A NONPARAMETRIC APPROACH TO SHORT-RUN PRODUCTION ANALYSIS IN A DYNAMIC CONTEXT

By

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ABSTRACT

A nonparametric approach to short-run production analysis from a cost and profit perspectives is developed in the context of an adjustment-cost model. The adjustment-cost hypothesis is incorporated in the theoretical framework in the form of the properties of the firm's technology with respect to the quasi-fixed factors. Nonparametric tests for consistency of a data series with short-run cost minimization and profit maximization are developed and the empirical implementation of these tests to U.S. agriculture is presented. The empirical evidence does not refute that agricultural producers behave collectively as if they were a short-run cost minimizing and profit maximizing firm.

Keywords: nonparametric revealed-preference approach, short-run production analysis, long-run adjustment, adjustment cost.

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1. Introduction

Varian (1984) proposes a unified nonparametric revealed-preference approach to static production analysis involving nonparametric tests for consistency with specific behavioral (e.g., cost minimizing behavior) and technological (e.g., homotheticity) assumptions as well as a nonparametric method to recover technological information from observed firm behavior. Nonparametric tests to check consistency of a data series with cost minimizing and profit maximizing behavior are presented in the form of a necessary and sufficient condition known as the Weak Axiom of Cost Minimization (WACM) and the Weak Axiom of Profit Maximization (WAPM), respectively (Varian, 1984). The WACM and WAPM are derived assuming all inputs are variable.

Ray and Bhadra (1993) modify Varian's nonparametric test for cost minimizing behavior to allow some inputs to be quasi-fixed. Using the reverse nestedness property of the conditional input requirement sets, Ray and Bhadra (1993) derive the Weak Axiom of Variable Cost Minimization (WAVCM) from Varian's WACM. However, the WAVCM proposed by Ray and Bhadra (1993) is developed in a static framework characterized by instantaneous adjustment in all factors of production and the lack of intertemporal linkage of production decisions, and which may be the sources of observed violations of the variable cost minimization hypothesis.

The adjustment-cost model of the firm developed initially by Eisner and Strotz (1963) and further elaborated, among others, by Lucas (1967), and Treadway (1969, 1970) provides a consistent dynamic theoretical framework to analyze the firm's behavior and the underlying production technology. The relative fixity of capital is incorporated explicitly in the firm's optimization problem in the form of an adjustment cost function where costs per unit of investment increases with the investment rate (Lucas, 1967; Treadway, 1970). The adjustment cost function incorporates all the forces that slow down the capital adjustment process. The impact of quasi-fixed factor adjustment on variable input use is extensively discussed in the dynamic literature.[1]

This paper incorporates the sluggish adjustment in some factors of production and the intertemporal linkage of production decisions in the theoretical framework by specifying the properties of the firm's production technology with respect to the dynamic factors. Nonparametric tests for consistency of a data series with short-run variable cost minimization and profit maximization are developed in a long-run adjustment context. The empirical implementation of these tests to U.S. agriculture is presented which permits investigating the validity of treating the collection of agricultural producers as if they were a short-run variable cost minimizing and profit maximizing firm.

2. General Representation of Dynamic Production Possibilities in the Short-Run

A well-behaved dynamic technology can be described in the short-run by a family of conditional input requirement sets or by a conditional production possibilities set satisfying some regularity conditions. Let $\{V(y(t): I(t), k(t))\}$, $t=1,...,T$, be a family of conditional input requirement sets describing the short-run technological limits on the actions of the firm. Define $V(y(t): I(t), k(t))$ as follows

$$(1) \qquad V(y(t): I(t), k(t)) = \{ x(t) : (x(t), I(t)) \in V(y(t): k(t)) \}$$

where x(t) is the perfectly variable input vector at time t; I(t) is the gross investment vector at time t; k(t) is the initial capital stock vector at time t; y(t) is the output level at time t; V(y(t): k(t)) is the input requirement set for *y(t)* given the initial capital stock vector *k(t)*; and $x(t) \in \Re^m_+$, $I(t) \in \Re^o_+$ and $k(t) \in \Re^o_+$.[2]

V(y(t): I(t), k(t)) is a subset of *V(y(t): k(t))* defined as the intersection of *V(y(t): k(t))* and a hyperplane. Consequently, the properties of the conditional input requirement set, *V(y(t): I(t), k(t))*, can be derived from the properties of *V(y(t): k(t))*. *{V(y(t): I(t), k(t))}, t=1,...,T*, satisfies the following regularity conditions: (a.1) closeness and non-emptiness; (a.2) nestedness in y(t); (a.3) positive monotonicity in x(t); (a.4) convexity in x(t); (a.5) nestedness in I(t); and (a.6) reverse nestedness in k(t).

Alternatively, a conditional production possibilities set can be used to describe the technological possibilities of the firm in the short-run. Let *T(I(t), k(t))* be the conditional production possibilities set at time t. Define the conditional production possibilities set as

$$(2) \qquad T(I(t),k(t)) = \{ (y(t),-x(t)) : (y(t),-x(t),-I(t)) \in T(k(t)) \}$$

where *T(k(t))* is the production possibilities set at time *t* given the initial capital stock vector, *k(t)*.[3]

T(I(t), k(t)) is a subset of *T(k(t))* defined as the intersection of *T(k(t))* and a hyperplane. As a result, the regularity conditions of *T(I(t), k(t))* are derived from the properties of *T(k(t))*. The conditional production possibilities set satisfies the following properties: (b.1) for each (I(t), k(t)), T(I(t), k(t)) is a non-empty closed subset of $\Re^{1+m}$; (b.2) negative monotonicity in y(t); (b.3) negative monotonicity in x(t); (b.4) for each (I(t), k(t)), the technology T(I(t), k(t)) is convex in (y(t), -x(t)); (b.5) reverse nestedness in I(t); (b.6) nestedness in k(t); (b.7) for each (I(t), k(t)), the T(I(t), k(t)) is bounded from above for each finite *x(t)*.

Property a.5 (b.5) is derived from the property of negative monotonicity of *V(y(t): k(t))* in *I(t)* (positive monotonicity of *T(k(t))* in *I(t)*), implying that current additions to the capital stock vector are output decreasing. However, current additions to the capital stock increase the future stock of capital leading to potential increases in output in the future (property a.6 or b.6). Properties a.5 and a.6 (or, b.5 and b.6) and convexity of *V(y(t): k(t))* in *I(t)* (or, convexity of

T(k(t)) in *I(t)*) reflect the presence of sluggish adjustment in the quasi-fixed factors and the intertemporal linkages of production decisions. Convexity of *V(y(t): k(t))* in *I(t)* (or, convexity of *T(k(t))* in *I(t)*) implies that the more rapidly the quasi-fixed factors are adjusted the greater the cost leading to sluggish adjustment in the quasi-fixed factors.[4]

3. Short-Run Variable Cost Minimization and Profit Maximization

Consider the following data set

$$S = \{ (y(t)^i, x(t)^i, I(t)^i, k(t)^i, w(t)^i, p(t)^i);\ i = 1,...,n;\ t = 1,...,T \}$$

where $y^i(t)$ is the output level for observation i at time t; $x^i(t)$ is the perfectly variable input vector for observation i at time t; $I^i(t)$ is the gross investment vector for observation i at time t; $k^i(t)$ is the initial capital stock vector for observation i at time t; $w^i(t)$ is the perfectly variable input price vector for observation i at time t; and $p^i(t)$ is the output price for observation i at time t.

3.1. Deterministic and Stochastic Tests for Short-Run Variable Cost Minimizing Behavior

It can be shown the following condition

$$(3) \qquad w^i(t)' x^j(t) \geq w^i(t)' x^i(t);\quad y^j(t) \geq y^i(t),\ k^i(t) \geq k^j(t),\ I^j(t) \geq I^i(t),$$

is necessary and sufficient for the data set S to be consistent with short-run variable cost minimization in a long-run adjustment context. Condition (3) is the Weak Axiom of Short-Run Variable Cost Minimization (WASRVCM) and can be used to test consistency of the data set S with short-run variable cost minimizing behavior in a long-run adjustment context.

Assuming observations are perfect measurements, the WASRVCM depends on directly observed variables. S is said to be consistent with short-run variable cost minimizing behavior if and only if the WASRVCM holds at all data points.

This test is deterministic since no probability assessments are implied and is a diagnostic procedure to check whether the data are fully consistent with the short-run variable cost minimization hypothesis. A stochastic test of the type proposed by Varian (1985) is conducted assuming observations are not perfect measurements.

Define the null hypothesis as the "true" data is consistent with short-run variable cost minimizing behavior. Assuming only variable input demand data is measured with error, the observed quantity variable $x^i(t)$ can be related to the "true" variable as follows

$$(4) \qquad \zeta_l^i(t) = x_l^i(t)(1 + \varepsilon_{vl}^i(t)),\ l=1,\ldots,m;$$

$i=1,\ldots,n;\ t=1,\ldots,T.$ $\zeta_l^i(t)$ is the "true" input demand of the variable input l for observation i at time t, and $\varepsilon_{vl}^{\,i}(t)$ is a random error assumed to be independently and identically distributed as $N(0,\sigma^2(t))$.

Given the assumption in (4), the WASRVCM depends on the observed variables $(w^i(t), y^i(t), I^i(t), k^i(t))$ and the unobservable variable $\zeta^i(t)$. The WASRVCM can be checked by running the following quadratic programming problem

$$(5) \qquad S(t) = \min_{\zeta_l^i(t)} \{ \sum_{i=1}^{n} \sum_{l=1}^{m} (\zeta_l^i(t)/x_l^i(t) - 1)^2 : \zeta^i(t) \geq 0;\ w^i(t)'\zeta^j(t) \geq w^i(t)'\zeta^i(t),$$
$$y^j(t) \geq y^i(t);\ k^i(t) \geq k^j(t);\ I^j(t) \geq I^i(t) \}$$

$t=1,\ldots,T.$ Rejection of the null hypothesis occurs when $S(t)/\sigma^2(t) > C_\alpha$, or $\sigma^2(t) < S(t)/C_\alpha$, where C_α is the α% critical value from the χ^2 table for (nm) degrees of freedom. Define $\overline{\sigma^2}(t) = S(t)/C_\alpha$ as the critical value of $\sigma^2(t)$, whose value is obtained after solving (5). If the error variance of the variable input demand data is known and less than $\overline{\sigma^2}(t)$, the null hypothesis is rejected. Thus, the stochastic test in (5) provides a range for the error variance over which the data set S is consistent with short-run variable cost minimizing behavior.

3.2. Deterministic and Stochastic Tests for Short-Run Variable Profit Maximizing Behavior

It can be shown the following condition

$$(6) \qquad p^i(t)y^i(t) - w^i(t)'x^i(t) \geq p^i(t)y(t) - w^i(t)'x(t)\,;\; k^i(t) \geq k(t);\; I(t) \geq I^i(t)$$

is necessary and sufficient for consistency of the data set S with short-run variable profit maximizing behavior in a long-run adjustment framework. Condition (6) is called the Weak Axiom of Short-Run Variable Profit Maximization (WASRVPM).

Assuming observations are perfect measurements, the WASRVPM depends on directly observed variables. S is said to be consistent with short-run variable profit maximizing behavior if and only if the WASRVPM holds at all data points.

A stochastic test can be conducted to account for the possibility of measurement errors in the data. Define the null hypothesis as the "true" data consistent with short-run variable profit maximizing behavior. Assuming only variable input demand data is measured with error, define the observed demand for each variable input as in (4). Given (4), the WASRVPM depends on the observed variables ($p^i(t)$, $w^i(t)$, $y^i(t)$, $k^i(t)$, $I^i(t)$) and on the unobservable variable $\zeta^i(t)$. The WASRVPM can be checked by running the following quadratic programming problem

$$(7) \qquad S(t) = \min_{\zeta_l^i(t)} \Big\{ \sum_{i=1}^{n} \sum_{l=1}^{m} (\zeta_l^i(t)/x_l^i(t) - 1)^2 \,:\, p^i(t)y^i(t) - w^i(t)'\zeta^i(t) \geq p^i(t)y^j(t) - w^i(t)'\zeta^j(t);\;\; k^i(t) \geq k^j(t);\; I^j(t) \geq I^i(t);\; \zeta^i(t) \geq 0 \Big\},$$

t=1,...,T.

The stochastic test in (7) is similar to the stochastic test in (5) providing a range for the error variance over which the data set *S* is consistent with short-run variable profit maximization.

4. Empirical Results

Annual data for aggregate U.S. agriculture over the time period 1948-1994 are taken from Ball *et al.* (1997). A detailed and exhaustive description of the data series and its construction is found in Ball *et al.* (1997).

The WASRVCM in (3) and WASRVPM in (6) are slightly modified to accommodate monotonic nonregressive technical change in time series data.[5] Tests for WAPM and WACM (Varian, 1984, 1985), and for the WAVCM (Ray and Bhadra, 1993) are also performed considering the possibility of nonregressive technical change. In addition, the Weak Axiom of Variable Profit Maximization (WAVPM) is also tested as a joint hypothesis with nonregressive technical change.[6]

The nonparametric tests for WASRVPM, WASRVCM, WAVPM, WAVCM, WAPM and WACM are conducted considering several combinations of output and input categories to investigate whether violations of the behavioral axioms and its degree of seriousness are sensitive to the number of output and input categories.[7] The combinations of outputs and inputs considered in the empirical analysis are: (i) twelve outputs, five variable inputs (agricultural chemicals, fuels and electricity, feed, seed and livestock purchases, other purchased inputs, hired labor) and four quasi-fixed factors (self-employed labor, durable equipment, real estate and farm inventories); (ii) twelve outputs, two variable inputs (intermediate inputs and hired labor) and four quasi-fixed factors; (iii) two outputs, two variable inputs and four quasi-fixed factors; (iv) two outputs, five variable inputs and four quasi-fixed factors.

The empirical results for all nonparametric tests are presented in Tables 1 and 2. The WASRVCM is fully satisfied in all cases. The nonparametric tests for WACM and WAVCM reveal the U.S. agricultural data do not violate these behavioral axioms in cases (i) and (ii) but violations are detected in cases (iii) and (iv). The estimates generated by the deterministic and goodness-of-fit tests for WACM and WAVCM are quite small, indicating the observed departures from the joint hypothesis of (total and variable) cost minimization and nonregressive technical change in the aggregate agricultural sector data are economically insignificant. Using the rejection criterion of 10% measurement error, it can be inferred from the stochastic test that violations of WACM and WAVCM are not statistically significant.[8]

Although the test results for the WACM and WAVCM indicate minor departures from the maintained hypothesis, the empirical results in Table 1 suggest the number of output categories are important to detect violations of the WACM and WAVCM. Violations of the WACM and WAVCM are detected in cases where two output categories are considered ((iii)-(iv)), and no violations are identified in cases with twelve output categories ((i)-(ii)).

The WASRVPM is fully satisfied in all cases. Violations of the WAPM are detected in all cases. The deterministic test for WAPM indicates that approximately 87% of the observation comparisons violate the joint hypothesis of total profit maximization and nonregressive technical change in all cases. Though the percentage of violations are similar in these cases, violations are economically more serious in cases (iii) and (iv) as indicated by the magnitude of average percentage error. The goodness-of-fit test indicates economically significant departures from WAPM. However, the stochastic test identifies standard errors in the variable input quantity data due to measurement error ranging between 2.08% and 3.45% across cases. Given the rejection criterion of 10% measurement error, the joint hypothesis is not rejected in all cases.[9]

Violations of the WAVPM are detected in all cases. The estimates generated by the deterministic and goodness-of-fit tests are quite small indicating the U.S. agricultural data do not violate seriously the WAVPM. The lower bound for the standard error of the input quantity data ranges between 1.10% and 1.68%. Employing the rejection criterion of 10% measurement error, the observed departures from WAVPM are not statistically significant.

The test results in Table 2 reveal the observed violations of WAPM and WAVPM and its degree of seriousness vary with the number of output categories. The percentage of violations of the WAPM and WAVPM is higher in cases with twelve outputs ((i), (ii)) than in cases with two output categories. However, violations of the WAPM are economically more serious in cases with two output categories ((iii) and (iv)) as indicated by the relative magnitude of the average percentage error. Violations of the WAVPM are economically more serious when twelve outputs are considered ((i) and (ii)). Departures from WAPM and WAVPM are statistically more serious in case (i).

5. Concluding Comments

The firm's short-run production decisions are clearly affected by the presence of sluggish adjustment in some factors of production. Nonparametric tests for consistency with short-run variable cost minimizing and profit maximizing behavior are developed in a long-run adjustment context. An empirical implementation of the nonparametric tests for WASRVCM and WASRVPM under the hypothesis of monotonic nonregressive technical change is presented for a time series of annual U.S. agricultural data over the time period 1948-1994. Nonparametric tests for WACM, WAPM, WAVCM and WAVPM subject to nonregressive technical change are also conducted.

The empirical results indicate that WASRVCM and WASRVPM are fully satisfied for each of the four aggregation levels considered, indicating that agricultural producers behave collectively as if they were a short-run cost minimizing and profit maximizing firm. The WACM and WAVCM are very sensitive to the number of output categories, since no violations are detected in cases with twelve outputs, regardless of the number of input categories. The WAPM and WAVPM are not fully satisfied for all aggregation levels.

Table 1. Nonparametric Test Results for Cost Minimization

Behavioral Objective	Deterministic Test		Stochastic Test
	Percentage of Violations	Average Percent Error	Critical Value of Error
(i)			
WACM	0.00	0.00	0.00
WAVCM	0.00	0.00	0.00
WASRVCM	0.00	0.00	0.00
(ii)			
WACM	0.00	0.00	0.00
WAVCM	0.00	0.00	0.00
WASRVCM	0.00	0.00	0.00
(iii)			
WACM	0.10	0.002	0.29
WAVCM	0.096	0.004	0.29
WASRVCM	0.00	0.00	0.00
(iv)			
WACM	0.10	0.002	0.28
WAVCM	0.096	0.004	0.49
WASRVCM	0.00	0.00	0.00

(a) Critical value of the standard error of the data is calculated at the 5% significance level.

Table 2. Nonparametric Test Results for Profit Maximization

Behavioral Objective	Deterministic Test		Stochastic Test
	Percentage of Violations	Average Percent Error	Critical Value of Error
(i)			
WAPM	87.4	305.5	3.45
WAVPM	1.44	0.082	1.68
WASRVPM	0.00	0.00	0.00
(ii)			
WAPM	87.4	306.6	2.18
WAVPM	1.44	0.081	1.10
WASRVPM	0.00	0.00	0.00
(iii)			
WAPM	87.3	486.7	2.08
WAVPM	1.06	0.073	1.10
WASRVPM	0.00	0.00	0.00
(iv)			
WAPM	87.2	485.3	3.30
WAVPM	1.15	0.072	1.65
WASRVPM	0.00	0.00	0.00

(a) Critical value of the standard error of the data is calculated at the 5% significance level.

FOOTNOTES

1. There are several studies emphasizing the theoretical linkages between adjustment costs and variable input use (e.g., Treadway (1970), McLaren and Cooper (1980), Epstein (1981), Caputo (1990)). Adjustment costs and the relation between quasi-fixed factors and variable inputs are the focus of several empirical studies (e.g., Vasavada and Chambers (1986), Howard and Shumway (1988), Luh and Stefanou (1991, 1996)).

2. The concept of *V(y(t): k(t))* and its regularity conditions are extensively discussed in Silva (1996) and Silva and Stefanou (1996a).

3. The concept of *T(k(t))* and its regularity conditions are explored in Silva (1996) and Silva and Stefanou (1996b).

4. Convexity of *V(y(t): k(t))* in *I(t)*, or, alternatively, convexity of *T(k(t))* in *I(t)* leads to sluggish adjustment in the quasi-fixed factors since it implies an increasing marginal cost of adjustment. For a detailed discussion of the regularity conditions of *V(y(t): k(t))* and *T(k(t))*, see Silva (1996) and Silva and Stefanou (1996a, 1996b).

5. The procedure adopted to incorporate nonregressive technical change is the one proposed by Fawson and Shumway (1988). As a result, the nonparametric test for WASRVCM (WASRVPM) determines the extent to which the aggregate sectoral data are consistent with the joint hypothesis of cost minimization (profit maximization) and nonregressive technical change. Furthermore, testing the consistency of the U.S. agricultural data series with short-run variable cost minimization and profit maximization requires information on the investment levels. For each quasi-fixed factor, investment is approximated as the difference between the current and lagged capital stock.

6. Modifying Varian´s nonparametric test for profit maximizing behavior to allow some inputs to be quasi-fixed permits derivation of the Weak Axiom of Variable Profit Maximization (WAVPM). The WAVPM is derived from Varian´s WAPM using the nestedness property of the conditional production possibilities set. The WAVPM is given as

$$p^{i}(t)y^{j}(t) - w^{i}(t)'x^{j}(t) \leq p^{i}(t)y^{i}(t) - w^{i}(t)'x^{i}(t),\ k^{i}(t) \geq k^{j}(t).$$

7. The joint hypothesis for each nonparametric test involves profit maximization or cost minimization and nonregressive technical change. As a matter of simplification, each test is denoted by the name of each behavioral axiom (e.g., WAPM).

8. No measurement error information is available for the specific data used in this study. In this case, it is common practice to adopt a standard error based on evidence of measurement error in other data series (e.g., Williams and Shumway (1998) adopt a 10% standard error).

9. There is an inconsistency between the results of the goodness-of-fit test and the stochastic test for WAPM. The goodness-of-fit test results indicate violations of WAPM are economically significant where the standard error generated by the stochastic test is small from a statistical viewpoint. This inconsistency can be attributed to the different nature of the goodness-of-fit test and the stochastic test. The goodness-of-fit test attributes violations to errors in optimization, and in the stochastic test violations are attributed to errors in measuring the quantity data.

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