternative Risk Preference Configurations Using MSU Utility

	DRRA	CRRA	IRRA
DARA	$q > 1, q > g$	$q > 1, q = g$	$q > 1, q < g$
CARA	$q = 1, q > g$	$q = 1, q = g$	$q = 1, q < g$
IARA	$q < 1, q > g$	$q < 1, q = g$	$q < 1, q < g$

Source: Saha.

Table 3. Maximum Likelihood Estimates of Price Distribution Parameters

Parameter	Dryland Wheat	Irrigated Corn
a_0	0.273	0.200
a_1	1.053	0.905
a_2	-0.542	
a_3	0.416	
d_0	-0.877	-2.08
d_1	-3.583	0.721

Table 4. Estimated Price moments for Dryland Wheat and Irrigated Corn

Year	Dryland Wheat		Irrigated Corn	
	$\overline{P}$	s_p	$\overline{P}$	s_p
1993	3.090	0.551	2.146	0.361
1994	2.904	0.636	2.562	0.382
1995	3.445	0.473	2.300	0.361
1996	4.555	0.062	3.132	0.486
1997	4.042	0.639	2.762	0.365

Note: Prices are in $/bu.

Table 5. Parameter Estimates and Results of Hypotheses Tests

Parameter	Description	Dryland Wheat	Irrigated Corn
b_0	Cost Function Paramerters[a]	7.8922* (6.51420)	-9.1848* (-10.316)
b_1		3.8941* (8.7104)	0.2204 (0.6283)
b_2		0.7388 (1.7021)	0.4127 (1.3711)
b_3		-5.5165* (-8.7575)	1.6036* (3.5918)
b_4		0.6805* (33.916)	0.9102* (63.551)
q	Uitility Function Parameters[a]	0.39429* (28.954)	1.3007* (21.288)
g		0.73617* (18.847)	1.4566* (20.669)
$H_0: q = g = 1$	Linear MSU model[b]	0.000	0.000
$H_0: q = 1$	CARA[b]	0.000	0.000
$H_0: q = g$	CRRA[b]	0.000	0.000
A(M,S)	Risk aversion measure[c]	22.757 ↔ 6839.7	0.015 ↔ 0.088

[a] T-ratios are in parentheses. [b]Asymptotic P-values. [c] Range of values. *Significant at 1%.

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