Storability on Modeling Commodity Futures Prices

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Econometric models of commodity prices have been estimated for more than 80 years, but both structural and time series models require ad hoc assumptions to capture all the features of commodity price series. Commodities can be broadly divided into two categories: storable and non-storable. The purpose of this study is to investigate the effects of storability on commodity futures pricing, especially whether meats can be reasonably approximated by storable commodity term structure models. From the empirical analysis of seven commodity futures prices, the two-factor Schwartz model is found to perform well for less storable commodities.

Keywords: commodity prices, storability, term structure model.

1. Introduction

Commodity prices have important characteristics due to the nature of the production technology. They are mean-reverting and gravitate towards a "normal" equilibrium level that is determined by the cost of production and level of demand. They are often "backwardated" in that futures prices decline with time to maturity. Many commodities also have strong seasonalities in both price levels and volatilities due to seasonal demand or production patterns.

Because prices of given commodities reflect the supply and demand for that commodity, prices for different commodities should be modeled differently (Tomek and Peterson, 2001). Commodities can be broadly divided into two categories: storable and non-storable. Storable goods like gold can be stored indefinitely with relatively little cost while non-storable goods such as electricity have storage costs so high that storage is impractical. However, most commodities fall between these two extreme cases. For example, grains can be stored but only for a matter of

months. Fresh meats, on the other hand, have very limited storability. Perishable goods can be regarded as being constantly consumed by time, whether or not it is actually used. Therefore, this important characteristic must be considered in modeling the price dynamics of different commodities.

In previous research, time series and structural models have been specified for the behavior of agricultural prices, but they all require *ad hoc* assumptions to capture all the features of commodity price series (Peterson and Tomek, 2005). For example, Deaton and Larocque (1992) use a rational expectations storage model. It adequately explains the high volatility and positive skewness historically exhibited by primary commodity prices, however, it fails to account for their high autocorrelation. Rui and Miranda (1995) employ a convex increasing marginal storage cost function and their analysis replicates the degree of autocorrelation in observed price distributions. They conclude that the discrepancy between their findings and Deaton and Larocque's are clearly attributable to differences in assumptions regarding the storage cost function.

Recently, Peterson and Tomek (2005) apply a rational expectations competitive storage model to the U.S. corn market. Compared to previous models, they add extensive realism to the model in terms of how production activities and storage costs are specified. They find that both the simulated cash and the futures prices are consistent with recent historical experience and reflect an efficient market with rational decision makers. They conclude that the simulations, therefore, are useful in generating prices that permit empirical analyses of the long-run impacts of economic and policy decisions.

An alternative method for modeling commodity prices uses the state variable approach. It begins by assuming a functional form for a set of underlying state variables. Since futures prices depend on them, an arbitrage relationship exists and it can be solved analytically under specific assumptions of the state variable process (e.g., Schwartz, 1997; Roberts and Fackler, 1999; Schwartz and Smith, 2000; Sorensen, 2002).

This study investigates the effects of storability on commodity futures pricing, especially whether meats can be reasonably approximated by storable commodity term structure models. In order to do this, two models are constructed. The first is a storage model of commodity prices, similar to Peterson and Tomek (2005), which is able to generate a term structure of futures prices and to accommodate different levels of storability. The second is the two-factor model in Schwartz (1997) with seasonality. After calibrating model one, simulations are generated from it. Then model two is estimated using the simulated data. The storability parameters are adjusted to note any divergence between models one and two, with particular attention paid to parameters that approximate meats. In this paper, an empirical application is employed to seven commodity futures, gold, crude oil, natural gas, corn, pork bellies, feeder cattle, and live cattle. The results show that the two-factor Schwartz model fits well for less storable commodities, but the importance of considering storability in modeling futures prices can not be ignored.

The remainder of this paper is as follows. In section 2 and 3, the modern storage model and the term structure model are reviewed. Section 4 gives a brief description of different markets covering metal, energy, grain, and livestock. Section 5 explains the data and the methods that are used in the estimation and provides a summary of the results. Section 6 gives concluding remarks.

2. The Modern Storage Model

Modern storage theory can trace its origins to the "supply of storage" theory first introduced by Williams (1936) and later extended by Kaldor (1939), Working (1948,1949) and Telser (1958). More recently, the storage framework appends demand and supply functions to the original supply of storage equation and adopts Muth's rational expectations hypothesis. Williams and Wright (1991) synthesize the modern theory of storage and show that the framework can explain seasonal and inter-year price patterns (Tomek and Peterson, 2001).

Commodities differ from financial assets in that they can be produced and consumed. Production must not match consumption in every period, but can be stored in the form of inventories (Nielsen and Schwartz, 2004). Usually storage is undertaken in the hope of price appreciation. Working (1948, 1949) proposed the price-of-storage theory to solve a problem presented by conflict of accepted theory with observed price behavior, why large quantities of wheat are stored in the absence of any observable return for storage. He recognized the existence of negative price-of-storage at low inventory levels and argued that stocks of a commodity below some fairly well recognized level carry what Kaldor (1939) termed a *convenience yield*. This convenience yield may offset the apparent loss from storage.

Williams and Wright (1991) developed the modern rational expectations commodity storage model. The central tenet of the modern storage model is that commodity price dynamics are governed mainly by the speculative and precautionary storage activity of rational commodity storers and processors (Rui and Miranda, 1995). Despite the intense interest in the rational expectations commodity storage model, econometric estimation and validation of the model has been hampered mainly by the absence of an analytical closed-form solution. Numerical methods

are instead used to estimate the rational expectations storage model and to assess its ability to explain the stylized facts of commodity price dynamics (e.g., Deaton and Larocque, 1992, 1996; Rui and Miranda, 1995; Routledge, Seppi, and Spatt, 2000; Peterson and Tomek, 2005).

Compared to previous models, Peterson and Tomek (2005) add extensive realism to the rational expectations competitive storage model, in terms of how production activities and storage costs are specified. They argue that this model can contribute to commodity price analysis by allowing primary parameters of price distributions to be recovered from a structural model. They also argue that given parameter estimates of price distributions, the model can generate intra- and inter-year price series similar to those faced by agents in the industry, mitigating the small sample sizes available for annually produced commodities and therefore permit the analysis of long-run economic consequences of risk management strategies. Based on their findings that both the simulated cash and the futures prices are consistent with recent historical experience and reflect an efficient market with rational decision makers, the Petersen and Tomek model will be used, but with a generalization to accommodate different levels of storability.

3. The Term Structure Model

Recent practice in financial econometrics has emphasized the use of models that utilize arbitrage relationships across collections of assets. Most fully developed for fixed income securities (e.g., Cox, Ingersoll, and Ross, 1985), the entire term structure is modeled in terms of a few underlying factors (Roberts and Fackler, 1999). Schwartz (1997) has applied the same approach to model the term structure of commodity futures prices. He utilizes up to three

stochastic factors: spot price, convenience yield, and interest rate, and compares the model performance using copper, oil, and gold data.

As Roberts and Fackler (1999) point out, this approach has some significant potential advantages over modeling commodity futures prices using univariate time series models. First, there is no need to artificially construct univariate series by rolling into new futures contracts as maturity is reached. Instead, all maturities are modeled simultaneously. Second, the resulting joint model for all maturities links each contract in a model free of arbitrage possibilities and hence the model incorporates restrictions consistent with economic equilibria. Third, due to its internal consistency, the model can be extrapolated with more confidence and thus utilized in applications beyond that of modeling futures prices. In particular, such a model can be used in evaluating investment projects that yield streams of returns over time that are linked to commodity prices.

Also this approach has benefited from the theory of storage in at least two ways as mentioned in Nielsen and Schwartz (2004). First, it has introduced convenience yield as an exogenous process, thus allowing the complete forward curve to be modeled. Second, by letting spot price and convenience yield be correlated, it has allowed for mean reversion without allowing arbitrage. The latter is important because, as documented by Bessembinder et al. (1995), commodity prices exhibit mean reversion as producers and consumers adapt their long term production and consumption plans.

Agricultural commodities have strong seasonal patterns in both price levels and volatilities due to the biological nature of the production process. Therefore incorporating

seasonality into the Schwartz model is potentially important. Roberts and Fackler (1999) demonstrate one way of modeling seasonality in a term structure model.

The state space approach starts by assuming a functional form for a set of underlying state variables, x, which are modeled as a continuous time diffusion process

$$dx = \mu(x,t)dt + \sigma(x,t)dz$$

The risk adjusted process for x can be written as

$$dx = [\mu(x,t) - \sigma(x,t)\theta(x,t)]dt + \sigma(x,t)d\hat{z}$$

where θ is the market price of state variable risk.

Futures prices are martingales with respect to the risk adjusted process and using Ito's Lemma they satisfy the partial differential equation

$$0 = F_t(x,t;T) + F_x(x,t;T)[\mu(x,t) - \sigma(x,t)\theta(x,t)] + \frac{1}{2}trace(\sigma^T(x,t)F_{xx}(x,t;T)\sigma(x,t)$$

subject to the boundary condition that the futures price equals the spot price at the maturity date, T.

In the two-factor Schwartz model, the first state variable is the spot price, $S(t)$. We treat it as a limit of an instantaneous futures price, i.e., $S(t) = x_1(t) = F(x,t;t)$. The second state variable is the stochastic convenience yield, δ, which is interpreted as the flow of benefits accruing to the holder of stocks but not the holder of futures (Hull, 2003). Specifically, the two-factor model is

$$dS = (R-\delta)Sdt + \sigma_1 S dz_1$$

$$d\delta = \kappa(\alpha-\delta)dt + \sigma_2 dz_2$$

with $dz_1 dz_2 = \rho dt$. R is the total return on the holding of the spot good, which consists of the spot price appreciation and the convenience yield δ.

From the no-arbitrage condition, $R = \mu + \delta = r + \sigma\theta$, where r is the risk-free interest rate, the risk adjusted drift of the spot price, $\mu - \sigma\theta$, can be replaced by $r - \delta$. Then futures price satisfies

$$F_T = (r-\delta)SF_S + (\kappa(\alpha-\delta)-\lambda)F_\delta + \frac{1}{2}\sigma_1^2 S^2 F_{SS} + \rho\sigma_1\sigma_2 SF_{S\delta} + \frac{1}{2}\sigma_2^2 F_{\delta\delta}$$

with the boundary condition $F(S,\delta;0) = S$. It can be verified that the solution is

$$F(S,\delta;T) = S\exp(A(T) - \delta\frac{1-e^{-\kappa T}}{\kappa})$$

Or, in log form:

$$\ln F(S,\delta;T) = \ln S - \delta\frac{1-e^{-\kappa T}}{\kappa} + A(T)$$

where

$$A'(t) = (\kappa\alpha - \lambda + \rho\sigma_1\sigma_2)\frac{1-e^{-\kappa(T-t)}}{\kappa} - \frac{\sigma_2^2}{2}(\frac{1-e^{-\kappa(T-t)}}{\kappa})^2 - r$$

Roberts and Fackler (1999) introduce seasonality into the model by making the parameters periodic functions of time, with a periodicity of one year. They argue that any seasonal variation in the model parameters other than κ would influence the futures price only through the $A(t)$ term. They propose a simple way to incorporate seasonality into the model by making δ be mean-reverting to a seasonal function rather than to a constant value. Specifically, $\alpha(t)$ is modeled as a truncated Fourier series, i.e.,

$$\alpha(t) = \eta_0 + \sum_i (\sin(\nu_i t)\eta_i + \cos(\nu_i t)\theta_i)$$

where $\nu_i = 2\pi i$.

Then it is easily to verify that

$$A(t) = (r + \frac{\lambda}{\kappa} + \frac{\sigma_2^2}{2\kappa^2} - \frac{\rho\sigma_1\sigma_2}{\kappa})(T-t) + \frac{\sigma_2^2}{4}\frac{1-e^{-2\kappa(T-t)}}{\kappa^3}$$

$$-(\frac{\lambda}{\kappa} + \rho\sigma_1\sigma_2 - \frac{\sigma_2^2}{\kappa})\frac{1-e^{-\kappa(T-t)}}{\kappa^2} + \int_t^T (1-e^{-\kappa(T-t)})\alpha(\tau)d\tau$$

The last term can be written as

$$\int_t^T (1-e^{-\kappa(T-t)})\alpha(\tau)d\tau = \eta_0(T-t) + \sum_i ([\sin(\nu_i T) - \sin(\nu_i t)]\frac{\eta_i}{\nu_i} - [\cos(\nu_i T) - \cos(\nu_i t)]\frac{\theta_i}{\nu_i})$$

$$-\frac{1}{\kappa}(m(T) - e^{-\kappa(T-t)}m(t))$$

where $m(t) = \kappa e^{-\kappa t}\int^t e^{\kappa\tau}\alpha(\tau)d\tau$.

4. Seasonality and Storability of Different Markets

4.1 NYMEX Gold Market

Following the California gold discovery of 1848, the United States has become one of the world's major gold suppliers. Gold has been coveted for its unique blend of rarity, beauty, and near indestructibility for centuries. Nations embrace gold as a store of wealth and a medium of international exchange; individuals seek to possess gold as insurance against the uncertainties of paper money (NYMEX, 2006a).

Gold is also a vital industrial commodity. It is an excellent conductor of electricity, is extremely resistant to corrosion, and is one of the most chemically stable of the elements, making it important in electronics and other high-tech applications. Today, gold prices float freely in accordance with supply and demand, responding quickly to political and economic events.

The gold futures and options at the New York Commodities Exchange (COMEX) Division of the New York Mercantile Exchange (NYMEX) are useful for hedging gold price risk. They also provide an important alternative to traditional means of investing in gold such as bullion, coins, and mining stocks.

4.2 NYMEX Energy Market

Energy is perhaps the most strategic material in world commerce and its price can be exceedingly volatile, especially during recent years. Countries that depend on the sale of energy resources have a vital interest in its price. The energy futures and options contracts listed on the

New York Mercantile Exchange (NYMEX) enable buyers and sellers of energy to manage their exposure to market fluctuations and reduce their risks.

Crude oil dominates the energy market, accounting for approximately 40% of world supply on an energy equivalent basis (NYMEX, 2006b). Since the introduction of the NYMEX light sweet crude oil futures contract in 1983, it has evolved into the world's most liquid forum for crude oil trading. In the United States, all but a handful of states are oil producers, while more than half of the world's economically recoverable reserves are found in the Middle East.

Crude oil is the raw material for gasoline, diesel (heating oil), jet fuel, boiler fuels, and thousands of petrochemicals. The oil market has experienced periods of extreme price volatility since the early 1970s, reacting to political and economic developments. The course of individual market trends, ranging from 6 to 18 months, has pushed prices up by more than twofold and caused them to plunge by almost two-thirds (NYMEX, 2006b). Since crude oil production involves extensive commitment of resources, often many years in advance, the Exchange's light sweet crude oil futures contract is the most far-reaching of its products, listing contracts up to seven years forward.

Natural Gas also plays a major role in the energy profile of the United States, where it accounts for almost a quarter of total energy consumption. Industrial users and electric utilities together account for approximately half of the market; commercial and residential users combined are approximately 40% (NYMEX, 2006b). To ensure reliable service, natural gas can be stored underground for use during peak demand, such as cold days. Underground storage accounts for about 20% of the natural gas consumed each winter, on average.

Since the enactment of the Natural Gas Policy Act of 1978, the industry has changed from one that is almost totally regulated to one that operates largely as a free market. The NYMEX launched the world's first natural gas futures contract in 1990, based on the delivery at the Henry Hub in Louisiana, the nexus of 16 inter- and intra-state pipelines. Today, it has become one of the most actively traded futures contracts for a physical commodity.

4.3 CBOT Grain Market

Temperature and precipitation are key factors in determining the supply of vital agricultural commodities such as corn, soybeans, wheat, oats, and rice. Grain and soybean supplies fluctuate continuously, and market demand for these commodities varies constantly. These uncertainties can cause grain and soybean prices to vary substantially. However, the existence of the Chicago Board of Trade (CBOT) futures markets on these commodities helps to stabilize food prices.

Grains feed our livestock. In both whole and processed forms, they provide nourishing food for our families. They are also used in an ever-increasing range of nonfood products. Taking corn as an example, its greatest use is feed for livestock and poultry. Corn also goes into many everyday food items—corn oil for margarine, cornstarch for gravy, corn sweeteners for soft drinks, etc. Nonfood uses of corn include alcohol for ethanol, absorbing agents for disposable diapers, and adhesives for paper products (CBOT, 2006).

In the United States, about 32% of planted acreage goes to corn. Corn is usually planted during April and harvested during October. In a cool year, when the corn matures more slowly, much of the crop is not harvested until November. The harvest time also depends on the different

corn hybrids. Even when plants are physically mature, farmers might wait to harvest them until corn kernels have dried further so that the corn can be stored for longer periods of time.

4.4 CME Livestock Market

The U.S. cattle and hogs industry is estimated to be worth $60 billion annually (CME, 2006a). Extremes in weather greatly affect the cost of feed, availability of forage, rates at which animals conceive and gain weight, and the number of animals that are brought to market. Disease is always an issue, as are shifting public tastes for consuming beef and pork.

The Chicago Mercantile Exchange (CME) offers a range of livestock futures including CME Feeder Cattle Futures and Options (young cattle), CME Live Cattle Futures and Options (market-ready animals), CME Lean Hog Futures and Options, and CME Frozen Pork Belly Futures and Options. The feeder cattle and lean hog contracts are settled in cash and not physically deliverable, while the live cattle and pork bellies are physically deliverable. CME livestock futures are traded electronically as well as on the trading floor (CME, 2006b).

The livestock industry can be divided into several basic phases that correspond to the animals' life cycle: 1) the production of young animals, 2) feeding the young animals to slaughter weight, and 3) slaughter and fabrication. Feeder cattle are young animals (650-849 pound feeder steers) sent to feedlots for finishing into "fed" cattle, the basis of CME Live Cattle contracts. The supply of beef cattle is a main fundamental focus for traders of cattle futures and supplies are largely influenced by weather conditions, profitability, and the price of feed. These factors also affect the supply of hogs. However, a hog is marketed about six months after birth, compared to a beef animal that is marketed about 18 months after birth. Once a hog has been slaughtered, the excess pork bellies that are made into bacon are often frozen during fall, winter,

and spring in order to meet summer's seasonal demand increase. CME Frozen Pork Belly Futures and Options were launched in 1961 and were the first futures on frozen meat products (CME, 2006c).

5. Empirical Data and Estimation Results

Wednesday closing futures prices of gold, crude oil, natural gas, corn, pork bellies, feeder cattle, and live cattle are used to estimate the parameters of the term structure model. The sample period is January 1, 1985 to November 14, 2005, except for natural gas, for which it starts from January 1, 1991. All futures contracts with maturities of two years or less were used, excluding observations in their delivery month.

Assuming a Gaussian model, maximum likelihood estimation was used, with the likelihood computed via the Kalman filter. As Roberts and Fackler (1999) mention that due to identification problems, it is not possible to obtain useful estimates of λ, the market price of convenience yield risk. Since it is expected to be small, it was set to zero in the estimation. The risk-free interest rate is assumed to be 5%.

Table 1 presents the parameter estimates for the seven commodities. For each commodity, four models with seasonal order from 0 (no seasonality) to 3 were estimated. The final order was chosen by considering both the Akaike information criteria (AIC) and the log-likelihood function. The order 2 model was found to have the appropriate balance between fit and parsimony for five out of seven commodities, while for crude oil the order 1 model was found appropriate. Seasonal patterns are strong in these markets as annual supply and demand forces repeat themselves. Gold has no seasonal effect in its prices as expected.

The seasonality functions in the convenience yield were plotted in figure 1. For corn, it has a peak around July or August. This corresponds to the pre-harvest period for corn, when large amount of uncertainty about the crop size generally causes the convenience yield to be high. For crude oil, the situation is complicated by the different seasonal demand patterns of two major oil products, gasoline and heating oil, however the magnitude of α is small. Natural gas has a seasonal peak during winter time as expected, while feeder cattle and live cattle have the opposite seasonal patterns. Pork bellies have two peaks around July and January, respectively. Figure 2 shows for each Wednesday the estimated state variables (the logarithm of the spot price and the instantaneous convenience yield) and the logarithm of the commodity futures price for the contract closest to maturity. The strong positive correlation between the spot price and the convenience yield (ρ is between 0.43 and 0.48 for all commodities except gold), is consistent with the stylized fact that convenience yield, like price, is high (low) when stocks are scarce (plentiful). The closeness between the log spot price and the log nearby futures price for most commodities can be observed from the figure which indicates a good fit of the term structure model.

For the model performance comparison, the root mean square errors (RMSE) range from 0.09% for gold, 0.63% for feeder cattle, 0.87% for crude oil, 1.30% for live cattle, 1.48% for corn, 2.06% for pork bellies, to 3.40% for natural gas. Or in absolute terms, they are about 32 cents for gold, 50 cents for feeder cattle, 21 cents for crude oil, 94 cents for live cattle, 4 cents for corn, 130 cents for pork bellies, and 11 cents for natural gas. The model fits best for gold, the most storable goods, but worst for natural gas, which is probably due to the available longer maturities of futures contracts causing relatively large estimation errors compared to the livestock market (the longest maturity available for feeder cattle, live cattle, and pork bellies are

approximately 11 months, 13 months, and 16 months, respectively). The model seems to perform well for non-storable goods, feeder cattle and live cattle. The RMSE is lower for corn than pork bellies, which may have relatively higher storage costs making its storability weaker than that of corn.

A further diagnosis of the in-sample prediction errors with respect to time to maturity and time of the year for each commodity was illustrated by figure 3. The errors are very small relative to its absolute price for gold, indicating the best fit. Crude oil has a complicated pattern in its prediction errors versus time to maturity, but generally its errors tend to be larger for contracts at the two ends. For natural gas, its errors are large relative to its absolute price and there are a few systematic errors which may due to futures price errors. It also has larger errors around November. In general, the energy market has large prediction errors during recent years, which is consistent with the empirical evidence. Corn has larger errors around July, while pork bellies seem to have smaller errors from September to December compared to other months. There are no obvious patterns in the prediction errors for feeder cattle and live cattle, but the errors of the latter are more spread out than the former. In sum, the diagnosis indicates that the prediction errors tend to be larger for longer maturity contracts and during price volatile periods. These results still provide support for the importance of considering storability in modeling futures prices, as seen from the large differences in model fit between the seven commodities.

6. Discussion and Conclusion

Schwartz has provided an integrated framework for modeling commodity futures prices that allows futures of all maturities to be used simultaneously to estimate model parameters. It also allows for fixed, deterministic seasonal effects to be incorporated and Roberts and Fackler

(1999) propose one way of modeling seasonality in the term structure model. However, the general framework does not necessarily capture individual attributes of the specific markets that may be important in empirical analysis, for example, the storability of different commodities. A possible consequence is that estimates of the model's parameters are not robust (Tomek, 2000).

This paper investigates the effects of storability on commodity futures pricing, especially whether meats can be reasonably approximated by storable commodity term structure models. First the two-factor model with seasonality was estimated using futures data from seven commodities, gold, crude oil, natural gas, corn, pork bellies, feeder cattle and live cattle. It is found that the model fit is good for less storable commodities but with large differences across the seven commodities. The results indicate that further investigation of the storability and seasonality for each market is necessary and more commodities may be added in the empirical analysis to better understand the differences.

As the next step of this study, a structural model will be constructed, which is similar to Peterson and Tomek (2005), but with a generalization to accommodate different levels of storability. After calibrating the model, simulated data will be generated to estimate the parameters of the two-factor Schwartz model to note any divergence between the two models.

References

Bessembinder, H., J.F. Coughenour, P.J. Seguin, and S.M. Smoller. 1995. "Mean Reversion in Equilibrium Asset Prices: Evidence from the Futures Term Structure." *Journal of Finance* 50: 361-375.

Chicago Board of Trade. 2006. "Grain and Soybean Futures and Options" CBOT Publications. Available at http://www.cbot.com/cbot/docs/62544.pdf.

Chicago Mercantile Exchange. 2006a. "An Introduction to CME Commodity Products." CME Brochures. Available at http://www.cme.com/files/intro_commodity.pdf.

Chicago Mercantile Exchange. 2006b. "An Overview of CME Commodity Futures for Hedgers." CME Brochures. Available at http://www.cme.com/files/Commodity_Futures_Hedgers.pdf.

Chicago Mercantile Exchange. 2006c. "CME Livestock Futures and Options: Introduction to Underlying Market Fundamentals." CME Brochures. Available at http://www.cme.com/files/LivestockFund.pdf.

Cox, J.C., J.E. Ingersoll, and S.A. Ross. 1985. "A Theory of the Term Structure of Interest Rates." *Econometrica* 53: 385-408.

Deaton, A., and G. Larocque. 1996. "Competitive Storage and Commodity Price Dynamics." *Journal of Political Economy* 104: 896-923.

Deaton, A., and G. Larocque. 1992. "On the Behavior of Commodity Prices." *Review of Economic Studies* 59: 1-23.

Hull, John. 2003. *Options, Futures, and Other Derivatives*, 5th edition. Prentice Hall.

Kaldor, N. 1939. "Speculation and Economic Stability." *Review of Economics Studies* 7: 1-27.

Muth, J.F. 1961. "Rational Expectations and the Theory of Price Movements." *Econometrica* 29: 315-335.

New York Mercantile Exchange. 2006a. "Gold Futures Contract Description and Specifications." Available at http://www.nymex.com/gol_fut_descri.aspx.

New York Mercantile Exchange. 2006b. "NYMEX Energy Complex" NYMEX Brochures. Available at http://www.nymex.com/media/NYMEX_energy_complex.pdf.

Nielsen, M.J., and E.S. Schwartz. 2004. "Theory of Storage and the Pricing of Commodity Claims." *Review of Derivatives Research* 7: 5-24.

Peterson, H.H., and W.G. Tomek. 2005. "How Much of Commodity Price Behavior Can a Rational Expectations Storage Model Explain?" *Agricultural Economics* 33: 289-303.

Roberts, M.C., and P. L. Fackler. 1999. "A Term Structure Model for Agricultural Futures." Selected Paper Presented at AAEA annual meeting.

Routledge, B.R., D.J. Seppi, and C.S. Spatt. 2000. "Equilibrium forward curves for commodities." *Journal of Finance* 55: 1297-1338.

Rui, X., and M. J. Miranda. 1995. "Commodity Storage and Interseasonal Price Dynamics." In: NCR-134 Conference Proceedings: 44-53.

Schwartz, E.S. 1997. "The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging." *Journal of Finance* 52: 923-973.

Schwartz, E.S., and J.E. Smith. 2000. "Short-term Variations and Long-term Dynamics in Commodity Prices." *Management Science* 46: 893-911.

Sorensen, C. 2002. "Modeling Seasonality in Agricultural Commodity Futures." *Journal of Futures Markets* 22: 393-426.

Telser, L. 1958. "Futures trading and the storage of cotton and wheat." *Journal of Political Economy* 66: 233–255.

Tomek, W.G. 2000. "Seasonality in Agricultural Commodity Futures: Comment." In Chicago Board of Trade 13th Annual European Futures Research Symposium, Glasgow Symposium Proceedings.

Tomek, W.G., and H.H. Peterson. 2001. "Risk Management in Agricultural Markets: a Review." *Journal of Futures Markets* 21: 953-985.

Williams, J.B. 1936. "Speculation and the Carryover." *Quarterly Journal of Economics* 50: 436-455.

Williams, J.C., and B.D. Wright. 1991. *Storage and Commodity Markets*. New York: Cambridge University Press.

Working, H. 1948. "Theory of the inverse carrying charge in futures markets." *Journal of Farm Economics* 30: 1–28.

Working, H. 1949. "The theory of price of storage." *American Economic Review* 39: 1254–1262.

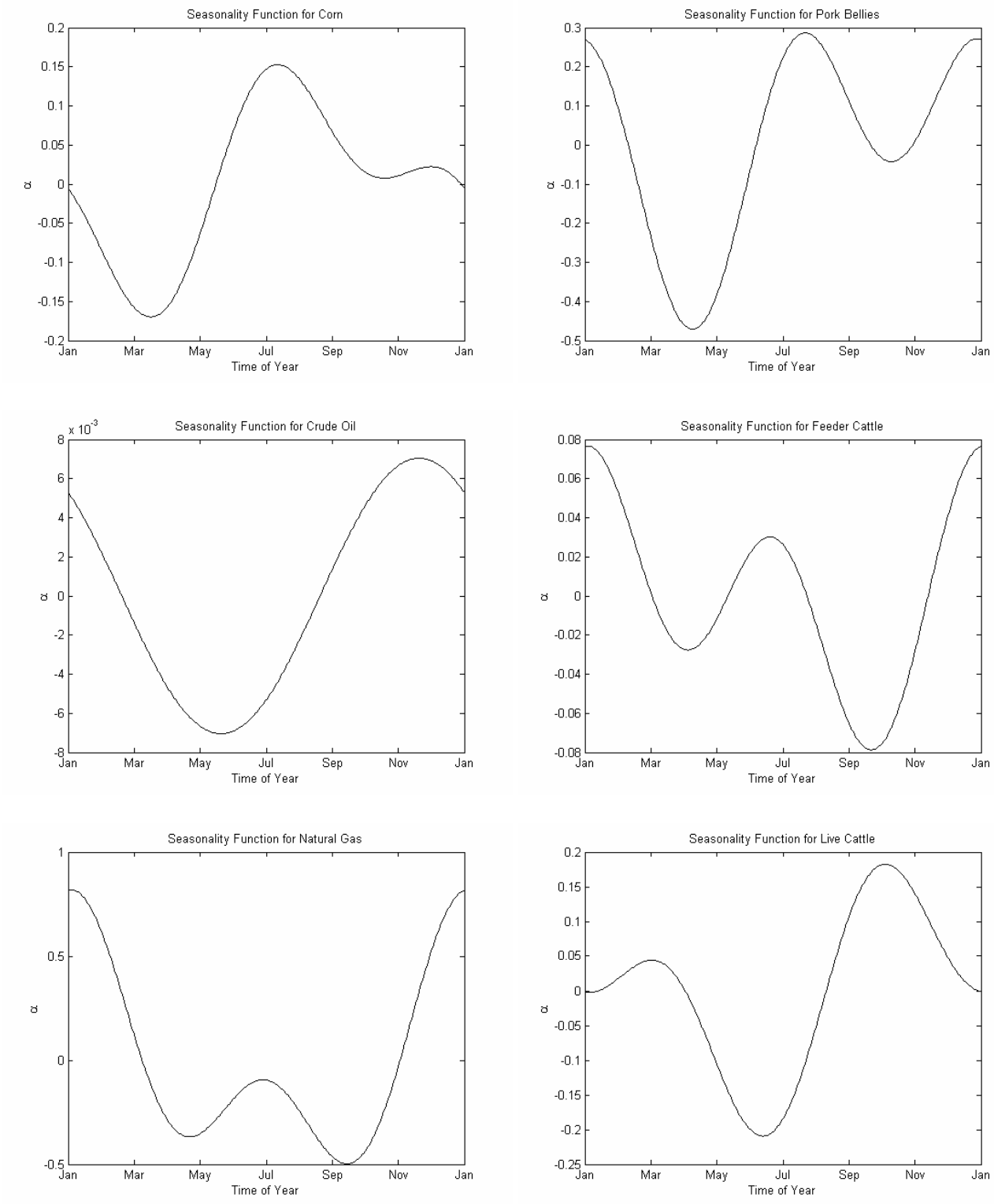
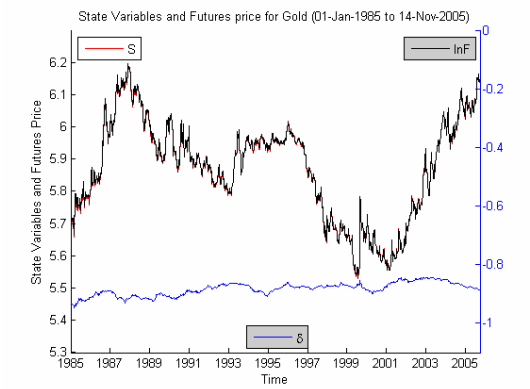
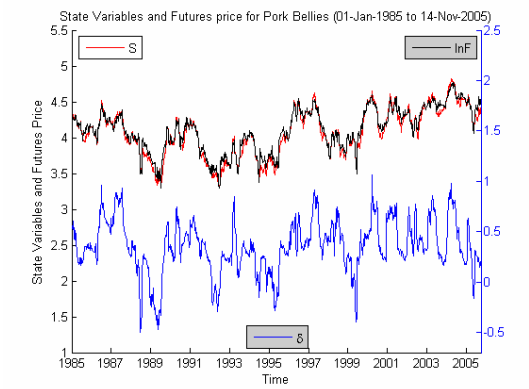
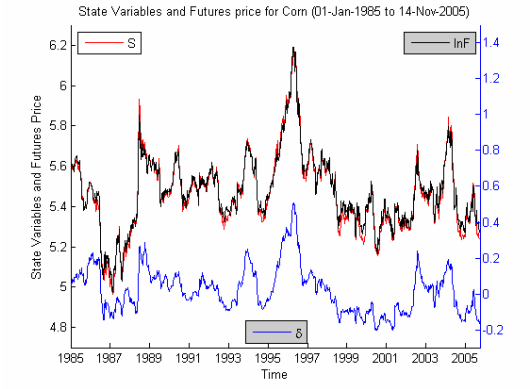
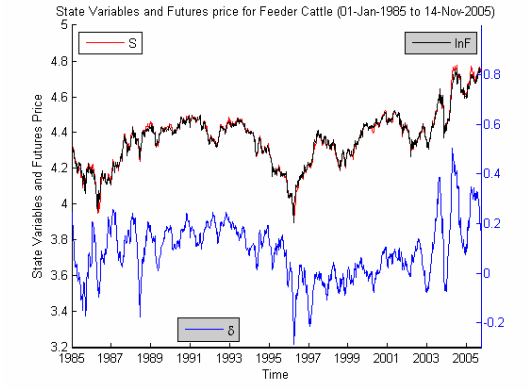


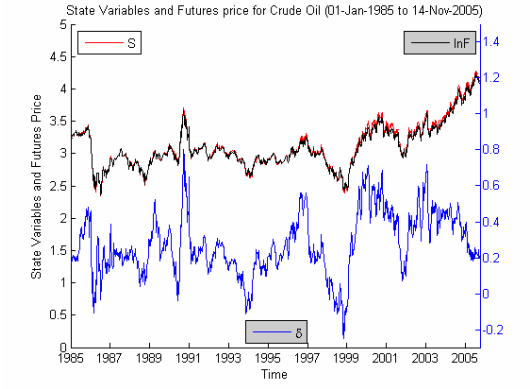
Figure 1. The figure shows the seasonality functions in the convenience yield for 6 commodities (except gold).

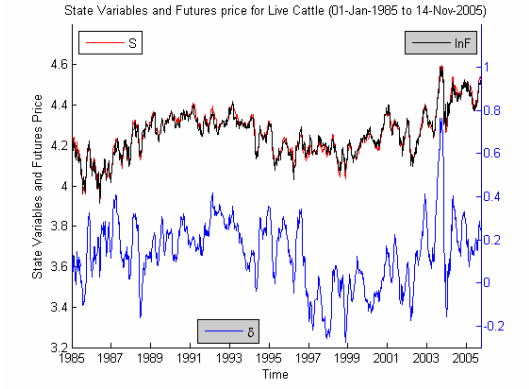












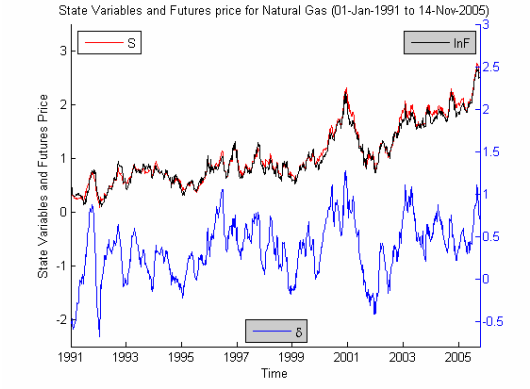
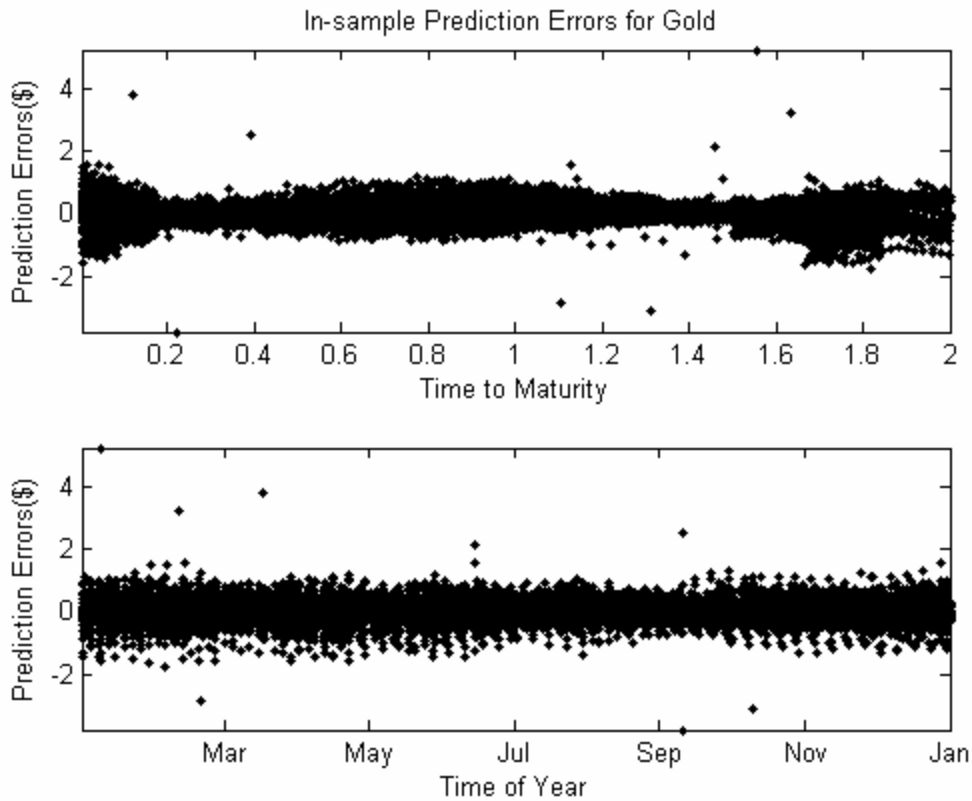
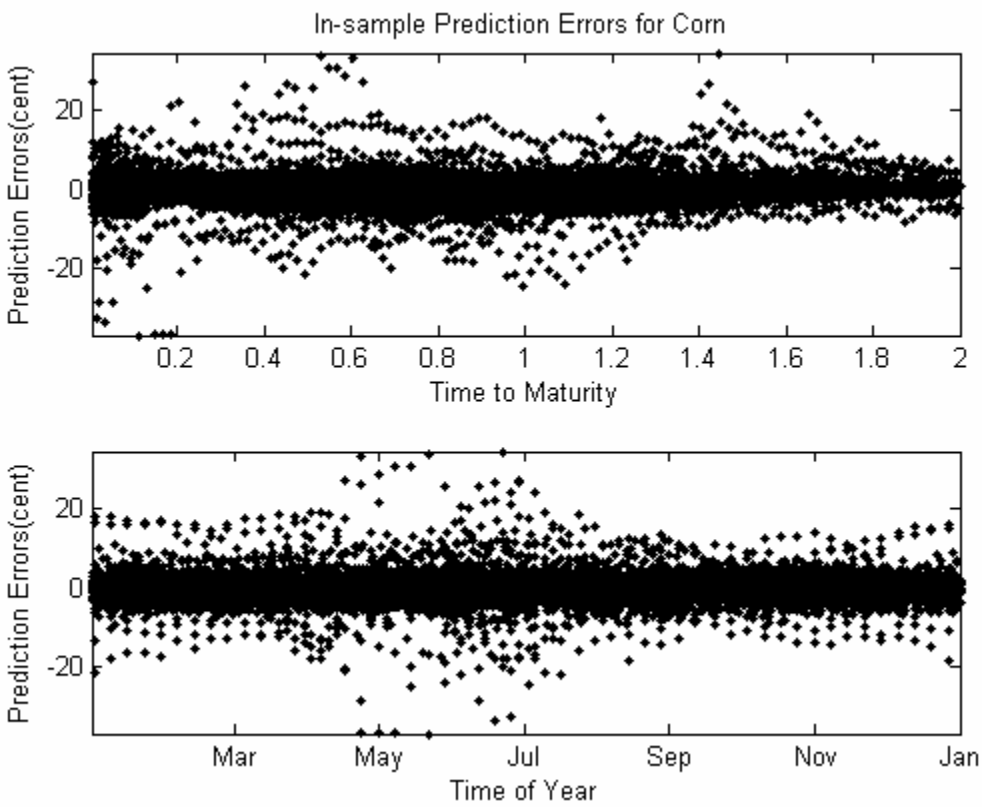
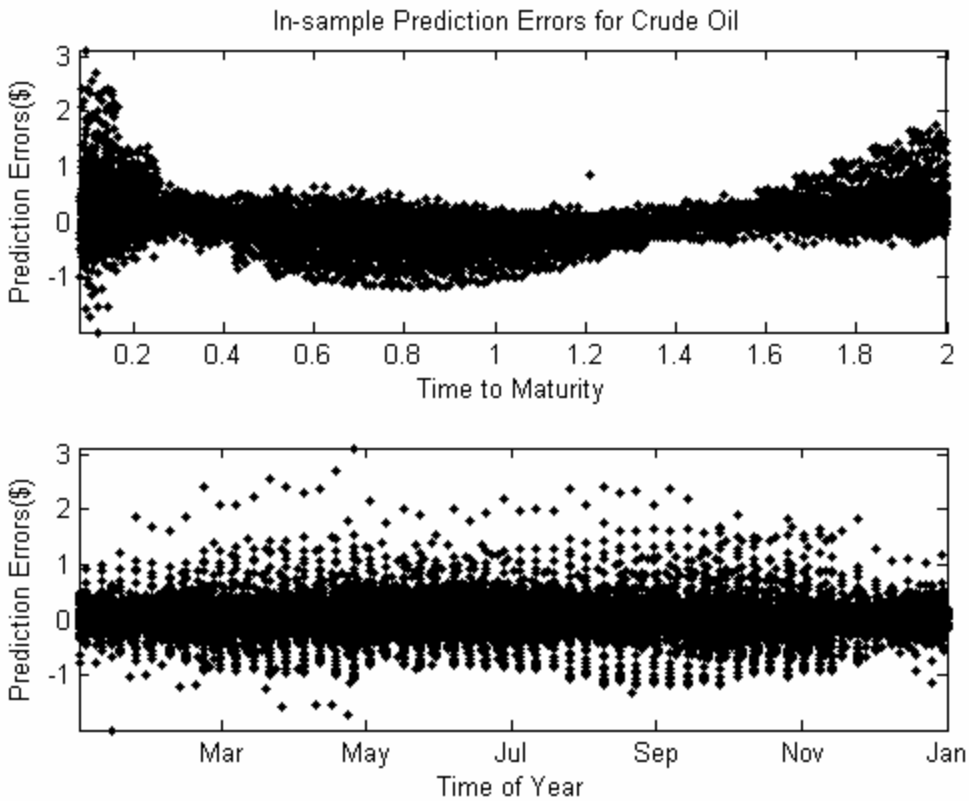
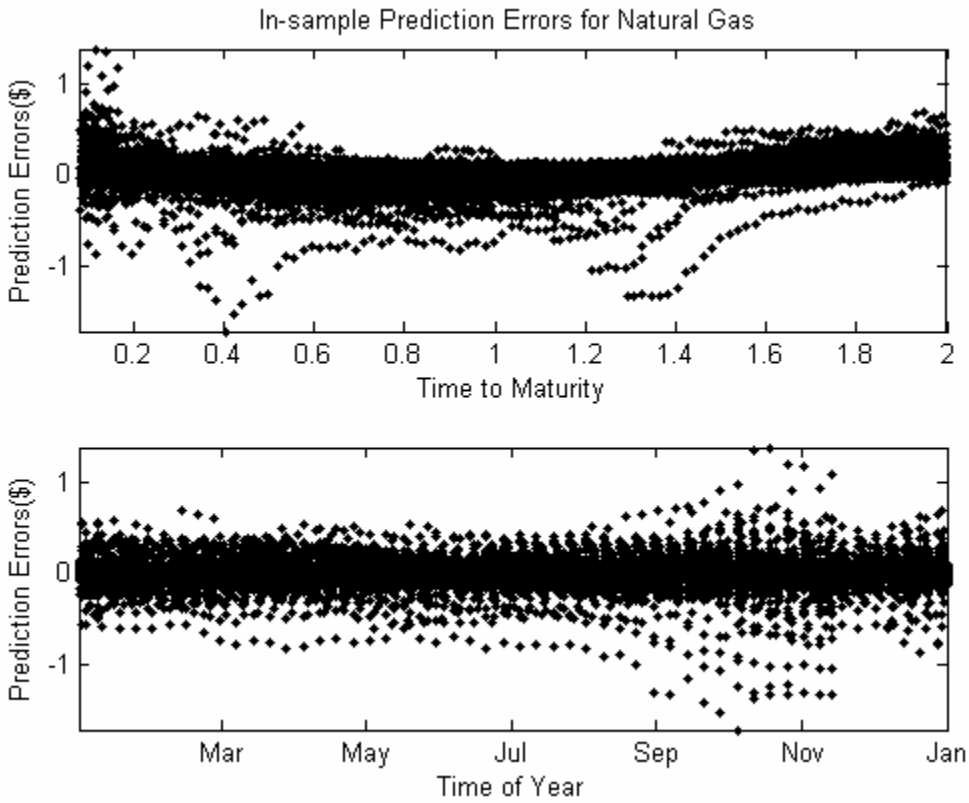


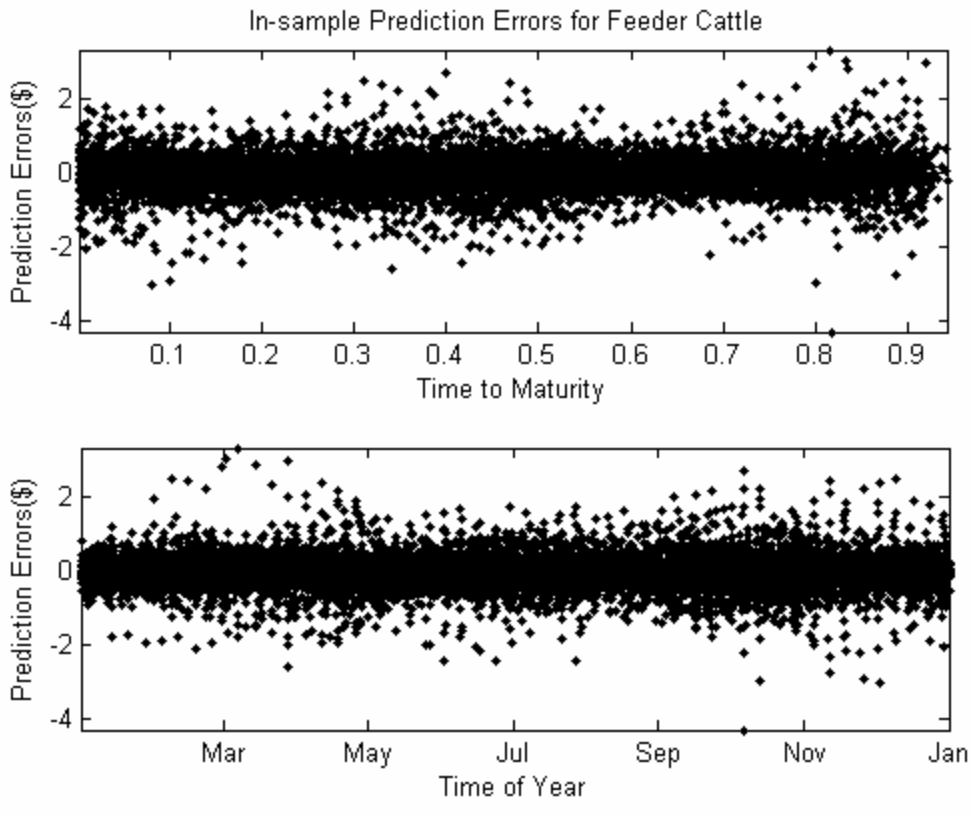
Figure 2. The figure shows for each Wednesday the estimated state variables (the logarithm of the spot price and the instantaneous convenience yield) and the logarithm of the futures price for the contract closest to maturity, starting from 1/1/1985 to 11/14/2005 (natural gas from 1/1/1991).

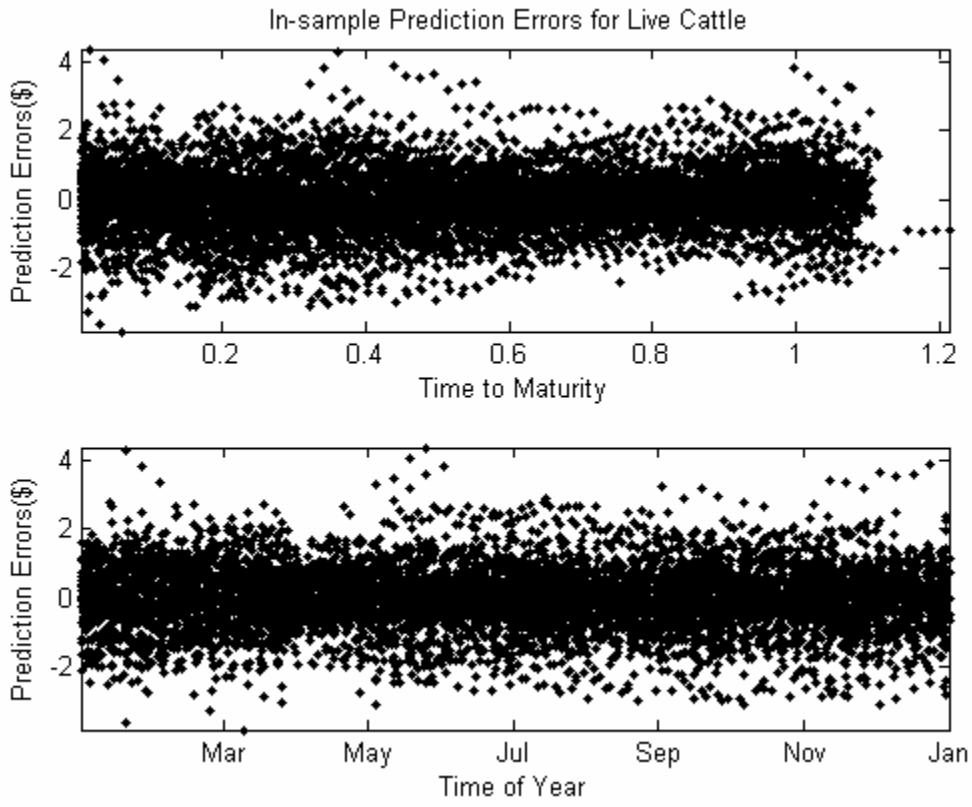

In-sample Prediction Errors for Gold
Prediction Errors($)
4
2
0
-2
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time to Maturity
Prediction Errors($)
4
2
0
-2
Mar
May
Jul
Sep
Nov
Jan
Time of Year


In-sample Prediction Errors for Corn
Prediction Errors(cent)
20
0
-20
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time to Maturity
Prediction Errors(cent)
20
0
-20
Mar
May
Jul
Sep
Nov
Jan
Time of Year


In-sample Prediction Errors for Crude Oil
Prediction Errors($)
3
2
1
0
-1
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time to Maturity
Prediction Errors($)
3
2
1
0
-1
Mar
May
Jul
Sep
Nov
Jan
Time of Year


In-sample Prediction Errors for Natural Gas
Prediction Errors($)
1
0
-1
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time to Maturity
Prediction Errors($)
1
0
-1
Mar
May
Jul
Sep
Nov
Jan
Time of Year


In-sample Prediction Errors for Feeder Cattle
Prediction Errors($)
2
0
-2
-4
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Time to Maturity
Prediction Errors($)
2
0
-2
-4
Mar
May
Jul
Sep
Nov
Jan
Time of Year


In-sample Prediction Errors for Live Cattle
Prediction Errors($)
4
2
0
-2
0.2
0.4
0.6
0.8
1
1.2
Time to Maturity
Prediction Errors($)
4
2
0
-2
Mar
May
Jul
Sep
Nov
Jan
Time of Year

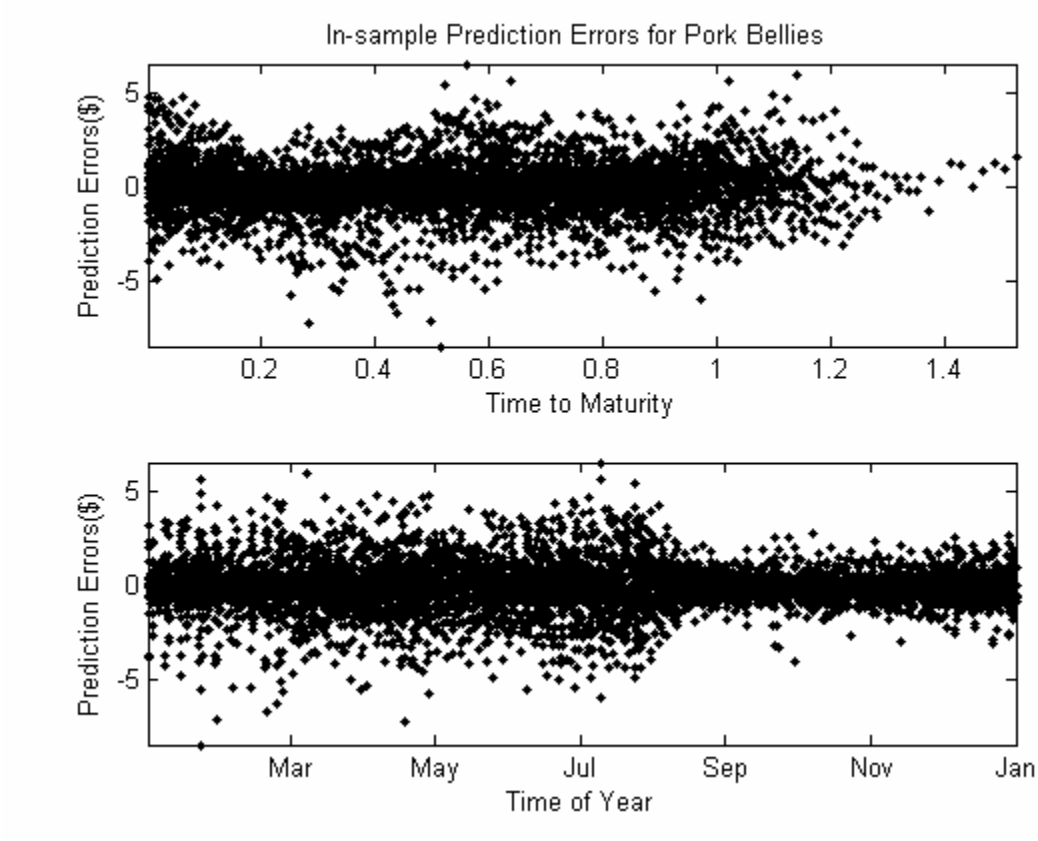


Figure 3. The figure shows the in-sample prediction errors vs. time to maturity (in years) and time of the year for each commodity.

Table 1. Parameter Estimates for the 7 Commodities

	Gold	Feeder Cattle	Crude Oil	Live Cattle	Corn	Pork Bellies	Natural Gas
μ	-0.7129***	-0.0044	0.1053	0.0028	-0.0713	0.1541	0.2047
α	-0.0521***	-0.0436***	-0.0012	-0.0249***	-0.1109***	-0.0204*	-0.0046
κ	0.0553***	1.5459***	0.9656***	2.0754***	0.5634***	1.0438***	1.4068***
σ_1	0.1341***	0.1431***	0.3370***	0.1615***	0.2326***	0.3947***	0.4360***
σ_2	0.0151***	0.1821***	0.2963***	0.3130***	0.1493***	0.5510***	0.6153***
σ_ε	0.0010***	0.0071***	0.0079***	0.0143***	0.0162***	0.0241***	0.0330***
ρ	0.0404*	0.4397***	0.4597***	0.4789***	0.4733***	0.4319***	0.4639***
η_1		-0.0241***	0.0046***	0.0892***	0.0861***	0.2130***	-0.0753***
θ_1		-0.0251***	-0.0053***	-0.0913***	0.0761***	-0.0225	-0.4553***
η_2		0.0069***		-0.0337***	0.0112	-0.0720**	-0.0115**
θ_2		-0.0514***		0.0928***	-0.0708***	-0.2463***	-0.3600***
RMSE(%)	**0.09**	**0.63**	**0.87**	**1.30**	**1.48**	**2.06**	**3.40**
LLK	71.5198	28.3686	74.3047	22.1252	25.4447	13.6580	54.9212
MEAN($)	359.67	79.87	23.81	72.17	2.41	63.04	3.38
S.D.	5.1625	1.1366	0.8293	1.2840	0.0581	2.7540	0.1890
OBS.	13308	7747	20805	7160	8528	5596	15974

- *** significant at 1%, ** significant at 5%, * significant at 10%.
- MEAN and S.D. are respectively the average sample mean and standard deviation of the month-before-expiry prices.