# Transformation of Fallow Systems under Population Pressure

by

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May 15, 1999

To be presented at the Summer 1999 Meetings of the American Agricultural Economics Association Meetings in Nashville, Tennessee, August 8-11, 1999

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50-Word Abstract:
In a fallow-cultivation model with biomass regeneration, we find the population-poverty-degradation linkage via the discount rate:  slight increases in the discount rate result in increased cropping frequency and much lower soil fertility.  Aggregating gives transitions equation declining in fertility and increasing in the fallow:cultivation ratio.

# Introduction

In many developing countries, farmers cultivate land for a year or more, and then let it lie in fallow for one or more years to restore soil fertility. This is called a fallow system. With rapid population growth causing shorter fallow periods, a natural concern is what the effect population growth will have on soil fertility—especially if one of the possible consequences is irreversible degradation—and whether the resulting land use patterns are in any sense sustainable. Given that perhaps as many as 300 million people rely on fallow systems (Brady and Weil), covering perhaps 30% of the world's arable soils (Sanchez), this is an extremely important issue for both the developing countries and the world as a whole, since one possible consequence of population growth is the cutting of forests, releasing greenhouse gases into the atmosphere. Furthermore, if population growth leads to more intensive cultivation, organic matter levels in the soil may fall. Since more carbon is sequestered in the organic matter contained in soils than is sequestered in forests (Brady and Weil), reduction in organic matter levels could have a major impact on levels of greenhouse gases in the atmosphere.

Economists have used two approaches to look at fallow systems. The first approach is that of Krautkraemer and others (Krautkraemer 1990, 1994; Barrett; Lewis and Schmalensee 1977, 1979) which assumes a single, indivisible plot. The second approach is that of López and others (López and Niklitschek; López 1994, 1995, 1997, 1998) which allows the agent to choose the amount of land to cultivate. The two models lead to different conclusions.

It is not clear, however, whether their different approaches lead to the different conclusions, or whether the conclusions differ because of assumptions incorporated in their models.

Both Krautkraemer and Barrett adopt McConnell's model of soil fertility, which is a model intended for analyzing optimal soil conservation, in which soil depth is an input to production. In that model, the state equation for soil depth has two components: an increase at a constant rate (due to new soil formation) and a decrease at a rate equal to the intensity of cultivation. I believe we must reject this type of dynamic for why farmers use fallow systems, since the rate of soil formation is so slow that it could only explain cycles that were in hundreds or thousands of years, rather than observed cycles that are between three and thirty years.

López bases his conceptual model on Sanchez's often-cited text on tropical agriculture. From this perspective, the reason for fallow is so that the natural vegetative regrowth (i.e., trees, bushes, and grasses) on the uncultivated plot—what López describes simply as "biomass"—can extract nutrients from the air and from deep in the soil where crops cannot reach, and store them in plant matter. Before the farmer begins cultivation, he extracts the nutrients in the biomass, usually by "slash and burn," a process in which trees are felled and all of the vegetation, including the bushes and grasses, are burned, releasing the nutrients they contain in the ash. Furthermore, the roots of the burned vegetation over time decay in the soil, forming organic matter. Organic matter has been credited with contributing to many positive aspects of soil fertility such as improving the soil physical structure (which contributes to water infiltration and retention) and strengthening the chemical properties of the soil (including nutrient retention and giving the soil a more favorable pH). Fallowing has also been cited in the agronomy literature as eliminating weed infestation that sometimes results from years of cultivation.[1]

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[1] In industrialized nations, farmers are able to compensate for most aspects of soil fertility decline caused by cultivation. For example, fertilizers add nutrients, lime increases pH, and specialized plowing can compensate for some types of poor soil

In the first section of this paper, I use the one-plot model of Krautkraemer, but adopt the more realistic explanations of López and Sanchez regarding fallow systems. In the second section of the paper, I use simulation techniques to aggregate plots at various stages within a given fallow system, in order to derive aggregate transition equations. I compare them to the ones used by López, and note the differences.

## Solution for One Indivisible Plot

Let $Y \equiv Y(Df,L)$ be a constant returns to scale production function with inputs of land, D, scaled by soil fertility, f; and labor, L. We define labor per unit of land $\ell \equiv L/D$, and define the yield function $y(f, \ell) \equiv Y(Df,L)/D$. If p is the price of agricultural output, and $\gamma$ is the amount of labor per unit of land required to prepare the land before planting, the instantaneous profit function can be written as

(1) $$\pi(t) = pDy(f(t), \ell(t) - \gamma).$$

Since biomass cannot regenerate under cultivation, the state equation must reflect one function, $g^c(f(t),\ell(t))$, under cultivation; and another function, $g^f(f(t),\ell(t))$, under fallow. Consistent with agronomic experiments, we assume that fertility declines under cultivation at a rate proportional to fertility—more fertility is lost to the crops under high fertility than low fertility. We also assume that in fallow, fertility regenerates as a logistic function, since biomass in forests have been observed to regenerate logistically after cutting or burning. We write

(2a) $$\dot{f}(t) = g^c(t) = -af(t)$$

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structure. These interventions are costly, often beyond what the smallholder farmers in developing countries can afford given their limited access to credit, their low education levels which hinder effective implementation of new technologies,

$$(2b) \qquad \dot{f}(t) = g^f(t) = bf(t)\left(1 - \frac{f(t)}{f_{max}}\right),$$

where a, b, and $f_{max}$ are positive constants, and where the dot over the f indicates a derivative with respect to time.[2] We say that *a* is the rate of fertility reduction, b is the rate of fertility regeneration, and $f_{max}$ is the maximum fertility obtainable.

López suggests that we need to include a clearing cost, φ, which would be incurred when the land is brought out of fallow into cultivation. This makes sense: if land clearing were without cost, then the farmer would always choose to cultivate a plot that had been in fallow, since the fallowed plot would have higher fertility than the plot just cultivated, and thus would yield higher profits for the same cost. In practice, farmers in many parts of the world cultivate for more than one season even though alternative land is available.[3]

If the initial fertility, $f(0) = f_0$, is high, we would expect the optimal solution for this problem to be as shown in Figure 1, where $T_0$ is the initial optimal time to keep the land under cultivation; $f_f$ is the level of fertility at which it is optimal to put the land into fallow; $T_f$ is the length of time to leave the land in fallow; $f_c$ is the level of fertility to begin cultivation; and $T_c$ is the amount of time to keep the land under cultivation for all remaining cycles.

Given Figure 1, we can set up the planner's problem in continuous time. The objective function is

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and poor infrastructure which raises input prices and lowers producer prices.

[2] While the profit function is the same as Krautkraemer's, the state equation is very different. His is given by $\dot{f}(t) = g(f(t)) - \ell(t)$, where g is a concave function which is equal to 0 at two positive values of its argument.

[3] Krautkraemer does not assume a clearing cost. However, the result of not having a clearing cost is that the optimal solution is to reach the optimal fertility level, and begin an infinitely repeated cycle of abandoning cultivation for an infinitesimally short period of time, and resume cultivation for an infinitesimally short period of time.

$$(3) \qquad W = -\varphi D + \int_0^{T_0} \pi(t)e^{-rt}dt - \frac{e^{-r(T_0+T_f)}}{1 - e^{-r(T_c+T_f)}}\left[\varphi D - \int_0^{T_c} \pi(t)e^{-rt}dt\right]$$

where $\pi(t)$ is as defined in (1), r is the discount rate, and the term $1 / (1 - e^{-r(Tc + Tf)})$ comes from

the discount factor and the infinite repetitions of the cycle.[4] The planner maximizes (3) by choos-

ing $\ell(t)$ for every t, $T_0$, and $T_f$, subject to (2), a non-negativity constraint on $\ell(t)$, another non-

negativity constraint on f(t), and a constraint on $\ell(t)$ limiting it to a maximum of the population, N,

divided by the land size, D. Note that $T_c$ is not a choice variable: given $T_0$, $T_f$, and $\ell(t)$ for every

t, $T_c$ is completely determined. This makes sense from Figure 1: $T_0$ and $\ell(t)$ determine the path

from $f_0$ to $f_f$, and the path from that point to $f_c$ is completely determined by $T_f$ and $\ell(t)$. The path

after $t = T_0 + T_f$ repeats states that already occurred before $T_0 + T_f$, so the same actions that were

optimal before must still be optimal. The difficulty in getting an analytical solution to this problem

arises because there are two separate integrals as well as two separate state equations,[5] and the

standard method of optimal control only has one of each. Fortunately, Kamien and Schwartz (pp

246-7) summarize Amit's result for problems with two separate integrals and two separate state

equations.[6] The Hamiltonians are given by

$$(4a) \qquad H^0 = \pi(t)e^{-rt} + \lambda^0(t)g^c(t) + \omega(t)\left(N/D - \ell(t)\right)$$

$$(4b) \qquad H^1 = \lambda^1(t)g^f(t)$$

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[4] Recall that $1 + a + a^2 + a^3 + ... = 1 / (1 - a)$. Here, $a = e^{-r(Tc + Tf)}$.

[5] Actually, there are an infinite number of integrals, but in (5) we have condensed them into 2.

[6] Amit showed that separate Hamiltonians can be written for each segment. Then he derived what are essentially boundary conditions for this type of problem. One condition requires that the costate variables be equal at the switch points except for an adjustment based on the cost of changing from one state equation to another (here, the clearing cost when going from fallow to cultivation, and nothing when going from cultivation to fallow), and the other condition requires that the Hamiltonians be equal at the switch points, except for an adjustment also based on the cost of changing from one state equation to another.

**Figure 1   Optimal cultivate-fallow cycle in the single, indivisible plot problem**
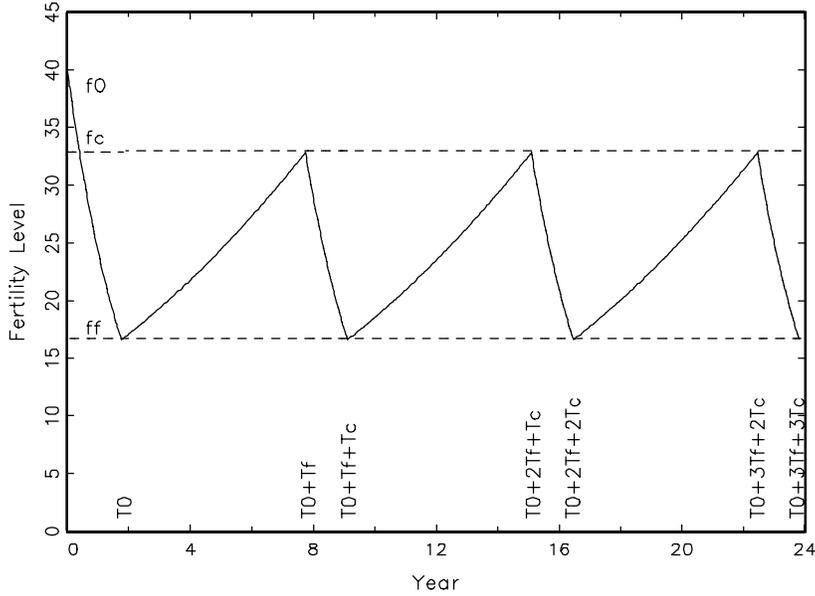


This problem has two cases.  The first is the one in which $\ell(t) < N/D$ for some t in which

cultivation takes place.  In such a case, $\omega(t) = 0$.  But if that is the case, then the first order condi-

tion with respect to $\ell$ implies that $pDy_\ell(f,\ell)e^{-rt} = 0$, which cannot occur in normal yield functions for

which we assume that $y_\ell > 0$, $y_f > 0$, $y_{\ell\ell} < 0$, and $y_{ff} < 0$ for all non-negative arguments.  Therefore,

it must be the case that $\ell(t) = N/D$.  Furthermore, we can solve the differential equations (2) for

f(t), conditional on $T_0$, $T_f$, and $T_c$.  The solutions are

(5a)     $f(t) = f_0 e^{-at}$                           for $0 \leq t \leq T_0$

(5b)     $f(t) = \dfrac{f_f f_{max}}{f_f + (f_{max} - f_f)e^{-b(t-T_0)}}$          for $T_0 \leq t \leq T_0 + T_f$

(5c)     $f(t) = f_c e^{-a(t-T_0-T_f)}$                for $T_0 + T_f \leq t \leq T_0 + T_f + T_c.$

At this stage, we must choose a functional form for the yield function.  However, when we

insert the yield function into the first order conditions, we get a system of two highly non-linear

equations in two unknowns, and we must use numerical methods to solve for $T_0$ and $T_f$. As an alternative, given that we were able to solve for closed form solutions for $f(t)$ and $\ell(t)$ at the beginning of the problem, we could use these solutions in (3), integrate, and do a grid search for $T_0$ and $T_f$, which I did using the GAUSS software package.[7] I solved the grid search multiple times, allowing different parameters to vary each time. This paper focuses on the effects of varying population, output price, and discount rate.

## Results of Analysis

In general, parameter changes had one of two effects on fertility levels: one effect was to have $f_f$ and $f_c$ move closer together (i.e., $f_c$ becoming smaller and $f_f$ becoming larger), as was the case for population and price increases; the other was for both $f_f$ and $f_c$ to become smaller, which was observed for increases in the discount rate. The results are found in Table 1.

I expected population to have a strongly negative effect on fertility. Instead, only discount rate had that effect. There are a number of models explaining how poverty leads to an increase in the discount rate.[8] What my results seem to show is that if population growth in already poor countries is bad for the environment, it is because of how rapid population growth reduces per capita income (see Table 1), which results in higher discount rates, which in turn results in soil degradation, as future earning potential is sacrificed for present income.

Table 1 also reports cropping frequency, $R = (100 * T_c) / (T_c + T_f)$. It was interesting to observe that cropping frequencies did not vary with parameter changes for population and price.

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[7] I chose parameters which I felt were reasonable, mostly because the parameter values had implications for biophysical systems, and published papers gave me a good idea what ranges these parameters ought to be in.

[8] See Lawrence or Becker and Mulligan.

This result is different from that of Krautkraemer (1994, p. 416), who found that with non-convexities in the production function (here, $\gamma \neq 0$), the cropping frequency would increase under population pressure. Cropping frequency, however, was found to optimally increase linearly with increases in the discount factor.[9]

**Table 1   Effects of population, price, and discount rate on fertility and income**

| Test | Population ('000s) | Price | Discount rate | $T_0$ | $T_f$ | $T_c$ | Cropping Frequency | $f_c$ | $f_f$ | Discounted profit (millions) | Per capita discounted profit ('000s) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Base | 2.5 | 0.25 | 0.05 | 3.02 | 5.99 | 1.36 | 18.5 | 32.9 | 16.6 | 2.56 | 1.02 |
| N | 1.5 | 0.25 | 0.05 | 3.08 | 6.48 | 1.48 | 18.6 | 33.6 | 16.1 | 2.03 | 1.35 |
| N | 2 | 0.25 | 0.05 | 3.04 | 6.19 | 1.41 | 18.6 | 33.1 | 16.4 | 2.33 | 1.17 |
| N | 2.5 | 0.25 | 0.05 | 3.02 | 5.99 | 1.36 | 18.5 | 32.9 | 16.6 | 2.56 | 1.02 |
| N | 3 | 0.25 | 0.05 | 3.00 | 5.85 | 1.33 | 18.5 | 32.6 | 16.8 | 2.75 | 0.92 |
| N | 3.5 | 0.25 | 0.05 | 2.98 | 5.74 | 1.31 | 18.6 | 32.5 | 16.9 | 2.91 | 0.83 |
| N | 4 | 0.25 | 0.05 | 2.97 | 5.64 | 1.29 | 18.6 | 32.3 | 17.0 | 3.05 | 0.76 |
| N | 4.5 | 0.25 | 0.05 | 2.96 | 5.57 | 1.27 | 18.6 | 32.2 | 17.1 | 3.19 | 0.71 |
| N | 5 | 0.25 | 0.05 | 2.95 | 5.50 | 1.25 | 18.5 | 32.1 | 17.2 | 3.30 | 0.66 |
| p | 2.5 | 0.05 | 0.05 | 3.67 | 10.50 | 2.40 | 18.6 | 39.6 | 12.0 | 0.49 | 0.20 |
| p | 2.5 | 0.1 | 0.05 | 3.33 | 8.22 | 1.87 | 18.5 | 36.2 | 14.2 | 1.01 | 0.40 |
| p | 2.5 | 0.15 | 0.05 | 3.18 | 7.14 | 1.63 | 18.6 | 34.6 | 15.3 | 1.52 | 0.61 |
| p | 2.5 | 0.2 | 0.05 | 3.08 | 6.47 | 1.47 | 18.5 | 33.6 | 16.1 | 2.04 | 0.82 |
| p | 2.5 | 0.25 | 0.05 | 3.02 | 5.99 | 1.36 | 18.5 | 32.9 | 16.6 | 2.56 | 1.02 |
| p | 2.5 | 0.3 | 0.05 | 2.97 | 5.63 | 1.28 | 18.5 | 32.3 | 17.0 | 3.07 | 1.23 |
| p | 2.5 | 0.35 | 0.05 | 2.93 | 5.35 | 1.22 | 18.6 | 31.9 | 17.4 | 3.59 | 1.44 |
| p | 2.5 | 0.4 | 0.05 | 2.90 | 5.11 | 1.16 | 18.5 | 31.5 | 17.6 | 4.11 | 1.64 |
| p | 2.5 | 0.45 | 0.05 | 2.87 | 4.91 | 1.12 | 18.6 | 31.2 | 17.9 | 4.63 | 1.85 |
| p | 2.5 | 0.5 | 0.05 | 2.85 | 4.74 | 1.08 | 18.6 | 31.0 | 18.1 | 5.14 | 2.06 |
| r | 2.5 | 0.25 | 0.01 | 1.72 | 5.16 | 0.92 | 15.1 | 50.3 | 31.8 | 9.07 | 3.63 |
| r | 2.5 | 0.25 | 0.02 | 1.97 | 5.26 | 1.00 | 15.9 | 46.1 | 28.0 | 4.93 | 1.97 |
| r | 2.5 | 0.25 | 0.03 | 2.27 | 5.42 | 1.10 | 16.9 | 41.8 | 24.2 | 3.58 | 1.43 |
| r | 2.5 | 0.25 | 0.04 | 2.61 | 5.65 | 1.22 | 17.8 | 37.4 | 20.4 | 2.93 | 1.17 |
| r | 2.5 | 0.25 | 0.05 | 3.02 | 5.99 | 1.36 | 18.5 | 32.9 | 16.6 | 2.56 | 1.02 |
| r | 2.5 | 0.25 | 0.06 | 3.52 | 6.49 | 1.56 | 19.4 | 28.2 | 12.9 | 2.32 | 0.93 |
| r | 2.5 | 0.25 | 0.07 | 4.18 | 7.26 | 1.84 | 20.2 | 23.3 | 9.3 | 2.16 | 0.86 |
| r | 2.5 | 0.25 | 0.08 | 5.11 | 8.56 | 2.29 | 21.1 | 18.3 | 5.8 | 2.05 | 0.82 |
| r | 2.5 | 0.25 | 0.09 | 6.67 | 11.30 | 3.16 | 21.9 | 12.9 | 2.7 | 1.98 | 0.79 |
| r | 2.5 | 0.25 | 0.1 | 8.44 | 13.20 | 3.82 | 22.4 | 7.4 | 1.1 | 1.92 | 0.77 |

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[9] In results not reported here, cropping frequency decreased hyperbolically with increases in the fertility reduction parameter. That means that marginal soils will optimally be cropped less frequently than normal soils, and that in fact differences in cropping frequencies are more noticeable on small differences on good soils rather than small differences on poor soils.

# Aggregate Model

In this section, I will briefly present the results of my analysis of the aggregated approach, in which plot-level dynamics are ignored, except to say that they, of course, form the underlying dynamics of the total and average fertility for the entire village, region, or country. López uses the aggregated approach in his models, and this sets him apart from all of the other authors who have looked at soil fertility dynamics in fallow systems. This approach is appealing because it is simpler to use than one-plot models; can be used in a two-sector, general equilibrium model; and is most readily adapted for data analysis and policy recommendations.

I used GAUSS to calculate what happens to fertility at the plot-level under a given regime, and then average the fertility levels of many plots at different stages of the $T_f$:$T_c$ cycle. This approach seems to be quite reasonable, since agronomists have developed models which make sense at the plot level regarding soil fertility and biomass, but know less about what happens in the aggregate. In order to examine the transition equation for average fertility, I assumed that each plot of land would change fertility based on (2). I assumed that all plots were of equal size, and that a regime would be chosen and used for whatever until steady state fertility was reached. Figure 2 shows mean fertility on cultivated plots, fallowed plots, and all plots under a 1:8 regime, starting with a fertility distribution which was uniformly distributed from 79.2% to 80% of maximum fertility.[10]

Mean fertility on fallowed land at a steady state equilibrium is given by

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[10] Recall that since I am using a logistic function for fertility recovery under fallows, I have implicitly assumed that fertility can reach a maximum level, which I have set equal to 100.

$$f_{fall}^{asym} = \frac{1}{T_f} \int_0^{T_f} \frac{f_c e^{-aT_c} f_{max}}{f_c e^{-aT_c} + (f_{max} - f_c e^{-aT_c})e^{-bt}} dt$$

where $f_c$ is the plot-level maximum fertility in steady state (see Figure 1), and is given by

$$f_c = \frac{f_{max}(e^{-aT_c} - e^{-bT_f})}{e^{-aT_c}(1 - e^{-bT_f})}$$

Upon evaluating the integral and substituting for $f_c$, we get

(6) $$f_{asy} = f_{max}\left(1 - \frac{a \cdot T_c}{b \cdot T_f}\right)$$

**Figure 2    Transition of mean fertility levels under a constant cultivate-fallow regime**

Mean Fertility Levels (Tc=1    , Tf=8    , Cult%=11, and timestep=0.1 )
Starting fert: min=79.2, max=80.0, mean=79.6, and std=00.3
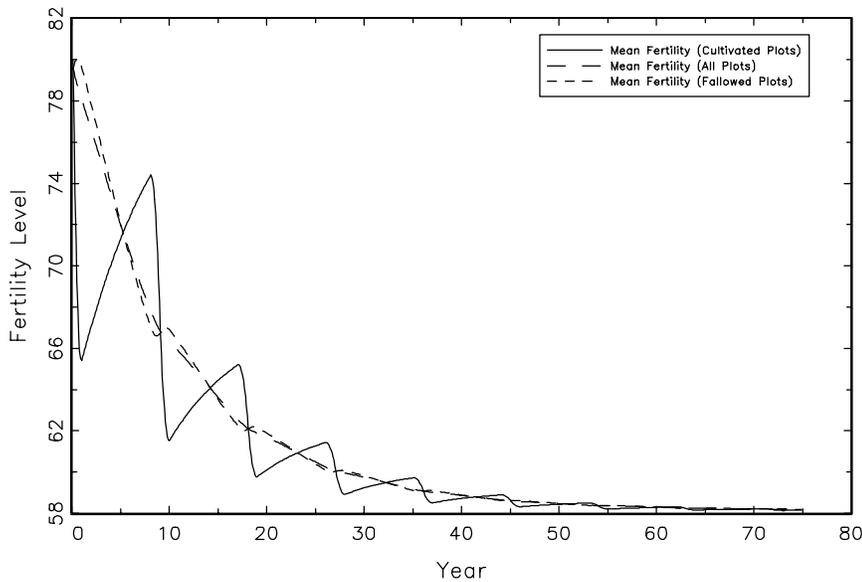


What I want to determine through this process is what would be a reasonable transition

equation in the aggregate.  We observe in Figure 2 an exponential transition from one steady state

to another.  One such differential equation which would generate that type of solution is

$\dot{f} = -b_0 f + b_1$, where $b_0$ and $b_1$ are constants.  Solving this differential equation, we get

$$f(t) = Ce^{-b_0 t} + \frac{b_1}{b_0}$$

where C is a constant of integration.  We can use $f(0) = f_0$ to find $C = f_0 - b_1/b_0$.  We can also find that $b_1/b_0 = f_{asy}$.

Analyzing the transitions from one steady state to another using GAUSS, it appears that $b_0 = T_f / (f_{max} \cdot T_c)$. Hence, the transition equation, given the underlying paramers, is

(7)
$$\dot{f} = -\frac{T_f}{f_{max} T_c} f + \frac{T_f}{T_c} - \frac{a}{b}$$

and the solution of this differential equation is

(8)
$$f(t) = f_0 \exp(-\frac{T_f}{f_{max} T_c} t) + f_{max} \left(1 - \frac{a \cdot T_c}{b \cdot T_f}\right) \left[1 - \exp(-\frac{T_f}{f_{max} T_c} t)\right]$$

Equation (8) tells us that given the parameters at the plot-level, if $T_f/T_c < a/b$ for too long, fertility will continue to decline until depleted.  Equation (7) tells us that high fertility levels require high fallow-to-cultivation ratios, and similarly that low fertility levels only require relatively low fallow-to-cultivation ratios so as not to decline.

We see that the coefficient on f in (7) is opposite in sign to that found in López (1994), which, when translated into the notation I am using would be $\dot{F} = \alpha F - \beta D_c$, where $F = fD$. A negative sign seems more plausible, because it agrees with a slowing down of fertility growth as fertility approaches its asymptote.[11]

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[11] In López and Niklitschek, the authors use $\dot{F} = k(D - D_c) - \phi(D_c)$, where k and $\phi$ are functions where k' > 0, k" < 0, $\phi$' > 0, and $\phi$" > 0.  This transition equation is highly restrictive in many ways.  First of all, fertility does not have an effect on the rate of change of fertility.  Secondly, the steady state for F determines the value for $D_c$, and is unaffected by any other values of variables.  That is, when dF/dt = 0, $D_c$ can take on only one unique value, since both k and $\phi$ are monotonic in their arguments.  Finally, one might argue that the assumed sign for the second derivative of $\phi$ is wrong, since farmers would first cul-

We note that the transition equation depends only on $T_c/T_f$, and not on the values of each component. What I did next was to compare profit functions for various regimes with a constant $T_c/T_f$. I computed the profit by running simulations, then regressed the log of profit per cultivated hectare on a logged Cobb-Douglas profit function and a quadratic of $T_c$ and $T_f$. What I found is that the profit function is influenced not only by the ratio of $T_c$ to $T_f$, but by the individual values. In other words, there are a unique values for $T_c$ and $T_f$ that maximize the profit function, subject to soil fertility transition equation. However, many places which use fallow systems use a one-year regime. In terms of theoretical models—and not in terms of applied models—given that the effects of $T_c$ and $T_f$ are small relative to the effects of quantity of land cultivated, labor, and fertility, we may be better off simplifying the problem and assuming that $T_c = 1$, as López consistently does.

# Conclusion

In this brief analysis, we have looked at the indivisible plot model of Krautkraemer, modified for more realistic dynamics. What we learned from that approach it that population increase alone has almost no effect on mean soil fertility. Instead, we found that when population growth reduces income, poverty's effect on soil fertility—through the discount rate—was substantial. We also saw that plot-level dynamics could be translated into aggregate-level dynamics, and that with aggregate soil fertility transition equations and profit functions based on micro-level models, we will be able to more confidently model policy choices.

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tivate the land with the highest fertility (here, biomass), and as more land was cultivated, less biomass per unit of land is destroyed, because land of lower biomass is brought under cultivation.

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