An Inverse Demand Approach to Recreation Fishing Site Choice and Implied Marginal Values

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Abstract

A distance function-motivated approach to the estimation of discrete recreation demand systems is proposed. The corresponding system of inverse demands leads naturally to a motivation of choice from among discrete consumption alternatives based on comparision of implicit price for the activity to actual market price. Systems of discrete choices for multiple activities (or recreation sites) are easily characterized and estimated as a difference in the multivariate cdf for the observed pattern of (integer-valued) trips. The model explains both participation (total number of trips) and the allocation of trips across sites within a coherent, utility-theoretic discrete choice framework. This helps resolve difficulties with recreation demand estimation, including the lack of consistency between discrete and continuous choice elements in the random utility framework, and the difficulty in imposing and evaluating preference restrictions such as weak complementarity between public and private goods.

A Distance Function Approach to Recreation Demand and Resource Valuation

Introduction

There has been substantial recent interest in the use of random utility models to characterize recreation demand decisions and welfare values, following the development of the approach by McFadden and its use in characterizing responses to contingent valuation surveys (Hanemann). Much of the appeal of the approach comes from the ease with which substitutes can be modeled and because of its natural fit with the decisionmaking behind discrete choices, which is the most realistic description of commodities that inherently-discrete commodities such as recreation trips. However, the approach has some limitations that have been hard to overcome when generalizing from a description of the process of choice made on a single choice occasion to models of choice over longer periods that include multiple choices. Perhaps the most arbitrary decision that must be made in this transition is the specification of the number of choice occasions a consumer has, because of the role played by the counterfactual alternative of taking no trips. Measurement of some variables applicable to choicemaking on single occasions (for example, the relevant budget constraint) can also be difficult, though these are second-order in importance compared to the choice occasion problem. Progress has been made in integrating participation (number of trips taken) models with site allocation models (the probability of visiting a given site), but there is not yet a model which rationalizes both within a utility-consistent structure (Parsons and Kealy; Hausman, McFadden, and Leonard; Smith 1996, 1997).

This paper proposes a model of recreation decisionmaking based on the consumer distance function and associated system of inverse demands. The model is fully utility consistent and can be defined for any length of decisionmaking period, thereby encompassing either single-occasion or multiple-occasion choices. The consumer decides on the number of trips to take to each of several sites during the choice period, based on a comparison of the implicit marginal prices of each site to their actual travel prices. While preferences for recreational activities are smooth and continuous, the quantities in which they come are integer-valued, resulting in the optimal number of trips to each site being determined as a discrete choice from among the available integer-valued alternatives to minimize the distance function. The model can be applied in either a single-site or multiple-site context.

The Modeling Approach

Both Hanemann and Cameron suggested alternative approaches to the evaluation of contingent valuation responses to proffered bids in contingent valuation studies of public good provision, based on the indirect utility function and the expenditure function, respectively. Using each approach it is easy to characterize the probability of observing a discrete response (yes or no), based on observed differences in the continous underlying primitive function and errors representing unobservables.

Optimal Consumption Choices in the Distance Function Framework

In contrast to these models, both of which are quantity-dependent, this paper develops an alternative motivation for discrete recreation choices based on the consumer's distance function and the associated system of inverse demands. Let d($\mathbf{x}$,u) be the consumer's distance function, with $\mathbf{x}=[x_1,...,x_n]$ a vector of consumption quantities and u the utility index. For a given set of consumption quantities $\mathbf{x}$, the distance function d($\mathbf{x}$,u) is defined as

$$d(\mathbf{x},u) = \min_{\mathbf{p}} \mathbf{p} \cdot \mathbf{x} + \phi[1 - e(\mathbf{p},u)] \tag{1}$$

where e($\mathbf{p}$,u) is the expenditure function from the price-independent version of the consumer choice model, $e(\mathbf{p},u) \equiv \min_{\mathbf{x}} \mathbf{p} \cdot \mathbf{x} + \lambda[u - u(\mathbf{x})]$, with prices $\mathbf{p}$ normalized on income M. The distance function is increasing, homogeneous of degree 1, and concave in **x**, and decreasing in u (Deaton and Muellbauer; Deaton; Kim). The solution to (1) defines the implicit prices $p_i(\mathbf{x},\mathrm{u})$, i=1,...,n, of all goods, and by the envelope theorem,

$$\partial \mathrm{d}(\mathbf{x},\mathrm{u})/\partial \mathrm{x}_i = p_i(\mathbf{x},\mathrm{u}) \tag{2}$$

yields the corresponding system of compensated inverse demands, analogously with the consumer's expenditure function and corresponding system of compensated direct demands. The correspondence between the distance function and the expenditure function is given by $\mathrm{e}(\mathbf{p},\mathrm{u}) \equiv \mathrm{M} \cdot \mathrm{d}(\mathbf{x}(\mathbf{p},\mathrm{u}),\mathrm{u})$. In the distance function formulation, the implicit "price" or marginal value of each good x_i is determined by consumption of x_i and all other goods.

The distance function can be used to tell an alternative, but equivalent, version of the standard consumer choice problem, where consumers face parametric prices **p** and choose quantities of each good to maximize their well-being. In Figure 1, at quantity x_1^1 of good 1, the consumer's marginal value of good 1 is $p_1(\mathrm{x}_1^1,\mathrm{x}_{-1},\mathrm{u})$, while the price of good 1 is p_1. Since $p_1(\mathrm{x}_1^1)$ exceeds p_1, the consumer gets a marginal surplus from consuming x_1^1, given consumption of all other goods x_{-1}. If consumption of good 1 were to increase, *ceteris paribus*, the consumer's welfare would increase, up to the point where $p_1(\mathrm{x}_1^1,\mathrm{x}_{-1},\mathrm{u}) = \mathrm{p}_1$ and marginal value of good 1 to the consumer just equals its cost (market price). Intuitively, the consumer's welfare would be maximized for the consumption bundle **x*** such that $p_i(\mathbf{x}^*,\mathrm{u}) = \mathrm{p}_i$ for all i. This, in fact, is the case, as can be seen from the primal-dual formulation of the consumer's distance function minimization problem (Silberberg),

$$\min_{\mathbf{x},\mathbf{p}} \ \mathbf{p} \cdot \mathbf{x} - \mathrm{d}(\mathbf{x},\mathrm{u}) + \psi[1 - \mathrm{e}(\mathbf{p},\mathrm{u})],$$

which is equivalent to

$$\mathcal{L} \equiv \max_{\mathbf{x},\mathbf{p}} \ \mathrm{d}(\mathbf{x},\mathrm{u}) - \{\mathbf{p} \cdot \mathbf{x} + \psi[1 - \mathrm{e}(\mathbf{p},\mathrm{u})]\}, \tag{3}$$

for which the necessary conditions with respect to **x** (comparative statics for parameters **x**) are

$$\partial \mathrm{d}(\mathbf{x^*},\mathrm{u})/\partial \mathrm{x}_i = \mathrm{p}_i \qquad \text{for all i.}$$ [1]

Equivalently, noting (2), one can write

$$p_i(\mathbf{x^*},\mathrm{u}) = \mathrm{p}_i \quad \text{for all i.}$$

That is, optimal quantity choices **x*** are those for which all implicit prices (marginal values) $p_i(\mathbf{x^*},\mathrm{u})$ of goods equal their marginal cost (market price p_i). The lower panel of Figure 1 illustrates the maximum of the primal-dual objective function $\mathcal{L}$ at x_1^*.

Note that this is a model of endogenous quantities (**x**) being chosen by a consumer responding to fixed, parametric prices of goods (**p**). The mechanism for that choice is a comparison of the implicit values of goods, resulting from selection of the vector **x**, with their resource cost to the consumer, their market prices **p**.

Choices with Discrete Consumption Alternatives

The logic of continuous consumer choices via the distance function extends readily to the case where consumption is integer-valued, as in recreation trips. The choice process is discussed first for the marginal decision of taking trips to a single recreation site during an observation period (e.g., a season or a year), then the model is generalized to consider joint choice of trips to multiple sites during that same period. The consumer is assumed to have continuous preferences for each of the goods, but they are available only in discrete quantities. The “market” price is the cost of access to the recreational activities at each site, or the travel cost

TC_j for each site j. The distance function-based choice process explains both the trips frequency and trips allocation decision within a single, coherent, utility maximization process.

Single Site Choice

Consider, first, the choice of how many trips to take to a single recreation site, site 1, conditional on given levels of trips taken to other sites x_{-1}. The consumer is assumed to have continuous preferences, but quantities that can be chosen are available only in integer values.

Continuing the analogy with consumer's surplus maximization, the optimal continuous choice is the value x_1^* for which, given $\mathbf{x}_{-1}$, the consumer's surplus area under the inverse demand for good 1 is maximized. However, only integer-valued consumption choices are available, and x_1^* will be one of these with probability zero. In Figure 2, it can be seen that the optimal discrete choice will be between trips x_1^1 and $x_1^2 = x_1^1 + 1$ such that

$$p_1(x_1^1,\mathbf{x}_{-1},u) > TC_1 > p_1(x_1^2,\mathbf{x}_{-1},u).^{2} \tag{4}$$

The reason is straightforward: trips x_1^1 dominates any smaller level of trips, because reducing trips taken below x_1^1 (e.g., to $x_1^1 - 1$) reduces net economic value. The cost of a trip saved (TC_1) is less than the value of the trip to the recreationist [$\int_{x_1^1-1}^{x_1^1} p_1(t,\mathbf{x}_{-1},u)dt$, or the area under the marginal value curve between $x_1^1 - 1$ and x_1^1]. Similarly, x_1^2 dominates all higher quantities of trips, because taking additional trips reduces net economic value: the cost of the additional trip (TC_1) exceeds its value [the area under $p_1(x_1,\mathbf{x}_{-1},u)$ from $x_1 = x_1^2$ to $x_1 = x_1^2 + 1$].

What the researcher *observes* is the actual number of trips taken, x_1^0, to site 1, along with the trips taken to other sites, x_{-1}^0. What is not known is whether x_1^0 was the best outcome in a comparison between x_1^0 and $x_1^0 - 1$, or in a comparison between x_1^0 and $x_1^0 + 1$. Thus, appending an additive, symmetric, zero-centered error to the marginal value of trips to site 1, the following probability statement can be made:

$$\text{Prob }[\mathrm{x}_1^0 \text{ chosen}] = \text{Prob}[p_1(\mathrm{x}_1^0 - 1,\mathbf{x}_{-1}^0,\mathrm{u}) + \epsilon_1 > \mathrm{TC}_1 > p_1(\mathrm{x}_1^0 + 1,\mathbf{x}_{-1}^0,\mathrm{u}) + \epsilon_1]$$

$$= \text{Prob}[\epsilon_1 > \mathrm{TC}_1 - p_1(\mathrm{x}_1^0 - 1,\mathbf{x}_{-1}^0,\mathrm{u}) \textit{ and } \epsilon_1 < \mathrm{TC}_1 - p_1(\mathrm{x}_1^0 + 1,\mathbf{x}_{-1}^0,\mathrm{u})]$$

$$= \mathrm{F}[p_1(\mathrm{x}_1^0 - 1,\mathbf{x}_{-1}^0,\mathrm{u}) - \mathrm{TC}_1] - \mathrm{F}[\mathrm{TC}_1 - p_1(\mathrm{x}_1^0 + 1,\mathbf{x}_{-1}^0,\mathrm{u})] \tag{5}$$

where F[$\cdot$] is the cdf of ϵ_1. This is a familiar form used frequently in double-bounded dichotomous choice CVM studies. If, alternatively, the errors associated with $p_1(\mathrm{x}_1^0 - 1,\mathbf{x}_{-1}^0,\mathrm{u})$ and $p_1(\mathrm{x}_1^0 + 1,\mathbf{x}_{-1}^0,\mathrm{u})$ were different (say ϵ_1 and ϵ_2) and assumed to be jointly normally distributed, a bivariate probit model would result.

To see how (5) can be estimated in a manner parallel to what is done in the existing literature, let the distance function be parameterized in a generalized quadratic form, as

$$\mathrm{d}(\mathbf{x},\mathrm{u}) = \alpha_0 + \sum_i \gamma_j \cdot \mathrm{x}_j + ½\sum_i\sum_j \gamma_{ij} \cdot \mathrm{x}_i \cdot \mathrm{x}_j + \mathrm{u} \cdot e^{\sum_i \beta_i \mathrm{x}_i} \tag{6}$$

with corresponding inverse compensated demands of the form

$$p_i(\mathbf{x},u) = \gamma_i + \sum_j \gamma_{ij} \cdot \mathrm{x}_j + \beta_i \cdot \mathrm{u} \cdot e^{\sum_i \beta_i \mathrm{x}_i}, \qquad \text{for i=1,...,n.} \tag{7}$$

To initialize the model, note that $\mathrm{d}(\mathbf{x},\mathrm{u}) = 1$ when all quantities are chosen optimally, which is the maintained hypothesis of the model; substituting this into equation (6), the term including utility is

$$\mathrm{u} \cdot e^{\sum_i \beta_i \mathrm{x}_i^0} = \{1 - [\alpha_0 + \sum_i \gamma_j \cdot \mathrm{x}_j^0 + ½\sum_i\sum_j \gamma_{ij} \cdot \mathrm{x}_i^0 \cdot \mathrm{x}_j^0]\}. \tag{8}$$

given the pattern of trips to all sites $\mathbf{x}^0 = [x_1^0,...,x_n^0]$ observed for the individual. Using (8) in (7), the inverse demand system expressed in terms of observables is

$$p_i(\mathbf{x}^0) = \gamma_i + \sum_j \gamma_{ij} \cdot x_j^0 + \beta_i \cdot \{1 - [\alpha_0 + \sum_i \gamma_j \cdot x_j^0 + ½ \sum_i \sum_j \gamma_{ij} \cdot x_i^0 \cdot x_j^0]\}, \text{ for i=1,...,n.} \quad (9)$$

Using (9) in (5), the probability statement for the observed trips x_1^0 to site 1, given $\mathbf{x}^0_{-1}$, is

$$\text{Prob } [x_1^0 | \mathbf{x}^0_{-1}] = \Phi[\gamma_1 + \gamma_{11}(x_1^0\text{-}1) + \sum_{j\neq 1} \gamma_{1j} x_j^0 + \beta_1 \{1 - [\alpha_0 + \gamma_{11}(x_1^0\text{-}1)$$
$$+ \sum_{j\neq 1} \gamma_{1j} x_j^0 + ½\gamma_{11}(x_1^0\text{-}1)^2 + \sum_{j\neq 1} \gamma_{1j} x_1^0 x_j^0 + ½ \sum_{i\neq 1} \sum_{j\neq 1} \gamma_{ij} x_i^0 x_j^0]\} - TC_1]$$

$$- \Phi[TC_1 - \{\gamma_1 + \gamma_{11}(x_1^0\text{+}1) + \sum_{j\neq 1} \gamma_{1j} x_j^0 + \beta_1 \{1 - [\alpha_0 + \gamma_{11}(x_1^0\text{+}1)$$
$$+ \sum_{j\neq 1} \gamma_{1j} x_j^0 + ½\gamma_{11}(x_1^0\text{+}1)^2 + \sum_{j\neq 1} \gamma_{1j} x_1^0 x_j^0 + ½ \sum_{i\neq 1} \sum_{j\neq 1} \gamma_{ij} x_i^0 x_j^0]\}\}]. \quad (10)$$

While (10) looks somewhat cumbersome, it should be pointed out that the only difference in the terms being subtracted is that one is evaluated at $x_1^0 + 1$ and the other is evaluated at $x_1^0 - 1$. If the single-site model is estimated without any information about other trips $\mathbf{x}^0_{-1}$ taken to other sites, a simpler form results. In this case, the implicit price function for site 1 is

$$p_1(\mathbf{x}^0) = \gamma_1 + \gamma_{11} \cdot x_1 + \beta_1 \cdot \{1 - [\alpha_0 + \gamma_1 \cdot x_1 + ½ \cdot \gamma_{11} \cdot (x_1)^2]\},$$

$$= [\gamma_1 + \beta_1 \cdot (1 - \alpha_0)] + (\gamma_{11} + \beta_1 \cdot \gamma_1) \cdot x_1 + ½ \cdot \beta_1 \cdot \gamma_{11} \cdot (x_1)^2]\}$$

$$= \gamma_1' + \gamma_{11}' \cdot x_1 + ½ \cdot \gamma_{11}'' \cdot (x_1)^2$$

where $\gamma' = [\gamma_1 + \beta_1 \cdot (1 - \alpha_0)]$, $\gamma'_{11} = (\gamma_{11} + \beta_1 \cdot \gamma_1)$, and $\gamma''_{11} = \beta_1 \cdot \gamma_{11}$. In the single site model with only information on trips to that site, not all parameters of the distance function are identified, and α_0 cannot be identified.

For this simpler case, the probability of observing x_1^0 is

$$\text{Prob}\,[x_1^0] = \Phi[\gamma'_1 + \gamma'_{11}(x_1^0\text{-}1) + ½ \cdot \gamma''_{11}(x_1^0\text{-}1)^2 - \text{TC}_1] - \Phi[\text{TC}_1 - \{\gamma'_1 + \gamma'_{11}(x_1^0\text{+}1) + ½ \cdot \gamma''_{11}(x_1^0\text{+}1)^2\}]. \tag{11}$$

Multiple Site Choice

While the distance function approach provides an alternative way of characterizing single-site demands, its real appeal is in its characterization of multiple-site demands. The model encompasses the two elements that have been notoriously hard to reconcile within utility-theoretic random utility model frameworks: the participation or total trips decision, and the site allocation decision. The present model provides a means of making probability statements about patterns of trips distributed among multiple sites for an observation period of arbitrary length, whether a year, a season, or a "single choice occasion" in the random utility framework, which is a special case when only one trip is taken to all sites.

The probability of observing a pattern of trips $\mathbf{x}^0 \equiv [x_1^0,...,x_n^0]$ can be stated concisely as the multivariate extension of equation (5),

$$\begin{aligned}\text{Prob}\,[\mathbf{x}^0 \text{ chosen}] &= \text{Prob}[p_1(\mathbf{x}^0\text{-}\mathbf{1},\text{u}) + \epsilon_1 > \text{TC}_1 > p_1(\mathbf{x}^0\text{+}\mathbf{1},\text{u}) + \epsilon_1,..., \\ &\qquad p_n(\mathbf{x}^0\text{-}\mathbf{1},\text{u}) + \epsilon_n > \text{TC}_n > p_n(\mathbf{x}^0\text{+}\mathbf{1},\text{u}) + \epsilon_n] \\ &= \text{F}[p_1(\mathbf{x}^0\text{-}\mathbf{1},\text{u}) - \text{TC}_1, ..., p_n(\mathbf{x}^0\text{-}\mathbf{1},\text{u}) - \text{TC}_n] \\ &\quad - \text{F}[\text{TC}_1 - p_1(\mathbf{x}^0\text{+}\mathbf{1},\mathbf{x}^0_{-1},\text{u}), ..., \text{TC}_n - p_n(\mathbf{x}^0\text{+}\mathbf{1},\mathbf{x}^0_{-1},\text{u})]\end{aligned} \tag{12}$$

where **1** is the unit n-vector. The arguments of the cdfs are differences between the market price (travel cost TC$_j$) and implicit price of trips (p_j) to each site j $=$ 1,...,n. Estimation of this model raises dimensionality issues similar to those which arise in multinomial probit models. Recent advances in estimation by simulation have rendered these problems tractable (e.g., Hajivassiliou; Hajivassiliou and Ruud). The advantage of multinomial probits in the distance function approach, however, over multinomial probits used in random utility, is that the model explains both participation and site allocation jointly within a single utility function.

Consumption of Single Trips per Period

When the total number of trips taken during the observation period is 1, the model takes a form similar to the random utility model. Since trips are, by definition, non-negative integer valued, the recreationist's choice is between 0 and 1 trips for all sites, with one site (k) selected. For this site, it must be true that $p_k(1,\cdot,\mathrm{u}) + \epsilon_k > \mathrm{TC}_k$, while for all other sites j $\neq$ k it must be true that $\mathrm{TC}_j > p_j(1,\cdot,\mathrm{u}) + \epsilon_j$. The probability statement simplifies to

$$\text{Prob [k chosen]} = \text{Prob}[\mathrm{TC}_1 > p_1(1,\cdot,\mathrm{u}) + \epsilon_1,\ldots,p_k(1,\cdot,\mathrm{u}) + \epsilon_k > \mathrm{TC}_k,\ldots,$$
$$\mathrm{TC}_n > p_n(1,\cdot,\mathrm{u}) + \epsilon_n]$$

$$= \mathrm{F}[\mathrm{TC}_1 - p_1(1,\cdot,\mathrm{u}), \ldots, p_k(1,\cdot,\mathrm{u}) - \mathrm{TC}_k,\ldots,\mathrm{TC}_n - p_n(1,\cdot,\mathrm{u})]$$

given symmetry of the multivariate distribution. While the functional form in the cdf arguments take a different form, one is left with a similar probability statement for observation of the location of a single trip. This is, of course, just a special case of the more general distance function formulation that explains the likelihood of observing multiple trips to all sites.

Conclusions

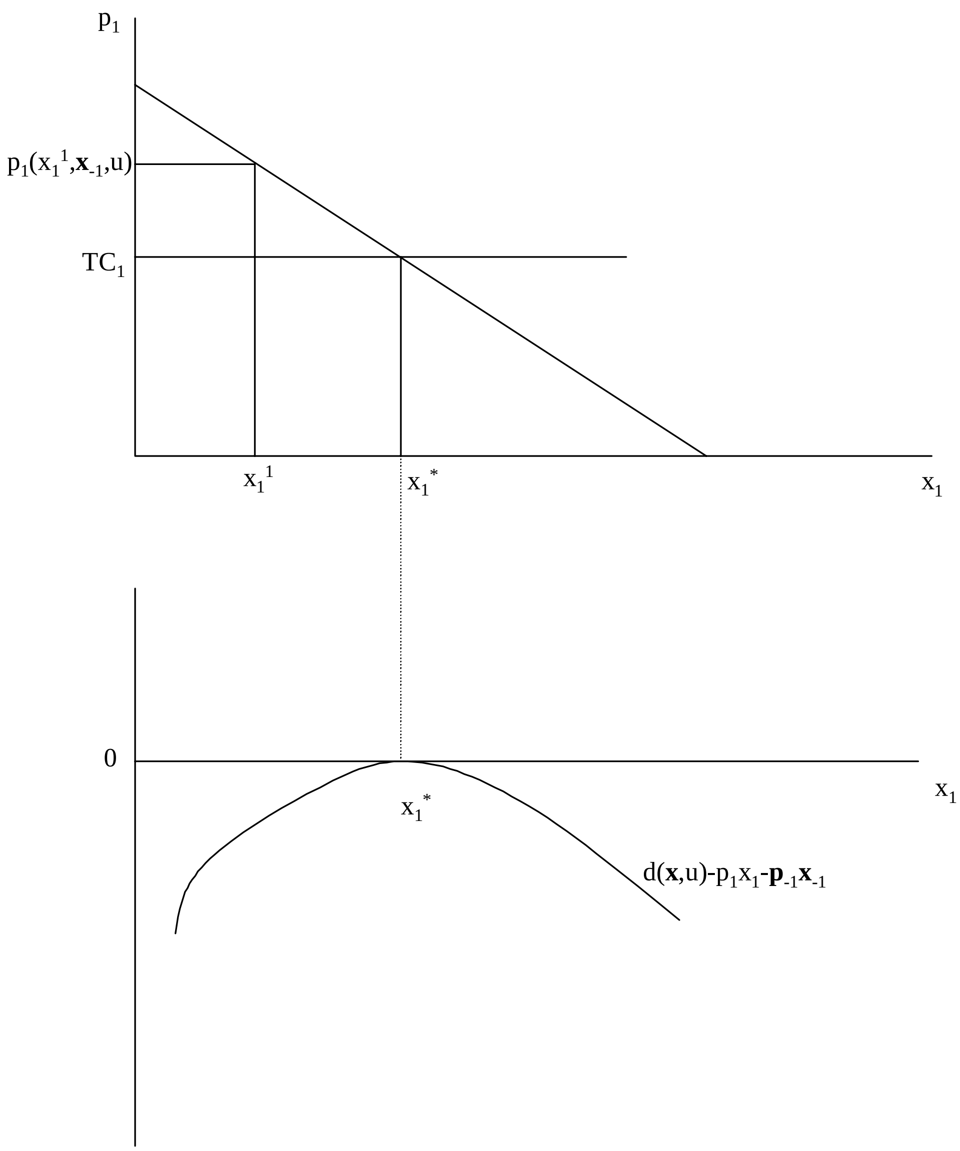
This paper has developed an alternative, utility-theoretic, approach to the determination of the number of trips taken and their allocation among sites in recreation demand analysis. The approach is based on the consumer's distance function and associated system of inverse demands for recreation at each site. The recreationist compares the implicit price, or marginal value, of each good (i.e., actitivity at each site) to its "market" price (the fixed travel cost of access) and chooses trips to all sites to equate (as nearly as possible given the discreteness of alternatives) the two. Because trips are integer-valued, the recreationist's decision is a discrete choice problem, the solution to which yields the number of trips to take to each site in the choice set per period. Because the period of observation can be any length, the model explains both the number of trips taken and their allocation to different sites. The model reconciles both the participation and site allocation decisions within a single, utility-theoretic framework.

Footnotes

1. The necessary conditions for **p**, $x_i = x_i^h(\mathbf{p},u)$, state that for the market price vector **p** and utility level u, the quantities **x** in the distance function should be the Hicksian demands $\mathbf{x}^h(\mathbf{p},u)$.

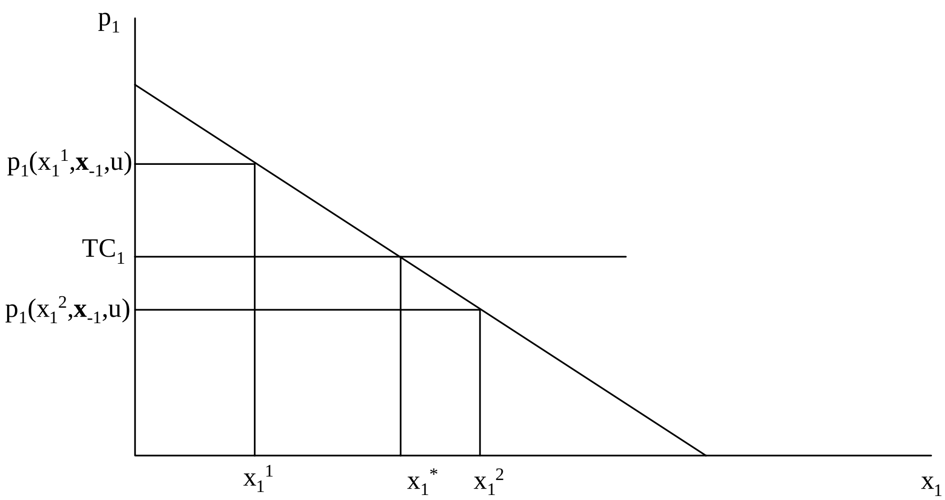
2. Or, equivalently, $x_1^1 < x_1^* < x_1^1 + 1$.

Figure 1. (a) The Choice of Trips When Trips Are Continuous



(b) The primal-dual distance objective function

Figure 2. The Optimal Discrete Choice of Trips



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