# Empirical Specification Requirements for Two-Constraint Models of Recreation Demand 

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#### Abstract

This paper develops the theoretical restrictions implied by the two versions of Roy's Identity when any consumption choice is made subject to two binding constraints. These restrictions are analogous to the Slutsky-Hicks equations of standard (single-constraint) consumer choice problems, though derived from a different conceptual basis in the choice problem. They provide the structure necessary to correctly specify two-constraint recreation demand models. We show the implications for specifying multiple-equation recreation demand models with endogenous values of time. Another implication is that many empirical models do not satisfy the required two-constraint conditions when time has an opportunity cost. Yet another implication is that properly specified models "reveal" the endogenous marginal value of time from coefficient estimates.


## Empirical Specification Requirements for Two-Constraint Models of Recreation Demand

There has been relatively little formal guidance about how to specify recreation demand models where time is an important constraint, beyond the basic case originally analyzed by Becker where time can be converted to money according to an exogenous labor supply function. The intuition behind the Becker analyis is that all demands should be functions of "full prices" and "full budgets," where time valued at the wage rate is included in the price and budget terms. One of the contributions of the Bockstael et al. paper was to point out that not all recreationists have the opportunity to "reveal" their marginal wage rate through participation in a discretionary labor activity, and that for these individuals the relevant value of time is endogenous. However, their paper does not provide any guidance on how to specify the value of time in such "corner solution" cases where the individual offers zero discretionary labor supply.

This paper develops the theoretical restrictions implied by the two versions of Roy's Identity when any consumption choice is made subject to two binding constraints. These restrictions are analogous to the Slutsky-Hicks equations of standard (singleconstraint) consumer choice problems, though derived from a different conceptual basis in the choice problem.

These results provide the structure necessary to correctly specify two-constraint recreation demand models. We show the implications for specifying multiple-equation recreation demand models with endogenous values of time. Another implication is that many empirical models using systems of demands, count data models, or random utility models are misspecified-- they do not satisfy the required two-constraint conditions when time has an opportunity cost. Yet another implication is that properly specified models "reveal" the endogenous marginal value of time from coefficient estimates.

## Two-Constraint Recreation Choice Models

The standard consumer choice problem with two binding constraints provides the appropriate theoretical foundation for developing the specification requirements for recreation demand models when time has an opportunity cost. Let $\mathbf{x} \equiv\left(\mathrm{x}_{1}, \ldots, \mathrm{x}_{n}\right)$ be consumption goods with corresponding non-negative money prices $\mathbf{p} \equiv\left(\mathrm{p}_{1}, \ldots, \mathrm{p}_{n}\right)$ and time prices $\mathbf{t} \equiv\left(\mathrm{t}_{1}, \ldots, \mathrm{t}_{n}\right)$, and choices are made subject to a money budget constraint $\mathbf{M}=\mathbf{p x}$ and a time constraint $\mathrm{T}=\mathbf{t x}$, both of which are strictly binding. The money and time budgets M and T can be thought of as resulting from a labor supply decision by the individual, which results in discretionary income and time to be allocated to leisure time activities and goods consumption.

Note that binding time and money constraints must characterize the model used whenever researchers argue that time spent in recreation has a "value" or opportunity cost. If the time constraint is non-binding, the marginal value of time is zero, the standard consumer choice problem results, and there is no bias to recreation benefit estimates from ignoring time. Intuitively, though, time must always be "spent" in some activity, so binding time constraints are highly plausible. Nonsatiation and the presence of numeraire activities with only one price (i.e, a positive money price and zero time price, or vice versa) are sufficient for both constraints to bind.

Consider a consumer with utility function $\mathbf{u}(\mathbf{x}, \mathbf{s})$, with $\mathbf{s}$ a vector of shift parameters. The primal version of the choice problem is solved by the Marshallian demands $\mathrm{x}_{i}=\mathrm{x}_{i}(\mathbf{p}, \mathbf{t}, \mathbf{s}, \mathrm{M}, \mathrm{T})$ which are functions of both time and money prices and time and money budgets. The indirect utility function $\mathrm{V}(\mathbf{p}, \mathbf{t}, \mathbf{s}, \mathrm{M}, \mathrm{T})$ for this problem is

$$
\begin{equation*}
\mathbf{V}(\mathbf{p}, \mathbf{t}, \mathbf{s}, \mathbf{M}, \mathbf{T}) \equiv \max _{\mathbf{X}} \mathbf{u}(\mathbf{x})+\lambda\{\mathbf{M}-\mathbf{p} \mathbf{x}\}+\mu\{\mathbf{T}-\mathbf{t} \mathbf{x}\} \tag{1}
\end{equation*}
$$

where, with both constraints binding, the ratio of the Lagrange multipliers on the time and money constraints, $\mu / \lambda=\mathrm{V}_{T}(\cdot) / \mathrm{V}_{M}(\cdot)^{1}$, is the money value of time.

## Empirical Implications of the Two Roy's Identities

The presence of an additional binding (time) constraint implies additional structure on the consumer choice problem. This structure can be developed by noting that with two constraints on choice, there are two versions of Roy's Identity, relating the price and budget slopes within each constraint.

From the envelope theorem applied to (1), $\mathrm{V}_{p_{j}}=-\lambda \mathrm{x}_{j}, \mathrm{~V}_{t_{j}}=-\mu \mathrm{x}_{j}, \mathrm{~V}_{M}=\lambda$, and $\mathrm{V}_{T}=\mu$, so for all goods in the estimated incomplete demand system one can write

$$
\begin{equation*}
\mathrm{x}_{j}(\mathbf{p}, \mathbf{t}, \mathbf{s}, \mathrm{M}, \mathrm{~T}) \equiv-\mathrm{V}_{p_{j}} / \mathrm{V}_{M} \equiv-\mathrm{V}_{t_{j}} / \mathrm{V}_{T}, \quad \text { for } \mathrm{j}=1, \ldots, \mathrm{n} . \tag{2}
\end{equation*}
$$

The two Roy's Identities in equation (2) are a source of parameter restrictions in the empirical demand system and prove useful for specification and identification of the marginal value of leisure time from demand system coefficients. ${ }^{2}$

## Cross-Price Restrictions

Differentiating (2) with respect to $\mathrm{p}_{i}$, one obtains two expressions for the Marshallian cross-money price slope $\partial \mathrm{x}_{j} / \partial \mathrm{p}_{i}$,

$$
\partial \mathrm{x}_{j} / \partial \mathrm{p}_{i}=-\left[\mathrm{V}_{T} \cdot \mathrm{~V}_{t_{j} p_{i}}-\mathrm{V}_{\mathrm{t}_{j}} \cdot \mathrm{~V}_{T p_{i}}\right] / \mathrm{V}_{T}^{2}=-\left[\mathrm{V}_{M} \cdot \mathrm{~V}_{p_{j} p_{i}}-\mathrm{V}_{p_{j}} \cdot \mathrm{~V}_{M p_{i}}\right] / \mathrm{V}_{M}^{2} .
$$

Noting that $\mathrm{V}_{T p_{i}} \equiv \mu_{p_{i}}$ and $\mathrm{V}_{M p_{i}} \equiv \lambda_{p_{i}}$, replacing the partial derivatives $\mathrm{V}_{M}$ and $\mathrm{V}_{T}$ with their respective shadow values $\lambda$ and $\mu$ from (1), and using (2), this can be simplified to

$$
\begin{equation*}
\partial \mathrm{x}_{j} / \partial \mathrm{p}_{i}=\left(\mathrm{V}_{t_{j} p_{i}}-\mathrm{x}_{j} \cdot \mu_{p_{i}}\right) / \mu=\left(\mathrm{V}_{p_{j} p_{i}}-\mathrm{x}_{j} \cdot \lambda_{p_{i}}\right) / \lambda \tag{3}
\end{equation*}
$$

Similarly, the expressions for the cross-time price derivative $\partial \mathrm{x}_{i} / \partial \mathrm{t}_{j}$ from (2) are

$$
\begin{equation*}
\partial \mathrm{x}_{i} / \partial \mathrm{t}_{j}=\left(\mathrm{V}_{t_{i} t_{j}}-\mathrm{x}_{i} \cdot \mu_{t_{j}}\right) / \mu=\left(\mathrm{V}_{p_{i} t_{j}}-\mathrm{x}_{i} \cdot \lambda_{t_{j}}\right) / \lambda \tag{4}
\end{equation*}
$$

Equations (3) and (4) can be solved for $\mathrm{V}_{p_{i} t_{j}}\left(\equiv \mathrm{~V}_{t_{j} p_{i}}\right.$ by Young's Theorem) and equated, yielding a restriction on the cross-time and cross-money prices,

$$
\begin{equation*}
\partial \mathrm{x}_{i} / \partial \mathrm{t}_{j}=(\mu / \lambda) \cdot \partial \mathrm{x}_{j} / \partial \mathrm{p}_{i}+\left(\mathrm{x}_{j} \cdot \mu_{p_{i}}-\mathrm{x}_{i} \cdot \lambda_{t_{j}}\right) / \lambda . \tag{5}
\end{equation*}
$$

Because of the unobservables, (5) is not directly useful as sources of empirical restrictions on two-constraint demand models. However, by comparing with cross-budget effects, it becomes possible to derive such restrictions.

## Cross-budget Restrictions

The Marshallian cross-budget effects are also derived by differentiating both versions of Roy's Identity in (2) with respect to M and T , yielding

$$
\begin{align*}
& \partial \mathbf{x}_{j} / \partial \mathbf{M}=-\left(\lambda_{t_{j}}+\mathbf{x}_{j} \cdot \mu_{M}\right) / \mu=-\left(\lambda_{p_{j}}+\mathrm{x}_{j} \cdot \lambda_{M}\right) / \lambda  \tag{6}\\
& \partial \mathbf{x}_{i} / \partial \mathrm{T}=-\left(\mu_{t_{i}}+\mathbf{x}_{i} \cdot \mu_{T}\right) / \mu=-\left(\mu_{p_{i}}+\mathbf{x}_{i} \cdot \lambda_{T}\right) / \lambda \tag{7}
\end{align*}
$$

Because $\mu_{M} \equiv \lambda_{T} \equiv \mathrm{~V}_{M T}$, when (6) is solved for $\mu_{M}$ and (7) for $\lambda_{T}$, the two expressions can be equated. When this equality is simplified, the result can be written as

$$
\begin{equation*}
\partial \mathrm{x}_{i} / \partial \mathrm{T}=(\mu / \lambda) \cdot\left(\mathrm{x}_{i} / \mathrm{x}_{j}\right) \cdot \partial \mathrm{x}_{j} / \partial \mathrm{M}-\left(1 / \mathrm{x}_{j}\right) \cdot\left(\mathrm{x}_{j} \cdot \mu_{p_{i}}-\mathrm{x}_{i} \cdot \lambda_{t_{j}}\right) / \lambda \tag{8}
\end{equation*}
$$

## Parameter Restrictions On Two-Constraint Demands

When (8) and (5) are compared, the general form of the Marshallian cross-equation restrictions in the two-constraint problem emerges as

$$
\begin{equation*}
\partial \mathrm{x}_{i} / \partial \mathrm{t}_{j}+\mathrm{x}_{j} \cdot \partial \mathrm{x}_{i} / \partial \mathrm{T}=(\mu / \lambda) \cdot\left[\partial \mathrm{x}_{j} / \partial \mathrm{p}_{i}+\mathrm{x}_{i} \cdot \partial \mathrm{x}_{j} / \partial \mathrm{M}\right], \tag{9}
\end{equation*}
$$

Equation (9) takes a form comparable to the Slutsky-Hicks equations from standard consumer theory, and express necessary conditions which follow from utility maximization subject to two binding constraints. They are conceptually distinct from, though closely related to, the two sets of Slutsky-Hicks equations that result from the two expenditure minimization problems dual to the two-constraint utility maximization problem. The advantage of casting the requirements of theory in a form such as (9), though, is that all quantities $\mathbf{x}_{i}(\mathbf{p}, \mathbf{t}, \mathbf{s}, \mathrm{M}, \mathrm{T})$ and $\mathbf{x}_{j}(\mathbf{p}, \mathbf{t}, \mathbf{s}, \mathrm{M}, \mathrm{T})$ are Marshallian, not Hicksian, so they represent directly observable levels and slopes of ordinary demand.

To complete the comparative statics, when cross-money price slopes are compared to cross-money budget slopes, and cross-time price slopes are compared with cross-time budget slopes, the cross-equation restrictions are

$$
\begin{align*}
& \partial \mathrm{x}_{i} / \partial \mathrm{p}_{j}+\mathrm{x}_{j} \cdot \partial \mathrm{x}_{i} / \partial \mathrm{M}=\partial \mathrm{x}_{j} / \partial \mathrm{p}_{i}+\mathrm{x}_{i} \cdot \partial \mathrm{x}_{j} / \partial \mathrm{M}  \tag{10}\\
& \partial \mathrm{x}_{i} / \partial \mathrm{t}_{j}+\mathrm{x}_{j} \cdot \partial \mathrm{x}_{i} / \partial \mathrm{T}=\partial \mathrm{x}_{j} / \partial \mathrm{t}_{i}+\mathrm{x}_{i} \cdot \partial \mathrm{x}_{j} / \partial \mathrm{T} . \tag{11}
\end{align*}
$$

The necessary conditions represented in (10) and (11) further illustrate the empirical advantages of developing the symmetry requirements of two-constraint choice theory from Roy's Identities. All terms are observable, so these conditions can be directly tested for or imposed in estimating empirical recreation demand models.

Equations (9)-(11) provide the general symmetry structure which empirical twoconstraint consumer models must follow. ${ }^{3}$

## Implications for Models with Endogenous Marginal Values of Leisure Time

Because equations (9)-(11) hold for general marginal value of leisure time functions $\mu / \lambda$, they describe the structure that must also apply to the system of demands $\mathrm{x}_{i}=\mathrm{h}_{i}^{C}(\mathbf{p}, \mathbf{t}, \mathbf{s}, \mathrm{M}, \mathrm{T})$ for those at corner solutions rather than interior solutions in the labor market. In this case, the marginal value of time $(\mu / \lambda)$ is an endogenous variable, which in general is a function of all parameters of the problem. What problems does the endogeneity of the marginal value of leisure time cause for specification of two-constraint demand systems?

Denoting this marginal value of leisure time function as $\mu / \lambda \equiv \rho(\mathbf{p}, \mathbf{t}, \mathbf{s}, \mathrm{M}, \mathrm{T})$, a set of sufficient conditions for (9)-(11) to hold is for the price and budget slopes to be related as

$$
\begin{array}{ll}
\partial \mathrm{x}_{i} / \partial \mathrm{t}_{j}=\rho(\mathbf{p}, \mathbf{t}, \mathbf{s}, \mathrm{M}, \mathrm{~T}) \cdot \partial \mathrm{x}_{j} / \partial \mathrm{p}_{i} & \text { for all } \mathrm{i}, \mathrm{j} \\
\partial \log \left(\mathrm{x}_{i}\right) / \partial \mathrm{T}=\rho(\mathbf{p}, \mathbf{t}, \mathbf{s}, \mathrm{M}, \mathrm{~T}) \cdot \partial \log \left(\mathrm{x}_{j}\right) / \partial \mathrm{M} & \text { for all } \mathrm{i}, \mathrm{j} . \tag{13}
\end{array}
$$

One might anticipate problems with models using full prices $\left[\mathrm{p}_{i}+\rho(\mathbf{p}, \mathbf{t}, \mathbf{s}, \mathrm{M}, \mathrm{T}) \cdot \mathrm{t}_{i}\right]$ and full budget $[\mathbf{M}+\rho(\mathbf{p}, \mathbf{t}, \mathbf{s}, \mathbf{M}, \mathrm{T}) \cdot \mathrm{T}]$, because of the dependence of $\rho(\cdot)$ on prices and budgets. In deriving the price and budget slopes in (12) and (13), terms involving changes in $\rho(\cdot)$ with those prices and budgets must be accounted for.

For the case of endogenous marginal value of leisure time, a demand equation of the form

$$
\begin{equation*}
\mathbf{x}_{i}=\mathrm{h}_{i}\left(\mathrm{p}_{1}+\rho(\cdot) \cdot \mathrm{t}_{1}, \ldots, \mathrm{p}_{n}+\rho(\cdot) \cdot \mathrm{t}_{n}\right) \cdot \mathrm{g}(\mathrm{M}+\rho(\cdot) \cdot \mathrm{T}, \mathbf{s}), \quad \text { for } \mathrm{i}=1, \ldots, \mathrm{n} . \tag{14}
\end{equation*}
$$

satisfies (12) and (13), which are sufficient conditions for (9)-(11) to hold, despite the dependence of $\rho(\mathbf{p}, \mathbf{t}, \mathbf{s}, \mathbf{M}, \mathbf{T})$ on the full set of prices and budgets. For this demand system, again assuming symmetric cross-partial price derivatives $\left(\partial \mathrm{h}_{j} / \partial \mathrm{p}_{i}=\partial \mathrm{h}_{i} / \partial \mathrm{p}_{i}\right)$, the price slopes and budget slopes are

$$
\begin{align*}
& \partial \mathrm{x}_{j} / \partial \mathrm{p}_{i}=\frac{\partial \mathbf{h}_{j}}{\partial \mathrm{p}_{i}} \cdot \mathrm{~g}+\frac{\partial \rho}{\partial \mathrm{p}_{i}} \cdot\left(\sum_{k} \mathrm{t}_{k} \cdot \frac{\partial \mathbf{h}_{j}}{\partial \mathrm{p}_{k}} \cdot \mathrm{~g}+\mathrm{h}_{j} \cdot \mathrm{~g}_{M} \cdot \mathrm{~T}\right)  \tag{15}\\
& \partial \mathrm{x}_{i} / \partial \mathrm{t}_{j}=\rho \cdot \frac{\partial \mathbf{h}_{j}}{\partial \mathrm{p}_{j}} \cdot \mathrm{~g}+\frac{\partial \rho}{\partial \mathrm{t}_{j}} \cdot\left(\sum_{k} \mathrm{t}_{k} \cdot \frac{\partial \mathbf{h}_{j}}{\partial \mathrm{p}_{k}} \cdot \mathrm{~g}+\mathrm{h}_{j} \cdot \mathrm{~g}_{M} \cdot \mathrm{~T}\right)  \tag{16}\\
& \partial \mathrm{x}_{j} / \partial \mathbf{M}=\mathrm{h}_{j} \cdot \mathrm{~g}_{M}+\frac{\partial \rho}{\partial M} \cdot\left(\sum_{k} \mathrm{t}_{k} \cdot \frac{\partial \mathbf{h}_{j}}{\partial \mathrm{p}_{k}} \cdot \mathrm{~g}+\mathrm{h}_{j} \cdot \mathrm{~g}_{M} \cdot \mathrm{~T}\right)  \tag{17}\\
& \partial \mathrm{x}_{i} / \partial \mathrm{T}=\rho \cdot \mathbf{h}_{i} \cdot \mathrm{~g}_{M}+\frac{\partial \rho}{\partial M} \cdot\left(\sum_{k} \mathrm{t}_{k} \cdot \frac{\partial \mathbf{h}_{j}}{\partial \mathrm{p}_{k}} \cdot \mathrm{~g}+\mathrm{h}_{j} \cdot \mathrm{~g}_{M} \cdot \mathrm{~T}\right) . \tag{18}
\end{align*}
$$

Homogeneity of degree zero of Marshallian demands in the price and budget arguments of each constraint imply that the term in parentheses in each of (15)-(18) is identically zero. The terms $\mathrm{h}_{i} \cdot \mathrm{~g}_{M}$ are the specific form of the income budget slope $\partial \mathrm{x}_{i} / \partial \mathrm{M}$ (for $\mathrm{i}=1, \ldots, \mathrm{n}$ ) for the multiplicative demand given in (14), while the terms $\left(\partial \mathrm{h}_{i} / \partial \mathrm{p}_{k}\right) \cdot \mathrm{g}$ are the money price slopes $\partial \mathrm{x}_{i} / \partial \mathrm{p}_{k}$ for all $\mathrm{i}, \mathrm{k}=1, \ldots, \mathrm{n}$. The term in parentheses is then

$$
\left(\sum_{k} \mathrm{t}_{k} \cdot \partial \mathrm{x}_{i} / \partial \mathrm{p}_{k}+\partial \mathrm{x}_{i} / \partial \mathrm{M} \cdot \mathrm{~T}\right) \equiv 0
$$

by homogeneity. ${ }^{4}$ Thus, for general value of time functions, (15)-(18) simplify to

$$
\begin{align*}
& \partial \mathrm{x}_{j} / \partial \mathrm{p}_{i}=\frac{\partial \mathrm{h}_{j}}{\partial \mathrm{p}_{i}} \cdot \mathrm{~g}  \tag{19}\\
& \partial \mathrm{x}_{i} / \partial \mathrm{t}_{j}=\rho \cdot \frac{\partial \mathrm{h}_{i}}{\partial \mathrm{p}_{j}} \cdot \mathrm{~g}  \tag{20}\\
& \partial \mathrm{x}_{j} / \partial \mathrm{M}=\mathrm{h}_{j} \cdot \mathrm{~g}_{M}  \tag{21}\\
& \partial \mathrm{x}_{i} / \partial \mathrm{T}=\rho \cdot \mathrm{h}_{i} \cdot \mathrm{~g}_{M}, \tag{22}
\end{align*}
$$

and these slopes satisfy the two-constraint choice restriction in equation (10).
Thus the endogeneity of the marginal value of leisure time in the general corner solution case causes no additional problems beyond those raised in the interior solution case. The two-constraint restrictions must hold, and equation (14) is an example of how these restrictions can be satisfied with Marshallian recreation demand functions. Equation (14) further suggests how researchers can incorporate hypotheses about the structure of the marginal value of leisure time, as it may depend on prices, budgets, and other shifters $\mathbf{s}$, directly into the demand model and estimate the marginal value of leisure time directly as part of the model.

## A Problem with Common Practice in Modeling Time

It is common in the literature to find recreation demand models that include a time price of recreation but no corresponding time budget variable. That is, full price (money cost plus time cost) and money income are included in the specification. The point which may not be fully appreciated is that omission of the time budget variable invalidates the use of full prices in the model.

The inconsistency of using full prices and money budget alone can be seen by recalling equation (11) for the single-equation demand model with exogenous marginal value of leisure time. This equation must hold in the empirical model if the researcher includes a time price (thereby invoking the maintained hypothesis of two constraints on choice). The rationale for omitting time budget must be an assumption that $\partial \mathrm{x}_{i} / \partial \mathrm{T}=0$, and when this is imposed on (11) the two-constraint restriction is

$$
\begin{equation*}
\partial \mathrm{x}_{i} / \partial \mathrm{t}_{i}=\rho \cdot\left[\partial \mathrm{x}_{i} / \partial \mathrm{p}_{i}+\mathrm{x}_{i} \cdot \partial \mathrm{x}_{i} / \partial \mathrm{M}\right] . \tag{23}
\end{equation*}
$$

If the money income effect on demand is nonzero, then a demand model based on full prices and budgets, such as (14), would not satisfy (23). An obvious problem is the dependence on a consumption quantity $\left(\mathrm{x}_{i}\right)$, but any term beyond $\partial \mathrm{x}_{i} / \partial \mathrm{p}_{i}$ on the right side invalidates the use of full prices.

The analysis for random utility and count data models is parallel, based on equation (2) for random utility models and (9)-(11) for count data models.

Time budgets play an integral role in the two-constraint recreation demand model, in maintaining the theoretical justification for the use of full prices. To avoid estimating incorrect models based on full prices and full budgets, they must be included in the empirical specification.

## Inferring the Marginal Value of Leisure Time from Utility-Theoretic Demands

A second empirical point is that the marginal value of leisure time can be measured from the demand coefficients of a properly-specified system. Perhaps the easiest way to make this point is with the "corner solution" version of the the empirical model of Bockstael et al., which is

$$
\mathbf{x}_{1}=\alpha+\gamma_{1} \cdot \mathbf{M}+\gamma_{2} \cdot \mathbf{T}+\beta^{\prime} \gamma_{1} \cdot \mathrm{p}_{1}+\beta^{\prime} \gamma_{2} \cdot \mathrm{t}_{1}+\gamma_{3} \cdot \mathrm{q}+\epsilon
$$

where q is an exogenous quality variable and $\beta^{\prime} \equiv \beta /\left(\gamma_{1}+\gamma_{2}\right)$. Because this system is utility-theoretic, it satisfies the two-constraint choice restriction in (11). From (22) and (23), it can be seen that the marginal value of time can be measured directly from the demand coefficients, as

$$
\rho=\left(\partial \mathrm{x}_{1} / \partial \mathrm{t}_{1}\right) /\left(\partial \mathrm{x}_{1} / \partial \mathrm{p}_{1}\right)=\left(\partial \log \left(\mathrm{x}_{1}\right) / \partial \mathrm{T}\right) /\left(\partial \log \left(\mathrm{x}_{1}\right) / \partial \mathrm{M}\right)
$$

For this model, $\partial \mathrm{x}_{1} / \partial \mathrm{p}_{1}=\beta^{\prime} \gamma_{1}, \partial \mathrm{x}_{1} / \partial \mathrm{t}_{1}=\beta^{\prime} \gamma_{2}, \partial \log \left(\mathrm{x}_{1}\right) / \partial \mathrm{M}=\gamma_{1} / \mathrm{x}_{1}$, and $\partial \log \left(\mathrm{x}_{1}\right) / \partial \mathrm{T}$ $=\gamma_{2} / x_{1}$, so (34) becomes

$$
\rho=\beta^{\prime} \gamma_{2} / \beta^{\prime} \gamma_{1}=\gamma_{2} / \gamma_{1} .
$$

Bockstael et al. estimated the money price slope to be $\hat{\gamma}_{1}=.024$, with a time price slope of $\hat{\gamma}_{2}=2.982$. Thus the marginal value of time in this model is a constant, $\rho \approx(2.982$ units $\mathrm{x} /$ hour $) /(.024$ units $\mathrm{x} / \$) \approx \$ 124 /$ hour. This contrasts with the estimate of the authors, who infer an estimate of $\$ 60 /$ hour for the marginal value of leisure time by comparing compensating variation estimates of welfare loss from eliminating the resource, denominated in dollar and time units.

## Conclusions

This paper develops a number of the structural requirements for the specification of recreation demand models where time is thought to be an important choice constraint. Coefficient restrictions take a form similar to the Slutsky-Hicks equations from standard consumer theory of choice subject to a single constraint, but arise from a different facet of the consumer choice problem when multiple constraints bind. The Slutsky-Hicks equations arise from the identity of Hicksian and Marshallian demands when income or utility is chosen appropriately, where the two-constraint restrictions arise from the equivalence of the two Roy's Identities that govern the response of Marshallian demands to parameter changes. Thus the two constraint restrictions relate observable Marshallian demand slopes and the generally-unobservable marginal value of leisure time. The restrictions relating cross-money price and money budget effects are fully observable, as are the restrictions relating cross-time price and time budget effects, so they can be implemented and tested for easily in practice. They provide guidance in two important
areas not addressed by the existing literature: specification of how time should enter systems of demand equations, and how to deal with endogenous marginal values of leisure time. The two-constraint requirements apply to all types of empirical demand models where time is a second constraint on choice, whether motivated as systems of continuous demands, count data models, or random utility models.

An important finding is that the basic intuition of the simple model where time is an exogenous function, and the resulting demand is a function of full prices and full budgets, carries through to models where the value of time is endogenous. This should enable researchers to estimate value of leisure time functions auxiliary to the recreation demand model of interest. Individuals with exogenous values of time (those at "interior solutions" in the labor market) represent a special case where the marginal value of time is a constant or a known exogenous function.

Use of the structure required by the hypothesis of choice subject to two binding constraints is also helpful in empirical practice. We show that the approach used by much of the current literature on valuing time, to include full price of the activity but only money income, cannot be consistent with the requirements of consumer theory. We also show how the theory can also be used to infer the marginal value of time from properly specified two-constraint models. Thus the empirical two-constraint restrictions should be of considerable use in specifying theoretically-consistent demand systems and in inferring marginal values of leisure time from their empirical implementation.

## Footnotes

1. Parameters appearing as subscripts refer to partial derivatives; e.g., $\mathrm{V}_{T p_{i}} \equiv$ $\partial^{2} \mathrm{~V}(\mathbf{p}, \mathrm{t}, \mathrm{z}, \mathrm{M}, \mathrm{T}) / \partial \mathrm{T} \partial \mathrm{p}_{i}$. The subscripts i and j index the consumption goods and their corresponding prices.
2. To minimize notational clutter, it is noted here that all restrictions developed below hold for goods $\mathrm{i}, \mathrm{j}=1, \ldots, \mathrm{n}$; that is, they are restrictions which must be accounted for in the estimated incomplete demand system.
3. The results we develop here have also been derived by Partovi and Caputo, who examine the implications of the general K-constraint consumer choice problem. They also prove the negative semidefiniteness and rank conditions for the matrix of cross-equation restrictions for the general K-constraint problem.
4. It is well-known that the two-constraint Marshallian demand functions are homogeneous of degree zero in the parameters of each constraint (Partovi and Caputo; Smith). For general two-constraint demands, zero-degree homogeneity implies $\mathbf{x}(\theta \mathbf{p}, \mathbf{t}, \mathbf{s}, \theta \mathrm{M}, \mathrm{T})=\mathbf{x}(\mathbf{p}, \mathbf{t}, \mathbf{s}, \mathrm{M}, \mathrm{T})$, and differentiation with respect to $\theta$ yields $\left(\sum_{k} \mathrm{p}_{k} \cdot \partial \mathrm{x}_{i} / \partial \mathrm{p}_{k}+\partial \mathrm{x}_{i} / \partial \mathrm{M} \cdot \mathrm{M}\right)=0$. For the two-constraint model with full prices and full budgets [which has, as a special case, equation (24)], scale both money and time prices and budgets by $\theta$ (which leaves the ratio of Lagrange multipliers, $\rho$, unchanged). Then homogeneity of degree zero implies $\mathbf{x}(\theta \mathbf{p}+\rho \cdot \theta \mathbf{t}$, $\mathbf{s}, \theta \mathbf{M}+\rho \cdot \theta \mathbf{T})=\mathbf{x}(\mathbf{p}+\rho \cdot \mathbf{t}, \mathbf{s}, \mathrm{M}+\rho \cdot \mathbf{T})$, which upon differentiation with respect to $\theta$ yields $\left(\sum_{k} \mathrm{p}_{k} \cdot \partial \mathrm{x}_{i} / \partial \mathrm{p}_{k}+\partial \mathrm{x}_{i} / \partial \mathrm{M} \cdot \mathrm{M}\right)+\rho \cdot\left(\sum_{k} \mathrm{p}_{k} \cdot \partial \mathrm{x}_{i} / \partial \mathrm{t}_{k}+\partial \mathrm{x}_{i} / \partial \mathrm{M} \cdot \mathrm{T}\right)=0$. Since the first term in parentheses must be zero by homogeneity in the money budget alone, the second term in parentheses must be zero also.

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