

Dynamics and Uncertainty in Environmental and Natural Resource Management Under Scarcity : The Case of Irrigation. *

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Abstract

The objective of this paper is to study the problem of management of irrigation water. We analyse farmers' economic rules of tactical irrigation scheduling in a certain or uncertain environment under water scarcity. The optimal plans depend on uncertain weather conditions and expectations.

We develop a dynamic economic model of irrigation water applications under certain or uncertain weather conditions and in the context of a limited total quantity of water. We program this economic model, introducing the agronomic model EPIC, which is used to obtain information on the crop yield. The model is used to explain the optimal irrigation management plan of corn in the area of Toulouse (France). This model improves water management and leads to substantial reduction of water consumed. As an example, we show that the total quantity of water needed for optimal irrigation scheduling in France is approximately a fourth of the applications recommended by agronomists.

Keywords : dynamic optimization, irrigation management, agroeconomic modeling, uncertainty.

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1 Introduction

Many decision problems occur in a dynamic and uncertain framework. Optimization under uncertainty is used in economics and finance when studying growth, investment, consumption and portfolio allocation rules but also natural and environmental resources management. Several aspects of dynamic problems can be considered as uncertain and are thus represented as random variables. These random variables affect the final result of the decision process. This paper deals with environmental and natural resources and more precisely with irrigation and the problem of crop returns that depend on uncertain weather conditions. Typically agents make decisions taking into account expectations on these random variables.

Irrigated areas have increased significantly during the last ten years in France. Agriculture is the main user of water. Actually, irrigation is the main water consumer in France, with approximately 40% of total consumption in net terms. It is mainly an agricultural practice used in order to insure farmers against the climatic risk. Irrigation is seen as a risk reducing input. Water scarcity can induce conflicts between users that lead governments to restrict or prohibit irrigation during summer. Water becomes a scarce resource that has to be better managed. Thus, the aim of this paper is to define the optimal management of this scarce resource.

Determining the optimal timing of irrigations over a season in an uncertain environment is a significant problem when water is scarce. We deal with the specific problem of finding the optimal allocation of a finite quantity of water over an irrigation season on a particular area of crop in the face of stochastically varying rainfall. That is : When to irrigate ? and How much of the total quantity available to apply each time ? Therefore the decision variables are the timing of irrigation scheduling, the water quantity to apply each time and the number of applications. These three parameters lead to a large set of choices for which decisions are irreversible.

The practice of timing crop irrigation has received considerable attention in the applied economics (Burt, 1968 ; Mc Guckin et al., 1987), and agronomics (Cabelguenne et al., 1993, 1994, 1995) literature. There exists many traditional methods of irrigation scheduling. One of them is a physical method such as tensiometer (Guide pratique du CEMAGREF, 1992, ; Riou et al., 1997). These methods are technical and need specific knowledge and data. Agronomic studies also give general recommendations about irrigation applications over a season (Cabelguenne et al., 1993, 1994, 1995 ; Monot et Flichman, 1994). Whenever management for irrigation has been discussed, most of the approaches provide only few accurate information on how to schedule single crop irriga-

tions under limited water resources. Hence, a simulation model can be an effective tool to support irrigation management decisions.

We develop a dynamic economic model that includes control variables for irrigation water to be applied each time over the decision making model. The model is used to define decision rules in order to support farmers' irrigation management. The dynamic problem of intraseasonal irrigation water allocation is modelled as a multistage decision problem. The agronomic model EPIC (Erosion Productivity Impact Calculator) is introduced in the economic model to obtain information on the soil-plant-climate parameters, and to simulate the effects of water stress during various stages of crop growth on yield. We then use the economic model to find irrigation decision rules in a deterministic context and in an uncertain one. In a deterministic environment, the farmer knows whether or not he can use various scenarios. In an uncertain environment, the model uses the farmers' expectations of weather conditions.

The model is used to study the optimal irrigation scheduling of corn with conditions prevailing in the area of Toulouse (France). We propose solutions to irrigation scheduling over all the seasons that make optimal use of the water resource while taking into account the climate variability. We appraise several irrigation schemes that can be applied in an uncertain context, over all stages of the decision model, taking into account decisions and expectations made by the farmer. Each time, the farmer can analyse any solution, calculate the effects on production of different irrigation levels, compare their economic results and evaluate the level of risk. This model is used to make important savings of resource and leads to a better water management.

The theoretical model is presented in section 2. The conditions which must hold for intertemporal optimality are derived using optimal control theory. In section 3, we present the highlights of the numerical method used to solve the problem. The results are given in section 4. Section 5 concludes the paper.

2 Optimal allocation of irrigation in a dynamic environment under water scarcity

As a useful benchmark, we first consider the problem of an optimal irrigation scheduling in a deterministic context. We then introduce random considerations in the analysis.

2.1 The deterministic model

The problem of an optimal allocation of irrigation in a production process can be modelled as discrete time optimal control problem. It can be represented by the set of equations (1), (2), (3) and (4).

The objective function is :

$$V(X_t, q_t, t) = - \sum_{t=1}^{T-1} C_t(q_t) + F(X_T) \quad (1)$$

$X_t = (m_t, v_t)$, represents the value obtained by the system at each period t . The two state variables are crop biomass, m_t , and soil water content, v_t . The one-component column vector of decision variable, q_t , is the level of water to be applied at each period t . The intermediate function, $C_t()$, represents the cost function for each period up to $T - 1$. The cost function is assumed linear : $cq_t + CF$ where c is the unit water cost and CF are fixed costs. $F()$ is the gross revenue or terminal state function. The final function is crop revenue, $pY(X_T)$, where p is the price of the output¹ and $Y(.)$ is the crop-water production function². We assume that Y is differentiable.

The objective function is subject to the following constraints:

$$X_{t+1} - X_t = f_t(X_t, q_t) \quad \text{for } t = 1, \dots, T - 1; \quad X_1 = \hat{X} \quad \text{given} \quad (2)$$

$$g_t(q_t) = Q - \sum_{i=1}^t q_i \geq 0 \quad \text{for } t = 1, \dots, T - 1 \quad (3)$$

$$\underline{q} \leq q_t \leq \bar{q} \quad \text{for } q_t > 0 \quad t = 1, \dots, T - 1 \quad (4)$$

Equation (2) represents the dynamic behavior of a deterministic system indicating that the change in the level of the state variable at any instant is a function³ of its present date, the decision taken and the time period, f_t . Equations (3) and (4) represent the constraints imposed on the control variables. (3) is the quantity of water available for the irrigation. Q is the limited total quantity of water available for the crop. The difficulty of applying small and high depths of water during irrigation is included in the model as an additional constraint (4). There are technical (irrigation practice, capacity) as well as economic motivations for these constraints.

The model formulation is conceptually similar to that of dynamic model used in Zavaleta et al.(1980), Johnson et al.(1991) and Vickner et al.(1998).

¹The farmer is "price taker".

²A production function is the relationship linking inputs to the output of a production process. Water is the input and crop yield is the output. Water is one of several inputs to crop production process. We assume that other inputs (nitrogen, phosphorus, potassium) have been applied at a level so that water is the limiting factor in crop production.

³ f_t^m is the transformation function of biomass whereas f_t^v is the transformation function of soil water content.

The control problem can be solved using either an optimal control approach or a dynamic programming approach. Dynamic programming methods are generally used to analyse dynamic decision problems. Yakowitz (1982) reviews dynamic programming models for water resource problems and examines computational techniques which have been used to find solutions to these problems. These methods proved a potential tool in defining irrigation management like reservoir operation model (Burt, 1968), multi-purpose reservoir model (Butcher, 1971 ; Dudley et al., 1971 ; Biere and Lee, 1972 ; Torabi and Mobasheri, 1973 ; Stedinger et al., 1984 ; Goulter and Tai, 1985 ; Vedula and Mujumdar, 1992), investment strategies (Burt and Stauber, 1971), optimal allocation of interseasonal irrigation water (Matanga and Miguel, 1979), and optimal allocation of intraseasonal irrigation water (Dudley et al., 1971 ; Yaron and Dinar, 1982 ; Tsakiris and Kiountouzis, 1984 ; Mc Guckin et al., 1987). No analytic results appear in all these studies. They only built numerical models in order to obtain solutions based on recursive functional equation.

We use optimal control method in order to describe analytical results on the optimal allocation of irrigation water under scarcity.

Let $(\lambda_t)_{t=1,\dots,T-1} = (\lambda_t^m, \lambda_t^v)_{t=1,\dots,T-1}$ be the sequence of Lagrange multipliers, which we rename adjoint variables, associated with the constraints (2) : λ_t^m is the multiplier associated with the biomass constraint and λ_t^v is the multiplier associated with the soil-water constraint. λ_t is the contribution which an additional unit of state variable would make to the change in the value function at the beginning of the period t . It is also referred as the shadow price of a unit of state variable. Similarly, let $(\nu_t)_{t=1,\dots,T-1}$ be the sequence of Lagrange multipliers associated with the constraints (3).

The Lagrangian function of this problem is :

$$L = - \sum_{t=1}^{T-1} (cq_t + CF) + pY(X_T) + \sum_{t=1}^{T-1} \lambda_{t+1} [f_t(X_t, q_t) - X_{t+1} + X_t] + \sum_{t=1}^{T-1} \nu_t g_t(q_t) - \underline{\nu} \underline{q} + \bar{\nu} \bar{q} \quad (5)$$

We define the Hamiltonien function to be :

$$H_t(X_t, q_t) = -cq_t + \lambda_{t+1} f_t(X_t, q_t) \quad (6)$$

Using (6) we can rewrite (5) as follows :

$$L = pY(X_T) + \sum_{t=1}^{T-1} [H_t(X_t, q_t) - \lambda_{t+1} (X_{t+1} - X_t)] + \sum_{t=1}^{T-1} \nu_t g_t(q_t) - \underline{\nu} \underline{q} + \bar{\nu} \bar{q} \quad (7)$$

Proposition 1 *The necessary conditions for q_t^* , $t = 1, \dots, T-1$, to be the optimal irrigation schedule are :*

$$-c + \lambda_{t+1}^m \frac{\partial f_t^m}{\partial m_t} + \lambda_{t+1}^v \frac{\partial f_t^v}{\partial v_t} - \nu_t - (\underline{\nu} - \bar{\nu}) = 0 \quad \text{for } t = 1, \dots, T-1 \quad (8)$$

$$\nu_t \geq 0, \quad \nu_t[g_t(q_t)] = 0 \quad \text{for } t = 1, \dots, T-1 \quad (9)$$

$$\underline{\nu} \geq 0 \quad \underline{\nu} \underline{q} = 0 \quad ; \quad \bar{\nu} \geq 0 \quad \bar{\nu} \bar{q} = 0 \quad (10)$$

$$\lambda_{t+1}^m - \lambda_t^m = -\lambda_{t+1}^m \frac{\partial f_t^m}{\partial m_t} \quad \text{for } t = 1, \dots, T-1 \quad (11)$$

$$\lambda_{t+1}^v - \lambda_t^v = -\lambda_{t+1}^v \frac{\partial f_t^v}{\partial v_t} \quad \text{for } t = 1, \dots, T-1 \quad (12)$$

$$\lambda_T^m = p \frac{dY}{dX} \big|_{X=X_T^*} \quad ; \quad \lambda_T^v = 0 \quad (13)$$

The conditions (8)-(10) are the maximum principle ; (11) and (12) are the adjoint equations ; (13) are the boundary conditions. These necessary conditions can also be derived by using a dynamic programming approach. The maximum principle states that the multiplier ν_t is zero if the constraint is not binding. In this case, the optimal path depends on the familiar economic relation between marginal cost and marginal revenue. The marginal revenue consists of the sum of the biomass marginal contribution, $\lambda_{t+1}^m \cdot \frac{\partial f_t^m}{\partial m_t}$, and the soil water marginal contribution, $\lambda_{t+1}^v \cdot \frac{\partial f_t^v}{\partial v_t}$. If the marginal revenue is greater than the marginal cost, then the farmer irrigates at time t , and otherwise, he does not irrigate. λ_T^m is the shadow price of having one unit more or less of crop at time T . The price λ_T^v of an additional unit of soil water at time T is zero.

2.2 The model under uncertainty

The model under uncertainty is conceptually the same as the deterministic model. The difference lies in the dynamic behavior of the system that now incorporates stochastic weather variables.

Equation (2) becomes then :

$$X_{t+1} - X_t = f_t(X_t, q_t, \epsilon_t) \quad (14)$$

with ϵ_t being the vector of stochastic factors⁴. In this case, the way of choosing q_t constitutes a stochastic problem. The stochastic control policies are defined according to the information on past and anticipated future observations available to the farmer. We assumed that the farmer has perfect information about the past. The only difference stands in the anticipation of future random climatic variables. Similarly to the deterministic case, the problem of the optimal allocation of irrigation water under uncertainty when water is scarce can be solved as a control problem. Then, the optimal irrigation scheduling under uncertainty satisfies the following conditions :

⁴The stochastic variables are assumed to be independently and identically distributed with repartition function $G(\cdot)$ in its support $[0, \bar{\epsilon}]$.

Proposition 2 *The necessary conditions for q_t^* , $t = 1, \dots, T-1$, to be the optimal irrigation schedule in a stochastic environment and under water scarcity are :*

$$-c + \int_0^{\bar{\epsilon}} (\lambda_{t+1}^m \frac{\partial f_t^m}{\partial m_t} + \lambda_{t+1}^v \frac{\partial f_t^v}{\partial v_t}) dG(\epsilon_t) - \nu_t - (\underline{\nu} - \bar{\nu}) = 0 \quad \text{for } t = 1, \dots, T-1 \quad (15)$$

$$\nu_t \geq 0, \quad \nu_t [g_t(q_t)] = 0 \quad \text{for } t = 1, \dots, T-1 \quad (16)$$

$$\underline{\nu} \geq 0 \quad \underline{\nu} \bar{q} = 0 \quad ; \quad \bar{\nu} \geq 0 \quad \bar{\nu} \bar{q} = 0 \quad (17)$$

$$\int_0^{\bar{\epsilon}} (\lambda_{t+1}^m - \lambda_t^m + \lambda_{t+1}^m \frac{\partial f_t^m}{\partial m_t}) dG(\epsilon_t) = 0 \quad \text{for } t = 1, \dots, T-1 \quad (18)$$

$$\int_0^{\bar{\epsilon}} (\lambda_{t+1}^v - \lambda_t^v + \lambda_{t+1}^v \frac{\partial f_t^v}{\partial v_t}) dG(\epsilon_t) = 0 \quad \text{for } t = 1, \dots, T-1 \quad (19)$$

$$\lambda_T^m = p \frac{dY}{dX} |_{X=X_T^*} \quad ; \quad \lambda_T^v = 0 \quad (20)$$

Relatively few optimal control problems can be solved in closed form⁵. The recursive equation for the state and adjoint variables are relatively complex functions and may be solved numerically.

3 Procedure and data

We present a new method for solving this problem based on simulations over all possible irrigation schedules. The solution generation uses a simplification of the general multistage decision problem.

The model has been applied to the analysis of the optimal irrigation scheduling of corn⁶ with conditions prevailing in Toulouse area, France.

3.1 The IRRi procedure

The integrated approach used in this analysis - the IRRi procedure (figure 1) - can be divided into two stages. The IRRi procedure (Bontemps et Couture, 1999) consists of (1) computation of all possible irrigation schedules and (2) computation of the optimal irrigation schedule. The mathematical formulation thus consists essentially of two models : model 1 : the agronomic model for calculating crop yields under different irrigation scheduling and conditions, and model 2 : the single crop irrigation scheduling model, for maximizing profits of individual crop for specified seasonal water supply. Model 1 provides the input to model 2.

⁵Some classes of problem with continuous state and decision variables can be solved analytically by dynamic programming.

These problems specify quadratic stage returns and linear state transformation (Kennedy, 1986).

⁶Corn is the only crop considered in this study. With 43 % of irrigated area in France, corn remains the main irrigated crop.

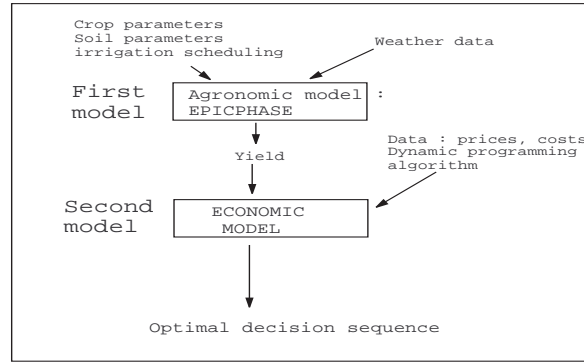


Figure 1: The algorithm of the IRRI procedure.

3.1.1 The agronomic model : EPICPHASE

We used the agronomic model, EPICPHASE (Cabelguenne and Debaeke, 1995), a version of EPIC (Sharpley and Williams, 1990 ; Williams et al., 1990) which has been adapted by INRA (Toulouse). The model can simulate crop growth involving relationship between plant growth and water use. EPICPHASE includes a general plant growth model that is used to simulate leaf interception of solar radiation, conversion to biomass, division to biomass into roots, above-ground biomass and yield, water use and nutrient uptake. Potential plant growth is simulated daily and constrained by the minimum of stress factors. EPICPHASE can be used in determining crop yield for various climates and irrigation scheduling. The output from the plant simulation model is used as input in the economic model.

3.1.2 The economic model :

The basic components of the economic model - the decision stages, states, decisions, return functions and transformations functions - must be discretized.

State variables : Two state variables are used in the decision process : the biomass and the available soil-water. Each variable is defined at the beginning of each decision period. Their values are given by the agronomic model at each stage.

Objective function : The objective is the maximisation of the expected crop profit function. The profit is defined as the crop price times the crop yield minus the variable water cost times the quantity of water applied minus the fixed cost. The values of the yield function are given by the agronomic model.

Decision process : It is assumed that the irrigation season begins on June 20th. The whole irrigation season is subdivided into ten irrigation intervals of 5 or 10 days duration which is a common practice (figure 2). The maximum amount of applications is fixed to be 5⁷. The farmer has to make a choice of five applications over ten possibilities. The quantity of water in each application is equal to the uniform repartition of the total quantity of water⁸.

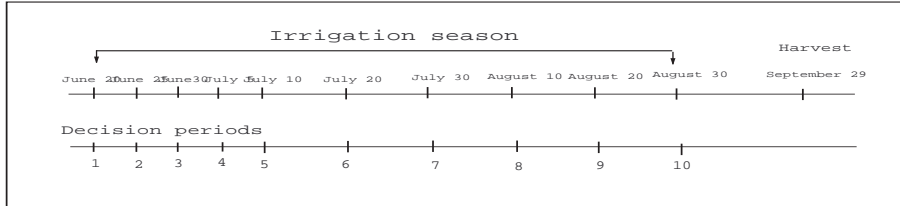


Figure 2: Decision process.

Uncertainty and expectations : The weather at each period is represented by a stochastic variable modifying the dynamic of yield formation. The farmer has no knowledge about the probabilities of future variables. He can anticipate the values of these variables according to past information. We assume only three classes of expectations : the farmer can expect a dry, normal or humid year. The farmer has more information to compute his expectations as soon as the number of decision stages decreases ; the set of available information increases with time. To define these expectations, we use actual daily average values and repartitions of weather variables.

The approach followed to obtain optimal irrigation scheduling under uncertainty is to solve a deterministic optimization problem for each period of the decision process. For each period, the deterministic model is formed by replacing random variables with their expected values which are conditioned on weather prior to the precedent period. The period decision is obtained by using expected future climate. Then, the level of the period irrigation is applied and the real weather between the two periods is used to compute the value for the remaining future periods. Finally, actual crop yield and profit are simulated with the irrigation levels for the ten periods.

The economic model identifies the optimal irrigation schedule by using yields predicted by the crop growth model from different irrigation strategies and by considering simulated profits. The formulation of this problem is based on the algorithm of search on all possible cases. It is conceptually similar to a dynamic programming solution algorithm. First, the solution process is

⁷This number corresponds to the average amount applications realized in Toulouse area (Enquête Agreste, 1996).

⁸This assumption is realistic because it is a common practice observed in the considered area.

formulated on recursive analysis of all available irrigation strategies. Second, this process can be stopped at each stage of the decision plan.

3.2 Data

The agronomic model is calibrated with experimental field data collected in Toulouse area, France. Data used to run EPICPHASE include weather variables (daily values of air temperature, solar radiation, precipitation, wind speed and relative humidity), soil variables, erosion variables, parameter values for crop, fertilization, pesticide and irrigation applications. The climates were derived from a 14-year series of weather variable data collected at Toulouse, France. The average price per ton for corn was 1440 Francs in Toulouse area. The crop price is known for each year. Costs equal the variable cost of irrigation water plus other fixed costs. The per-unit cost of irrigation water is estimated as 0,25 F per hectare. Fixed costs evaluated at 2150 F per hectare are composed of fertilizer, nitrate, seed and hail insurance costs.

4 Results

The results are presented in the deterministic context and then under uncertainty.

4.1 Optimal irrigation water allocation with deterministic environment

We have restricted our attention to optimal water allocation with only three climates : 1989, dry year, 1991, normal year, and 1992, humid year, to account for weather variability in the studied area. We assume that the fixed total available quantity of water is $1500 \text{ m}^3/\text{ha}$. The model is used to analyse the impact of weather variables on yields and profits. The results are presented in the table 1. The table contains water applications, profits and yields for the no-irrigation case, agronomically optimal case (potential case), and optimal case obtained by the model. The agronomically optimal case provides a benchmark against which the effects of alternative water application strategies can be evaluated. It is clear from the table 1 that moving the optimal timing of water applications results in less total water consumed, relatively high yields and profits for all three climates. Thus, if application amounts and timing predicted by the model were followed, producers could reduce water needs⁹ (table 1). The timing of water applications typically depends on weather events (table

⁹In Toulouse area, the average total quantity of irrigation water used by the farmers is $1800 \text{ m}^3/\text{ha}$ and the average yield is $7,58 \text{ T}/\text{ha}$ (Enquête Agreste, 1996).

2). The primary change in management observed in the optimal solution is the period of first water application. Under dry weather, the farmer must irrigate earlier than under normal or humide ones. The drier the weather, the earlier the optimal irrigation scheduling. If the farmer follows the schedule recommended by the optimization model, he makes important savings of resource and he improves water management. These analyses for managed irrigation water applications with limited

	1989	1991	1992
Profit without irrigation (F/T ^a) (yield, T/ha ^b)	5529 (7,32)	6600 (8,43)	7481 (9,51)
Potential profit (F/T) (yield, T/ha)	10141 (12,90)	7875 (10,53)	7995 (10,85)
Total water quantity needed (m^3/ha)	4970	3620	3370
Profit (yield, T/ha) obtained with optimal irrigation scheduling (F/T)	9676 (11,63)	8384 (10,51)	8179 (10,57)
Difference between without irrigation and potential profits (F/T) (yield, T/ha)	-4612 (-5,58)	-1275 (-2,10)	-514 (-1,34)
Difference between optimal and potential profits (F/T) (yield, T/ha)	-465 (1,27)	+509 (0,02)	+184 (0,28)
Reductions of water consumed (m^3/ha^c)	3470	2120	1870

^a1 F= 0,163 US\$.

^b1 T/ha = 14,87 bushels per acre.

^c1 m^3/ha = 0,0159 inch per acre.

Table 1: Profits and yields simulated for three climates with a uniform repartition of the fixed total available quantity of water equal to $1500 m^3/ha$.

	Stages of decision									
Year	1	2	3	4	5	6	7	8	9	10
1989	30		30	30		30	30			
1991					30	30	30	30	30	
1992						30	30	30	30	30

Table 2: Optimal irrigation scheduling for three climates with a uniform repartition of the fixed total available quantity of water equal to $1500 m^3/ha$.

water supply can be accomplished with little loss in yield. Specifically, by optimizing the timing and application rates of water, profits for some years increase while reducing total application levels.

4.2 Optimal irrigation water allocation under uncertainty

Under uncertainty, the farmer chooses the optimal irrigation scheduling maximizing expected profit according to weather risk and expectations. We have restricted our attention to the optimal irrigation water allocation to the sole dry-year 1989. We assume that the fixed total available quantity of water is $1500 \text{ m}^3/\text{ha}$. Then, we applied the principle of solution generation with two scenarios. In the first case, we assume that the farmer anticipates each period dry weather conditions ; therefore, the expected climate is close to the real one. On the contrary, in the second case, the farmer makes humid year expectations at each period. The results of the stochastic control model are summarized in the table 4 and are compared to the deterministic case. The values for the perfect knowledge environment are $12,90 \text{ T/ha}$ for potential yield and $11,63 \text{ T/ha}$ for maximal yield. The results

	Decision periods									
Optimal irrigation scheduling	1	2	3	4	5	6	7	8	9	10
Deterministic	30		30	30		30	30			
Dry expectations (case 1)	30		30	30	30	30				
(case 2)	30	30		30	30	30				
Humid expectations		30			30	30		30	30	

Table 3: Optimal irrigation scheduling under uncertainty with a uniform repartition of the fixed total available quantity of water equal to $1500 \text{ m}^3/\text{ha}$.

	Expectations	
	dry year	humid year
Deterministic potential profit (F/T) (yield ,T/ha)	10141 (12,90)	
Deterministic maximal profit (F/T) (yield ,T/ha)	9676 (11,63)	
Expected maximal profit (yield ,T/ha)	9424 (case 1 : 11,39)	8868 (10,86)
depending on expectations (F/T)	9414 (case 2 : 11,38)	

Table 4: Simulated profits and yields with a uniform repartition of the fixed total available quantity of water equal to $1500 \text{ m}^3/\text{ha}$.

obtained for yields with the stochastic model slightly differ from those obtained in the deterministic case. In the uncertain case with dry year expectations, two different irrigation plans (table

3) are optimal and give almost the same yield and profit ; there appears small differences because of average values used to compute expectations. With humid year expectations, yield and profit are relatively high. The optimal irrigation scheduling from the uncertain model is different from the one obtained in the deterministic case. In this case, the farmer delayed irrigation applications because he anticipated rainfalls in the near future. Optimal irrigation plans and the corresponding yield and profit are depending on expectations. The profits are diminished between the two cases. These differences could be considered to be the cost of not possessing complete information and not revising expectations.

5 Conclusion

This paper describes a new method for solving the problem of optimal intraseasonal irrigation water allocation under uncertainty and scarcity. We develop a dynamic economic model that includes control variables for irrigation water to be applied each time over the decision making model and two state variables. We obtain some analytical results using optimal control theory to solve this problem. We then obtain empirical solutions using a numerical technique based on a new approach, the IRRI procedure. The mathematical formulation of IRRI consists essentially of : (1) the agronomic model, EPIC, and (2) the single crop irrigation scheduling model for maximizing profits or expected profits of an individual crop for a specified seasonal water supply. The aim of the numeric procedure is to propose solutions to irrigation water scheduling providing the optimal use of the resource, under uncertainty and scarcity. These optimal water applications over a season generate relatively high profits despite a limited water supply. This model leads to important savings of resource and a better management of water. Three extensions can be proposed to this model. Firstly, by a microeconomic approach of water management, the model can be used to obtain crop-level irrigation water demand and then can be extended to an analysis of a farm-level aggregated water demand. Secondly, optimization was made with respect to the quantity of water to be applied and assumes that the timing of irrigation and the number of applications are fixed. These assumptions can be relaxed. Finally, because of uncertainty, the farmer could be assumed risk averse.

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