

## **Endogenous Quality and Agricultural Policy Analysis**

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The typical analysis of agricultural policy assumes that the commodity of interest is homogeneous, and that the nature of the commodity does not change as a result of the implementation of the policy. In many cases, these may be appropriate assumptions. In a number of instances, though, commodities are quite heterogeneous. An assumption that all units of a commodity are of identical quality is quite restrictive. While most economists recognize that commodities are not perfectly homogeneous, the assumption is rarely mentioned or justified in studies of commodity policy. In some cases, the homogeneity assumption may be based on the belief that quality effects are not important, so that modeling the market for a commodity as if it were homogeneous closely approximates reality. In other cases it may be believed that quantities and prices are properly aggregated to reflect the distribution of quality, or some “representative” quality, and that this is sufficient to accommodate quality issues.

Whether the assumption is one of homogeneity or one of perfect aggregation methods, representing a commodity as if it were homogeneous fails to account for changes in quality (or average quality) that may occur as a result of the implementation of policy. Alchian and Allen (1964) and Barzel (1976) make persuasive arguments for the existence of such quality changes, and many instances may be found where quality varies and responds to policy. Nevertheless, quality responses have yet to be incorporated formally into the analysis of agricultural commodity policy. Rather, economists have presumed implicitly that explicitly modeling quality responses to policy is unnecessary. However, it is impossible to determine how closely a homogeneous goods model approximates the actual policy impacts until a more complete model has been developed and implemented.

This paper will account for quality responses in a very simplified framework, where a commodity is available in two qualities: high and low. The focus here will be on taxes, though the results may be easily re-interpreted as applying in the context of production quotas and subsidies. Subsidies, of course, are the equivalent of negative taxes, so that the effects of per unit and ad valorem subsidies will simply be the negatives of the effects of the corresponding taxes. The effects of a per unit tax are directly analogous to those of a freely transferable quota, though in this context, the quota rent per unit is exogenous, and the quota quantity is endogenously determined. The quota rent per unit, then, is just the difference between the consumer price and the marginal cost of production at the equilibrium quantity, as is the case with a per unit tax.

## Quality Responses

The Alchian and Allen theorem was introduced in 1964 as an heuristic example of the law of demand, but has since been coined as the third law of demand (Bertonazzi, Maloney, and McCormick 1993). The theorem postulates the effects of transportation costs on the relative consumption of high-quality and low-quality goods. The original example given by Alchian and Allen (1964) concerned “good” and “bad” grapes grown in California. They noted that the cost of transporting grapes to, say, New York is the same for all shipments of grapes, regardless of their quality. From an individual consumer’s perspective, prices are fixed so that the price of each quality of grapes increases by the same amount for consumers in New York. Thus, good grapes become *relatively* cheaper for a consumer in New York, and hence, a New Yorker will consume a larger proportion of good grapes relative to a person in California with identical preferences.

While the Alchian and Allen theorem is intuitively appealing, it is theoretically provable only under very restrictive conditions. Gould and Segall (1968) showed that when a fixed cost per unit is added, the individual consumer unequivocally increases relative consumption of the high-quality good only in a two-good world with no income effects. Introduction of either an income effect or a third good renders the change ambiguous. Borcharding and Silberberg (1978) argue that while it is possible for the Alchian-Allen theorem to be negated with the introduction of a third good, unless the high- and low-quality products have very different consumption relationships with the third good, the standard Alchian-Allen result will hold.

The typical per unit cost introduced to produce the Alchian-Allen effect is a transportation cost. The hypothesized increase in the consumption share of high-quality goods could occur as a result of many other types of per unit costs. Umbeck (1980) discusses the nature of these costs, and points out that many examples proposed by Borcharding and Silberberg (1978) to demonstrate the Alchian-Allen effect fail to do so because of the nature of the per unit costs considered. The primary criteria for a cost to generate the Alchian-Allen result are that the cost does not change the good itself, and that it does not have any inherent economic value in and of itself—i.e., it acts just like a per unit tax.

Because the analysis is at the individual firm level, though, prices are exogenous. When either transportation costs or per-unit taxes are introduced at the market level, these costs are shared by consumers and producers, and theory has little to say about the relative changes in production and consumption of low- and high-quality commodities. While the Alchian-Allen effect has not been proven theoretically at the market level, it is a convincing empirical regularity (Bertonazzi, Maloney, and McCormick 1993). This suggests that the Alchian-Allen effect may be more compelling as a conjecture about market-level relationships than it is regarding individual behavior.

Barzel (1976) addresses a similar phenomenon at the market level in his alternate approach to taxation. Barzel notes that every commodity is more or less a bundle of characteristics. Because an ad valorem tax applies to the commodity's entire value, it essentially taxes all of its characteristics. In contrast, if a per unit tax is imposed, the tax statute will use a subset of characteristics to define the commodity, assuming that an exhaustive description is either impossible or very costly. As a result, the per unit tax is actually taxing the defining characteristics. In maximizing their profits subject to the tax constraint, producers may alter the characteristics included in their units of production in order to minimize their costs of the per unit tax. Barzel (1976) showed that a predictable outcome is that the quantity of the defining characteristics (specified in the tax statute) will decrease, and the additional characteristics, which are not subject to the tax because they are not specified in the statute, will increase on a per unit basis.

### **A Two-Market Model**

The previous section presented several arguments for quality responses to taxes. This section develops a simple model in an attempt to quantify the extent of these quality responses. The effects of taxes are modeled by specifying an equilibrium displacement model, as used by Muth (1964), Buse (1958), Perrin (1980), Alston (1985), and others (see Piggott (1992) for a review). A two-market model is specified for the most simple case of high- and low-quality products related in consumption and production.

A supply and demand function is specified for each market. Because the two qualities are related in consumption, the quantity demanded of each quality will depend on its own price, and the price

of the other quality, as well as other shift variables. Similarly, the quantity supplied of each quality will depend on its own price and the price of the other quality and supply-shift variables. The equilibrium displacement model does not require specification of functional form, so that these supply and demand relationships can be written in general form as:

$$\begin{aligned}
 (1) \quad Q_L^D &= Q_L^D(P_L, P_H, a_L) \\
 (2) \quad Q_L^S &= Q_L^S(P_L, P_H, b_L) \\
 (3) \quad Q_H^D &= Q_H^D(P_L, P_H, a_H) \\
 (4) \quad Q_H^S &= Q_H^S(P_L, P_H, b_H)
 \end{aligned}$$

where Q and P denote quantities and prices, subscripts L and H denote the low- and high-quality markets, respectively, and superscripts D and S denote quantities demanded and supplied, respectively. The last argument of each function summarizes the effects of all relevant shift variables, which are assumed to be exogenous.

Totally differentiating equations (1) through (4) and converting to matrix notation yields:

$$\begin{bmatrix} 1 & 0 & -\zeta_{LL} & -\zeta_{LH} \\ 0 & 1 & -\zeta_{HL} & -\zeta_{HH} \\ 1 & 0 & -\hat{a}_{LL} & -\hat{a}_{LH} \\ 0 & 1 & -\hat{a}_{HL} & -\hat{a}_{HH} \end{bmatrix} \begin{bmatrix} EQ_L \\ EQ_H \\ EP_L \\ EP_H \end{bmatrix} = \begin{bmatrix} \hat{a}_L \\ \hat{a}_H \\ \hat{a}_L \\ \hat{a}_H \end{bmatrix}$$

where E denotes a proportional change in the price or quantity term, for instance,  $EQ_L = d \ln Q_L = dQ_L/Q_L$  is the proportional change of the quantity sold in the low-quality market. Coefficients on the  $EP_i$  terms are elasticities:  $\zeta_{ij}$  is the elasticity of demand for quality i with respect to the price of quality j, and  $\hat{a}_{ij}$  is the elasticity of supply for quality i with respect to the price of quality j. The  $\hat{a}_i$  terms represent the proportional changes in the quantity of quality i from an exogenous demand shift in that market, and the  $\hat{a}_i$  terms represent the proportional change in the quantity of quality i from an exogenous supply shift in the i-quality market. The four equations implicitly define the four endogenous variables, the proportional changes in prices and quantities in each of the two markets, as functions of the four exogenous shift variables. These shift variables can be used to represent the effects of alternative types of taxes, or other government policies, on the market equilibrium.

Inverting the coefficient matrix and post-multiplying by the vector of exogenous shifters yields expressions of the endogenous variables as functions of the exogenous shifters:

$$\begin{bmatrix} EQ_L \\ EQ_H \\ EP_L \\ EP_H \end{bmatrix} = \frac{1}{D} \begin{bmatrix} \hat{a}_{LH}(\zeta_{HL} - \hat{a}_{HL}) + \hat{a}_{LL}(\hat{a}_{HH} - \zeta_{HH}) & \zeta_{LH}\hat{a}_{LL} - \zeta_{LL}\hat{a}_{LH} & \zeta_{LH}(\hat{a}_{HL} - \zeta_{HL}) + \zeta_{LL}(\zeta_{HH} - \hat{a}_{HH}) & \zeta_{LL}\hat{a}_{LH} - \zeta_{LH}\hat{a}_{LL} \\ \zeta_{HL}\hat{a}_{HH} - \zeta_{HH}\hat{a}_{HL} & \hat{a}_{HL}(\zeta_{LH} - \hat{a}_{LH}) + \hat{a}_{HH}(\hat{a}_{LL} - \zeta_{LL}) & \zeta_{HH}\hat{a}_{HL} - \zeta_{HL}\hat{a}_{HH} & \zeta_{HH}(\zeta_{LL} - \hat{a}_{LL}) + \zeta_{HL}(\hat{a}_{LH} - \zeta_{LH}) \\ \hat{a}_{HH} - \zeta_{HH} & \zeta_{LH} - \hat{a}_{LH} & \zeta_{HH} - \hat{a}_{HH} & \hat{a}_{LH} - \zeta_{LH} \\ \zeta_{HL} - \hat{a}_{HL} & \hat{a}_{LL} - \zeta_{LL} & \hat{a}_{HL} - \zeta_{HL} & \zeta_{LL} - \hat{a}_{LL} \end{bmatrix} \begin{bmatrix} \hat{a}_L \\ \hat{a}_H \\ \hat{a}_L \\ \hat{a}_H \end{bmatrix}$$

$$\text{where } D = (\zeta_{LL} - \hat{a}_{LL})(\zeta_{HH} - \hat{a}_{HH}) + (\zeta_{LH} - \hat{a}_{LH})(\hat{a}_{HL} - \zeta_{HL})$$

In order to determine the direction of change for each endogenous variable, the signs, and in some cases, the magnitudes, of the supply and demand elasticities must be determined. Determining the sign of the own-price elasticities is straightforward. The cross-price elasticities may be estimated empirically, but a strictly theoretical approach would leave their signs unknown (since they are Marshallian, and include income effects). More importantly, the link between the results from this two-market specification and a single-market representation that assumes a homogeneous good is unclear. The next section develops a means for simplifying the terms in the above matrix and for linking the two-market results to those from a single-market representation.

### An Armington Approach

Many of the problems with analyzing the results from the model presented above can be alleviated by interpreting the results in the context of an Armington model. This model was originally designed to represent the demand for internationally traded commodities. Armington (1969) noted that a single commodity may be produced in many different countries or geographic regions, but the nature of the commodity would vary, depending on the country of origin. By imposing some restrictions on the relationships among commodities of the same type but different origins, Armington (1969) reduced the number of demand parameters to be estimated.

First, the marginal rate of substitution between any two commodities of the same type and different origin is independent of the consumption of other commodity types. This restriction implies that commodities of the same type comprise a weakly separable group of products, and allows for the budgeting process to be represented in two stages. In the first, total utility is expressed as a function of aggregated quantities of commodity types, and the budget is allocated among the commodity types. In the second stage, the expenditures for each particular commodity type are allocated among the quantities of that commodity from different origins. The function used to aggregate quantities can also

be thought of as a sub-utility function that is maximized in the second stage of the budgeting process, subject to the expenditure allocation from the first stage. The second important restriction of the Armington model is that the aggregation (or sub-utility) functions are homogeneous of degree one. Armington specified them as constant elasticity of substitution (CES) utility functions.

The relationship of the Armington model to the problem at hand is relatively straightforward. Rather than differing by country of origin, the commodities considered here are of the same general type, but vary in quality. If the high- and low-quality varieties of the commodity under consideration constitute a separable group of goods, then in the first stage of the budgeting process, the consumer maximizes utility derived from consumption of the aggregate commodity and all other goods, subject to the budget constraint. This determines the quantity aggregate,  $Q$  (the absence of a subscript denotes aggregated quantity or price). In the second stage, total expenditure on the commodity is allocated between the high- and low-quality varieties. Thus, demand functions for each quality can be expressed as functions of the aggregate quantity, the aggregate price, and the prices of the low- and high-quality goods.

Imposing the CES sub-utility function, the elasticities of demand for the individual qualities with respect to individual prices can be expressed as:

$$(5) \quad \zeta_{LL} = -s_H \acute{o} + s_L \zeta$$

$$(6) \quad \zeta_{LH} = s_H (\acute{o} + \zeta)$$

$$(7) \quad \zeta_{HH} = -s_L \acute{o} + s_H \zeta$$

$$(8) \quad \zeta_{HL} = s_L (\acute{o} + \zeta)$$

where  $s_i = \frac{P_i Q_i}{PQ}$  is the value-share of quality  $i$ . The decomposition of the elasticities of demand with respect to prices is very similar to the standard Slutsky decomposition. The substitution effect is given by the  $\acute{o}$  term, where  $\acute{o}$  is the elasticity of substitution between the two goods. The scale effect is given by the  $\zeta$  term, where  $\zeta$  is the overall elasticity of demand, defined as the elasticity of the aggregate quantity with respect to the aggregate price. Thus, for example, when the price of the low-quality good increases, the quantity demanded of the high-quality good increases through the substitution effect, since  $\acute{o} > 0$ , and decreases through the scale effect, since the increase in  $P_L$  increases the aggregate price,  $P$ , and thus decreases the aggregate quantity,  $Q$ .

A similar representation of the individual firm's profit maximization problem can be specified in order to derive expressions for the supply elasticities:

$$(9) \quad \hat{a}_{LL} = -s_H \tau + s_L \hat{a}$$

$$(10) \quad \hat{a}_{LH} = s_H (\tau + \hat{a})$$

$$(11) \quad \hat{a}_{HH} = -s_L \tau + s_H \hat{a}$$

$$(12) \quad \hat{a}_{HL} = s_L (\tau + \hat{a})$$

These elasticities can be decomposed similarly to the elasticities of demand. The first term gives the substitution effect, where  $\tau$  is the elasticity of transformation in the production process, and is negative. The second term gives the scale effect, where  $\hat{a}$  is the overall supply elasticity, or the elasticity of aggregate quantity with respect to aggregate price. When the price of the low-quality product increases, the quantity of high-quality product will decrease through the substitution effect, and increase through the scale effect.

The differentiated goods model laid out above provides a means of relaxing the assumption of product homogeneity while managing the number of parameters to be estimated. In addition, the explicit modeling of the two-stage budgeting process clarifies the consequences of aggregating commodities. As described above, a change in the price of one quality will change the aggregate price of the commodity group. In the first stage of the budgeting process, the change in the aggregate price will alter the consumer expenditure allocated to the group of commodities and the producer revenue from that commodity group. These first-stage effects are the scale effects represented by the overall demand and supply elasticities. In the second stage of the budgeting process, the individual price change alters the relative prices of the commodities within the group. As a result, consumers will change the mix of commodities consumed, and producers will alter the mix of commodities produced (providing they are not perfect complements). These second-stage effects are the substitution effects represented by the  $\hat{a}$  and  $\tau$  terms discussed above. An aggregate analysis of a group of commodities would examine only the first stage of the budgeting process, ignoring the substitution effects from the second stage.

Substituting the expressions for the elasticities of demand and supply into the solution of the two-market equilibrium displacement model yields the following solution:



$$\begin{bmatrix} EQ_L \\ EQ_H \\ EP_L \\ EP_H \end{bmatrix} = \frac{1}{D} \begin{bmatrix} s_H \tau (\zeta - \hat{a}) + s_L \hat{a} (\acute{o} - \tau) & s_H (\hat{a} \acute{o} - \zeta \tau) & s_H \acute{o} (\hat{a} - \zeta) + s_L \zeta (\tau - \acute{o}) & s_H (\zeta \tau - \hat{a} \acute{o}) \\ s_L (\hat{a} \acute{o} - \zeta \tau) & s_L \tau (\zeta - \hat{a}) + s_H \hat{a} (\acute{o} - \tau) & s_L (\zeta \tau - \hat{a} \acute{o}) & s_L \acute{o} (\hat{a} - \zeta) + s_H \zeta (\tau - \acute{o}) \\ s_L (\acute{o} - \tau) + s_H (\hat{a} - \zeta) & s_H (\acute{o} + \zeta - \tau - \hat{a}) & s_L (\tau - \acute{o}) + s_H (\zeta - \hat{a}) & s_H (\tau + \hat{a} - \acute{o} - \zeta) \\ s_L (\acute{o} + \zeta - \tau - \hat{a}) & s_H (\acute{o} - \tau) + s_L (\hat{a} - \zeta) & s_L (\tau + \hat{a} - \acute{o} - \zeta) & s_H (\tau - \acute{o}) + s_L (\zeta - \hat{a}) \end{bmatrix} \begin{bmatrix} \acute{a}_L \\ \acute{a}_H \\ \hat{a}_L \\ \hat{a}_H \end{bmatrix}$$

where  $D = (\zeta - \hat{a})(\tau - \acute{o}) > 0$ .

This substitution achieves three goals. First, it reduces the number of parameters appearing in the solution matrix. The eight elasticities of supply and demand are replaced with a value share ( $s_H$ , noting that  $s_H = 1 - s_L$ ), the elasticity of substitution in consumption ( $\acute{o}$ ), the overall elasticity of demand ( $\zeta$ ), the elasticity of transformation in production ( $\tau$ ), and the overall elasticity of supply ( $\hat{a}$ ). Second, all of the parameters are of known sign. Third, evaluating the expressions as either  $\acute{o}$  or  $\tau$  approach zero yields the effects on prices and quantities that would result from a single-market approach.

### Price and Quantity Effects of a Supply Shift

Using the solution to the equilibrium displacement model specified above, the changes in the price and quantity of each quality in response to a policy can be found by appropriately specifying values for the exogenous shift variables. A tax can be represented as an upward shift in supply, where the resulting equilibrium price would be the consumer price, or as a downward shift in demand, where the resulting equilibrium price would be the producer price (net of the tax). Here, a tax is modeled as a supply shift, and  $\acute{a}_L$  and  $\acute{a}_H$  are set to zero. In general, for supply shifts in both the low- and high-quality markets, the proportional changes in the quantities and prices are:

$$(13) \quad EQ_L = \frac{\zeta \hat{a}_L}{(\zeta - \hat{a})} + \frac{s_H (\hat{a} \acute{o} - \zeta \tau) (\hat{a}_L - \hat{a}_H)}{(\zeta - \hat{a})(\tau - \acute{o})}$$

$$(14) \quad EQ_H = \frac{\zeta \hat{a}_H}{(\zeta - \hat{a})} - \frac{s_L (\hat{a} \acute{o} - \zeta \tau) (\hat{a}_L - \hat{a}_H)}{(\zeta - \hat{a})(\tau - \acute{o})}$$

$$(15) \quad EP_L = \frac{\hat{a}_L}{(\zeta - \hat{a})} + \frac{s_H (\zeta + \acute{o} - \hat{a} - \tau) (\hat{a}_L - \hat{a}_H)}{(\zeta - \hat{a})(\tau - \acute{o})}$$

$$(16) \quad EP_H = \frac{\hat{a}_H}{(\zeta - \hat{a})} - \frac{s_L (\zeta + \acute{o} - \hat{a} - \tau) (\hat{a}_L - \hat{a}_H)}{(\zeta - \hat{a})(\tau - \acute{o})}$$

In examining each of the effects, it is useful to note that in a single-market model, the proportional changes in quantity and price from a supply shift are defined as:

$$(17) \quad E\tilde{Q} = \frac{\zeta \hat{a}}{(\zeta - \hat{a})}$$

$$(18) \quad E\tilde{P} = \frac{\hat{a}}{(\zeta - \hat{a})}$$

where tildas ( $\sim$ ) denote that the result is derived from a single-market representation, and  $\hat{a}$  defines the proportional shift of supply in a single-market representation.

Given equations (17) and (18), it is clear that the first term in each of equations (13) through (16) is analogous to the single-market effect, and represents the effect of a supply shift if the same proportional supply shift were applied in each market. So, if  $\hat{a}_L = \hat{a}_H$ , then the second terms in each of equations (13) through (16) would be zero, and the proportional changes in the quantities and prices of both qualities would be the same as in the single-market representation, where  $\hat{a} = \hat{a}_L = \hat{a}_H$ . However, if the two supply shifts differ, then the second terms will not be equal to zero, and will adjust each proportional change for the differential shifts between the two markets. The direction of the adjustment to each proportional change for the differential supply shifts depends on the relative sizes of the two supply shifts ( $\hat{a}_L - \hat{a}_H$ ), and the supply and demand parameters.

Implicit in the single-market model is the assumption of constant quality. More specifically, a single-market model would predict the same proportional change in the quantity of each quality, i.e.,  $E\tilde{Q} = EQ_L = EQ_H$ . However, if the proportion of units of high-quality product relative to the quantity of low-quality product is used as a measure of average quality sold, then the difference between the proportional quantity changes,  $EQ_H - EQ_L$ , measures the change in the average quality resulting from the tax policy. So, if  $EQ_H - EQ_L > 0$ , then the quantity sold in the low-quality market would be reduced by a larger proportion than in the high-quality market, and the average quality would decrease as a result of the tax. If the inequality were reversed, then average quality would increase. Similarly, the price premium for high-quality could be expressed as the price of high-quality product relative to the price for low quality. Thus, the proportional change in the price premium will equal the difference between the proportional price changes,  $EP_H - EP_L$ . If this difference is positive (negative), then the price premium for high quality would increase (decrease) as a result of the tax.

The differences between the proportional quantity and price changes are:

$$(23) \quad EQ_H - EQ_L = -\left[\frac{\acute{o}}{\acute{o} - \tau}\right](\hat{a}_L - \hat{a}_H)$$

$$(24) \quad EP_H - EP_L = \left[\frac{1}{\acute{o} - \tau}\right](\hat{a}_L - \hat{a}_H)$$

The changes in average quality and the quality premium hinge on the relationship between the sizes of the two supply shifts. In the case where the proportional supply shift in the low-quality market is larger than that in the high-quality market (i.e.,  $\hat{a}_L > \hat{a}_H$ ), average quality decreases, and the quality premium increases relative to the no intervention case. The directions of the quality changes are reversed when the shift in the high-quality market is larger than that in the low-quality market. These changes in the distribution of quality and the quality premium are discussed in more detail for each of the tax policies.

#### *Per Unit Taxes*

In the case of a tax of  $T$  per unit, the supply functions in each market shift up by  $T$ . The tax is represented in proportional terms by dividing by  $P_L$  for the shift in the low-quality market, and by  $P_H$  in the high-quality market. Because the supply shift variables are proportional changes in the quantity direction, the proportional tax is translated into a proportional quantity shift by multiplying by the negative of the relevant own-price elasticity. Thus, the  $\hat{a}_i$  variables are

$$\hat{a}_L = \frac{-T\acute{a}_{LL}}{P_L} = \frac{-T(-s_H\tau + s_L\acute{a})}{P_L} \quad \text{and} \quad \hat{a}_H = \frac{-T\acute{a}_{HH}}{P_H} = \frac{-T(-s_L\tau + s_H\acute{a})}{P_H},$$

and the proportional changes in the quantity and price in each market are:

$$(19) \quad EQ_L = T \left[ \frac{\zeta\acute{a}_{LL}}{P_L(\acute{a} - \zeta)} + \frac{s_H(\zeta\tau - \acute{a}\acute{o})(P_H\acute{a}_{LL} - P_L\acute{a}_{HH})}{P_L P_H(\zeta - \acute{a})(\tau - \acute{o})} \right]$$

$$(20) \quad EQ_H = T \left[ \frac{\zeta\acute{a}_{HH}}{P_H(\acute{a} - \zeta)} - \frac{s_L(\zeta\tau - \acute{a}\acute{o})(P_H\acute{a}_{LL} - P_L\acute{a}_{HH})}{P_L P_H(\zeta - \acute{a})(\tau - \acute{o})} \right]$$

$$(21) \quad EP_L = T \left[ \frac{\acute{a}_{LL}}{P_L(\acute{a} - \zeta)} - \frac{s_H(\zeta + \acute{o} - \acute{a} - \tau)(P_H\acute{a}_{LL} - P_L\acute{a}_{HH})}{P_L P_H(\zeta - \acute{a})(\tau - \acute{o})} \right]$$

$$(22) \quad EP_H = T \left[ \frac{\acute{a}_{HH}}{P_H(\acute{a} - \zeta)} + \frac{s_L(\zeta + \acute{o} - \acute{a} - \tau)(P_H\acute{a}_{LL} - P_L\acute{a}_{HH})}{P_L P_H(\zeta - \acute{a})(\tau - \acute{o})} \right]$$

If the two qualities were actually a single homogeneous product, then their prices and elasticities of supply would be the same, the second terms in each of equations (19) through (22) would vanish, and the proportional quantity and price effects would be equal in the two markets (i.e.,  $EQ_L = EQ_H$  and

$EP_L = EP_H$ ), as they would be in a single-market representation. However, when the two qualities are distinct, their prices and supply elasticities will differ. As a result, the values of the first terms in equations (19) and (20) will differ, and the second terms in each equation will adjust the proportional quantity changes for the difference between the two supply shifts (and similarly for the proportional price changes in equations (21) and (22)). For example, the first term in equation (19) represents the proportional change in the quantity of low-quality product that would result from supply shifts of  $\hat{a}_L = \frac{-T\hat{a}_{LL}}{P_L}$  in both the low-quality and high-quality markets. The second term, then, adjusts for the different supply shift in the high-quality market,  $\hat{a}_L = \frac{-T\hat{a}_{HH}}{P_H}$ . The other equations can be interpreted similarly.

What happens to the average quality sold in the two markets when a per unit tax is imposed?

The differences between the proportional quantity and price changes are:

$$(23) \quad EQ_H - EQ_L = T \left[ \frac{\acute{o}(P_H\hat{a}_{LL} - P_L\hat{a}_{HH})}{P_L P_H (\acute{o} - \tau)} \right] = \left[ \frac{-\acute{o}}{\acute{o} - \tau} \right] \left[ \frac{T}{P_L P_H} \right] [\hat{a}(s_L P_H - s_H P_L) + \tau(s_L P_L - s_H P_H)]$$

$$(24) \quad EP_H - EP_L = T \left[ \frac{-(P_H\hat{a}_{LL} - P_L\hat{a}_{HH})}{P_L P_H (\acute{o} - \tau)} \right] = \left[ \frac{1}{\acute{o} - \tau} \right] \left[ \frac{T}{P_L P_H} \right] [\hat{a}(s_L P_H - s_H P_L) + \tau(s_L P_L - s_H P_H)]$$

In the case where  $\frac{P_H}{P_L} > \frac{\hat{a}_{HH}}{\hat{a}_{LL}}$  (i.e.,  $\hat{a}_L > \hat{a}_H$ ) average quality increases, and the quality premium decreases, relative to the no intervention case.

### *Ad Valorem Taxes*

An ad valorem tax is expressed as a percentage of the value of the product. Here, if an ad valorem tax of 100t percent is imposed, then that will define the magnitude of the supply shift in the price direction. As with the per unit tax, the proportional supply shift in the price direction is converted into a proportional shift in the quantity direction by multiplying by the negative of the own-price elasticity of supply. Thus, for an ad valorem tax, the supply shifters are

$$\hat{a}_L = -t * \acute{a}_{LL} = -t(-s_H \tau + s_L \acute{a}) \quad \text{and} \quad \hat{a}_H = -t * \acute{a}_{HH} = -t(-s_L \tau + s_H \acute{a}),$$

and the proportional changes in quantities and prices are:

$$(25) \quad EQ_L = t \left[ \frac{\zeta \hat{a}_{LL}}{(\hat{a} - \zeta)} + \frac{s_H (\zeta \tau - \hat{a} \acute{o}) (\hat{a}_{LL} - \hat{a}_{HH})}{(\zeta - \hat{a})(\tau - \acute{o})} \right]$$

$$(26) \quad EQ_H = t \left[ \frac{\zeta \hat{a}_{HH}}{(\hat{a} - \zeta)} - \frac{s_L (\zeta \tau - \hat{a} \acute{o}) (\hat{a}_{LL} - \hat{a}_{HH})}{(\zeta - \hat{a})(\tau - \acute{o})} \right]$$

$$(27) \quad EP_L = t \left[ \frac{\hat{a}_{LL}}{(\hat{a} - \zeta)} - \frac{s_H (\zeta + \acute{o} - \hat{a} - \tau) (\hat{a}_{LL} - \hat{a}_{HH})}{(\zeta - \hat{a})(\tau - \acute{o})} \right]$$

$$(28) \quad EP_H = t \left[ \frac{\hat{a}_{HH}}{(\hat{a} - \zeta)} + \frac{s_L (\zeta + \acute{o} - \hat{a} - \tau) (\hat{a}_{LL} - \hat{a}_{HH})}{(\zeta - \hat{a})(\tau - \acute{o})} \right]$$

In the case of an ad valorem tax, the relationship between the two supply shifts is determined entirely by the own-price elasticities of supply. If the supply of low-quality product is more elastic than supply of the high-quality commodity (with respect to their own prices), then  $\hat{a}_{LL} - \hat{a}_{HH} > 0$  and  $\hat{a}_L > \hat{a}_H$ . This condition interacts with the relative substitution and scale effects of supply and demand to determine the direction of adjustment to the proportional changes in quantity and price to account for the different supply shifts.

The changes in average quality and the quality premium are calculated in the same manner as above to obtain:

$$(29) \quad EQ_H - EQ_L = t \left[ \frac{\acute{o} (\hat{a}_{LL} - \hat{a}_{HH})}{(\acute{o} - \tau)} \right]$$

$$(30) \quad EP_H - EP_L = t \left[ \frac{-(\hat{a}_{LL} - \hat{a}_{HH})}{(\acute{o} - \tau)} \right]$$

As in the case of the per unit tax, the change in average quality and the quality premium are entirely determined by the relative sizes of the two supply shifts. When  $\hat{a}_{LL} > \hat{a}_{HH}$ , the supply shift in the low-quality market is larger than the one in the high-quality market, average quality increases, and the price premium decreases.

## Conclusion

While the assumption of product homogeneity is convenient and at times necessary (as dictated by the nature of data available), it is important to recognize the effects the assumption may have on results. The simple model presented here exemplifies some of those effects. First, the same supply-shifting policy implemented in two related markets will have different effects in each market if the sizes of the

shifts in supply differ. In the case of the two tax policies considered, the differences in the supply shifts reduce to differences in supply elasticities. Second, the model shows that to the extent that these supply shifts differ, average quality and the quality premium will change when the policy is implemented. These changes will not be accounted for in a single-market model of a homogeneous good.

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