Teaching sustainable resource management in uncertain environments

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Abstract

Dynamic evolutions of resource stocks with stochastic elements in the transition equation are in general very difficult to master. Their handling requires a deep understanding of control theory,[1] probability theory and sometimes even of game theory due to strategic interaction of 'agents'. But without strong mathematical backgrounds, students from adjacent research fields have a hard time with control theory. The same is true for probability theory and game theory. One way to avoid this problem is to change the aim: instead of target function optimization, guarantee the continuance of the system within certain boundaries. The latter relates to Viability theory.[2] Unfortunately, even Viability theory requires more mathematics than the 'average' student is prepared for.

The paper at hand will demonstrate how Excel can help here. Excel is applied since it is a widespread tool and most students are familiar with its basic features. Therefore students can concentrate on how to implement a dynamic system in a spreadsheet and how to simulate probability distributions and approximate the distribution of the target function - given different control rules. This enables them to assess opportunities and risks associated with these control rules.

One topic appropriate to demonstrate the idea is renewable resource management. As many studies state, there is a deficit in sustainable learning not only in economics (Salemi and Siegfried 1999; Walstad and Allgood 1999)[3], but particular in system dynamic models (Moxnes, E. 2000; Pala and Vennix 2005). This is due to the complexity associated with long run- and feedback effects, and the complexity becomes even harder when stochastic development is included. The purpose of this paper will be to inspire students and to encourage them to solve stochastic dynamic problems later on their own – with the simple tools at hand presently.

JEL references: A22, C73, Q30

Key Words: Viability theory; resource management, uncertainty.

Introduction

Excel is a tool not tailored to system dynamics[4], dynamic optimisation or game theory[5]. But Excel is very flexible, able to use adequate add-ins and Macros and to be managed via VB programs, respectively. Furthermore it is equiped with random number generation according to the most relevant probability distributions. But above all, it is widespread. Therefore, more and more resource economic textbooks and papers work with Excel spreadsheets. To name just some, Conrad (1999) employs Excel for fishery- and forestry models as well as for exhaustible resources and pollution management. Buongiorno and Gilles (2003) use Excel in the context of forest management. Their spreadsheets take into account constraints through environmental policy, biodiversity requirements and integer variables. Examples of adjacent fields where Excel is also used, are: Kirschke and Jechlitschka (2002) and Ragsdale (2001). The former deal with interventions in agricultural markets. The latter concentrates on business and organisational problems.

Examples of studies in resource economics using Excel to refer to are the work of Gerking et al. (2002) and the study paper of Caplan (2004). Gerking et al. (2002) look at the effects of decreasing tax rates and increasing environmental requirements on oil and gas drilling and coal mine production. Caplan (2004) deals with extraction from a mine.

But there is one sub area rarely addressed in the literature mentioned above: renewable resource management in a stochastic dynamic system. The situation modelled here is inspired by Béne, Doyen and Gabay (2001), who deal with viability analysis, yet it allows for stochastic elements in the dynamic development up to a defined period and assumes to stay in a steady state thereon.

Viability analysis replaces the wide-spread target to maximize a net present value of resource usage induced profits through the target to keep the system viable. Aubin (1990) introduced the concept of the Viability kernel. In the 2002 "Introduction to Viability Theory and the management of renewable resources" he defines "*viable evolution*" and "*viable evolution capturing the target*" in the following way: The first denotes an evolution $x(t)$ not leaving a certain subset K of the state space X. The latter denotes a viable evolution $x(t)$ arriving at a target C within finite time[6]. The viability kernel is then defined as the initial states $x_0 \in K$ for which either a viable evolution exists or an evolution exits which is viable in K till it reaches the target C in finite time. Evolutions fulfilling the latter of the two conditions are in the capture basin, which is ergo a subset of the viability kernel. To manage a resource in such a way, that the corresponding evolution is viable, does neither mean to implement a state-independent management-rule, nor to implement a management rule, which is state-dependent to a certain "degree", but which does not include a reaction to the arrival at the kernel boundaries. It means to react when necessary.

The concept might become more perspicuous, when one tries to get to the bottom of the results of famous studies concerning the environmental and economic future of the world. One famous report is "Our common future" – a report from the World Commission on Environment and Development. It was presented in 1987 and became well-known under the name Brundtland-report. At the beginning the Commission states: "… we see .. the possibility for a new era of economic growth, one that must be based on policies that sustain and expand the environmental resource base. … hope for the future is conditional on decisive political action now to begin managing environmental resources to ensure both sustainable human progress and human survival" (World Commission on Environment and Development 1987, p. 18). In the concept of viability theory, the evolution so fare was viable, but we arrived at the boundary of the viability kernel and we have to react on this bang: change our management rule. And another issue becomes perspicuous here: the boundary is not defined by nature and limits of renew ability solely, but by the economic requirements, too.

Two more famous studies on the environmental and economic future of the world are the "Limits to growth" from 1972 and the presentation of revised results 20 years later in 1992 with "Beyond the limits". In the "Limits to Growth" the authors state, that if all our "management rules" stay the same in the next decades, mankind will reach the limits of growth within the next century. In the concept of viability theory the authors announced the bang on the boundary of the viability kernel. And they emphasized the existence of growth rates (for population, capital stock, food production, ...) – i.e. management rules - such that a long-run economical and ecological equilibrium exists. Yet, we have to react to our impending approach to the boundaries and we have to change our management rules. Further, the authors remarked, the sooner we turn towards these 'sustainable' growth rates, the more likely we can implement this equilibrium. This can be interpreted as the advise not to wait till the bang is there, because then the danger of an irreversible step out of the viability kernel is serious. Yet, in the revised version 1992, the authors still saw an opportunity to switch to 'sustainable' life.

The concept of a capture basin as a subset of the viability kernel might become more perspicuous, when one looks again in the Brundland-report. Concerning world population size two statements link the year global growth rate reaches the replacement-level to the resulting stable world population. In case fertility achieves replacement-level in 2010, 50 years later population will stabilize at 7.7 billion. But if it achieves the level not before the year 2065, the population will increase up to be 14.2 billion at the end (World Commission on Environment and Development 1987, p. 106). The global population after stabilization is an example for the target to be reached, and the capture basin represents population levels for which viable evolutions exist, such that the target is met within finite time.

Summarizing, in contrast to dynamic control theory, viability theory does not look for an intertemporal optimum. It asks for the existence of controls, such that an evolution currently in the viability kernel, will stay in the kernel. As long as the evolution is not in danger to step beyond the boundaries, the control might be represented by a rule of low complexity (state-independent or state-dependent), but in case of a bounce at the boundary, there has to exist an adjustment of the rule preventing the system from leaving the viability kernel.

An important aspect emphasised by Aubin, is the non-deterministic character of dynamics on earth. We want to introduce this aspect into the adoption of viability theory in the model of Béne, Doyen und Gabay (2001). They analyse a setting with a renewable resource and economic requirements adding the boundaries of viability. To keep the analysis as simple as possible we will leave out capacity aspects they include.

The Model

Following Béne, Doyen und Gabay (2001) we analyse the management of a renewable resource and ask for viability. The renewable resource has a logistic growth function. Yet the intrinsic growth rate we apply is uniform distributed within given boundaries; i.e.:

$$f(x(t)) = r \cdot x(t) \cdot \left(1 - \frac{x(t)}{L}\right) \tag{1}$$

with

$$r \sim [r_{\min}; r_{\max}];\ 0 < r_{\min} \leq r_{\max} \tag{2}$$

Figures 1 visualises the stochastics; the displayed curves between the two limiting curves are all similar probable.

FIGURE 1 ABOUT HERE

The natural growth $f(x(t))$ lessened by the harvest $h(e(\cdot), x(t))$ gives the net growth:

$$\dot{x}(t) = f(x(t)) - h(t) \tag{3}$$

with

$$h(e(\cdot), x(t)) = q \cdot e(t) \cdot x(t) \tag{4}$$

The functional form is known from the Gordon-Schaefer model (1954). Within the harvest function e denotes the adduced effort and q is an efficiency parameter. Thus, the harvest is proportional to effort and stock size $x(t)$. Now, it is intuitive, that an effort rule like $e \equiv 0$ will leave the evolution of the resource stock a priori viable, since the stochastic in the natural growth does not foreclose the approximation to the carrying capacity. But as Aubin (2002) cites Monod who cites Democritus "Everything that exists in the universe is due to change and necessity" (Democritus , 460–370 BC). And the necessity is introduced in the Béne-Doyen-Gabay model through economic requirements which will forbid a choice of $e \equiv 0$. But before these requirements are introduced too, we look at effort rules, since they link the economic perspectives of renewable resource management to the evolution of the state.

Two simple effort rules in the stochastic setting

Within our stochastic setting there will exist effort rules – other than $e \equiv 0$ – which guarantee for the conservation of the resource – as long as the initial values (x_0, e_0) are "adequate". To clarify the issue, assume $x_0 = 20;\ r_{\min} = 0.5;\ r_{\max} = 1; \cdots L = 100$ as in figure 1. Under these assumptions, the intrinsic growth of the first period will be a random variable uniform distributed on the interval $[8,16]$. The natural growth in the second period is a random variable depending on the realization of the random natural growth in the first period and on the effort role chosen. In case $e(x(t)) = r_{\min} \cdot (1 - x(t)/L)/q$ we have $h(t) = q \cdot e(x(t)) \cdot x(t) = \ r_{\min} \cdot x(t) \cdot (1 - x(t)/L) \leq f(x(t))$ i.e. harvest will never exceed natural growth and therefore the resource stock will increase. The following figure 2 displays some random trajectories for the first 30 periods. Even within this rather limited time horizon we see an increase of the resource stock up to the carrying capacity with high probability, and for all the 5 trajectories displayed in figure 2.

FIGURE 2 ABOUT HERE

The arrival at the carrying capacity in the long-run can not astonish, due to the overcautious decision never to harvest more than the natural growth in the worst case.

With a rigid effort rule like $e = r_{\min} \cdot (1 - x_0 / L)/q$ effort is state-independent, solely determined by initial conditions. Here we are on the safe side, too. Figure 3 presents some trajectories of the resource stock for the same initial conditions as before, but the rigid effort rule.

FIGURE 3 ABOUT HERE

With the rigid rule from figure 3, the stock has to increase in the first periods. Figure 3 displays strictly increasing trajectories till the limit value of 80 is reached. Thereafter - i.e. as soon as stock size arrives at 80 ($80 = L - x_0$) - negative net growth becomes possible due to the fact that the natural growth might be lower for a stock of size over 80 than the rigid harvest, which emanates from the rigid effort.

Changing the initial stock to a smaller value ($0 < x_0 < 20$) keeps the harvest at a lower level for the rigid rule and therefore generates a higher and less volatile figure of trajectories.
On the other hand for the overcautious effort rule and a smaller initial stock, on average it will take more time to approximate to the carrying capacity. The lower the initial stock, the greater the difference between L and x_0.

Now, changing the initial stock to a higher value ($20 < x_0 < 50$) does not really “endanger” the stock for neither rule. For the first, flexible rule an excess of harvest compared to natural growth was impossible. And for the second, rigid rule, even starting with $x_0 = 50$ and therefore with the highest harvest possible does not mean that one will eradicate the resource. A secure degradation is only given for stock sizes below 14.65 ($12.5 = r_{\max} \cdot 14.65 \cdot (1 - 14.65 / L)$ and $12.5 = r_{\min} \cdot 50 \cdot (1 - 50 / L)$; see the most left arrow in figure 4). And degradation will impend for a stock size of about 21 ($12.5 \approx r_{avg} \cdot 21 \cdot (1 - 21 / L)$; i.e. at stock size 21 the probability of a positive and of a negative net growth are both equal; i.e. they are 0.5; see the commensurate brackets in figure 4). Yet, starting at $x_0 = 50_0$, the probability of a negative net growth is diminishing. Thus, the development will induce an increase in the stock. A stock size above 50 will allow for a negative net growth, but the negative growth will not be large enough in size to jump on the increasing segment of the growth curves, where a stochastic decline of the stock would become possible (see Appendix). Figure 4 demonstrates the relation between the values:

FIGURE 4 ABOUT HERE

There is one other rule, we tested: the average rule (avg rule). Its structure is the same as for the overcautious rule, i.e. it utilizes the current stock. But instead of the minimum growth rate it employs the average growth rate.

The economic perspective

Now, does the rigid effort rule raise risks higher than what we feel up to except? And how do we evaluate the risk? So far, economic issues are neglected and therefore what was interpreted as the “necessity” in the citation of Democritus. To introduce the economic perspective, go back to Béne, Doyen, and Gabay, and assume that the sales price (p) for the resource as well as the unit costs for effort (c) are both fix, and in addition to the variable costs, fix costs C exist; thus the period-profit is given by the following expression:

$$R(x(t),e(t)) = (p \cdot q \cdot x(t) - c) \cdot e(t) - C \tag{5}$$

The "margin" – relating to the effort - is a function of the actual stock size.

To be in line with Béne, Doyen, and Gabay, ask for non-negative profits in all periods as long as possible without putting the resource at risk. Depending on the parameter constellation, effort rules generating viability exist in the deterministic setting of Béne, Doyen, and Gabay.

As long as the intrinsic growth rate is not to low everlasting viability can be implemented; i.e. identify effort–stock–combinations generating non-negative profits and at the same time no-degrading the resource stock:

$$R(x(t),e(t)) = (p \cdot q \cdot x(t) - c) \cdot e(t) - C = 0 \Leftrightarrow e = \frac{C}{(p \cdot q \cdot x(t) - c)} \tag{6}$$

$$\dot{x}(t) = f(x(t)) - h(x,e) = r \cdot x(t) \cdot \left(1 - \frac{x(t)}{L}\right) - q \cdot e \cdot x(t) = 0 \Leftrightarrow e = \frac{r}{q} \cdot \left(1 - \frac{x(t)}{L}\right) \tag{7}$$

$$\frac{C}{(p \cdot q \cdot x(t) - c)} = \frac{r}{q} \cdot \left(1 - \frac{x(t)}{L}\right) \quad \Leftrightarrow \quad x_{1;2} = \frac{1}{2}\left(L + \frac{c}{p \cdot q}\right) \pm \sqrt{\left(\frac{1}{2}\left(L + \frac{c}{p \cdot q}\right)\right)^2 - \frac{L}{p \cdot q}\left(c + \frac{q}{r}C\right)} \tag{8}$$

The radicant below the root is positive as long as:

$$\left(\frac{1}{2}\left(L + \frac{c}{p \cdot q}\right)\right)^2 - \frac{L}{p \cdot q}\left(c + \frac{q}{r}C\right) > 0 \quad \Leftrightarrow \quad r > q \cdot C \frac{4 \cdot p \cdot q \cdot L}{(p \cdot q \cdot L - c)^2} \tag{9}$$

Figure 5 demonstrates the situation: each effort-stock combination above the $R(x,e) = 0$-curve has non-negative profits and all combinations above the $\dot{x}(x,e) = 0$-curve give a decline in the stock size[7]. For a very small initial x_0 no effort rule exists ensuring a viable evolution right from the beginning. The abolishment of the economic perspective – suspension from harvest and therefore no revenue through resource use in the first periods – is the only opportunity.

For a sufficiently high initial x_0 effort rules exist ensuring a viable evolution. For example, for some "medium" $x_0 \in [x_-, x_+]$, rigid rules like $e = r \cdot (1 - x_0 / L)/q$ or $e = \bar{e} \in [e_+, e_-] \cap \{e : R(x_o, e) \geq 0\}$

keep the evolution viable. The difference between these two rigid rules lies in the profit path. The first rule keeps the profits constant, while the second one will induce an initial decrease in the profits during the first periods, in case we choose $\bar{e}$ above the $\dot{x}(x,e)=0$-curve, and an initial increase of the profits, in case we choose $\bar{e}$ from the area bounded by the $\dot{x}(x,e)=0$-curve and the $R(x,e)=0$-curve.

For $x_0 \in [x_+, L]$ the rule $e=\bar{e}\in[e_+, e_-]$ will always generate an initial decrease in profits, as the harvest will decrease with the decrease of the stock, and therefore revenue will decrease leaving the cost side unaffected.

It is intuitive clear, more viable rules that the two rigid rules discussed exist for $x_0 \in [x_-, L]$. But leaving the deterministic setting, they differ in their probability of viable evolutions.

FIGURE 5 ABOUT HERE

In our stochastic setting, the location of the $\dot{x}(x,e)=0$-curve from figure 5 is not fix any longer. The sustainable effort becomes random:

$$\dot{x}(t)=\tilde{r}\cdot x(t)\cdot\left(1-\frac{x(t)}{L}\right)-q\cdot e\cdot x(t)=0 \Leftrightarrow e=\frac{\tilde{r}}{q}\cdot\left(1-\frac{x(t)}{L}\right) \tag{10}$$

It rotates around its intersection with the axis of abscissae[8]. See figure 6 for demonstration:

FIGURE 6 ABOUT HERE

The overcautious effort rule $e(x(t))=r_{\min}\cdot(1-x(t)/L)/q$ as well as the rigid rule $e=r_{\min}\cdot(1-x_0/L)/q$ both generate a random revenue flow. Therefore, there is no longer a guarantee that they can keep the evolution viable for an arbitrary initial stock x_0.

For the overcautious rule, the effort path is a random process, as it takes into account the actual stock size $x(t)$, which is random itself[9]. But since it generates a stock evolution towards the carrying capacity, the final revenue will diminish. Therefore no x_0 exists, thus that positive fix costs can be covered forever. See figure 7 for demonstration in case of initial positive profits[10][11]:

FIGURE 7 ABOUT HERE

Next, the rigid rule generates a random revenue flow, too, although the effort stays the same all the time. But the harvest as a product of effort, stock size and efficiency parameter, is random. From the previous argumentation, we know that the stock fluctuates in the long run. Therefore, there might exist a region for x_0 without losses during the dynamic process. The region will be influenced negative by the cost parameters, as a matter of course[12]. See figure 8 for demonstration:

FIGURE 8 ABOUT HERE

Summing up, the overcautious rule does not induce a viable evolution. But the rigid effort rule $e = r_{\min} \cdot (1 - x_0 / L)/q$ might be part of a viable evolution, depending on the initial stock x_0 in relation to the cost parameters c and C. The distribution of minimum profits fits for the corresponding analysis. And furthermore, we are able to evaluate the system beyond the first 50 periods. The cut after period 50 is arbitrary. It is a simplification of the capture basin aspect, as we force our system to conserve the stock of period 50 with the sustainable management rule: $e(t) = r \cdot (1 - x_t / L)/q;\ t > 50$. In order to keep things easy, world is deterministic thereafter. With these simplifications the probability of an initial stock reaching a defined target is tractable (given a certain management rule). Though it is less than to identify the capture basin, it is a step in direction to capture basins.

The Excel spreadsheet

Figures 9a-c display parts of the Excel spreadsheet. In cells A4:A15 are parameter names and B4:B15 contain the corresponding values; B4:B15 got their names from the cells in the A-column. In our simulation runs we chose a parameter value r of 1, in order to have as same stochastic setting as we discussed under Chapter 2.

A22:A71 display the period numbers; the next column (B22:B71) calculates the evolution of the resource stock with the formula $= \mathrm{WENN(B22 + C22 - q * D22 * B22 > 0; B22 + C22 - q * D22 * B22; 0)}$ for the second period. The evolution of the stock will follow the time-discrete version of formula (3) as long as it stays non-negative; negative values are excluded; C22:C71 contain the natural growth due to formula (1) as random variables. The random element of the natural growth bases on the random growth rate which is even distributed on $[\mathrm{random\_low; random\_up}]$ (see formula (2)). The corresponding expression is $\mathrm{(random\_low + F22 * (random\_up - random\_low)) * r}$. Due to our parameter choice it generates a random variable with an even distribution on $[0.5;1]$. Multiplication with the expression $x(t) \cdot (1 - x(t)/L)$ completes the formula for natural growth $= \mathrm{(random\_low + F22 * (random\_up - random\_low)) * r * B22 * (1 - B22/l)}$ of the first period.

Column D contains the effort rule. The rigid rule is $= \mathrm{random\_low * (1 - x\_0/l)/q}$ in the first period, respectively $= \mathrm{WENN(B23 > q * D22 * B23; \$D\$22; 0)}$ in the second period. I.e. be pessimistic in the first period and make sure that harvest will not exceed natural growth, and as long as possible do not change effort. Only in case harvest would exceed the actual stock, suspend resource usage for the actual period. As soon as possible, start usage again at the level of period 1.

Alternatively, $= \text{random\_low} * (1 - \text{B22/l})/\text{q}$ characterises the overcautious rule in the first period and $= \text{WENN}(\text{B23} > \text{q} \cdot (\text{random\_low} * (1 - \text{B23/l})/\text{q}) \cdot \text{B23}; \text{random\_low} * (1 - \text{B23/l})/\text{q};0)$ in the second period. I.e. as long as possible chose effort such that harvest represents the natural growth of the current period in the worst case. If this is not possible due to resource degradation, suspend effort. The latter regulation is never relevant as we demonstrated in chapter 2.

Finally, we look at an effort rule focusing on the expected average natural growth of the corresponding period. The regulation is given by $= (\text{random\_low} + (\text{random\_up} - \text{random\_low})/2)$ $*\text{r} * (1 - \text{B22/l})/\text{q}$ in the first period and by $= \text{WENN}(\text{B23} > \text{q} \cdot ((\text{random\_low} + (\text{random\_up} - \text{random\_low})/2)) \cdot$ $(*\text{r} * (1 - \text{B23/l})/\text{q}) \cdot \text{B23};\ (\text{random\_low} + (\text{random\_up} - \text{random\_low})/2)/\ *\text{r} * (1 - \text{B23/l})/\text{q};0)$ in the second period.

Further, column E calculates the profit corresponding to formula (5), and column F the random variables.

FIGURE 9a-c ABOUT HERE

Figures 10a-b present the last 10 periods and the long-run-steady state - accessible after period 50.

FIGURE 10a-b ABOUT HERE

To simulate a run with random growth rate, press button F9 on the keyboard. To generate a Makro simulating a series of simulation runs, just use the Makro recorder: start a simulation and copy the main results of the current run; stop the recorder session, edit the Makro and insert the program segment in a loop like “for i = 1 to X ---- .. next i”. Finally, adjust the relevant pieces of the program according to the requirements and the loop design.

The following chapter presents some results for simulation-series length of 30.

The Results

Data Analysis

Concerning the rigid rule the simulation results are:

Due to the increase of the stock during the first periods, the minimum stock of the relevant time horizon of 50 periods is identical to the initial stock. Further, as the effort path is determined by the initial stock, and harvest grows as the stock grows by time, the minimum profit is the profit of the first period. Therefore, the simulation results concerning minimum stock and minimum profit match our argumentation, and the rigid rule might result in a viable evolution, or not. Higher fix costs endanger the viability; as already expressed in formula (9). An illustration is dispensable.

In contrast, the steady state stock and the steady state profit are non-degenerated random variables and an approximation of their distribution is given through the data generated with an Excel Makro. See the following two figures for the result:

FIGURE 11 ABOUT HERE

FIGURE 12 ABOUT HERE

For the overcautious rule the simulation results are what were expected:

The initial resource stock is neither relevant for the steady state stock nor for the steady state profit: the stock will increase near the carrying capacity level and economic losses occur due to uncovered fix costs. The rule is not adequate for viable evolutions.
An illustration of these results is dispensable.

The last rule analysed was the average rule:
It focuses on the expected average growth within a period. The average rule (avg rule) allows for a resource decrease right form the beginning; therefore, the distribution of the minimum stock size and the minimum profits become relevant. Figures 13 and 14 display the distribution:

FIGURE 13 ABOUT HERE

FIGURE 14 ABOUT HERE

As expected, for small initial stocks, there are periods of usage suspension. Accordingly, losses occur.
Further as expected, the higher the initial stock, the higher the average of minimum stock generated in the simulation runs. Intuitively clear, the upper bound of the minimum stock is identical to the initial stock.
More of interest, even for an initial stock of 50, the stock size in the next 50 years can decrease to nearly zero, and losses occur. In two of 30 simulations (with $x_0 = 50$) periods of usage suspension occur – i.e. the evolution displayed a bounce on the boundary of the viability kernel.
There is another important result form figure 14: the rule induces a serious danger of economic losses for high initial stocks. These losses are no consequence of usage suspension

– minimum stock size stays clearly positive. They result from the low natural growth near the carrying capacity. For high initial resource stocks the average rule is therefore inadequate as it induces probably not a viable evolution.

FIGURE 15 ABOUT HERE

FIGUR 16 ABOUT HERE

Figure 15 displays the distribution of the steady state stock and figure 16 of the steady state profit. As already seen from the distribution of the minimums in the first 50 periods, as long as the initial stock is less than half of the carrying capacity, there exists danger of resource degradation at the end.

Decision support opportunity

Resource management has to put weights on their targets concerning the viability of an evolution. Additional to the distribution of minimum stock size or the steady state stock size the management might have other targets. Further an evaluation of distributions is necessary. One example is to search for stochastic dominance of distributions.

The data generated allow for an approximation of density functions and cumulated distribution functions. Figure 17 presents a comparison between the rigid rule and the average rule for $x_0 = 50$. Content is the steady state stock. As expected, the rigid rule is preferable here. The distribution is almost stochastic dominant in the first degree: the distribution curve of the rigid rule stays nearly strict below the distribution curve of the average rule for $x_0 = 50$. But other initial values demonstrate a different picture. And other content does, too. Further, other criteria (e.g. μ-σ- criterion) exist to evaluate distributes.

Furthermore it is fundamental to analyse more rules, e.g. the rule presented in Béne, Doyen, and Gabay, which takes into account additional economic boundaries due to slow capacity adjustments.

Last but not least, the effect of various parameter values determining the stochastics (the upper and the lower bound of the growth rate) should be included in the decision.

Conclusion

The implementation of the simplified Béne-Doyen-Gabay model presented an example of how to introduce viability concepts to students without mathematical background. Viability theory is applicable to many problems related to the management of renewable resources. Special advantage consists in the simple manner stochastic dynamics can be included in the analysis as stochastic aspects are a fundamental aspect for most renewable resources. Simulation runs then generate the distribution of relevant outcomes. As it keeps the necessary time effort low, it allows to “play” with various control rules. The user obtains experience with the design of controls and gains useful time to deal with decision concepts.

Appendix

Define Δ as the difference between an x_t and 50, the stock size with the highest natural regrowth – given any growth curve possible. As we concentrate on $\Delta > 0$, the proposition is, that the stock size cannot jump below 50. I.e. the jump width cannot exceed Δ in case of a jump to the left. Since the harvest is 12.5 (the minimum natural growth at stock size 50) and the natural growth is at minimum given through $0.5 \cdot (50+\Delta) \cdot (1-(50+\Delta)/100)$, the corresponding jump width is $12.5 - 0.5 \cdot (50+\Delta) \cdot (1-(50+\Delta)/100)$ and the constraint to be proven is:

$$12.5 - 0.5 \cdot (50+\Delta) \cdot \left(1 - \frac{50+\Delta}{100}\right) < \Delta \qquad (*)$$

I.e. we concentrate on the worst case possible in the comedown of the resource (LHS of (*)) and ask whether the worst case still means that we do not loose more than we are afar from 50. With other words (*) expresses we would stay right from 50 even in the worst case.

(*) can be converted in the following equivalent conditions:

$$\begin{aligned}
&12.5 - 25 - \frac{\Delta}{2} + \frac{0.5}{100} \cdot (50+\Delta)^2 < \Delta \\
&-12.5 - \frac{\Delta}{2} + \frac{0.5}{100} \cdot \left(\Delta^2 + 2 \cdot 50 \cdot \Delta + 50^2\right) < \Delta \\
&-12.5 - \frac{\Delta}{2} + \frac{0.5}{100} \cdot \Delta^2 + 0.5 \cdot \Delta + 12.5 < \Delta \qquad \text{q.e.d..} \\
&\frac{0.5}{100} \cdot \Delta < 1 \\
&\Delta < 50
\end{aligned}$$

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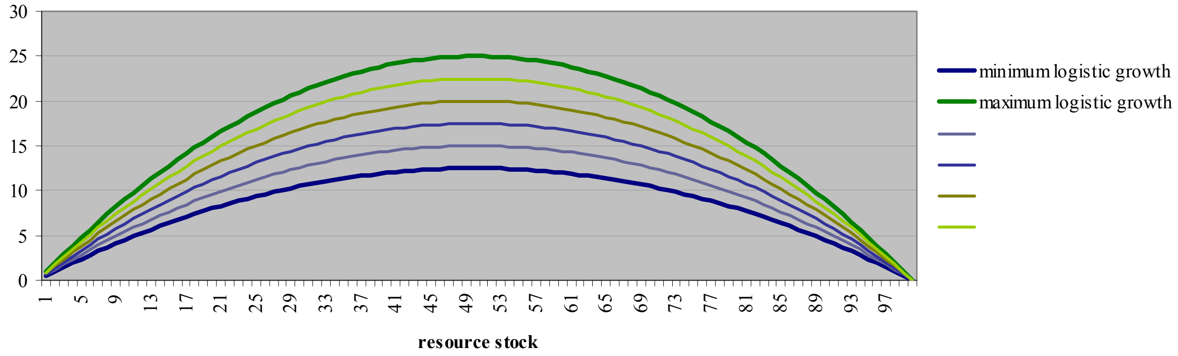
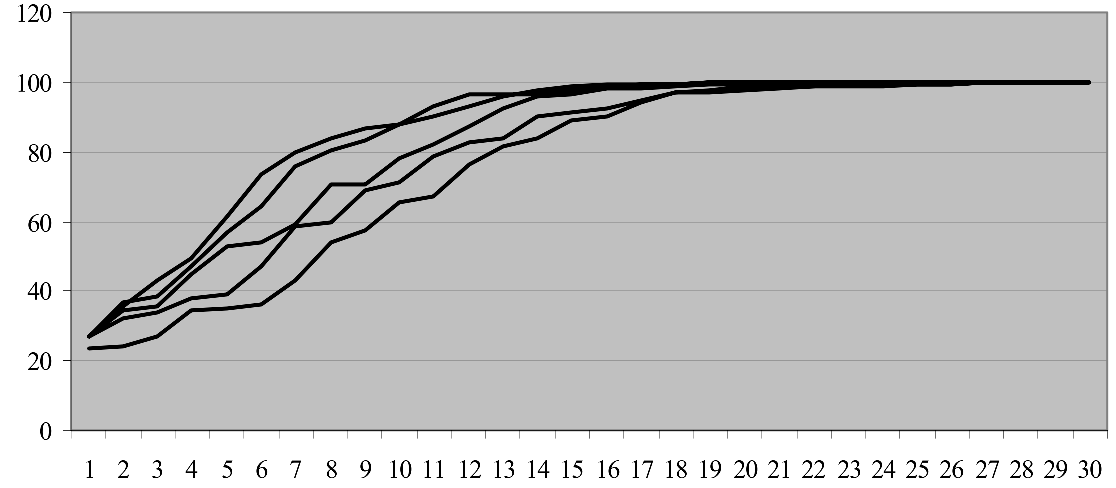
Figures 1. Range of Growth Curves



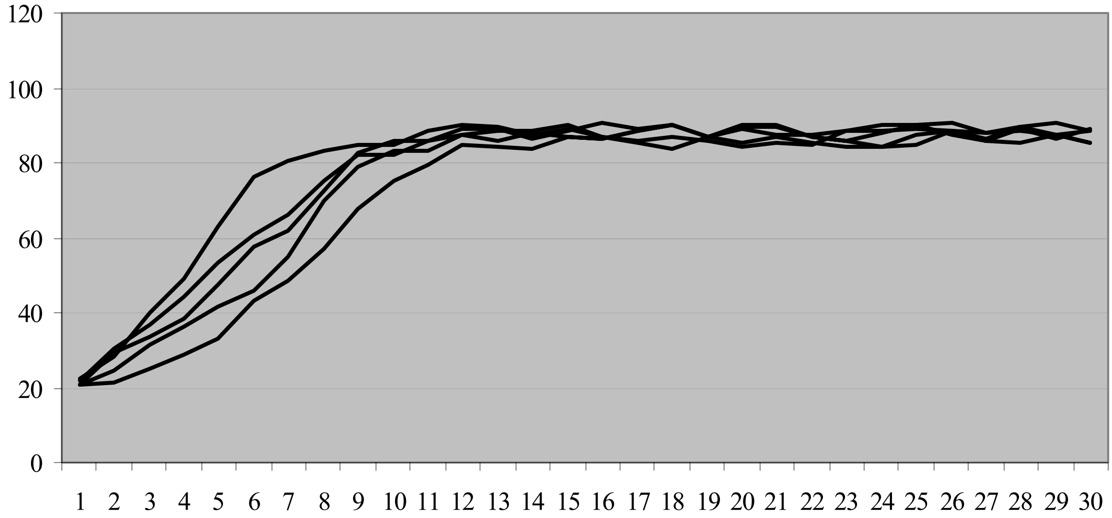
Source: own illustration; parameters: $r_{\min} = 0.5; \quad r_{\max} = 1; \quad L = 100$

Figure 2. 5 Trajectories given x_0 = 20 and e(x(t)) =r_{min}·(1-x(t)/L)/q



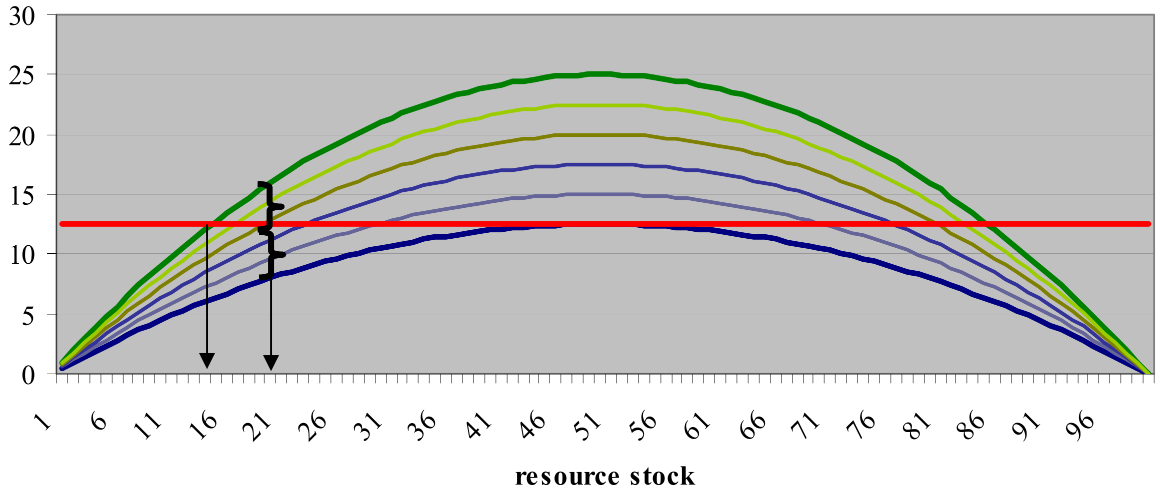
Source: own illustration; parameters: $r_{\min} = 0.5; \quad r_{\max} = 1; \quad L = 100, \quad x_0 = 20 \text{ and } e(x(t)) = r_{\min} \cdot (1 - x(t)/L)/q$

Figure 3. 5 Trajectories given x_0 = 20 and e =r_{min}*(1-x_0/L)/q



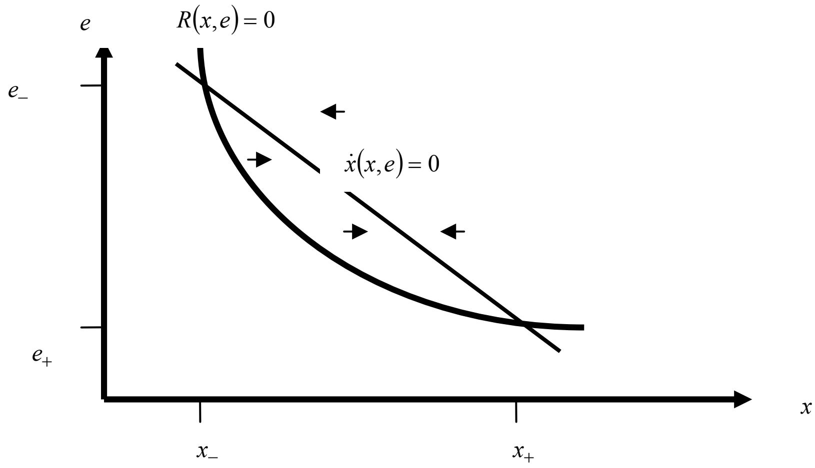
Source: own illustration; parameters: $r_{\min} = 0.5; \quad r_{\max} = 1; \quad L = 100, \quad x_0 = 20 \; and \; e = r_{\min} \cdot (1 - x_0 / L) / q$

Figure 4. Growth Curves and the Rigid Rule for $x_0 = 50_0$



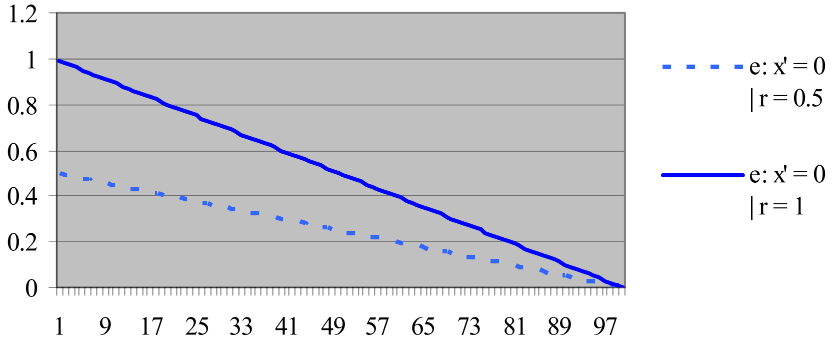
Source: own illustration; parameters: $r_{\min} = 0.5; \quad r_{\max} = 1; \quad L = 100$

Figure 5. Dynamics



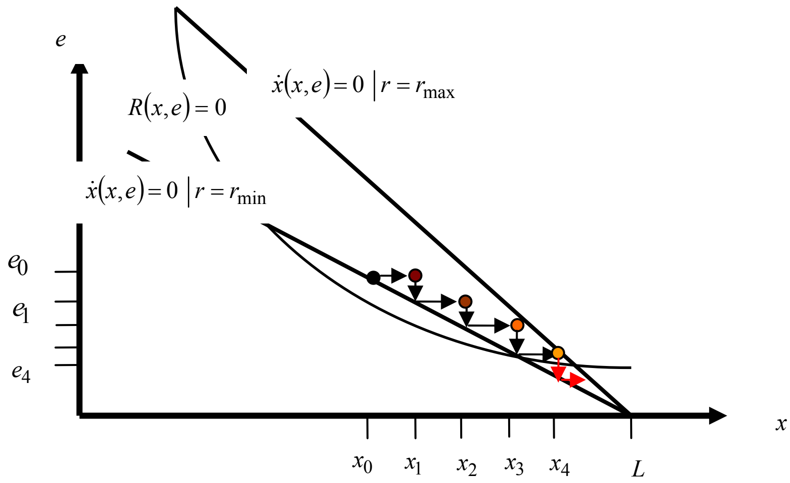
Source: simplification of Fig 2 from Béne, Doyen, and Gabay

Figure 6. the $\dot{x}(x,e)=0$ -Curves for $r_{\min}=0.5$ and $r_{\max}=1$



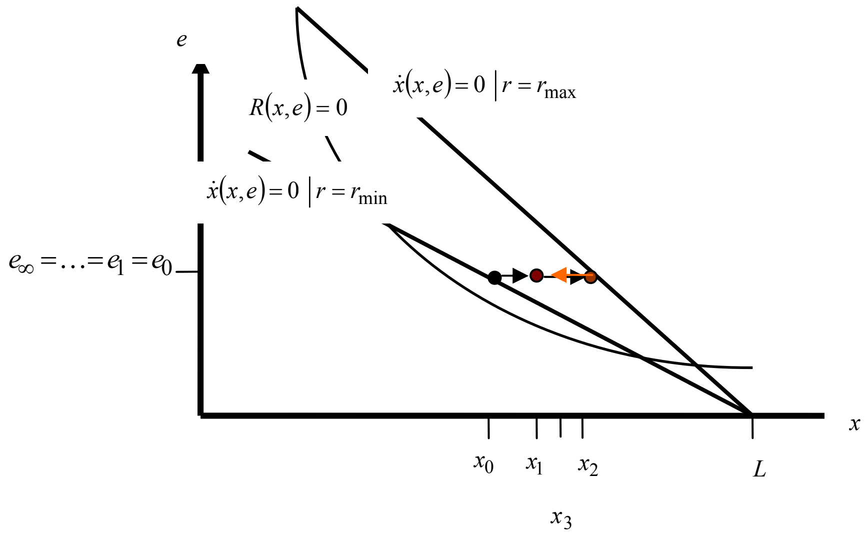
Source: own illustration; parameters: $r_{\min}=0.5$; $r_{\max}=1$; $L=100$; $q=1$

Figure 7. Evolution for the Overcautious Rule $e(x(t)) = r_{\min} \cdot (1 - x(t)/L)/q$



Source: adaptation of Fig 2 from Béne, Doyen, and Gabay

Figure 8. Evolution for the Rigid Rule $e = r_{\min} \cdot (1 - x_0 / L) / q$



Source: adaptation of Fig 2 from Béne, Doyen, and Gabay

Figure 9a-c. View of Parameter Area, the first 10 Periods

	A	B
3		
4	x_min	75
5	r	1
6	l	100
7	q	1
8	c	1
9	p	1
10	e_max	
11	C	2
12	x_0	10
13		
14	random_low	0.5
15	random_up	1

	A	B	C
16		minimum stock	minimum growth
17		=MIN(B22:B71)	=MIN(C22:C71)
18		steady state stock	steady state growth
19		=B73	=C73
20			
21	t	x	f(x)
22	1	=x_0	=(random_low+F22*(random_up-random_low))*r*B22*(1-B22/l)
23	2	=WENN(B22+C22-q*D22*B22>0;B22+C22-q*D22*B22;0)	=(random_low+F23*(random_up-random_low))*r*B23*(1-B23/l)
24	3	=WENN(B23+C23-q*D23*B23>0;B23+C23-q*D23*B23;0)	=(random_low+F24*(random_up-random_low))*r*B24*(1-B24/l)
25	4	=WENN(B24+C24-q*D24*B24>0;B24+C24-q*D24*B24;0)	=(random_low+F25*(random_up-random_low))*r*B25*(1-B25/l)
26	5	=WENN(B25+C25-q*D25*B25>0;B25+C25-q*D25*B25;0)	=(random_low+F26*(random_up-random_low))*r*B26*(1-B26/l)
27	6	=WENN(B26+C26-q*D26*B26>0;B26+C26-q*D26*B26;0)	=(random_low+F27*(random_up-random_low))*r*B27*(1-B27/l)
28	7	=WENN(B27+C27-q*D27*B27>0;B27+C27-q*D27*B27;0)	=(random_low+F28*(random_up-random_low))*r*B28*(1-B28/l)
29	8	=WENN(B28+C28-q*D28*B28>0;B28+C28-q*D28*B28;0)	=(random_low+F29*(random_up-random_low))*r*B29*(1-B29/l)
30	9	=WENN(B29+C29-q*D29*B29>0;B29+C29-q*D29*B29;0)	=(random_low+F30*(random_up-random_low))*r*B30*(1-B30/l)
31	10	=WENN(B30+C30-q*D30*B30>0;B30+C30-q*D30*B30;0)	=(random_low+F31*(random_up-random_low))*r*B31*(1-B31/l)

	D	E	F
16	minimumeffort	minimum Profit	
17	=MIN(D22:D71)	=MIN(E22:E71)	
18	steady state effort	steadystate profit	
19	=D73	=E73	
20			
21	e	R	random_variable
22	=random_low*(1-x_0/l)/q	=p*q*D22*B22-c*D22-Fixcosts	=ZUFALLSZAHL()
23	=WENN(B23>q*D22*B23;D22;0)	=p*q*D23*B23-c*D23-Fixcosts	=ZUFALLSZAHL()
24	=WENN(B24>q*D23*B24;D22;0)	=p*q*D24*B24-c*D24-Fixcosts	=ZUFALLSZAHL()
25	=WENN(B25>q*D24*B25;D22;0)	=p*q*D25*B25-c*D25-Fixcosts	=ZUFALLSZAHL()
26	=WENN(B26>q*D25*B26;D22;0)	=p*q*D26*B26-c*D26-Fixcosts	=ZUFALLSZAHL()
27	=WENN(B27>q*D26*B27;D22;0)	=p*q*D27*B27-c*D27-Fixcosts	=ZUFALLSZAHL()
28	=WENN(B28>q*D27*B28;D22;0)	=p*q*D28*B28-c*D28-Fixcosts	=ZUFALLSZAHL()
29	=WENN(B29>q*D28*B29;D22;0)	=p*q*D29*B29-c*D29-Fixcosts	=ZUFALLSZAHL()
30	=WENN(B30>q*D29*B30;D22;0)	=p*q*D30*B30-c*D30-Fixcosts	=ZUFALLSZAHL()

Source: own work

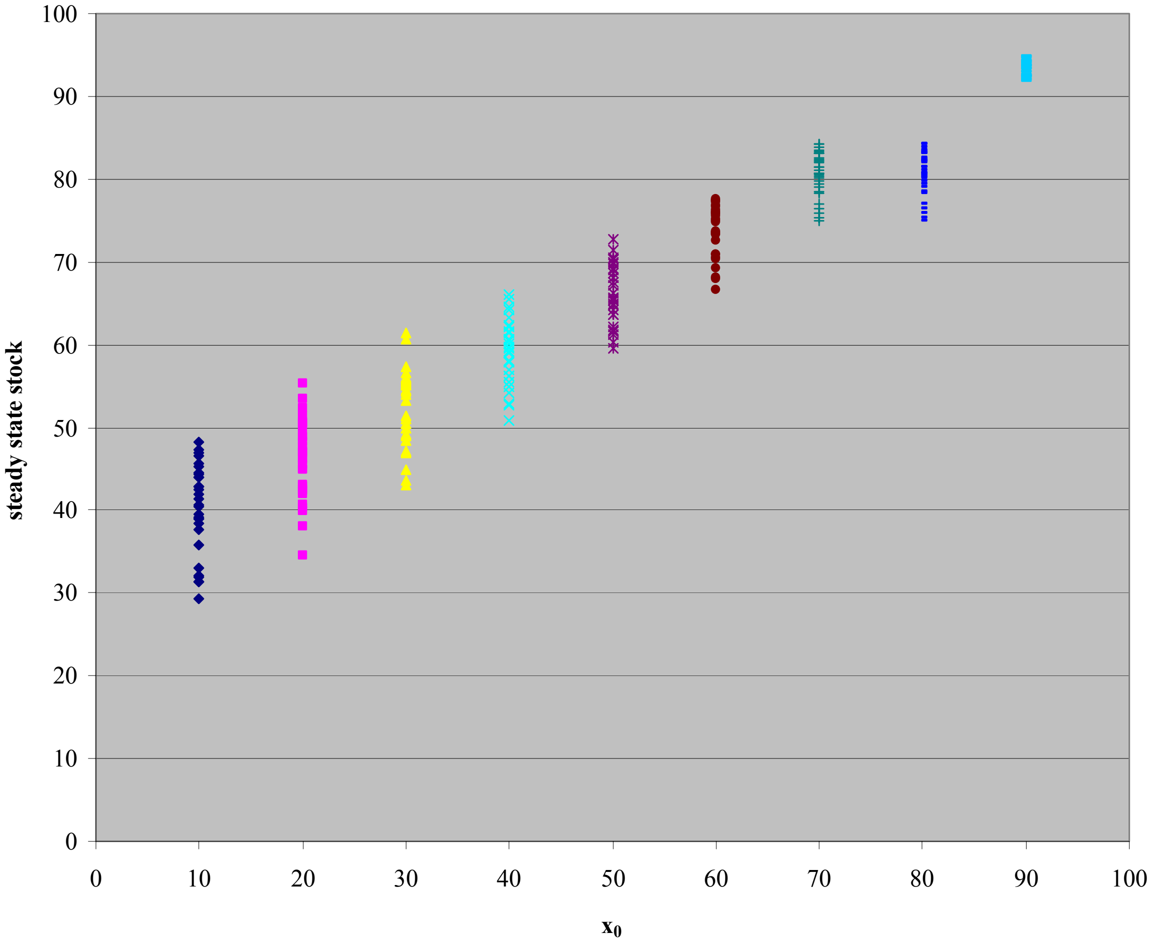
Figure 10a-b. View of the last 10 Periods with the Additional Steady State Calculation

	A	B	C
62	41	=WENN(B61+C61-q*D61*B61>0;B61+C61-q*D61*B61;0)	=(random_low+F62*(random_up-random_low))*r*B62*(1-B62/l)
63	42	=WENN(B62+C62-q*D62*B62>0;B62+C62-q*D62*B62;0)	=(random_low+F63*(random_up-random_low))*r*B63*(1-B63/l)
64	43	=WENN(B63+C63-q*D63*B63>0;B63+C63-q*D63*B63;0)	=(random_low+F64*(random_up-random_low))*r*B64*(1-B64/l)
65	44	=WENN(B64+C64-q*D64*B64>0;B64+C64-q*D64*B64;0)	=(random_low+F65*(random_up-random_low))*r*B65*(1-B65/l)
66	45	=WENN(B65+C65-q*D65*B65>0;B65+C65-q*D65*B65;0)	=(random_low+F66*(random_up-random_low))*r*B66*(1-B66/l)
67	46	=WENN(B66+C66-q*D66*B66>0;B66+C66-q*D66*B66;0)	=(random_low+F67*(random_up-random_low))*r*B67*(1-B67/l)
68	47	=WENN(B67+C67-q*D67*B67>0;B67+C67-q*D67*B67;0)	=(random_low+F68*(random_up-random_low))*r*B68*(1-B68/l)
69	48	=WENN(B68+C68-q*D68*B68>0;B68+C68-q*D68*B68;0)	=(random_low+F69*(random_up-random_low))*r*B69*(1-B69/l)
70	49	=WENN(B69+C69-q*D69*B69>0;B69+C69-q*D69*B69;0)	=(random_low+F70*(random_up-random_low))*r*B70*(1-B70/l)
71	50	=WENN(B70+C70-q*D70*B70>0;B70+C70-q*D70*B70;0)	=(random_low+F71*(random_up-random_low))*r*B71*(1-B71/l)
72			
73	infinity	=B71+C71-q*D71*B71	=r*B73*(1-B73/l)

	D	E	F
61	=WENN(B61>q*D60*B61;D22;0)	=p*q*D61*B61-c*D61-Fixcosts	=ZUFALLSZAHL()
62	=WENN(B62>q*D61*B62;D22;0)	=p*q*D62*B62-c*D62-Fixcosts	=ZUFALLSZAHL()
63	=WENN(B63>q*D62*B63;D22;0)	=p*q*D63*B63-c*D63-Fixcosts	=ZUFALLSZAHL()
64	=WENN(B64>q*D63*B64;D22;0)	=p*q*D64*B64-c*D64-Fixcosts	=ZUFALLSZAHL()
65	=WENN(B65>q*D64*B65;D22;0)	=p*q*D65*B65-c*D65-Fixcosts	=ZUFALLSZAHL()
66	=WENN(B66>q*D65*B66;D22;0)	=p*q*D66*B66-c*D66-Fixcosts	=ZUFALLSZAHL()
67	=WENN(B67>q*D66*B67;D22;0)	=p*q*D67*B67-c*D67-Fixcosts	=ZUFALLSZAHL()
68	=WENN(B68>q*D67*B68;D22;0)	=p*q*D68*B68-c*D68-Fixcosts	=ZUFALLSZAHL()
69	=WENN(B69>q*D68*B69;D22;0)	=p*q*D69*B69-c*D69-Fixcosts	=ZUFALLSZAHL()
70	=WENN(B70>q*D69*B70;D22;0)	=p*q*D70*B70-c*D70-Fixcosts	=ZUFALLSZAHL()
71	=WENN(B71>q*D70*B71;D22;0)	=p*q*D71*B71-c*D71-Fixcosts	=ZUFALLSZAHL()
72			
73	=r*B73*(1-B73/l)/q	=p*q*D73*B73-c*D73-Fixcosts	

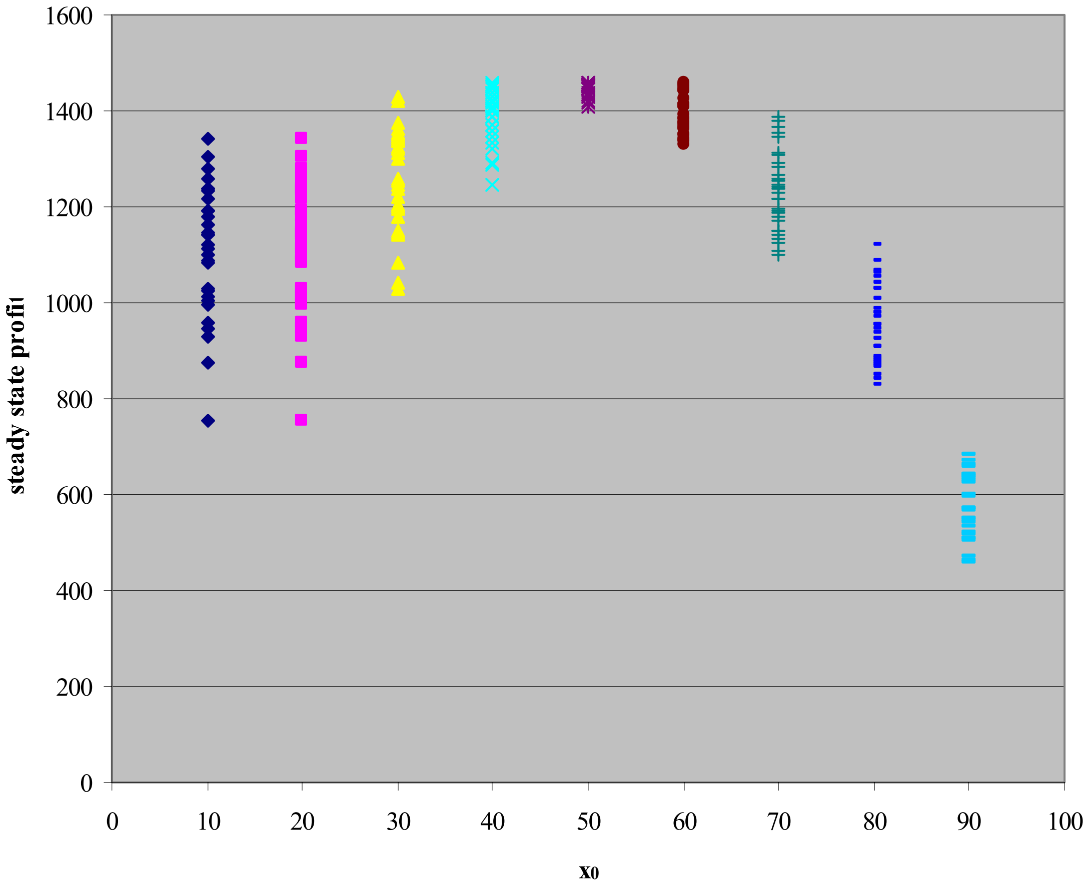
Source: own work

Figure 11. Distribution of Steady State Stock for $r_{min} = 0.5$ and $r_{max} = 1$ and Various Initial Stocks x_0 – the Rigid Rule



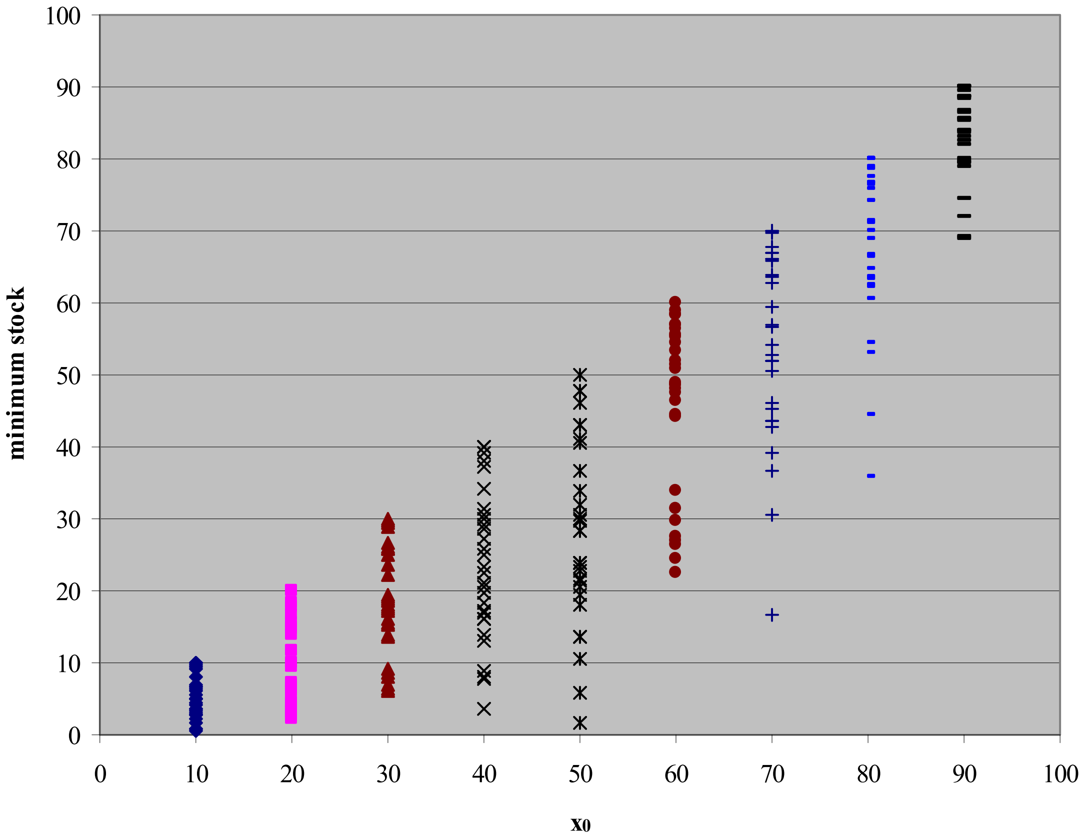
Source: own computation

Figure 12. Distribution of Steady State Profit for r_{min} = 0.5 and r_{max} = 1 and Various Initial Stocks x_0 – the Rigid Rule



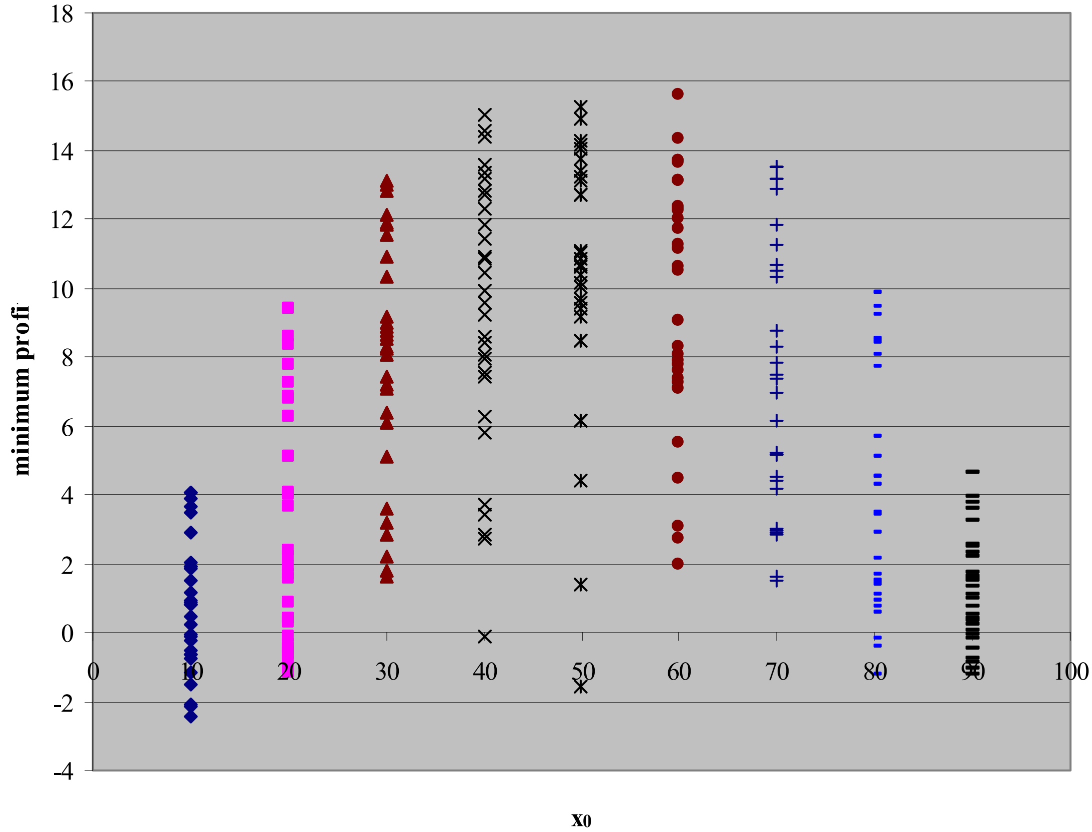
Source: own computation

Figure 13. Distribution of the Minimum Stock during the 50-Years-Horizon given r_{min} = 0.5 and r_{max} = 1 and Various Initial Stocks x_0 – the Avg Rule



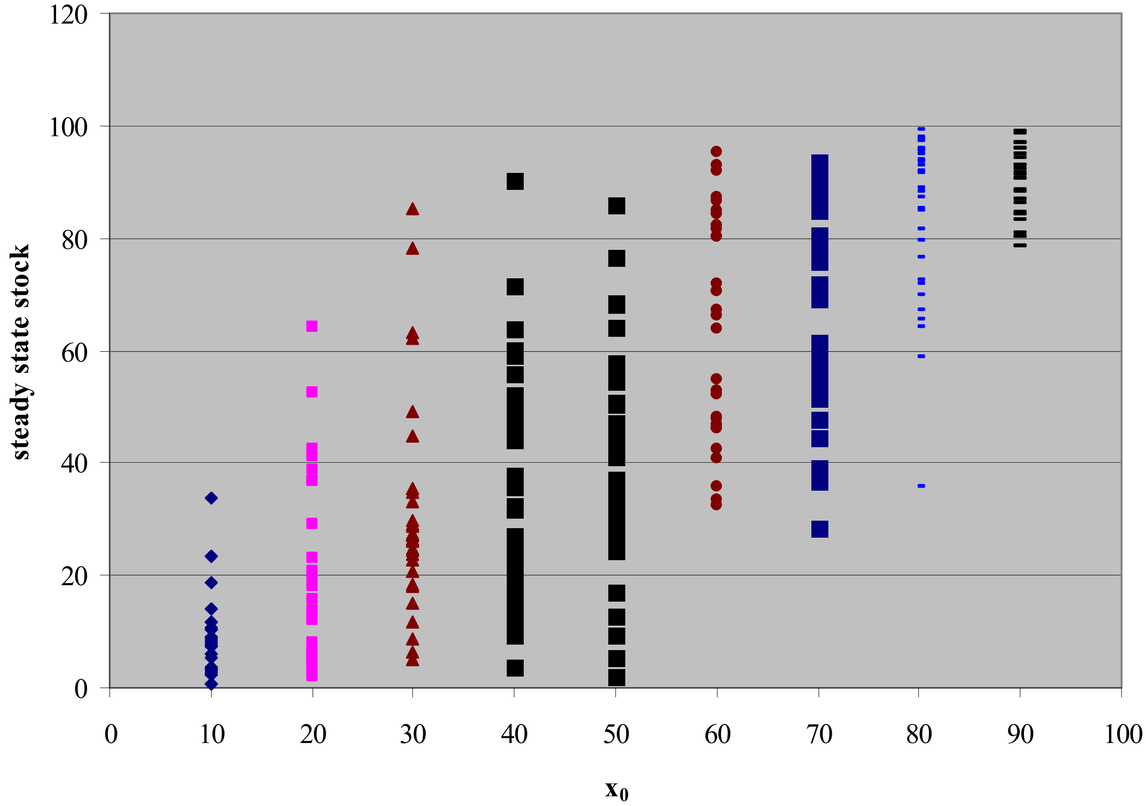
Source: own computation

Figure 14. Distribution of the Minimum Profit during the 50-Years-Horizon given r_{min} = 0.5 and r_{max} = 1 and Various Initial Stocks x_0 – the Avg Rule



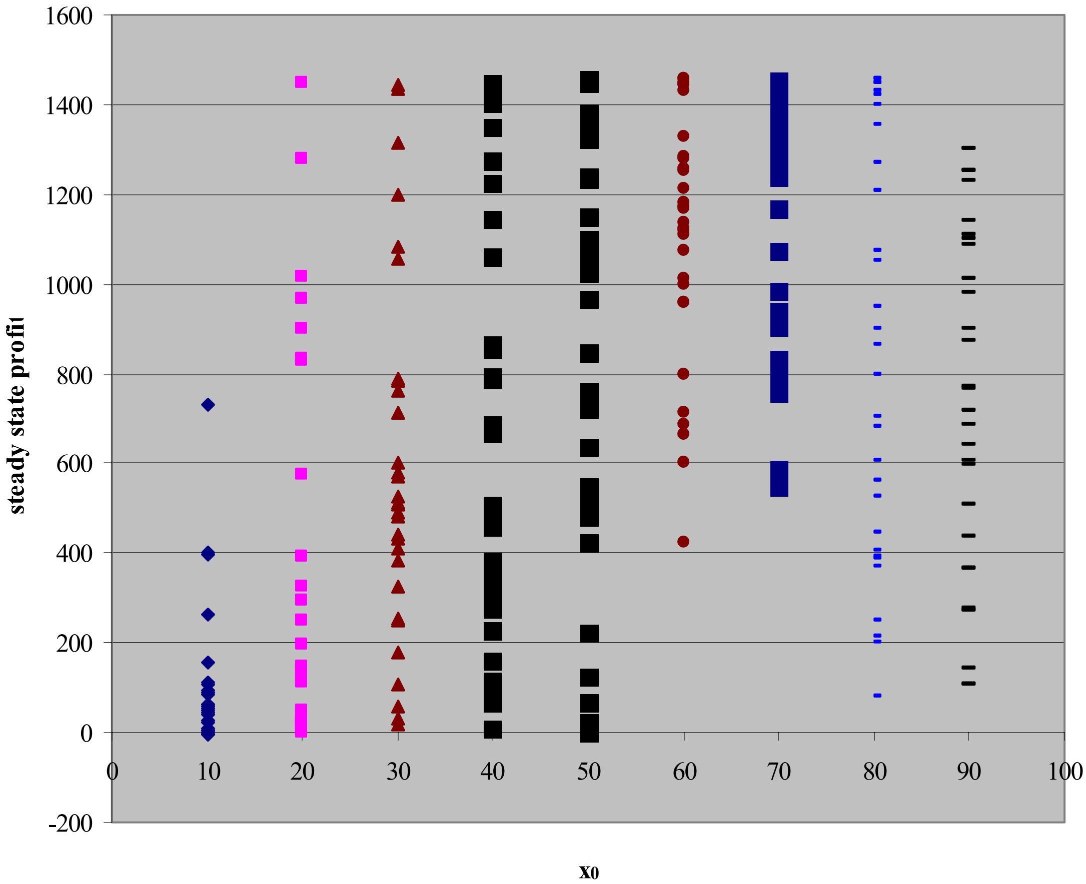
Source: own computation

Figure 15. Distribution of Steady State Stock for r_{min} = 0.5 and r_{max} = 1 and Various Initial Stocks x_0 - the Avg-Rule



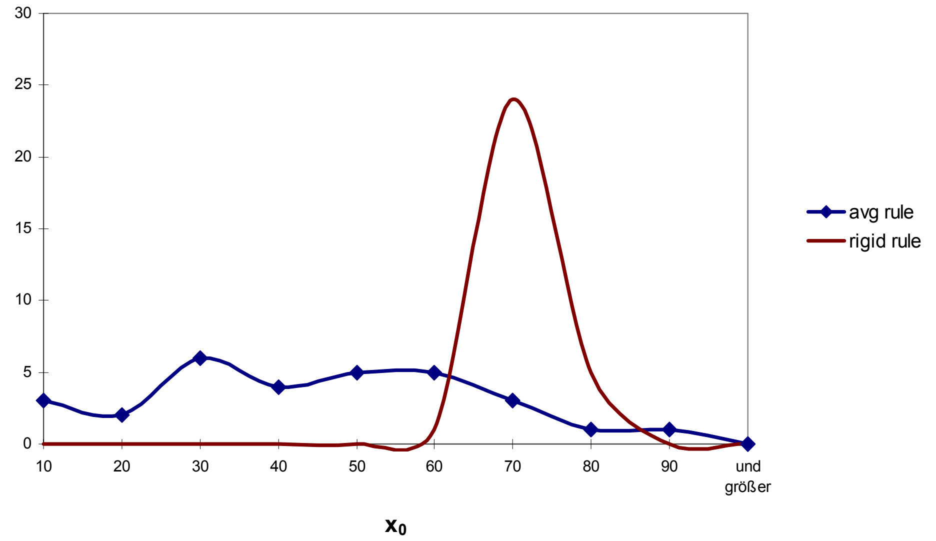
Source: own computation

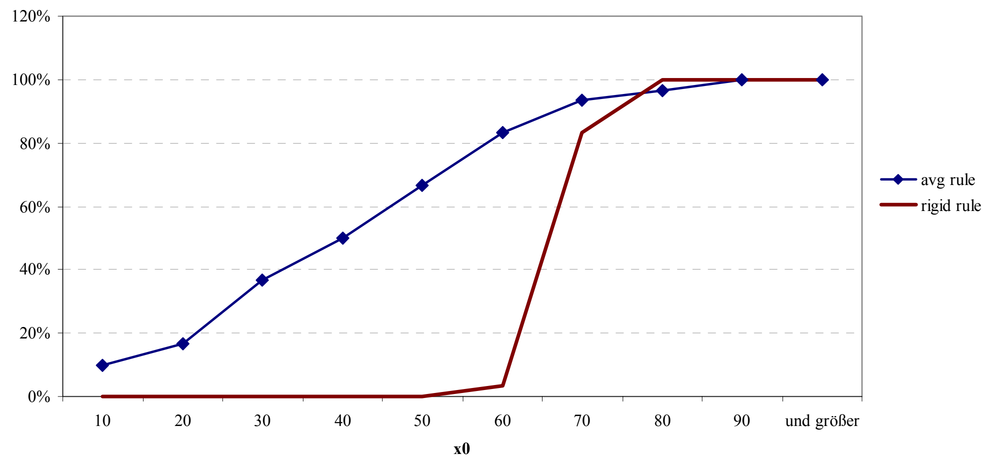
Figure 16. Distribution of Steady State Profit for r_{min} = 0.5 and r_{max} = 1 and Various Initial Stocks x_0 – the Avg Rule



Source: own computation

FIGURE 17. Comparison of Rigid Rule and Avg Rule concerning the approximated Density and the approximated Distribution function of the Steady State Stock for r_{min}=0.5 and r_{max}=1 given x_0=50





[1] Control theory is a comparatively young field within mathematics (For its history, see Fernández-Cara and Zuazua (0000)). In the fifties and sixties of the last century Calculus of Variation enhanced more and more; at the same time Bellman equations and the Maximum Principle of Pontryagin were introduced (Calculus of Variation, Bellmann equations, and the Maximum Principle, see Kamien and Schwartz (1981), Intriligator (1971), Chiang (1992.).

[2] As stated in Aubin (2002) the purpose is to identify viable evolutions governed by nondeterministic dynamics.

[3] Walstad and Allgood (1999) found a shortfall in economic understanding in a sample of college seniors as well as in another sample of former students with Major Field Test in Business II. Salemi and Siegfried (1999) see a need to enlarge the methods and media used in lectures, e.g. to use technology, in order to improve long-run economic understanding.

[4] Examples for system dynamic tools are Dynamo, SIMPAS, DynSim, VenSim and PowerSim or Stella; concerning history and features of the software see Gilbert and Troitzsch (1999), chap. 3. These tools allow the definition of stocks and flows, to control feedback effects, and so on. They ease forecasting the development of variables linked through a system of differential equations.

[5] An example for a freeware game theory tool is gambit.

[6] The letter C will be employed in another context latter in the model. It is used here only to be in line with Aubin's notation.

[7] For more details concerning the sustainable effort and the effort yield curve see Wacker and Blank.

8 In case of good luck and a high intrinsic growth rate, effort is allowed to be higher than in case of bad luck and a low intrinsic growth rate, as a matter of course. The difference between the highest and the lowest effort is large in case of a small stock size due to the following reason: the derivative of effort as a function of the quantity to be harvested decreases with the square of x in the denominator:

$$h = q \cdot e \cdot x \Leftrightarrow e = \frac{h}{q \cdot x} \Rightarrow \frac{\partial e}{\partial h} = -\frac{q}{x^2} \tag{11}$$

The smaller x, the larger the reaction of the effort to changes in the required harvest quantity.

[9] And it gets multiplied by the product of $x(t)$ and the efficiency parameter, in order to calculate the harvest.

[10] There is no guarantee that initial profits will be positive. Very high fix costs can induce a $R(x,e)=0$ -curve strictly above the $\dot{x}(x,e)=0 \mid r = r_{\max}$ -curve. In this case the viability kernel is empty.

[11] Concerning changes in c and C, the line of argumentation is: the tangent of the $R(x,e)=0$ -curve is $x = c / p \cdot q$; thus a higher c shifts the tangent to the right; and C shifts the $R(x,e)=0$ -curve upwards. With

higher C or c, the interval $[\tilde{x}_-, \tilde{x}_+]$ is smaller, and therefore there are less x_0 inducing positive profits in the beginning. At the end, revenue will never cover the fix costs, as already stated.

[12] Again, there is no guarantee that initial profits will be positive. For example, assume the $\dot{x}(x,e) = 0 \mid r = r_{\min}$ -curve stays strictly below the $R(x,e) = 0$ -curve. Then, the rule $e = r_{\min} \cdot (1 - x_0 / L)/q$ can never induce a viable evolution – independent from x_0 (profits will stay negative forever). Negative initial profits will appear even if the three curve relate to each other like in figure 8, but the x_0 lies left from the intersection of $\dot{x}(x,e) = 0 \mid r = r_{\min}$ and $R(x,e) = 0$.