Selected Paper

Dynamic Analysis with Time Series Models: Simulation and Empirical Evidence

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Abstract.

The performance of the FPE, AIC, HQ and SC criteria in choosing lag-length, and the effect on the impulse-response functions, are studied in a Monte Carlo simulation. The experiments include stationary, cointegrated, and mixed unit root VAR and MA cases.

Keywords: Statistical selection criteria, cointegration, mixed unit roots, impulse response functions, small sample properties.

I. Introduction

Impulse response functions (IRF) are often used in agricultural economics to study price dynamics, market integration, and linkages between the macroeconomy and the agriculture, among other areas. However, macroeconomic data sets for the postwar era are comparatively short, which has led observers to question the statistical reliability of impulse response estimates from unrestricted vector autoregressions --VARs (Lutz Kilian, 1998).

Confidence bands for impulse response estimates are often based on Lütkepohl's (1990) asymptotic normal approximation, Runkles's (1987) nonparametric bootstrap method, or the parametric Monte Carlo integration procedure of Doan (1990). More over, if the variables in the system are cointegrated, the procedure described by Lütkepohl and Reimers (1992) can be followed and which verifies well known asymptotic properties. In the presence of unit roots (or mixed unit roots), Phillips (1998) describes the impulse response asymptotic distribution which closely resembles the cointegrated case.

Although the recent developments in time-series econometrics (units-roots, cointegration, etc.) have helped improve our understanding of nonstationarity and its role in time series modeling, however some questions remain. One that has not been studied much relates to the effect of chosen lag-length through alternative statistical selection criteria (SSC). More importantly, limited work is available on this issue and its relationship to various orders of integration when modeling a system such as a vector autoregression (VAR). Thus, the main objective of this paper is to present and discuss the results of some relevant empirical evidence attained through Monte Carlo experiments, in choosing lag-length through alternative SSC and its effect on the IRFs. The “what if” scenarios considered, due to their empirical relevance for VARs, are: (a) all I(0) series, (b) all I(1) series, and (c) some I(0) and others I(1) series.

The paper is organized as follows. Section II describes the notation and models adopted, Section III introduces details of the simulation design. Section IV presents the results in two separate subsections, one related to the SSC results and the other on the small sample properties of IRFs in presence of mixed unit roots. Last Section V is discussion.

II. Notation and main results used.

The VAR model

To focus the discussion and establish notation, first a VAR model is described. Consider a k-variate data series y of length $T+p$, generated by a covariance stationary VAR(p) process with intercept v,

$$y_t = v + A_1\, y_{t-1} + \ldots + A_p\, y_{t-p} + u_t\ , \qquad (1)$$

where the lag order p is assumed to be finite and known, u_t is a identically and independently distributed white noise disturbance, with mean zero and covariance matrix S_u.

Criteria for VAR Order Selection

When the lag order p is unknown, and if forecasting (for analysis purposes) is the objective, it makes sense to choose the order p such that a measure of forecast precision is minimized (Lütkepohl, 1993). Akaike (1969) suggested to base the VAR choice on the approximate 1-step forecast mean squared error (MSE) $S_{\hat{y}}(1)=\frac{T+Km+1}{T}\, S_u$, where m denotes the order of the VAR process fitted to the data. To make this criterion operational Akaike suggests using the LS estimator for S_u,with degrees of freedom adjustment, and taking the determinant of the resulting expression. The resulting criteria is called the *final prediction error* (FPE) *criterion*, that is,

$$\text{FPE}(m) = \left(\frac{T+Km+1}{T-km-1}\text{a}\right)^K det(\tilde{S}_u), \qquad (2)$$

where $\tilde{S}_u$ stands for the MLE estimate of S_u (see for instance, Lütkepohl (1993), eq 4.2.11).

Based on the FPE criterion the estimate $\hat{p}$(FPE) of p is chosen such that,

$$\text{FPE}[\hat{p}(\text{FPE})] = min\{\text{FPE}(m)|m = 0, 1, \ldots, M\}. \qquad (3)$$

That is, VAR models of orders m=0, 1, …, M are estimate and the corresponding FPE(m) values are computed. The order minimizing the FPE values is the chosen as estimate for p.

Akaike (1974), based on a quite different reasoning, derived a similar criterion, abbreviated as AIC (*Akaike's Information Criteria*). For a VAR(m) process this criterion is

$$\text{AIC}(m) = ln\,|\tilde{S}_u(m)| + \frac{2mK^2}{T}. \qquad (4)$$

The estimate $\hat{p}$(AIC) for p is chosen so that this criterion is minimized.

Before presenting other two criteria, it is worth mentioning that the limiting probability for underestimating the VAR order is zero for both $\hat{p}$(AIC) and $\hat{p}$(FPE) so that they overestimate the true order positive probability. However, the limiting probability for overestimating the order declines with increasing dimension K, and is negligible for $K \geq 5$ (Paulsen and Tjostheim, 1985).

Two consistent criteria that have been quite popular in recent applied work are the HQ (Hannan and Quinn, 1979) and the SC (Schwarz, 1978) criteria, defined as

$$\text{HQ}(m) = ln\,|\tilde{S}_u(m)| + \frac{2\,ln\,ln\,T}{T}\,mK^2, \qquad (5)$$

and

$$\text{SC}(m) = ln\,|\tilde{S}_u(m)| + \frac{ln\,T}{T}\,mK^2, \qquad (6)$$

respectively. As before, the order $\hat{p}$(HQ) is such that minimizes HQ(m) for m=0,1,…, M, and $\hat{p}$(SC) the one that minimized SC(m) for m=0,1,…, M.

The impulse response functions

The system's responses to disturbances (expressed in reduced-form), are obtained by the recursion

$$\Phi_i = \sum_{j=1}^{i} \Phi_{i-j}\,A_j, \quad i=1,\,2,\,\ldots \qquad (7)$$

where $\Phi_0 = I_T$ and $A_j = \mathbf{0}$ for $j>p$.

The orthogonal impulse responses are defined as

$$\Theta_i = \Phi_i\,P, \qquad i=0,\,1,\,\ldots \qquad (8)$$

where P is the lower triangular matrix of the Cholesky decomposition, thus satisfying $PP'=S_u$.

The functions of interest are the entries of the orthogonal impulse responses Θ_i, say $\theta_{lm,i}$, and are interpreted as the response of variable l to a one-time impulse in variable m, i periods ago.

The asymptotic distributions of Impulse Responses

Before concentrating in the small-sample statistical properties of the impulse response functions, it is important to introduce the asymptotics distributions of the IRFs under different VARs specifications.

If the VAR(p) data generating process (DGP) is stationary and stable, and if

$$\sqrt{T}\begin{pmatrix}\hat{\alpha}-\alpha \\ \hat{\sigma}-\sigma\end{pmatrix} \xrightarrow{d} N\left(\mathbf{0}\ ;\begin{pmatrix} S_{\hat{a}} & 0 \\ 0 & S_{\hat{s}}\end{pmatrix}\right)$$

then (Lütkepohl, 1990)

$$\sqrt{T}\, vec(\hat{\Phi}_i-\Phi_i) \xrightarrow{d} N(\mathbf{0},\, G_i S_{\hat{a}} G_i'), i=1, 2,\ldots \qquad (9)$$

and

$$\sqrt{T}\, vec(\hat{\Theta}_i-\Theta_i) \xrightarrow{d} N(\mathbf{0},\, C_i S_{\hat{a}} C_i' + \bar{C}_i S_{\hat{s}} \bar{C}_i), \qquad i=1, 2,\ldots \qquad (10)$$

where the hat on a parameter represents the unrestricted least square estimate of that parameter, $\alpha=vec(A_1, \ldots, A_1)$, *vec* denotes the column stacking operator, $\sigma=vech(S_u)$, *vech* is the operator that stacks the elements on and below the diagonal only, $Gi = \frac{\partial\, vec(\Phi_i)}{\partial \alpha'}$, $C_0 = \mathbf{0}$, $C_i = (P'\otimes I_K)G_i$, i=1, 2, …, $\bar{C}_i = (I_K\otimes \Phi_i)H$, i=0, 1, 2, …, and $H=\frac{\partial\, vec(P)}{\partial \sigma'}$.

For a researcher who does not know the true structure of the DGP it is possible to base an impulse response analysis on an approximating finite order VAR process. The consequences of such an approach can be found in Lütkepohl (1993, pp314). It the VAR(p) DGP is cointegrated or having unit roots or near unit roots, equivalent expressions to the stationary case (equations (0 and (10)) can be found in Lütkepohl and Reimers (1992) and Phillips (1998) respectively.

III. The simulation design.

A Monte Carlo experiment is designed to study the performance of SSC in choosing lag-length for various sample sizes. Additionally, stochastic properties of the simulated bi-variate series cover scenarios that are reported in empirical work: a) stationarity, b) integrated series with cointegration, and c) mixed unit roots. The performance of IRFs for each of these scenarios in classical VAR and MA effects is evaluated by comparing the asymptotic IRFs and their standard errors to the simulated ones. Table 1 contains all simulated model structures.

Part I: Evaluating the statistical selection criteria

The performance of the FPE, AIC, HQ, and SC criteria is evaluated for the following model specifications: (i) Stationary VAR (StatVAR), (ii) Stationary MA (StatMA), (iii) Cointegrated VAR (CoiVAR), (iv) Cointegrated MA (CoiMA), (v) Mixed Unit Roots VAR (MixVAR), (vi) Mixed Unit Roots MA (MixMA). These six variations combined with four different sample sizes of T=25, 50, 75, and 200, give a total of twenty-four scenarios; each scenario is replicated 1000 times.

Table 1: Population models adopted for the six variations considered, with K=2, and T=25, 50, 75, 200. For the VAR specifications p=2, for the MA specifications p=50.

Model	Specifications
StatVAR[1]	$y_t = v + A_1 y_{t-1} + A_2 y_{t-2} + u_t, \quad t=1,\ldots,T$ $u_t \sim N\left(\begin{bmatrix}0\\0\end{bmatrix};S_u\right), S_u=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, v=\begin{bmatrix}0\\0\end{bmatrix}, A_1=\begin{bmatrix}-.319 & .959\\.044 & -.264\end{bmatrix}, A_2=\begin{bmatrix}-.160 & .932\\.050 & -.022\end{bmatrix}.$
StatMA[2]	$y_t = M_1 u_{t-1} + u_t = \Sigma_{\infty}^{i=1} A_i y_{t-i} + u_t, \quad t=1,\ldots,T$ $u_t \sim N\left(\begin{bmatrix}0\\0\end{bmatrix};S_u\right), S_u=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, M_1=\begin{bmatrix}-.310711 & .8355649\\-.00978 & -.275499\end{bmatrix}, A_i=-(-M_1)^i$
CoiVAR[3]	$y_t = v + A_1 y_{t-1} + A_2 y_{t-2} + u_t, \quad t=1,\ldots,T$ $u_t \sim N\left(\begin{bmatrix}0\\0\end{bmatrix};S_u\right), S_u=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, A_1=\begin{bmatrix}.5 & .3\\.3 & .5\end{bmatrix}, A_2=\begin{bmatrix}-.5 & -.3\\-.3 & .5\end{bmatrix}.$ Note that $\lvert I_k - A_1 - A_2\rvert=0$.
CoiMA[4]	$y_t = y_o + \delta t + \psi(1)\left(\Sigma_{\infty}^{i=1} e_i\right) + h_t - h_o, \quad t=1,\ldots,T$ $e_i \sim N\left(\begin{bmatrix}0\\0\end{bmatrix};I_k\right)$ independent of $h_t \sim N\left(\begin{bmatrix}0\\0\end{bmatrix};I_k\right)$; $\delta = \begin{bmatrix}1\\0.8670793\end{bmatrix}$ is such that $A'\delta=0$, with A a matrix verifying that $A'\psi(1)=0$; and $\psi(1) = I_K + M_1$, with $\lvert\psi(1)\rvert=0$, $M_1 = \begin{bmatrix}-.310711 & .8355649\\.5976682 & -.275499\end{bmatrix}.$
MixVAR[5]	$y_t = v + A_1 y_{t-1} + A_2 y_{t-2} + u_t, \quad t=1,\ldots,T$ $u_t \sim N\left(\begin{bmatrix}0\\0\end{bmatrix};S_u\right), S_u=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, v=\begin{bmatrix}0\\0\end{bmatrix}, A_1=\begin{bmatrix}1 & 0\\0 & 0.3\end{bmatrix}, A_2=\begin{bmatrix}-.1 & 0\\0 & -0.3\end{bmatrix}.$
MixMA[6]	$y_t = y_o + \delta t + \psi(1)\left(\Sigma_{\infty}^{i=1} e_i\right) + h_t - h_o, \quad t=1,\ldots,T$ e_i and h_t as in CoiMA; $\delta=\begin{bmatrix}0.7\\0\end{bmatrix}$; $\psi(1)=I_K + M_1$, $\lvert\psi(1)\rvert\neq 0$, $M_1=\begin{bmatrix}.9925923 & -.067217\\.1475988 & .159295\end{bmatrix}$

[1] The specification for this variation closes resembles the example in Lütkepohl (1993), pp 79.
[2] The specification for the matrix M_1 of this variation is derived from the MA representation of the VAR(2) specification in the StatVAR case, as in Lütkepohl (1993), pp. 28, eq. [6.2.2].
[3] This specification follows Hamilton (1994), pp. 580, eq. [19.1.41]. In the Monte Carlo sample generation process, the Johansen-Juselius (1990) test was performed to test the hypothesis of cointegration. If rejected, that sample was discharged (co-integration pre-test).
[4] This specification follows Hamilton (1994), eq. [19.1.13], pp. 575. Monte Carlo samples were also pre-tested .
[5] This specification follows Phillips (1998). The Monte Carlo samples were tested with the Dickey-Fuller (1979) test for the presence of unit roots.
[6] Note that, for this specification, setting the second entry of the vector δ equals to zero makes y_{2t} be stationary, while y_{2t} still has a unit roots. The Monte Carlo samples are also accordingly pre-tested.

Part II: Small-sample properties of IRFs

This second part of the study evaluates the effective coverage rate of the nominal 95% asymptotic confidence intervals for the orthogonal IRFs (Lütkepohl and Reimers, 1992) and compares them to the Monte Carlo integration intervals. The confidence intervals are constructed for samples of size T=25, 50, 75 and 200, while the Monte Carlo experiments are based on 1000 replications.

The effective coverage is defined as the relative frequency at which the confidence interval covers the true. The true orthogonal IRFs are derived, accordingly, from the models described in Table 1. As the simulation results are based on 1000 trials, then the Monte Carlo standard error for the coverage estimate are less than 0.01 (Kilian, 1998).

In the framework provided by the test of hypothesis theory, errors Type I and II play an important role. The effective coverage rates can be interpreted as an estimate of the probability of committing one or the other of these errors. If the model adopted is the “correct” one, then the effective coverage rate provides an estimate of the error Type I probability, meanwhile if the adopted model is “wrong” the coverage rate provides an estimate of the error Type II probability. Table II describes the different experiments considered to obtain effective coverage rates to estimate the probabilities of Type I and II errors.

For the case where the adopted model is STAT, it is meant that the estimation procedure adopted is the standard LS procedure, with distribution of the orthogonal IRFs estimates as described in equation (10). For the cases COI and MIX, the estimation procedure adopted is the restricted maximum likelihood, with distributions as in Lütkepohl and Reimers (1992) and Phillips (1998). In all cases, it is assumed that the right lag-length has been chosen.

Table 2. Experiments considered for the evaluation of the orthogonal IRFs via the effective coverage rates.

	MODEL ADOPTED		
TRUE	Stationary (STAT)	Cointegrated (COI)	Mixed Unit Roots (MIX)
STAT	STAT:STAT(*)		
COI	COI:STAT(**)	COI:COI(*)	
MIX	MIX:STAT(**)	MIX:COI(**)	MIX:MIX(*)

A (*) indicates a correct decision in adopting a model for inferences purposes; the effective coverage rate in that cases estimates the probability of the error Type I. A (**), on the contrary, indicates that the efective coverage rate estimates the probability of the error Type II.

The cases STAT:COI and STAT:MIX where not considered in this study because they are situations that in practice does not occur, meanwhile the cases COI:STAT and MIX:STAT are the most frequent mistakes observed in the real world of the VARs applications.

IV. Results

On the performance of the SSC

We present the results of a bivariate model with a true lag-length p=2 for the VAR and p=1 for the MA specifications. The main results on the use of SSC to choose lag-length are presented in Table 3, with stationarity (upper block), cointegration (middle block), and mixed unit roots (bottom block). We summarize the main results as follows. The stationary case, which is the benchmark, generates the expected results; all SSC choose the correct lag-length for the VAR(2), and at least the correct, or an extended, lag-length for the MA(1). This result is consistent with other simulation works. The results are encouraging. All SSC chose an appropriate lag-length around the true; to our knowledge this is first hand simulation evidence in experiments that use mixed unit roots as the general setup. For the most complex cases, the mixed unit roots, we were surprised to find that all SSC remained parsimonious, particularly so for the MA(1) effects; however, note that the nominal size increases in all cases, and that this is also true for all experiments where there is a MA(1) effect.

On the small sample properties of the IRFs when unit roots are present

While this experiment was in progress, Phillips (1998) published some theoretical results and Monte Carlo evidence on IRFs for non-stationary models. One important addition in our experiment is that we fixed the lag-length to the one most frequently identified by the SSC (p=2 for the VAR models, and p=1 if MA effects are present) in the previous section. To save space, we only report that the ***stationary*** case results in this experiments reproduce the well known theory as in Lütkepohl (1993), that is for a bi-variate VAR the coverage rates for the simulated confidence intervals were almost identical to the asymptotic ones, and sometimes smaller. For instance, the IRF of y_2 on y_1 had coverage rates of (0.043, 0.045, 0.052) for one period ahead at samples of sizes 25, 75 and 200; increasing the horizon to nineteen periods, generated coverage rates of size (0.009, 0.067, 0.153) for the three sample sizes. When MA effects are present in the VAR models, the SSC most frequently identified a lag-length of k=1; this is an unexpected result given that MA effects imply infinite AR lags. It is also found that the coverage rates deteriorate very quickly as the number of IRs is increased. For example, for the same cases discussed above, the set of coverage rates is (0.077, 0.111, 0.227) and (1, 1, 1).

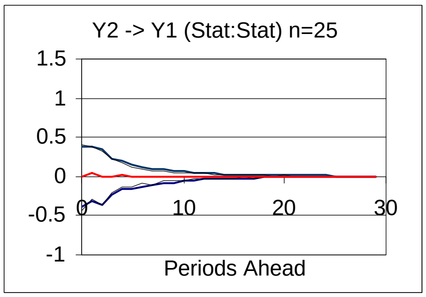
Table 3. Lag-length chosen by the SSC, sample size T=75. Numbers in parentheses are percentages on 1000 replications.

Model	FPE	AIC	HQ	SC	Property
		Sample Size:	***n*=25**		
VAR(2)	2 (56.5)	2 (45.5)	2 (50.3)	2 (51.8)	Stationary
	3 (12.8)	5 (23.0)	5 (16.8)	0 (22.5)	
MA(1)	1 (49.5)	1 (44.2)	1 (56.3)	1 (61.0)	Stationary
	2 (23.9)	2 (21.5)	2 (18.1)	0 (28.1)	
VAR(2)	2 (57.2)	2 (44.8)	2 (50.9)	2 (58.7)	Cointegrated
	3 (12.6)	5 (26.5)	5 (20.3)	2 (20.6)	
MA(1)	1 (59.5)	1 (50.1)	1 (68.5)	1 (91.3)	Cointegrated
	2 (15.8)	6 (15.8)	2 (12.1)	2 (6.1)	
VAR(2)	2 (45.7)	2 (35.8)	2 (40.7)	2 (54.5)	Mixed Unit Roots
	3 (12.8	5 (28.5)	5 (22.0)	1 (27.3)	
MA(1)	1 (64.1)	1 (54.2)	1 (72.3)	1 (92.9)	Mixed Unit Roots
	2 (12.7)	6 (17.5)	2 (9.4)	2 (4.2)	
		Sample Size:	***n*=75**		
VAR(2)	2 (84.6)	2 (83.7)	2 (96.9)	2 (99.3)	Stationary
	3 (8.4)	3 (8.6)	3 (2.4)		
MA(1)	2 (50.8)	2 (49.6)	1 (48.6)	1 (78.6)	Stationary
	1 (20.3)	1 (19.9)	2 (44.4)	2 (20.3)	
VAR(2)	2 (86.0)	2 (85.2)	2 (96.5)	2 (99.5)	Cointegrated
	3 (9.1)	3 (9.2)	3 (3.1)	1 (0.5)	
MA(1)	1 (58.2)	1 (56.5)	1 (83.4)	1 (95.9)	Cointegrated
	2 (30.1)	2 (29.1)	2 (14.7)	2 (4.1)	
VAR(2)	2 (81.7)	2 (81.3)	2 (95.7)	2 (99.8)	
	3 (10.6)	3 (10.6)	3 (3.6)	3 (0.2)	Mixed Unit Roots
MA(1)	1 (52.9)	1 (52.3)	1 (81.0)	1 (95.6)	
	2 (33.0)	2 (31.7)	2 (17.4)	2 (4.4)	Mixed Unit Roots
		Sample Size:	***n*=200**		
VAR(2)	2 (88.5)	2 (88.5)	2 (98.6)	2 (99.8)	Stationary
	3 (7.2)	3 (7.2)	3 (1.3)	3(0.2)	
MA(1)	2 (49.5)	2 (49.5)	2 (79.4)	2 (67.7)	Stationary
	3 (36.1)	3 (35.8)	3 (13.8)	1 (31.2)	
VAR(2)	2 (87.6)	2 (87.4)	2 (99.2)	2 (99.9)	Cointegrated
	3 (8.9)	3 (9.0)	3 (0.7)	3(0.10)	
MA(1)	2 (55.7)	2 (55.6)	1 (67.1)	1 (93.4)	Cointegrated
	1 (30.2)	1 (30.2)	2 (31.7)	2 (6.6)	
VAR(2)	2 (87.4)	2 (87.4)	2 (99.5)	2 (100)	
	3 (8.6)	3 (8.6)	3 (0.5)		Mixed Unit Roots
MA(1)	2 (70.9)	2 (70.8)	2 (62.9)	1 (73.6)	
	3 (15.3)	3 (15.3)	1 (34.3)	2 (26.3)	Mixed Unit Roots

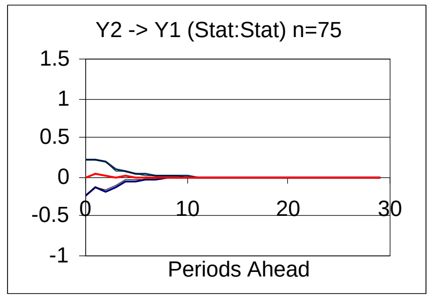
The results of the ***cointegrated*** models in this experiments are as reported in Lütkepohl and Reimer (1992) for sample sizes greater than T= 50 and a bivariate VAR. The coverage rates for the simulated confidence intervals were almost identical to the asymptotic ones. For the smallest sample size (T=25), the simulated confidence interval is wider than the theoretical, as can be seen in Figure 1, panel (c), meanwhile the coverage rates are above the nominal size. For instance, the IRF of y_2 on y_1 had coverage rates of (0.123, 0.062, 0.057) for one period ahead at samples of sizes 25, 75 and 200; increasing the horizon to nineteen periods, generated coverage rates of size (0.15, 0.09, 0.08) for the three sample sizes.

For the ***mixed unit root*** models the most salient feature of the IRFs, at any sample size, is that coverage rates deteriorate considerably as the response horizon is lengthened. For example, the coverage rate of the IRFs of y_2 on y_1 were (0.168, 0.074, 0.072) and (0.86, 0.896, 0.92) at horizons 1 and 19, respectively, and sample sizes of 25, 75 and 200. In each of these cases the true model was a VAR.

Figure 1: visual perspective on the behavior of the IRFs and the asymptotic confidence intervals (thick black lines) versus the simulated one -average results are shown (thin gray lines).



(a)



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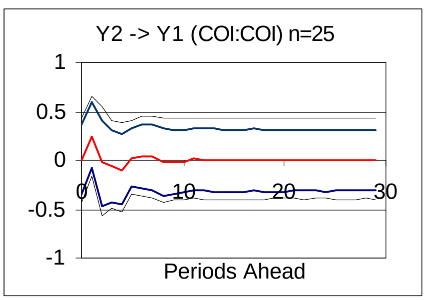

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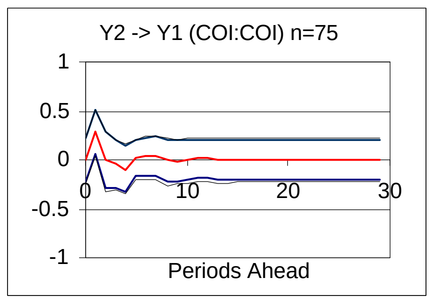

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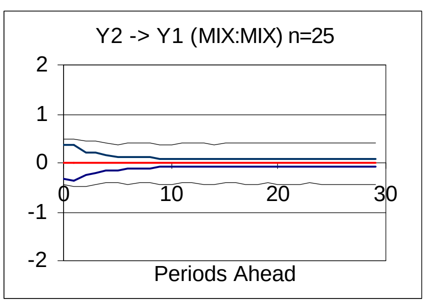

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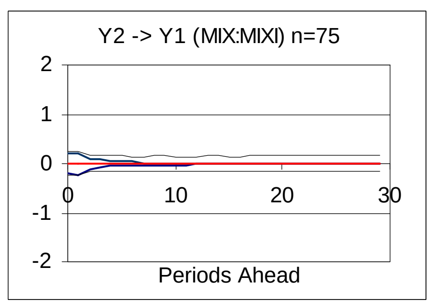

(f)

Results on miss specification. The most interesting findings arise from evaluating what happens when a) a cointegrated model is estimated as if it were stationary (using a classical VAR on levels), and b) a mixed unit roots model is estimated as stationary. The results for the estimation of IRFs of y_2 on y_1 are reported for brevity in table 4. First, the coverage rates, when a stationary VAR is estimated on cointegrated data, are close to the nominal size for short horizons (less than ten periods ahead); then they deteriorate some (Table 4a). Second, a similar result is found for the estimation of a VAR in levels for a mixed unit roots model; however, the deterioration of the coverage rates occurs at shorter horizons –less than nine periods (Table 4b). Third, estimation of a VAR in levels when the variables are cointegrated and have MA effects, the coverage rate deteriorates right at the first period ahead, at all sample sizes; a similar result occurs when the true model is mixed.

Table 4. Coverage rates of the IRFs confidence intervals, nonstationary cases modeled as stationary.

	COI:Stat (a)			MIX:Stat (b)		
IR: Y2->Y1	n=25	n=75	n=200	n=25	n=75	n=200
0	0.093	0.058	0.055	0.102	0.067	0.051
1	0.056	0.068	0.065	0.085	0.05	0.054
2	0.072	0.078	0.1	0.092	0.081	0.154
3	0.027	0.06	0.072	0.059	0.078	0.153
4	0.015	0.046	0.05	0.012	0.09	0.2
9	0.003	0.027	0.066	0.197	0.566	0.816
19	0.385	0.192	0.149	0.828	0.939	0.995
29	0.622	0.343	0.224	0.923	0.973	1

V. Conclusions.

This study raises some important issues about what we can expect to learn from empirical research related to the study of impulse response functions in presence of mixed unit roots and small sample sizes.

First of all it is possible to conclude that all SSC choose the correct lag-length for the VAR(2). Although this result is consistent with other simulation works, it is worth mentioning that to our knowledge this is first hand simulation evidence in experiments that use mixed unit roots. For the most complex cases, the mixed unit roots, all SSC remained parsimonious, particularly so for the MA(1) effects. The nominal size increases in all cases where there is a MA(1) effect.

Secondly, when the stationary case is modeled correctly trough a VAR in levels, the IRFs confidence intervals are not affected seriously by the sample size. It is not the case for the cointegrated or mixed unit roots models when modeled correctly via RML, for which the small sample size deteriorates somewhat the estimates. Although these results are as expected, further research is needed in order to provide a more formal characterization of this behavior. If the variables are cointegrated, and a VAR in levels is used to estimate the IRFs, although the samples size does play an important role as is already known is not as important as in the case of mixed unit roots.

It is important to mention that our results are in line with those in Phillips (1998) for the estimated IRFs, which are shown to be inconsistent at long horizons in unrestricted VARs with some unit roots. In contrast, RMLE provides consistent estimates that are asymptotically optimal.

Further research is needed on the impact of MA effects on the IRFs. The results here suggests a considerable deterioration in the reliability of the estimated IRFs for either cointegrated or mixed unit root models.

Some caveats of this study are mainly related with the population specification chosen (see Table 1). As has been shown by Spencer (1989) and Todd (1990), the estimated impulse responses can be very sensitive to changes in the VAR specification, such as the inclusion of trends and additional variables. Thus, further research has to be conducted in order to provide more insight into this problem. Phillips (1998) provides theoretical results than should be considered for these purposes.

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