Incorporating Quadratic Scale Curves in Inverse Demand Systems

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Abstract:
In this paper we introduce inverse demand systems that include quadratic scale terms. These systems are similar to regular quadratic demand systems introduced by Howe, Pollack, and Wales. A unique feature of these specifications is that they maintain linear scale curves as a special case. For illustrative purposes we estimate the Normalized Quadratic Inverse Demand-Quadratic Scale System using monthly South Atlantic fish landings and valuation data, 1980-1996. In estimation concavity is maintained locally, and the rank reduction procedures advocated by Diewert and Wales (1988b) are employed. The estimated model is then used to obtain welfare estimates associated with catch restrictions.

Key Words: Antonelli matrix, inverse demands, negative semidefinite, quadratic scale curves

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1. Introduction

In recent years, there has been a resurgence of interest in inverse demand systems, especially in the agricultural economics literature. Studies that report estimates of inverse demand models include Barten and Bettendorf; Thurman and Easley; Park Thurman, and Easley; Eales, Durham and Wessels; and Holt and Bishop for fish; Moschini and Vissa; Eales and Unnevehr; Eales; Holt and Goodwin; and Kesavan and Buhr for U.S. meat demand; Brown, Lee, and Seale concerning the demand for oranges; and Huang for composite food and nonfood commodities. In inverse demand systems, quantities are exogenous and prices (marginal valuations) are the dependent variables as opposed to 'direct' demand systems where quantities are endogenous. It is advantageous to treat quantities as fixed in cases where quantities can not adjust in the short run or for non-market goods where prices are not readily available.

While there have been several prior efforts to estimate inverse demand systems for various commodities, some of these earlier studies have employed systems that implicitly assume linear scale curves (e.g., Holt and Bishop). In this paper, a functional form that includes quadratic scale terms is developed, the Normalized Quadratic Inverse Demand–Quadratic Scale System (NQID-QSS).[1] Linearity in scale is then a special case of the more general form.

The price equations reported below in (1) are indicative of inverse demand systems occasionally reported in the literature (e.g., Holt and Bishop).

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[1] Two additional functional forms including quadratic scale terms were developed (the Generalized Leontief Inverse Demand – Quadratic Scale System and the Direct Translog – Quadratic Scale System). However, due to space constraints, only the NQID-QSS will be presented here.

(1) $$\pi_i = a_i(\underline{q}) + \frac{b_i(\underline{q})}{b(\underline{q})}\left[1 - a(\underline{q})\right], \quad i = 1, \ldots, n,$$

where $\pi_i = p_i / y$ denotes the ith normalized price, with y denoting expenditure and p_i the nominal money price, and $a_i(\underline{q})$ and $b_i(\underline{q})$ denoting the ith first partial derivatives of $a(\underline{q})$ and $b(\underline{q})$, respectively, where $a(\underline{q})$ and $b(\underline{q})$ are linear homogeneous and concave functions.

As with direct demand systems derived from expenditure functions linear in u, (1) is a potentially restrictive specification. Specifically, if $\underline{q}$ is scaled by an arbitrary constant, $(1/\tau)$, say, than π_i will be linear in τ–a linear scale curve.[2] The parallel in direct demand systems is quasi homotheticity, where Engle curves are straight lines that do not necessarily emanate from the origin.

In direct demand systems, restricting demand functions to be linear in expenditure implies that consumers will purchase the same proportion of each commodity at every income level. Introduced to combat this potential limitation, systems with quadratic Engle curves were first explored by Howe, Pollak, and Wales. Gorman independently introduced demands quadratic in income. Comparable to linear Engle curves, linear scale curves imply the potentially implausible result that marginal valuations associated with consuming proportionally more of all goods in the bundle will be the same irrespective of the size of the initial bundle.

[2] Similarly, if all prices in an expenditure function that is linear in u are scaled by the same (positive) factor of proportionality, $(1/\tau)$, the resulting demand curves will be linear in τ–a system of linear Engle curves.

2. Quadratic Scale Systems

Following Ryan and Wales (1996), it may be shown that quadratic demand systems are generated from indirect utility functions of the form

(2) $$\varphi(\underline{p}, y) = -\frac{g(\underline{p})}{y - f(\underline{p})} - h(\underline{p})$$

where $y = \sum_k p_k q_k$ is total expenditure, $\underline{p} = (p_1, \ldots, p_n)^T$ is a n-vector of money prices, f and g are homogeneous of degree one in prices, and h is homogeneous of degree zero in prices. Equation (2) can be rearranged to solve for the expenditure function, yielding

(3) $$e(\underline{p}, u) = -\frac{g(\underline{p})}{u + h(\underline{p})} + f(\underline{p}).$$

At present we are interested in creating a distance function correspondent to (3) in order to generate inverse demands. For a given level of utility u, a vector of consumption quantities $\underline{q} = (q_1, \ldots, q_n)^T$, and a representative consumer's indirect preference function $\varphi(\underline{\pi})$, the distance function is defined as

(4) $$D(u, \underline{q}) = \min_{\underline{\pi}} \{\underline{\pi}^T \underline{q} : \varphi(\underline{\pi}) \geq u\},$$

where, D is an ordinal measure of 'distance,' and $\underline{\pi} = (\pi_1, \ldots, \pi_n)^T$ denotes a vector of normalized prices. As discussed by Deaton and Deaton and Meullbauer, distance functions are non-increasing in u and are increasing, homogeneous of degree one, and concave in $\underline{q}$. Intuitively, the distance function is the amount by which $\underline{q}$ must be divided in order to bring it on to indifference surface u.

Application of the Shephard-Hannoch lemma to distance function (4) recovers compensated inverse demands. That is,

(5) $$\underline{\pi}^a(u,\underline{q}) = \nabla_{\underline{q}} D(u,\underline{q}),$$

where $\underline{\pi}^a(\cdot)$ is a vector valued function of quantities and the utility target *u*. Taking the second derivatives of the distance function yields the Antonelli matrix,

(6) $$A = \nabla_{\underline{q}} \underline{\pi}^a(u,\underline{q}) = \nabla^2_{\underline{qq}} D(u,\underline{q}).$$

Antonelli matrix *A* is symmetric, negative semidefinite, and, due to the homogeneity condition, is at most of rank n–1.

A distance function specification that corresponds to the expenditure function in (5) and that, moreover, would be associated with quadratic scale terms is simply:

(7) $$D(u,\underline{q}) = -\frac{g(\underline{q})}{u + h(\underline{q})} + f(\underline{q}),$$

where *f*, *g*, and *h* are now functions of quantities and unknown parameters.

When quantities are such that utility level *u* is attained the useful, albeit arbitrary, normalization $D(u,\underline{q}) = 1$ is typically applied. This implies that the distance function may be inverted, thereby solving for the direct utility function. Performing the required manipulations in the present case gives:

(8) $$U(\underline{q}) = -\frac{g(\underline{q})}{1 - f(\underline{q})} - h(\underline{q}),$$

which is the direct utility function corresponding to (7).

By applying the Shephard-Hannoch lemma to (7), compensated inverse (Antonelli) demands of the form

$$(9) \qquad \pi_i(u,\underline{q}) = \frac{\partial}{\partial q_i} D(u,\underline{q}) = -\frac{\left(u+h(\underline{q})\right)g_i(\underline{q}) - g(\underline{q})h_i(\underline{q})}{\left(u+h(\underline{q})\right)^2} + f_i(\underline{q}), \quad i = 1,\ldots, n$$

are obtained, where f_i, g_i, and h_i are first partial derivatives of f, g, and h, respectively. To acquire uncompensated inverse demand functions, (8) is substituted into (9) to eliminate the unobservable utility index, generating

$$(10) \qquad \pi_i(\underline{q}) = \frac{h_i(\underline{q})}{g(\underline{q})}\left[1 - f(\underline{q})\right]^2 + \frac{g_i(\underline{q})}{g(\underline{q})}\left[1 - f(\underline{q})\right] + f_i(\underline{q}).$$

Several observations are in order regarding (10). First, if all quantities are scaled by the same factor (1/τ), then the resulting scale curves clearly involve terms that are both linear *and* quadratic in τ. That is, (10) is consistent with an inverse demand system that implies quadratic scale curves and, therefore, no longer restricts marginal valuations associated with consuming proportionally more of all goods in the bundle to be invariant to initial bundle size. Second, if $h_i(\underline{q}) = 0 \; \forall i$ and $\forall \underline{q} \in \Re^n_{++}$, then (10) assumes a form that is observationally equivalent to (3). The implication is that with a suitable parameterization for h, (10) may be used to test for linearity in scale response.

3. Normalized Quadratic Inverse Demand – Quadratic Scale System

For the NQID-QSS, f, h, and g are specified as

$$(11) \qquad f(\underline{q}) = \sum_k q_k d_k,$$

$$(12) \qquad h(\underline{q}) = \sum_k a_k \ln q_k, \qquad \sum_k a_k = 0, \text{ and}$$

$$(13) \qquad g(\underline{q}) = \sum_k q_k b_k + \tfrac{1}{2}\left(\frac{\sum_k \sum_j B_{kj} q_k q_j}{\eta}\right),$$

where $\eta = \sum_k \alpha_k q_k$.

From (12) if $a_k = 0\ \forall k$, then $h_k(\underline{q}) = 0\ \forall k$. We therefore have a direct way of testing for linearity of scale curves by using either a Wald or Likelihood Ratio (LR) test.

By substituting (11)–(13) into (10), the normalized quadratic inverse demands may be written as

(14) $$\pi_i(q) = \frac{a_i}{q_i g}\left[1 - \sum_k q_k d_k\right]^2 + \frac{\left(b_i + \eta^{-1}\sum_k B_{ik} q_k - \frac{1}{2}\alpha_i \eta^{-2}\sum_k\sum_j B_{kj} q_k q_j\right)}{g}\left[1 - \sum_k q_k d_k\right] + d_i,$$

where a_k , b_k , d_k , and B_{kj} are unknown parameters, and $\alpha_k > 0$ are predetermined parameters, $k, j = 1,...,n$. We choose a reference vector of quantities $\underline{q}^* = \left(q_1^*,\ldots,q_n^*\right)^T$.

As well, we assume that the nxn matrix B with kjth element B_{kj} satisfies the n restrictions

(15) $$B\underline{q}^* = 0, \qquad B = B^T.$$

In addition, we assume that the following restrictions hold at the reference bundle $\underline{q}^*$:

(16) $$\underline{\alpha}^T \underline{q}^* = 1, \qquad \underline{\alpha} \geq \underline{0}_n,$$

(17) $$\underline{d}^T \underline{q}^* = 0, \text{ and}$$

(18) $$\underline{b}^T \underline{q}^* = 1.$$

These normalizations are necessary for the parameters in the system to be fully identified (estimable) and to ensure that $g(\underline{q})$ is homogeneous of degree one in $\underline{q}$. The inverse demand system defined by (14)-(18) is of special interest because it nests the normalized quadratic inverse demand (NQID) system of Holt and Bishop.

There is no reason *a priori* to believe that curvature requirements will be satisfied spontaneously by the estimated NQID-QSS, even at $\underline{q}^*$; curvature may, however, be imposed at a point (locally) through a Cholesky decomposition of the Antonelli matrix.[3] This procedure introduces additional non-linearity into the estimating equations, but with the significant advantage of guaranteeing that the curvature conditions are satisfied locally.

4. Empirical Application of Quadratic Scale Models

As an illustration of the applicability of quadratic scale models, we estimate inverse ex-vessel demands for finfish landed in the South Atlantic region of the U.S. We restrict attention to the NQID-QSS.[4] The data, compiled from National Marine Fisheries Service data on monthly finfish landings and total value of landings, cover the period January 1980–December 1996, for a total of 204 observations.

Data for all reported species were aggregated into nine categories comprised of bluefish, dolphinfish, other finfish, flounder, groupers, scups, trout, snappers, and tilefish/triggerfish. There is tremendous variation in landings of each of these species over time. It is typical for these fish categories to have their minimum and maximum values differ by an order of magnitude for both shares and quantities.

As indicated previously, prior to estimation all quantities are normalized to have a unit mean. To estimate the NQID-QSS, we convert equation (16) to share form by multiplying both sides of equation (16) by q_i and then appending a stochastic disturbance

[3] See Moschini and Ryan and Wales (1998) for more details on the use of Cholesky decompositions.

term. Maximum likelihood estimates are obtained by using the Davidon-Fletcher-Powell algorithm, and, because the contemporaneous covariance matrix, Ω, is singular, one equation is deleted from the system.

Because the quantity data are scaled to have unit means, we follow Moschini in choosing the reference bundle $\underline{q}^*$ to equal $\underline{\iota}_n$, the unit vector. That is, the means of the scaled data are used as the reference point. There is apparently no obvious method for selecting the predetermined values for $\underline{\alpha}$. We therefore simply follow Diewert and Wales (1998a, 1998b, 1993) in defining $\alpha_i = (1/n)\ \forall\ i$.

The unrestricted NQID-QSS (without concavity imposed) does not have a negative semidefinite Antonelli matrix; there are two positive eigenvalues at the point of approximation. Imposing concavity does not change the numerical values of negative eigenvalues by much. And while positive eigenvalues become negative after imposing curvature restrictions, their values typically lie very close to zero, suggesting that the rank reduction procedures of Diewert and Wales (1988b) may be justified.

Single-equation R^2 values are in the range 0.67-0.93, revealing that each equation in the model fits the data fairly well, especially considering that the dependent variables are expenditure shares and that the data are monthly. Imposing concavity on each model has little effect on individual equation R^2 values. However, an examination of log likelihood values reveals that the imposition of concavity does diminish the explanatory power of the model somewhat.

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[4] The GLID-QSS and DTL-QSS were also estimated, but results are not reported in the interest of brevity. Results are available upon request. While the results obtained from these models were similar to those for

Following Moschini, LR tests are used to determine how far the rank of the Antonelli matrix may be reduced before causing a significant decline in model fit. LR test results indicate that the rank of the NQID-QSS may be reduced to K = 5 with no significant change in log likelihood function values. The rank K = 5 model is therefore the version of the NQID-QSS used to calculate welfare loss estimates in the following section. By using the rank K = 5 model, the number of estimated parameters is reduced from 60 to 54.

As indicated previously, an interesting question is whether or not the estimated a_i terms are significantly different from zero. If, in fact, $a_i = 0 \; \forall i$, then quadratic terms no longer appear in the estimated equations. Furthermore, if these restrictions hold the NQID-QSS model reduces to the globally concave NQID model (with linear scale curves) of Holt and Bishop. LR tests reveal that the null hypothesis of linear scale curves may be rejected at the 1% level for the unrestricted, restricted, and K = 5 models. Therefore, there is strong empirical evidence that linearity in scale is not a viable assumption in the present application.

5. Estimated Welfare Losses

Not only does imposing curvature satisfy economic theory, it also enables us to obtain consistent money-metric estimates of welfare losses caused by quantity reductions. By using the theory presented in Kim, the estimated inverse demand system may be used to examine welfare costs associated with forced reductions in fish landings.

Compensating variation (CV) is defined as the amount of additional (normalized)

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the NQID-QSS, the NQID-QSS nonetheless provided the best overall fit to the data.

outlay necessary for the consumer to achieve u^0 while facing the new quantity vector $\underline{q}^1$. Positive values for $\overline{CV}$ indicate that consumers are worse off when facing the new quantities, $\underline{q}^1$, than they were with the initial quantities, $\underline{q}^0$.

In a similar manner, (normalized) equivalent variation (*EV*) associated with a change in quantities from $\underline{q}^0$ to $\underline{q}^1$ represents the amount of additional (normalized) expenditure that is necessary for the consumer to accept utility level u^1 while continuing to face the original quantity vector, $\underline{q}^0$. As for $\overline{CV}$, positive $\overline{EV}$ values indicate consumers are made worse off under $\underline{q}^1$ than under the base quantity vector, $\underline{q}^0$. For non-homothetic preferences, *CV* will be less than *EV* for a decrease in the quantity of a single good (Kim).

To obtain money metric measures for *CV* and *EV*, we simply multiply the normalized *CV* and *EV* values by total expenditure, yielding:

(19) $$CV = m^0\left[D(u^0,\underline{q}^1) - D(u^0,\underline{q}^0)\right]$$, and

(20) $$EV = m^0\left[D(u^1,\underline{q}^1) - D(u^1,\underline{q}^0)\right]$$,

where m^0 represents total base expenditure. By using equations (19) and (20) along with the estimated distance functions defined by (7) and (13)-(18), we are able to estimate welfare losses associated with an arbitrary reduction of the quantity landed for a particular finfish species. To calculate estimates of *CV* and *EV*, the semiflexible model with the smallest rank that is supported empirically is used. Therefore, the K = 5 SNQID-QSS model is used to obtain empirical estimates. The welfare loss estimates at the sample means and in 1996 are reported in Table 1.

6. Conclusions

In this paper, the methods of Ryan and Wales (1996) for estimating functional forms with quadratic Engel curves were applied to inverse demand systems in order to develop analogous systems with quadratic scale curves. Curvature restrictions were placed on the Antonelli matrix in order to satisfy economic theory, as well as allowing us to perform welfare evaluations. Because several eigenvalues of the Antonelli matrix are close to zero following the imposition of concavity, rank reduction is used to decrease the number of parameters estimated without significantly harming the fit of the models. The result of this rank reduction is a set of semiflexible inverse demands.

The model developed here was used to estimate a system of inverse demands for finfish landed commercially in the South Atlantic from 1980-1996. Importantly, we found empirical support for including quadratic scale terms; a Likelihood Ratio test of the model that maintained linear scale curves (i.e., Holt and Bishop's NQID) indicates that this model is clearly rejected. The empirical results also suggest that reduced rank models may be used without a significant loss in fit, with the K = 5 SNQID-QSS appearing to have the most empirical support. This model was used to obtain compensating and equivalent variation estimates associated with an arbitrary ten percent reduction in the quantity landed for individual species. Overall, it appears that including quadratic terms in inverse demand specifications offers an improvement in modeling systems in which quantities are taken as exogenous and may prove beneficial in future applications to inverse demand models.

Table 1. Compensating and Equivalent Variations for a 10 Percent Reduction in Catch for Selected Categories of South Atlantic Fish (K=5 SNQID-QSS Model)

Fish Category	CV($)	%CV[i]	EV($)	%EV	Total Value ($)
		1996			
Bluefish	1,262,361	1.98	1,281,407	2.01	13,049,400
Dolphinfish	1,289,856	2.02	1,308,752	2.05	10,017,600
Other Finfish	1,393,120	2.18	1,435,323	2.25	12,933,400
Flounder	1,194,401	1.87	1,219,327	1.91	14,274,400
Grouper	414,834	0.65	418,215	0.66	4,554,193
Scups	210,549	0.33	211,179	0.33	2,325,790
Trout	278,613	0.44	279,961	0.44	2,666,190
Snapper	265,493	0.42	266,440	0.42	2,642,197
Tile/Triggerfish	133,867	0.21	134,175	0.21	1,368,399
					63,831,569
		Sample Means			
Bluefish	1,276,728	2.17	1,298,053	2.21	12,593,953
Dolphinfish	1,110,701	1.89	1,126,181	1.92	11,339,527
Other Finfish	941,960	1.60	963,192	1.64	8,999,917
Flounder	1,037,957	1.77	1,058,963	1.80	9,968,914
Grouper	345,102	0.59	347,675	0.59	3,815,150
Scups	280,740	0.48	282,055	0.48	2,892,517
Trout	479,438	0.82	484,078	0.82	4,335,100
Snapper	281,571	0.48	282,699	0.48	2,997,860
Tile/Triggerfish	161,688	0.28	162,171	0.28	1,803,673
					58,746,611

[i] %CV and %EVdenote CV and EV respectively as a percentage of total expenditure on South Atlantic fish.

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