Modeling Price Impacts of Backward Vertical Integration in the US Pork Industry

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The U.S. pork sector is evolving from an industry of small, independent firms vertically linked by spot markets to one of substantially larger firms vertically connected through contractual agreements and integration. Potential consumer benefits of tighter vertical arrangement include higher quality pork products (Martinez et al.) sold at a lower prices, although the nature of this benefit is still debated [Abiru, Greenhut and Ohta (1976, 1979), Quirmbach, Warren-Boulton, and Westfield]. There is concern that a highly consolidated, vertically coordinated industry may lead to market foreclosure, a situation in which independent producers no longer have open markets in which to sell their output (Salinger). Consequently, the evolution of the pork industry presents a dilemma for policy makers (Barkema and Cook).

The objective of this paper is to develop an econometric model of backward integration by oligopsonistic pork processing firms into an upstream hog production stage, and to simulate the price impacts of vertical integration at the consumer and producer price level. Following Perry, backward integration is defined as the fraction of the upstream limiting production factor (e.g., farm land, feedlot facilities and water supply) that downstream processors control. Perry's measure was first exploited empirically by Azzam in his study of the fed cattle industry, though the current study differs substantially from Azzam's pioneering effort. First, Azzam presumes monopsony in regional markets while the current study considers an oligopsonistic, national pork market. Secondly, a multi-output, variable proportions technology is posited for the downstream processor in lieu of a single output, fixed proportions production technology. Finally, the current study explicitly accounts for the difference between the total slaughter hog quantity and open market hog quantities. Theoretical restrictions derived from the model transform the variable associated with unavailable open market hog quantities in order to utilize USDA slaughter data.

Before proceeding, a brief discussion will relate the conceptual model to the various forms of vertical arrangements actually observed in U.S. pork processing. In the model, two extreme cases of vertical relationships between hog producers and processors are considered; vertical integration and open market transaction. A parameter λ is introduced to represent the proportion of hogs produced via vertical

integration, and 1- λ is the non-integrated portion of the industry. Of particular interest is what the parameter λ captures when the pork sector is organized according to a continuum of vertical arrangements including both vertical integration and coordination (e.g., contracting), with the latter predominating (Lawrence et al.). Importantly, a specific economic outcome is observed in time series data which arises from existing vertical arrangements. The modeling strategy is to find a single parameter, λ, which groups the continuous spectrum of vertical arrangements into the two polar cases and still reproduces the same economic outcome. Consequently, the vertical integration parameter is the weighted average of the two vertical arrangements. As an example, a λ of 0.25 indicates the economic outcome observed in the time series data is consistent with an industry that is 25% vertically integrated. Such an interpretation of λ is in the spirit of the conjectural variations parameter commonly found in the New Empirical Industrial Organization (NEIO) literature when studying market power. Ultimately, this parameter may be increased to simulate increasing levels of vertical integration, and resulting price impacts may be discerned.

Conceptual Framework

The model is composed of an upstream production stage and a downstream processing stage. In the upstream production stage, hogs are produced using farm inputs, while in the downstream stage the intermediate hog input is combined with other inputs to create a variety of processed pork products. The upstream stage consists of many individual producers who take farm input and hog output prices as given. The downstream stage is composed of a handful of pork processors who act competitively in all but the intermediate hog input market. The downstream oligopsonists acquire the intermediate hog input from both independent producers and from their own upstream subsidiaries; two groups of producers that are assumed to share the same production technology. This hog production technology will be discussed first, followed by a discussion of an individual processor's maximization problem from which one derives the optimal pork output supply, total hog input demand, and open market hog input purchases. The behavioral equations of the individual processor are then aggregated and may be estimated econometrically in conjunction with the pork product demand equations of consumers and the open market hog supply

equation of independent hog producers. The aggregate econometric model is then used as a basis for simulating price and welfare effects of vertical integration.

Defining Vertical Integration

Following Perry, the technology of upstream hog producers is assumed to be constant returns to scale for all output levels, but the marginal production cost is increasing due to the limited availability of a specialized factor.

Let $C(x, \lambda)$ be the variable cost function of any subset of hog producers who produce x with a fraction λ of the limiting factor $\{0 < \lambda \leq 1\}$. Constant returns to scale in the production technology implies that $C(x, \lambda)$ is linearly homogenous in its arguments. In particular,

$$(1) \quad C(x,\lambda) = \lambda * C\left(\frac{x}{\lambda}, 1\right).$$

The technology is such that producing x units of the hog output with λ of the limiting factor costs a fraction λ of the expense of producing the quantity x/λ with all of the limiting factor.[2] Notice that $C(\bullet, 1)$ is equivalent to the industry's variable cost function.

Differentiating (1) with respect to x, it is apparent that the marginal cost of producing x units for the subset of producers who own λ of the limiting factor is equivalent to the industry's marginal cost of producing x/λ units of the output. That is:

$$(2) \quad \frac{\partial C(x,\lambda)}{\partial x} = \frac{\partial C\left(\frac{x}{\lambda}, 1\right)}{\partial x}.$$

Given the assumption that hog producers are price takers in their output market, (2) indicates that the hog supply function for producers owning λ of the limiting factor is equivalent to the industry supply function shifted horizontally by λ. Thus, the upstream industry supply curve can be continuously and horizontally partitioned into supply curves of producers owning fractions of the limiting factor, and a downstream

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[2] For example, producing x with one third of the limiting factor is one third as costly as producing three times x with all of the limiting factor.

processor's backward integration can be defined in terms of the fraction of the upstream limiting factor that he owns (Perry). Specifically, let λ_i be the fraction of the upstream limiting factor owned by the ith downstream processor and let $\lambda = \Sigma\lambda_i$. Then, as defined by Perry, λ is the extent of backward vertical integration in the industry. A λ near zero suggests the industry is relatively more disintegrated whereas a λ near one suggests the industry is nearly integrated.

The Processor's Profit Maximization Problem

The ith processor produces various pork outputs using a hog input (x_i) and other processing inputs (l_i). The hog input is obtained from two sources, open market purchases $\left(x_i^o\right)$ and internal production $\left(x_i - x_i^o\right)$ of the processor's upstream subsidiary which produces the needed hogs at a minimum cost. The processor treats the price of pork products (P) and the price of non-hog processing inputs (S_l) as parameters. Given the extent of backward integration (λ_i), the processor maximizes profit by choosing total hog input quantity, open market hog purchases and the quantity of non-hog processing inputs. The maximization problem is written as:

$$(3a) \quad \max_{x_i, l_i, x_i^O} \left\{ R\left(x_i, l_i \big| P\right) - S_l \ l_i - C\left(x_i - x_i^o \Big| \lambda_i, S_f\right) - W\left(x^O\right) x_i^O \right\},$$

where R is the revenue function for pork products, and C is the cost function of the upstream subsidiary who treats as parameters the farm input prices (S_f). Notice that the oligopsonistic processor can influence the open market price of hogs (W) via the linkage between x_i^o and x^o where $\Sigma_i \ x_i^o = x^o$. A unique feature of (3a) is that the revenue function treats hogs as an input while the subsidiary cost function treats hogs as an output; consequently, analysis is always focused on the intermediate hog product. Also, the revenue function specification is conducive for examining the multiproduct case considered in this study as, via Shepherd's lemma, $\partial R/\partial P_i$ is the processor's conditional output supply of pork product i. Without loss of generality, the optimization in (3a) can be decomposed into a revenue maximization problem and a hog

input expenditure minimization problem where the latter involves an optimal choice between open market purchases and internal production of the hog input. Specifically,

$$(3b) \quad \max_{x_i, l_i} \left\{ \begin{array}{l} R\left(x_i, l_i \mid P\right) - S_l\, l_i \\[2ex] \quad - \min_{x_i^o} \left\{ \Gamma_i \equiv C\left(x_i - x_i^o \mid \lambda_i, S_f\right) + W\left(x^o\right)\, x_i^o,\ given\ x_i \right\} \end{array} \right\}.$$

Hog expenditure minimization is considered first. Given the total hog input x_i, the processor allocates hog input expenditures to balance the marginal production cost of his upstream subsidiary with the marginal open market outlay which includes both the prevailing hog price and an oligopsony markdown. Differentiate Γ_i in (3b) with respect to x_i^o and set the resulting expression equal to zero to obtain the first order condition:

$$(4) \quad \frac{\partial\, C\left(x_i - x_i^o \mid \lambda_i, S_f\right)}{\partial x_i^o} = W + \frac{\partial W}{\partial x^o} \frac{\partial x^o}{\partial x_i^o}\, x_i^o = W(1 + \theta_i\, \varepsilon),$$

where
$$\theta_i \equiv \frac{\partial x^o}{\partial x_i^o} \frac{x_i^o}{x^o}, \quad \varepsilon \equiv \frac{\partial W}{\partial x^o} \frac{x^o}{W}.$$

As in the NEIO literature, the term $\partial x^o / \partial x_i^o$ in (4) captures the conjectures that the processor has for competitors' responses to marginal changes in his own open market purchases. The term $\frac{\partial W}{\partial x^o} \frac{\partial x^o}{\partial x_i^o}\, x_i^o$ represents the oligopsony markdown while $\frac{\partial\, C(\bullet)}{\partial x_i^o}$ is the marginal production cost of the upstream subsidiary. The right hand side of (4) expresses the first order condition in terms of the conjectural elasticity (θ_i) of the processor and the inverse supply elasticity or price flexibility (ε) of independent hog producers. From (4), the ith oligopsonist's demand for open market hogs can be written as:

$$(5) \quad x_i^o \;=\; x_i^o\left(x_i \mid W, \lambda_i, \varepsilon, \theta_i, S_f\right) \;\equiv\; x_i^o\left(x_i \mid W, \kappa_i\right)$$

Equation (5) illustrates how the oligopsonist divides total hog input procurements between open market purchases and production by its own upstream subsidiaries. Throughout the remainder of this paper, (5) will be referred to as the hog input allocation rule. Using (5), x_i may be substituted for x_i^o in the objective function (3b).[3] Before continuing with the optimization problem, the previous derivation will be made more concrete by considering a particular functional specification for the upstream cost function.

Specific Functional Form

Following Azzam, specify the cost function for any hog producer whose output is q and owns z fraction of the limiting factor as:

$$(6) \quad C(q, z, S_f) \;=\; \frac{v}{\varepsilon+1}\left(\frac{q}{z}\right)^{\varepsilon} q,$$

where v captures the impact on C of a set of supply shifters (e.g., input price, S_f) and ε is the inverse supply elasticity as defined previously. The marginal production cost for the ith processor's upstream subsidiary can now be written as:

$$\frac{\partial\, C\left(x_i - x_i^o \mid \lambda_i, S_f\right)}{\partial\left(x_i - x_i^o\right)} = v\left(\frac{x_i - x_i^o}{\lambda_i}\right)^{\varepsilon}.$$

The first-order condition in (4) is then specialized to:

$$(7) \quad v\left(\frac{x_i - x_i^o}{\lambda_i}\right)^{\varepsilon} \;=\; W\left(1 + \varepsilon\, \theta_i\right),$$

where θ_i is the conjectural elasticity as defined before. Solving equation (7) for x_i^o in terms of x_i, one obtains:

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[3] Expressing hog input usage in terms of x_i instead of x_i^o is conducive for empirical analysis because aggregate data are reported in terms of total hogs killed, Σx_i.

$$(8) \quad x_i^o \quad = \quad x_i - \lambda_i \left(\frac{W}{v} \left(1 + \varepsilon \, \theta_i \right) \right)^{\frac{1}{\varepsilon}}.$$

Notice in (8) that as λ_i approaches zero then x_i^o approaches x_i suggesting that all of the processor's hog

input is acquired from the open market. Equation (8) is the specific counterpart of the hog input allocation

rule in (5).

The Optimal Input Demand and Output Supply

Substituting (5) into Γ_i in the second part of equation (3b) provides the minimum hog expenditures:

$$(9) \quad E\left(x_i \big| \kappa_i \right) \quad \equiv \quad C\left(x_i - x_i^o\left(x_i \big| W, \kappa_i \right) \big| \lambda_i, S_f \right) + W \, x_i^o\left(x_i \big| W, \kappa_i \right).$$

The profit maximization problem in (3b) then involves a revenue function as well as the hog input

expenditure function:

$$(10) \quad \max_{x_i, l_i} \left\{ R\left(x_i, l_i \big| P \right) - S_l \, l_i - E\left(x_i \big| W, \kappa_i \right) \right\}.$$

Differentiating the processor's objective function in (10) with respect to the remaining two choice variables,

and setting the resulting expressions equal to zero, will yield first order conditions for intermediate hog

input and other processing inputs demands:

$$(11a) \quad \frac{\partial R\left(x_i, l_i \big| P \right)}{\partial x_i} - \frac{\partial E\left(x_i \big| W, \kappa_i \right)}{\partial x_i} \quad = \quad 0, \text{ and}$$

$$(11b) \quad \frac{\partial R\left(x_i, l_i \big| P \right)}{\partial l_i} - S_l \quad = \quad 0.$$

Solving the first order conditions in (11), the optimal demand for the hog input and other processing inputs

can be expressed as:

$$(12a) \quad x_i = x_i\left(P, S_l, w, \kappa_i \right), \text{ and}$$

$$(12b) \quad l_i = l_i\left(P, S_l, w, \kappa_i \right).$$

One also seeks the output supply of pork products. Applying Shepherd's lemma to the revenue function obtains the conditional output supply vector:

$$(13a) \quad y_i^c \left(P \mid x_i, l_i \right) = \frac{\partial R\left(x_i, l_i \mid P\right)}{\partial P} \ .$$

The Marshallian supply vector can then be found by substituting the optimal input demands of (12) into (13a) for x_i and l_i:

$$(13b) \quad y_i \quad = \quad y_i\left(P, x_i\left(P, S_l, w, \kappa_i\right), l_i\left(P, S_l, w, \kappa_i\right)\right).$$

To aggregate the previous model, follow Appelbaum's approach of assuming that the conjectural elasticities are the same for all processors $\left(\theta_i = \theta, \forall i\right)$. It is also assumed that each processor owns the same fraction of the upstream limiting factor $\left(\lambda_i = \overline{\lambda}, \forall i\right)$, and denote the sum of $\overline{\lambda}$ for all processors is simply λ. Given these assumptions, the parameter κ_i in equation (5) becomes the same for all i; that is, $\kappa_i = \kappa \equiv \left\{\lambda, \varepsilon, \theta, S_f\right\}$. It follows that the aggregate versions of first order conditions (11), optimal input demand equations (12), and optimal pork supply equations (13) can be obtained by suppressing the i subscripts in those equations.

Similarly, the processors' hog input allocation rule in (5) can be aggregated as:

$$(5') \quad x^o \quad = \quad x^o\left(x \mid W, \kappa\right).$$

Substituting the aggregate version of $(12a)$ into $(5')$ for the optimal demand for hog input (x) gives the open market demand for hogs:

$$(14) \quad x^o \quad = \quad x^o\left(x\left(P, S_l, W, \kappa\right), W, \kappa\right),$$

which has to be balanced by the output of independent hog producers whose supply curve, given the price taking assumption, coincides with their marginal cost curve:

$$(15) \quad W = \frac{\partial\, C\left(x^o, 1-\lambda, S_f\right)}{\partial\, x^o}.$$

From (14) and (15) the equilibrium open market hog quantity (x^o) and hog price (W) are determined, given the required oligopsonistic markdown ($W\theta\varepsilon$) as observed in (4).

The equilibrium conditions in the pork product market can be obtained by introducing the consumer demand for the products:

$$(16) \quad y = y\left(P, \Psi\right),$$

where Ψ is a vector of demand shifters. Given the assumption of perfect competition in final output markets, equilibrium pork prices and quantities can be obtained by solving the aggregate version of $(13b)$ and (16) simultaneously.

To summarize, the aggregate model of the pork industry has six equations including the total hog input demand in $(12a)$, the open market hog demand in (14), the open market hog supply in (15), the pork product supply in $(13b)$, the retail pork product demand in (16), and the optimal demand for other processing inputs in $(12b)$. There are six endogenous variables including the total hog input (x), open market hog quantity (x^o) and price (W) [and hence the oligopsony markdown ($W\theta\varepsilon$)], the pork product quantity (y) and price (P), and other processing input usage (l).

The equations can, in principle, be estimated jointly as a system with the neoclassical restrictions associated with the revenue function in (10) tested or imposed. In practice, however, the aggregate input demand equations $(12a)$ and $(12b)$ are exceedingly complex as they are the simultaneous solutions of the first order conditions in $(11a)$ and $(11b)$, and this complexity makes them difficult to estimate. This limitation can be overcome by estimating the first order conditions instead because all the parameters in (12) also appear in (11). Likewise, the empirical expression for the aggregate output supply equation in $(13b)$ is very complex because it includes the simultaneous solution of the first order condition (11).

Instead of direct estimation, one can estimate the conditional output supply in $(13a)$ and then use the estimated parameters to recover the Marshallian supply equation of $(13b)$.

Empirical Framework

The empirical specification for the upstream variable cost function is in (6) and the subsequent hog input allocation rule is in (8). The aggregate version of (8) is:

$$(8') \quad x^o \;\; = \;\; x - \lambda \; \left(\frac{W}{v} (1 + \varepsilon \; \theta) \right)^{\frac{1}{\varepsilon}}.$$

Given the empirical specification of the cost function for the upstream production stage, the empirical counterpart of the open market hog supply function in (15) can be expressed as:

$$(17) \quad\quad x^o \;\; = \;\; \left(\frac{W}{v} \right)^{\frac{1}{\varepsilon}} (1 - \lambda),$$

where, as mentioned in the discussion of (6), v captures the impact on upstream production costs of a set of supply shifters. The supply shifters include the price of feed, the price of feeder pigs, cost of labor, and dummy variables accounting for seasonal variation in factors related to hog production. The open market supply equation in (17), however, is not amenable for empirical estimation because the data for x^o are not available; the slaughter hog series published by the USDA pertains to total hog kills, x. This data limitation can be addressed by making use of the allocation rule $(8')$ derived from the hog expenditure minimization problem. Substituting $(8')$ into (17), the supply of open market hogs is transformed into:

$$(18) \quad\quad x \;\; = \;\; \left(\frac{W}{v} \right)^{\frac{1}{\varepsilon}} \left(1 - \lambda + \lambda \left(1 + \theta \; \varepsilon \right)^{\frac{1}{\varepsilon}} \right),$$

which can be estimated using available data.

The empirical model also requires the specification of the processor's revenue function in (10). For ease of exposition and to be consistent with the aggregate model, subscripts will be used to denote processing inputs and outputs rather than individual processing firms. Three pork outputs (hams, loins, and bacon) are considered with their prices denoted by P_1 for ham, P_2 for loins, and P_3 for bacon. The

processing inputs include labor (l_1), energy (l_2), transportation services (l_3), and the intermediate hog input (x). Let z be a vector containing the four input quantities; $z = (l_1, l_2, l_3, x)$. Consistent with the notation in the conceptual framework, let the first three input prices be denoted as S_{l_i}, $i = 1,2,3$ and the intermediate hog input price as W. A quadratic form is chosen for the revenue function with P_1 and P_2 being normalized by P_3; the normalization imposes linear homogeneity on the revenue function and eases estimation burden. Let $\tilde{P}_j$ denote the normalized price for P_j, $j = 1,2$ and let $\tilde{R}$ be the corresponding normalized revenue:

$$\tilde{R}(z, P) = \alpha_0 + \sum_{m=1}^{4} \alpha_m \, z_m + \sum_{j=1}^{2} \beta_j \, \tilde{P}_j + \frac{1}{2} \sum_{m=1}^{4} \sum_{m'=1}^{4} \alpha_{mm'} \, z_m \, z_{m'}$$

$$+ \frac{1}{2} \sum_{j=1}^{2} \sum_{j'=1}^{2} \beta_{jj'} \, \tilde{P}_j \, \tilde{P}_{j'} + \sum_{j=1}^{2} \sum_{m=1}^{4} \delta_{jm} \, \tilde{P}_j \, z_m,$$

where α's, β's, and δ's are parameters.

With the above empirical specifications for the cost function and the revenue function, imposing the symmetry condition that $\alpha_{mm'} = \alpha_{m'm}$ and aggregating, one obtains the empirical counterpart of the first order conditions in ($11a$) and ($11b$):

$$(19a) \quad \alpha_4 + \sum_{m=1}^{4} \alpha_{4m} \, z_m + \sum_{j=1}^{2} \delta_{4j} \, \tilde{P}_j - W\left(2 + \theta \varepsilon\right) = 0, \quad \text{and}$$

$$(19b) \quad \alpha_i + \sum_{m=1}^{4} \alpha_{im} \, z_m + \sum_{j=1}^{2} \delta_{ij} \, \tilde{P}_j - S_{l_i} = 0 \qquad i = 1, 2, 3.$$

The estimated coefficients of ($19a$) and ($19b$) are then used to construct the empirical counterparts of equations ($12a$) and ($12b$) which are the optimal input demands.

Similarly, the conditional supply function of the jth pork product can be derived by differentiating the revenue function with respect to its output price, imposing the symmetry condition that $\beta_{jj'} = \beta_{j'j}$ and aggregating:

$$(20) \quad \frac{\partial \tilde{R}}{\partial \tilde{P}_j} = \beta_j + \sum_{j'=1}^{2} \beta_{jj'} \, \tilde{P}_{j'} + \sum_{m=1}^{4} \delta_{jm} \, z_m \qquad j = 1,2.$$

Equation (20) is the empirical version of (13a) and its estimated parameters will be substituted into the empirical counterpart of (13b) to obtain the estimated form of the Marshallian supply equation.

Finally, the empirical counterpart of the consumer demand equation for pork products in (16) is specified in double logarithmic form as:

$$(21) \quad \ln y_j = \gamma_o + \sum_{i=1}^{4} \gamma_i \ln P_i + \gamma_5 \ln P_o + \gamma_6 \ln y_{j_{-1}} + \gamma_7 \ln M$$

$$+ \gamma_8 \ SEAS + \gamma_9 \ TREND + \gamma_{10} \ POP \qquad j = 1,2,3.$$

where y_j is the demand for the jth pork product and P_j is its price, P_o is the price vector of other food items and non-food items, M is income, $y_{j_{-1}}$ is the lagged demand accounting for habit formation of consumers, *SEAS* is a vector of seasonal dummy variables, *POP* is a population variable, and *TREND* captures the impact of consumption trends such as the shift to leaner diets.

Equations (18) - (21) can be estimated as a system using Three Stage Least Squares to account for endogeneity, contemporaneous correlation, and cross equation restrictions. Symmetry and price homogeneity conditions of the revenue function are imposed in the above derivation. The unrestricted version of the equation system can be estimated and the restrictions tested. The convexity of the revenue function can also be verified via an LDL decomposition procedure as outlined in Moschini and others. Once the estimated versions of equations (18) - (21) are derived, appropriate simulation techniques will be used to determine the impacts of increasing vertical coordination on the slaughter hog price/quantity and producer pork price/quantity.

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