Alternative Soil Fertility Management Options in Malawi:

An Economic Analysis

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Abstract

In this paper, we analyze the factors that influence the productivity of maize among smallholder farmers, given that unfavourable output and input market conditions throughout the 1990s have compelled smallholder farmers into unsustainable agricultural intensification. We use farm-household survey data in order to compare the productivity of smallholder maize production under integrated (ISFM) and chemical-based soil fertility management using a normalized translog yield response model. The results indicate higher maize yield responses for integrated soil fertility management options after controlling for the intensity of fertilizer application, labour intensity, seed rate, land husbandry practices as well as selected policy factors. The estimated model is highly consistent with theoretical conditions. Thus we conclude that the use of ISFM improves maize productivity, compared to the use of inorganic fertilizer only. Since most farmers in the maize-based farming systems are crowded out of the agricultural input market and can hardly afford optimal quantities of inorganic fertilizer, enhancement of ISFM is likely to increase their maize productivity. We finally highlight areas of policy support needed to enhance ISFM uptake in smallholder maize-based farming systems.

Keywords: Malawi, smallholder agriculture, soil fertility management, yield response model

1. Introduction

Maize is the dominant crop in most smallholder farming systems in Africa south of the Sahara. In Malawi, it is the main staple crop, estimated to be grown on over 70% of the arable land and nearly 90% of the cereal area, making Malawi the world's highest consumer of maize at 148 kg per capita per year (Smale and Jayne

2003). Thus, maize will remain a central crop in the food security equation of Malawi even if the agricultural economy is diversified. The dominance of maize as a staple crop mainly emanates from self-sufficiency policy which the Government adopted after independence in the mid 1960s. This resulted from the need to produce enough food to feed the growing rural population as well as keep staple food prices low.

Many studies conducted in Malawi indicate declining levels of maize productivity that poses serious food security concerns, since maize is the staple crop for most of the country (Kydd, 1989; Smale and Jayne 1995; Chirwa, 2003). In this paper, we analyze the factors that influence productivity of maize among smallholder farmers, given that unfavourable output and input market conditions throughout the 1990s, have compelled smallholder farmers into unsustainable agricultural intensification. Currently, the most comprehensive studies of smallholder productivity in Malawi have been conducted by Chirwa (1996), Chirwa (2003) and Edriss et al. (2004). The first two studies have used data collected from a sample of farmers from Machinga Agricultural Development Division (ADD). Edriss et al. (2004) used national level data to analyze the levels of maize productivity given the labour market liberalization. All these studies use parametric approaches to estimate the efficiency of Malawian smallholder farmers in maize production. Our study complements these studies in a number of ways. First, the first two studies have been restricted to only one agro-ecological zone and their results may not be applicable to other agro-ecological zones, whereas our sample is drawn from three agro-ecological

zones and thus accounts for agro-ecological variations. Secondly, both studies did not account for the theoretical regularity conditions in their analysis. Therefore it is highly likely that policy conclusions drawn from these studies may have been flawed due to lacking regularity of the estimated functions. Thirdly, our study considers the productivity effect of alternative soil fertility management options available to smallholder farmers. This is important because while many alternative soil fertility management options have been developed for smallholder farmers, very little is known about their impact on improving smallholder farmers' productivity. The obvious weakness of the study by Edriss et al. (2004) is the use of national level data that masks the farm-level variations. We improve on that by using farm-level data.

The rest of the paper is arranged as follows: the next section presents a review of maize productivity, drawing from previous studies conducted in Sub-Saharan Africa, in general and Malawi, in particular, with specific reference to the smallholder sector. This is followed by the discussion of the theoretical model on which the analytical model presented in section four is based. Section five describes the data and the analysis. Section six concludes with main findings and their policy implications.

2. Review of smallholder maize productivity in Malawi

Despite the central role that maize plays in food security in Malawi, its productivity has not been impressive especially from the early 1990s when stagnation in maize yield led to frequent food security problems. Smale and Jayne (2003) have

attributed the decline in maize yield to four main reasons: (i) removal of subsidies; (ii) devaluation of the Malawi Kwacha; (iii) increase in world fertilizer prices; and (iv) low private market development because fertilizer dealers require substantial risk premiums to hold and transport fertilizer in an inflationary economy with uncertain demand (Conroy, 1997; Diagne and Zeller 2001; Benson, 1997; 1999). The situation is exacerbated because maize price changes follow export parity while fertilizer price changes reflect full import costs. Since most fertilizer in Malawi is used on maize (and tobacco), the removal of implicit subsidies in the form of over-valued exchange rates had a strong negative effect on fertilizer use. Furthermore, since almost all of Malawi's fertilizer supply is imported, the depreciation of the real exchange rate has also invariably raised the nitrogen to grain price ratios (Minot, Kherallah and Berry 2000; Heisey and Smale, 1995). All these factors, along with shifts in relative prices of competing crops, as well as the unfavorable weather patterns may have contributed to the major fluctuations in the maize yield and production through the 1990s as shown in Figure 1.

Figure 1: Maize Productivity in Malawi (1980-2002)
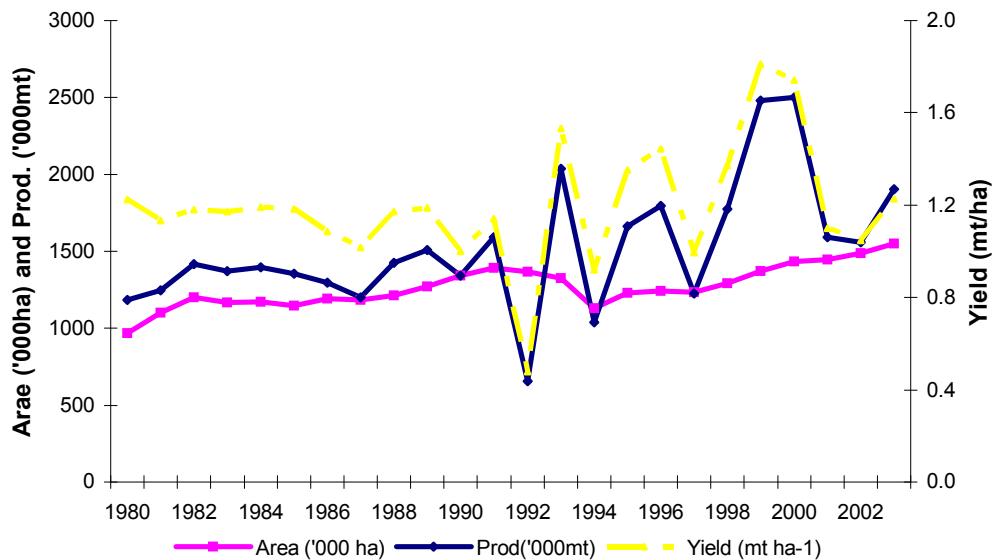
One critical consequence of the increase in fertilizer prices relative to maize grain prices is that most farmers over the past decade have continued to over-exploit the natural soil fertility. This is because the improved maize varieties released by the National Agricultural Research (i.e. MH17 and MH18) proved to yield more than local maize without fertilizer at the seed prices that prevailed through the early 1990s. This implies that it made economic sense for farmers to grow hybrids even if they could not apply fertilizer (Heisey and Smale 1995; Benson, 1999). This has resulted in soil fertility mining, leading to unsustainability, as the inherent soil fertility is no longer capable of supporting crop growth at a rate that is required to feed the growing population. This calls for concerted efforts to promote smallholder soil fertility management using relatively more sustainable options such as integrated

soil fertility management (ISFM) i.e. involving incorporation of grain legumes and inorganic fertilizer in maize production systems. However, farmers' choice of the available soil fertility management options depends to a large extent on the relative returns of the options.

Our study aims at assessing the productivity (and thus profitability) of maize production using ISFM or inorganic fertilizers only. This in order to assist farmers making rational soil fertility management choices, given the options made available by agronomy research.

3. Theoretical Review

A number of functional forms have been used to specify yield response functions, most commonly the Cobb-Douglas, quadratic, square root, translog, Mitscherlich-Baule (or MB) as well as the linear and non-linear Von-Liebig functions. The rationale for choosing a particular functional form depends on the research questions and the underlying production processes to be modeled (Nkonya, 1999). Furthermore, the choice of a functional form should be based on the need to ensure rigorous theoretical consistency and factual conformity within a given domain of application as well as flexibility and computational ease (Lau, 1986; Sauer et al., 2004). For example, while the Cobb-Douglas is simpler and easier to estimate, it assumes invariant returns to scale and does not ensure the attainment of a yield response plateau, thereby resulting in an overestimation of the optimal input quantities

(Ackello-Ogutu et al. 1985). While the polynomial functions (i.e. the quadratic and square root) do allow for the diminishing marginal returns of inputs as well as flexible input substitution, they are also lacking when it comes to the yield response plateau. The non-linear Von-Liebig and MB functions are the most widely used functions, especially in the field of agronomy. However, because they are highly non-linear, especially when a number of inputs are involved, their estimation is cumbersome and liable to several parametric restrictions. The other weakness of the MB function is that it may not be appropriate for modeling farm production in developing countries because it is only appropriate for stage II production (where marginal product increases at a decreasing rate). But research shows that most constrained farmers in developing countries still largely operate within stage I where marginal product increases at an increasing rate (Franke et al. 1990; Keyser, 1998).

We maintain the assumption that farmers' choice of a soil fertility management option is based on the desire to increase the profit derived from increased crop yield. As such the underlying problem is that of optimizing profit, given the technology and soil fertility management options available. Thus given the production function:

$$h(q, x, z) = 0 \qquad\qquad [1]$$

where q is the vector of output, x is the vector of variable inputs and z is a vector of fixed factors. If we let p and c be the output and input prices respectively, the farmer's restricted profit becomes[1]:

$$\pi = p^{'}q - c^{'}x \tag{2}$$

The farmer is thus assumed to choose a combination of variable inputs and outputs that will maximize restricted profit subject to the production technology constraint:

$$\underset{x,q}{Max}\ p^{'}q - c^{'}x \quad s.t.\ h(q,x,z) = 0 \tag{3}$$

The solution to this profit maximization problem becomes a set of input demand and output supply functions of the form:

$$x = x(p,c,z)\ \text{and}\ q = q(p,c,z) \tag{4}$$

If we substitute the expressions in [4] into the restricted profit equation given by [2], we obtain a profit function specification which gives the maximum profit the farmer could obtain given the output and input prices, the availability of fixed factors and the production technology.[2] Hence it is possible to derive the optimal level of inputs, which when substituted into the corresponding production function yield the

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[1] Profit is restricted because only the variable costs are subtracted from the gross revenue. The restricted profit equation uses $p^{'}$ and $c^{'}$ to denote the transposition of vectors (see Sadoulet and de Janvry 1995).

[2] Due to duality theory a well behaved dual profit function is a "sufficient statistic" of the underlying well-behaved production function (see McFadden, 1978). According to economic theory a well-behaved production/profit function has to be non-negative, monotonically increasing (decreasing) in output (input) prices, concave/convex, homogeneous of degree zero in all prices and if the production function displays constant returns to scale, homogeneous of degree one in all fixed factors (Sadoulet and de Janvry 1995).

optimum level of output which is consistent with the optimal level of restricted profit. The following analysis uses a primal production function rather than the dual profit function as the latter is conditioned on prices. Relevant prices in the study area suffer from a considerable bias of aggregation as it is fairly difficult to capture the variation in prices on household level. Given further the uncertainties in expected agricultural prices and production, it is unlikely that the correspondence between expected prices and production would give a good model fit.

4. The Empirical Model

In this analysis, we use a normalized translog functional form because we assume that yield response depends on nitrogen use efficiency and a second order polynomial function can approximate such a relationship. The normalized translog models have been widely used for describing the crop response to fertilization and tend to statistically perform better than other functional forms. Belanger et al. (2000) compared the performance of three functional forms (quadratic, exponential and square root) and concluded that although the quadratic form is the most favoured in agronomic yield response analysis, it tends to overstate the optimal input level, and thus underestimating the optimal profitability. Other studies that have reached similar conclusions include Bock and Sikora (1990), Angus et al. (1993) and Bullock and Bullock (1994). Our choice of the normalized translog is based on two further reasons: First, it is the best-investigated second order flexible functional form and

certainly one with the most applications (Sauer et al. 2004); secondly, this functional form is convenient to estimate and proved to be a statistically significant specification for economic analyses as well as a flexible approximation of the effect of input interactions on yield.

The normalized translog maize production model can be expressed as:

$$\ln(\frac{q}{q'}) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln(\frac{x_i}{x_i'}) + \frac{1}{2} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \beta_{ij} \ln(\frac{x_i}{x_i'}) \ln(\frac{x_j}{x_j'}) + \sum_{k=1}^{m} \gamma_k z_k + \varepsilon_i \quad \varepsilon_i \quad N(0, \sigma^2) \quad [5]$$

Where q is the yield (kg/ha), x_i are the variable inputs (fertilizer, labour and seed), z is a vector of productivity shifters such as land husbandry practices (i.e. weeding and date of planting) as well as rainfall. All variables are normalized to the sample mean by dividing by the mean value (q', xi', xj'). We also include a dummy variable for soil fertility management (i.e. integrated management or use of inorganic fertilizer only) in order to assess the impact of soil fertility management choice on yield response as well as other control variables. α_i are the linear input parameters, β_{ij} are the quadratic and interaction parameters, δ_k are the parameters for the productivity shifters and ε_i is the error term assumed to be randomly distributed with zero mean and constant variance σ^2.

In the case of a (single output) production function monotonicity requires positive marginal products with respect to all inputs and thus non-negative elasticities. With respect to the normalized translog production model the marginal product of input i

is obtained by multiplying the logarithmic marginal product with the average product of input i. Thus the monotonicity condition given holds for our translog specification if the following equation is true for all inputs:

$$\frac{d\left(\frac{q}{q'}\right)}{d\left(\frac{x_i}{x_i'}\right)} = \left(\frac{q}{q'}\right)\frac{d\ln\left(\frac{q}{q'}\right)}{\left(\frac{x_i}{x_i'}\right)d\ln\left(\frac{x_i}{x_i'}\right)} = \frac{\left(\frac{q}{q'}\right)}{\left(\frac{x_i}{x_i'}\right)}\left(\alpha_i + \sum_{j=1}^{n}\beta_{ij}\ln\left(\frac{x_i}{x_i'}\right)\right) > 0 \qquad [6]$$

Since both (q/q') and (x_i/x_i') are positive numbers, monotonicity depends on the sign of the term in parenthesis, i.e. the elasticity of (q/q') with respect to (x_i/x_i').[3]

By further adhering to the law of diminishing marginal productivities, marginal products, apart from being positive should be decreasing in inputs implying the fulfillment of the following expression:

$$\frac{d^2\left(\frac{q}{q'}\right)}{d\left(\frac{x_i}{x_i'}\right)^2} = \left[\alpha_{ii} + \left(\alpha_i - 1 + \sum_{j=1}^{n}\beta_{ij}\ln x_j\right)\left(\alpha_i + \sum_{j=1}^{n}\beta_{ij}\ln x_j\right)\right]\left[\frac{\left(\frac{q}{q'}\right)}{\left(\frac{x_i}{x_i'}\right)^2}\right] < 0 \qquad [7]$$

Again, this depends on the nature of the terms in parenthesis. These should be checked a posteriori by using the estimated parameters for each data point. However,

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[3] If it is assumed that markets are competitive and factors of production are paid their marginal products, the term in parenthesis equals the input i's share of total output, s_i.

both restrictions (i.e. $\left[\partial(q/q')/\partial(x_i/x_i')\right]>0$ and $\left[\partial^2(q/q')/\partial(x_i/x_i')^2\right]<0$)

should hold at least at the point of approximation.

The necessary and sufficient condition for a specific curvature consists in the semi-definiteness of the bordered Hessian matrix as the Jacobian of the derivatives $\partial(q/q')/\partial(x_i/x_i')$ with respect to x_i: if $\nabla^2 Y(x)$ is negatively semi-definite, Y is quasi-concave, where ∇^2 denotes the matrix of second order partial derivatives with respect to the normalized translog production model. The Hessian matrix is negative semi-definite at every unconstrained local maximum[4]. The conditions of quasi-concavity are related to the fact that this property implies a convex input requirement set (see in detail e.g. Chambers 1988). Hence, a point on the isoquant is tested, i.e. the properties of the corresponding production function are evaluated subject to the condition that the amount of production remains constant. With respect to the translog production function curvature depends on the specific input bundle X_i, as the corresponding bordered Hessian **BH** for the 3 input case shows:

$$BH = \begin{pmatrix} 0 & b_1 & b_2 & b_3 \\ b_1 & h_{11} & h_{12} & h_{13} \\ b_2 & h_{21} & h_{22} & h_{23} \\ b_3 & h_{31} & h_{32} & h_{33} \end{pmatrix} \qquad [8]$$

where b_i is given in [6], h_{ii} is given in [7] and h_{ij} is:

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[4] Hence, the underlying function is quasi-concave and an interior extreme point will be a global maximum. The Hessian matrix is positive semi-definite at every unconstrained local minimum.

$$\frac{d^2\left(\frac{q}{q'}\right)}{d\left[\left(\frac{x_i}{x_i'}\right)\left(\frac{x_j}{x_j'}\right)\right]} = \left[\alpha_{ij} + \left(\alpha_i + \sum_{j=1}^{n}\beta_{ij}\ln\left(\frac{x_j}{x_j'}\right)\right)*\left(\alpha_j + \sum_{i=1}^{n}\alpha_{ij}\ln\left(\frac{x_i}{x_i'}\right)\right)\right]*\left(\frac{\left(\frac{q}{q'}\right)}{\left(\frac{x_i}{x_i'}\right)\left(\frac{x_j}{x_j'}\right)}\right) < 0 \quad [9]$$

Given a point x^0, necessary and sufficient for curvature correctness is that at this point

v'Hv ≤ 0 and **v's** = 0 where **v** denotes the direction of change.[5] For some input bundles

quasi-concavity may be satisfied but for others not and hence what can be expected is

that the condition of negative semi-definiteness of the bordered Hessian is met only

locally or with respect to a range of input bundles. The respective bordered Hessian is

negative semi-definite if the determinants of all of its principal submatrices are

alternate in sign, starting with a negative one (i.e. $(-1)^j D_j \geq 0$ where D is the

determinant of the leading principal minors and j = 1, 2, ..., n).[6] Hence, with respect

to our normalized translog production model it has to be checked a posteriori for

every input bundle that monotonicity and quasi-concavity hold. If these theoretical

criteria are jointly fulfilled the obtained estimates are consistent with microeconomic

theory and consequently can serve as empirical evidence for possible policy measures.

With respect to the proposed normalized translog production model quasi-

concavity can be imposed at a reference point (usually at the sample mean) following

Jorgenson and Fraumeni (1981). By this procedure the bordered Hessian in [8] is

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[5] Which implies that the Hessian is negative semi-definite in the subspace orthogonal to s ≠ 0.

[6] Determinants of the value 0 are allowed to replace one or more of the positive or negative values. Any
negative definite matrix also satisfies the definition of a negative semi-definite matrix.

replaced by the negative product of a lower triangular matrix Δ times its transpose Δ' (see appendix A1). Imposing curvature at the sample mean is then attained by setting

$$\beta_{ij} = -(\Delta\Delta')_{ij} + \alpha_i\lambda_{ij} + \alpha_i\alpha_j \qquad [10]$$

where i, j = 1, …, n, λ_{ij} = 1 if i = j and 0 otherwise and $(\Delta\Delta')_{ij}$ as the ij-th element of $\Delta\Delta'$ with Δ a lower triangular matrix.[7] As our point of approximation is the sample mean all data points are divided by their mean transferring the approximation point to an (n + 1)-dimensional vector of ones. At this point the elements of **H** do not depend on the specific input price bundle. The estimation model of the normalized translog production function is then reformulated as follows:

$$
\begin{aligned}
\ln(\frac{q}{q'}) &= \alpha_0 + \alpha_1\ln(\frac{x_1}{x_1'}) + \alpha_2\ln(\frac{x_2}{x_2'}) + \alpha_3\ln(\frac{x_3}{x_3'}) + \frac{1}{2}\left(-\delta_{11}\delta_{11} + \alpha_1 - \alpha_1\alpha_1\right)\ln(\frac{x_1}{x_1'})^2 + \frac{1}{2}\left(-\delta_{12}\delta_{12} - \delta_{22}\delta_{22} + \alpha_2 - \alpha_2\alpha_2\right)\ln(\frac{x_2}{x_2'})^2 \\
&+ \frac{1}{2}\left(-\delta_{13}\delta_{13} - \delta_{23}\delta_{23} - \delta_{33}\delta_{33} + \alpha_3 - \alpha_3\alpha_3\right)\ln(\frac{x_3}{x_3'})^2 + \frac{1}{2}\left(-\delta_{12}\delta_{11} - \alpha_1\alpha_2\right)\ln(\frac{x_1}{x_1'})^2\ln(\frac{x_2}{x_2'})^2 + \frac{1}{2}\left(-\delta_{13}\delta_{11} - \alpha_1\alpha_3\right)\ln(\frac{x_1}{x_1'})^2\ln(\frac{x_3}{x_3'})^2 \quad [11] \\
&+ \frac{1}{2}\left(-\delta_{13}\delta_{12} - \delta_{23}\delta_{22} - \alpha_2\alpha_3\right)\ln(\frac{x_2}{x_2'})^2\ln(\frac{x_3}{x_3'})^2 + \sum_{k=1}^{m}\gamma_k z_k + \varepsilon_i
\end{aligned}
$$

However, the elements of Δ are nonlinear functions of the decomposed matrix, and consequently the resulting normalized translog model becomes nonlinear in parameters. Hence, linear estimation algorithms are ruled out even if the original function is linear in parameters. By this "local" procedure a satisfaction of consistency at most or even all data points in the sample can be reached. The transformation in [11] moves the observations towards the approximation point and thus increases the likelihood of getting theoretically consistent results at least for a range of observations

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[7] Alternatively one can use Lau's (1978) technique by applying the Cholesky factorization Δ = -**LBL**' where **L** is a unit lower triangular matrix and **B** as a diagonal matrix.

(see Ryan and Wales 2000). However, by imposing global consistency on the translog functional form Diewert and Wales (1987) note that the parameter matrix is restricted leading to seriously biased elasticity estimates. Hence, the translog function would lose its flexibility. By a second analytical step we finally (a posteriori) check the theoretical consistency of our estimated model by verifying that the first derivatives of [6] are positive (monotonicity) the own second derivatives are negative and finally the Hessian is negative semi-definite (concavity).

Using equation [5], the optimal level of x_i is obtained by setting the marginal productivity (i.e. the first order condition) equal to the input/output price ratio. Using the predicted yield response at the optimum level of x_i, predicted profit levels are compared between the two soil fertility management practices. The predicted profit equation is given as:

$$\pi = p.q - \sum_{i=1}^{j} cx_{ij} \qquad [12]$$

where p and c are output and input prices. Assuming that all farmers face the same output and input prices, then profit will solely depend on the yield response function given by the marginal productivity of the input. Thus:

$$\frac{\partial \pi}{\partial x_i} = \bar{p} * \frac{\partial q}{\partial x_i} - \bar{c} \qquad [13]$$

Therefore, substituting the optimal level of x_i into equation [12], and solving for q, keeping all the other variables at the mean, results in the optimal yield, which is

then used in calculating the level of profit. This procedure is performed for all alternative soil fertility management options and the levels of optimal yield and profit are then compared. Similarly, we also compute the average total costs for maize production using the two soil fertility management practices.

4. Data

The data used for analysis in this study were based on a farm household survey administered to a stratified sample of 376 farmers. These farmers were randomly drawn from those that have been participating, more or less consistently, in the soil fertility management efforts involving public research institutions, donor organizations and NGOs for at least the last 5 seasons. The farmers were sampled from Blantyre, Lilongwe and Mzuzu Agricultural Development Divisions (ADD) using the stratified random sampling approach. See Figure 2 for the location of Malawi and the agroecological zones from which data were collected.

Figure 2: Map of Malawi
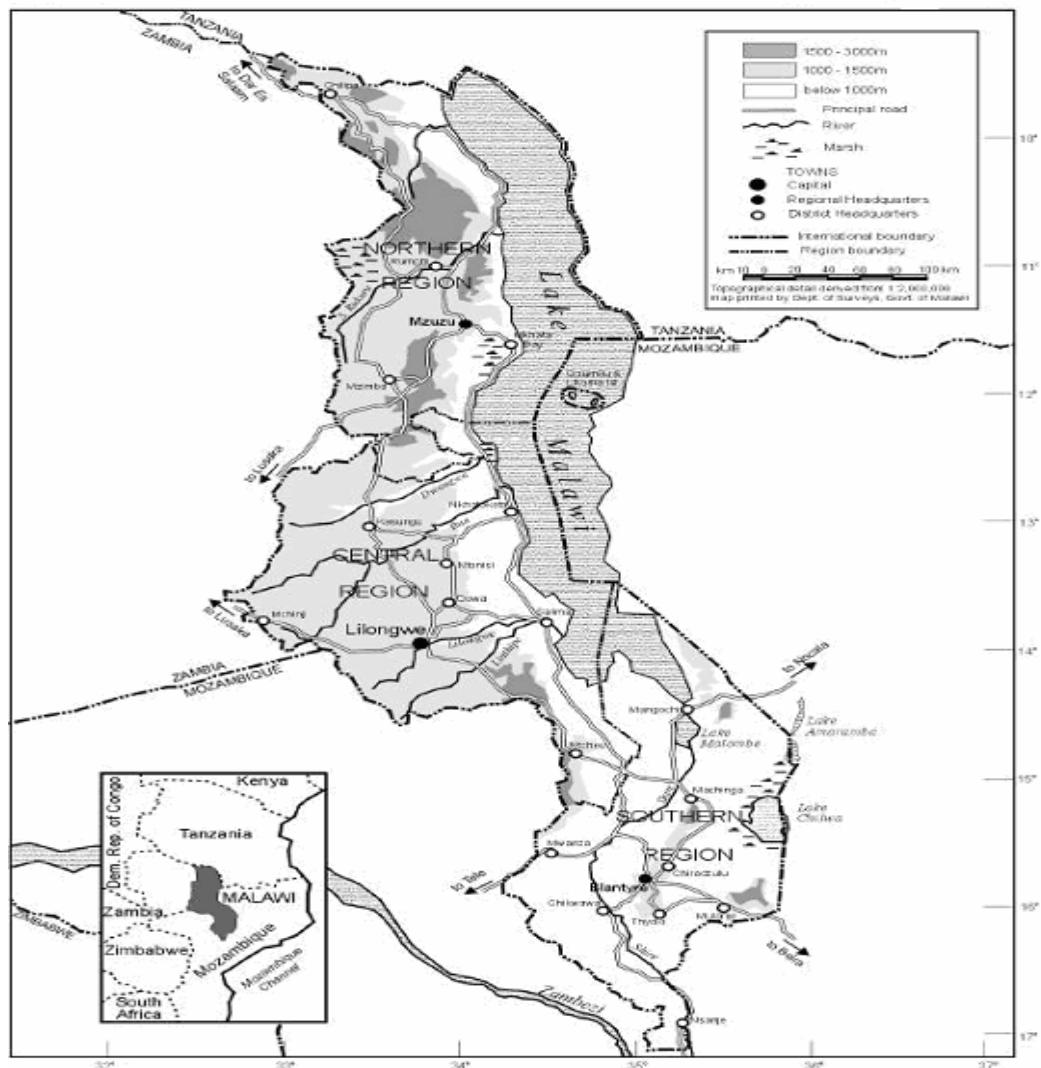
From these farmers, maize technology information related to variety grown, rate of
input application, other soil fertility options applied as well as the general husbandry
practices applied to the crop were collected and used in the analysis. The sample used

for the analysis comprises of 253 plots (out of a total of 573 plots) on which hybrid maize was grown as the main crop[8].

To validate the performance of various soil fertility management practices, we compared the farmers' yields with those obtained from two on-farm trails. The first is the area-specific fertilizer recommendation trail conducted by the Maize Productivity Task Force (MPTF) of the Ministry of Agriculture in which 1750 demonstrations were laid out on farmers' fields in all the agricultural extension sections in the country in the 1997/98 season. The second data set is also a Nationwide Best-bet Trial that was implemented on 1400 on-farm sites by the Malawian Extension Service in 1998/99, using the same set-up as the Area-specific Fertilizer Recommendation Trail. The objective was to compare the maize yield responses of fertilized and unfertilized legume cropping systems. In total six treatments were included in the experiment: (i) green legume rotation involving either soybean or groundnuts; (ii) *Mucuna pruriens* rotation; (iii) maize pigeon pea intercrop; (iv) fertilized maize; (v) unfertilized maize; and (iv) local maize (fertilized and unfertilized) as the control. The fertilized option involved either 35 or 69 kg ha^{-1} of N fertilizers (urea or 23:21:0+4S) depending on the area-specific fertilizer recommendations. In all treatments except the control, the same maize varieties i.e. MH17 and MH18 were planted depending on the altitude of an area.

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[8] We only analyze the productivity of hybrid maize mainly because government policy only promotes farmer' adoption of hybrid and not low yielding local maize varieties.

In comparing the on-farm trail results with those estimated from the farm household survey, the on-farm trials data were adjusted downwards by a total of 26% comprising a 7.5% adjustment to account for a higher than standard grain moisture and an additional 20% to reflect the difference in yields from the trial plots and that which the majority of farmers achieve on larger plots under comparable management.

Apart from the key inputs such as fertilizer, seed and labour, the specification of the productivity model includes also a number of important control variables that substantially affect yields, especially in the smallholder farming systems. These include rainfall and its variation, crop husbandry practices such as weeding frequency and date of planting as well as the critical policy variables i.e. frequency of extension visits, access to seasonal agricultural credit, access to product and factor markets and agro-ecological dummies. We also incorporate a soil fertility management dummy (either fertilizer only or integrated soil fertility management (ISFM) involving fertilizer and grain legume intercrops for biological nitrogen fixation). The descriptive statistics for all the variables that were included in the productivity model are presented in Table 1.

Table 1: Descriptive Statistics

VARIABLE	DESCRIPTION	MEAN	STD.
YIELD	Hybrid maize yield (kg/ha)	914.9	886.6
FERTILIZER	Fertilizer intensity (kg/ha)	30.9	38.3
LABOUR	Labour intensity (mandays/ha/month)	67.3	34.8
SEED	Seed intensity (kg/ha)	25.7	15.6
SFM	Soil fertility management (1=ISFM;0=fert)	0.6	0.5
WEEDING	Frequency of weeding	1.4	0.8
PLANTING	Date of planting (1=early; 0=later than first rains)	1.7	0.5
RAIN	Rainfall in mm	899.1	59.0
EXT_FREQ	Frequency of extension visits per month	0.8	1.0
CREDIT	Access to credit (1=yes; 0=no)	0.4	0.5
MACCESS	Market access (1=accessible; 0=remote)	0.4	0.5

Source: Own survey (2003)

5. Discussion of the Results

The estimation results are shown in Table 2. Given the cross-sectional data set and the imposed regularity constraints the overall model fit is significant at the 1%-level (P<0.000). Nearly 87% of all observations are consistent with the regularity conditions of monotonicity, diminishing marginal returns and quasi-concavity respectively. Refer to appendix AII for the numerical details of the regularity tests performed. The subsequent discussion is based on the theoretically consistent range of observations in the sample.

Except for seed, all input parameters show the expected sign. Among the inputs, fertilizer, its quadratic and seed interaction terms are highly significant. The

parameter on soil fertility management is highly significant implying that the use of integrated soil fertility practices significantly influences maize yield.

Table 2: Estimation Results

PARAMETER	COEFF.		SE	T-VALUE	P-VALUE
Constant	-1.349	4.019		-0.336	0.737
ln(labour)	0.108	0.101		1.074	0.284
ln(fertilizer)***	0.428	0.105		4.067	0.000
ln(seed)	0.493	0.390		1.265	0.207
ln(labour_sq)	0.007	0.082		0.088	0.930
ln(fertilizer_sq)***	-0.014	0.004		-3.654	0.000
ln(seed_sq)	0.005	0.535		0.009	0.993
ln(labour)X ln(fertilizer)	0.004	0.011		0.361	0.719
ln(labour) X ln(seed)	-0.034	0.315		-0.107	0.915
ln(fertilizer) X					
ln(seed)***	0.156	0.027		5.795	0.000
SFM***	0.042	0.013		3.126	0.002
Rainfall	0.245	0.594		0.412	0.681
Weeding frequency	0.005	0.008		0.537	0.592
Planting date	0.034	0.121		0.278	0.781
Market access	0.007	0.008		0.909	0.364
Extension frequency**	0.013	0.007		2.001	0.046
Credit access	0.007	0.006		1.205	0.229
ADJ. R^2	0.708	MONOTONICITY (%)	86.9		
F-VALUE	335.577	DIM. MARGINAL RETURNS (%)	86.9		
PROB>F	0.000	QUASI-CONCAVITY (%)	86.9		
# OBS.	253	REGULAR (%)	86.9		

Note: *** $P<0.000$; **$P<0.05$; *$P<0.10$

Although the parameters for rainfall, weeding frequency and planting dates show the expected signs, they are all insignificant. Among the policy variables, extension frequency is positively and significantly (P<0.05) related to maize productivity, while market and seasonal agricultural credit access are positively related to maize productivity, but are both insignificant. While we would expect significant influences of rainfall and its variation on maize yield, given the rainfed systems, the insignificance may be attributed to two reasons: First, hybrid varieties e.g. MH18 are bred specifically for drought resistance among other aspects and in Malawi most of these are particularly recommended for areas that are prone to intermittent droughts. Secondly, we attribute the insignificance to the way the rainfall data were collected. Rainfall figures are collected at an Extension Planning Area (EPA) level and thus do not reflect the actual variations experienced by different farms within an EPA. The husbandry practices are all positively related to yield for both varieties but are not significant.

The elasticities presented in Table 3 indicate that, keeping all factors constant, a unit increase in seed, fertilizer and labour will result in a 0.43%, 0.42% and 0.11% increase in maize yield respectively. Hence smallholder farmers are not producing at their optimal point with respect to the usage of variable inputs: The relative input usages could be radially increased to increase the maize output. The use of integrated soil fertility management improves the yield of maize by 4.2% on average, compared to the use of inorganic fertilizer only. The elasticity of maize yield with respect to the

amount of rainfall further indicates a relatively importance of climatic factors. The unit input effect of the other control and policy variables on maize yield is finally quite low as shown in Table 3:

Table 3: Mean Output Elasticities

VARIABLE	ELASTICITY $\left(\partial \ln\left(\frac{q}{q'}\right) / \partial \ln\left(\frac{x_i}{x_i'}\right) \right)$
Labour***	0.106 (0.0077)
Fertilizer***	0.420 (0.0613)
Seed***	0.428 (0.1621)
Soil fertility management♣	0.042
Rainfall	0.245
Weeding Frequency	0.005
Planting date	0.034
Market access	0.007
Extension Frequency	0.013
Credit access	0.007

Note: *** P<0.000; **P<0.05; *P<0.10

♣: invariant over observations as linear added control variables for SFM to Credit access

In Table 4, we compare the returns to scale associated with smallholder maize production using alternative soil fertility management options. The results indicate that smallholder farmers exhibit considerable returns to scale, consistent with other previous studies (Kamanga et al. 2000). This is because most smallholder farmers operate in a region of the production function where marginal productivity of inputs is increasing (stage I in figure 2). However, returns to scale for farmers using integrated soil fertility management practices are significantly higher (P<0.000) than

for farmers using only inorganic fertilizer. The relatively higher returns to scale for integrated soil fertility management options imply that there is still scope for smallholder farmers to improve maize productivity by an increase of their production: ISFM options improve the soil fertility and hence enhance the efficiency of inputs.

Table 4: Returns to Scale by Soil Fertility Management Option

SOIL FERTILITY MANAGEMENT OPTION	RTS	RTS RANGE	
		MIN.	MAX.
INORGANIC FERTILIZERS ONLY	1.12 (0.07)	0.98	1.35
INTEGRATED SOIL FERTILITY MANAGEMENT	1.50 (0.12)	1.09	1.71
TOTAL SAMPLE	1.31 (0.22)	0.98	1.71

Note: Returns to scale (RTS) difference between soil fertility management options is significant at ($P<0.000$), Figures in parentheses are standard errors.

These results imply that assuming constant maize/fertilizer price ratios, the optimal yield response for inorganic fertilizer (as well as other inputs) is higher in the case of integrated soil fertility management, due to the significance of the SFM parameter. Thus, with farmers facing more or less the same maize price and input cost, the profitability of smallholder maize production is likely to be higher when farmers integrate inorganic fertilizers with grain legumes. This is illustrated by figure 2:

Figure 2: Average and Marginal Products
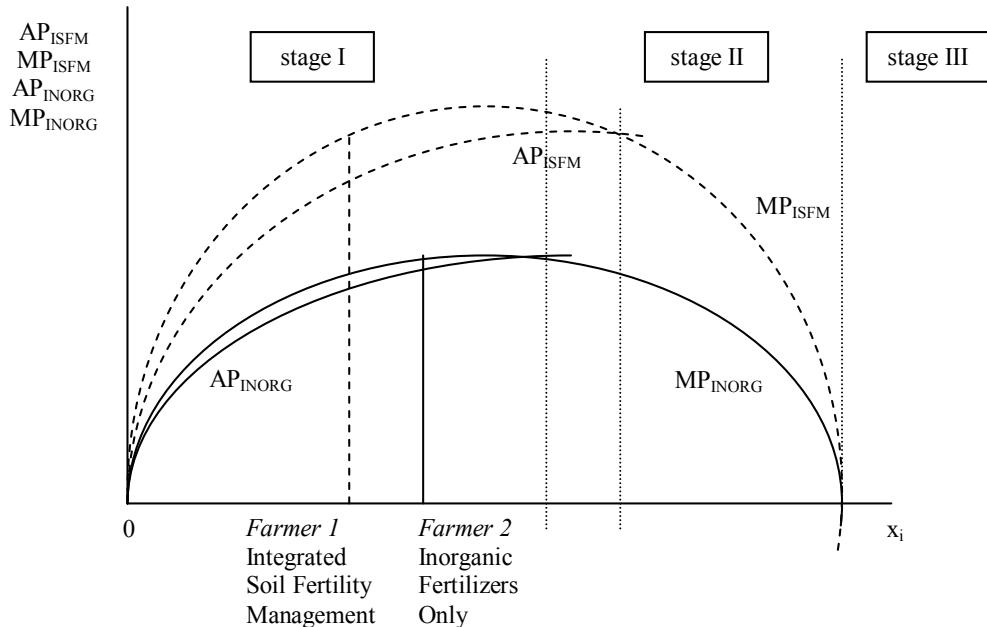
Farmer 1 as the average farmer using integrated soil fertility management enjoys a higher marginal product (MP_{ISFM}) as well as average product (AP_{ISFM}) than farmer 2 as the average farmer applying inorganic fertilizers only (MP_{INORG}, AP_{INORG}). As depicted by figure 2 both smallholder farmers experience increasing returns to scale and hence could enhance the production of maize, however, the average returns to scale for farmer 1 are relatively higher than those for farmer 2 (space inbetween the MP and AP curve).

Although the yield effect implied by the elasticity of SFM is somehow low (at 4.2% on average), given the low yields experienced by smallholder farmers, if we account for other bonus crops such as grain legumes (groundnuts, soya and pigeon peas), the overall additional yield effect of ISFM is quite substantial. In fact it is likely

to be higher among farmers which are unable to afford optimal quantities of inorganic fertilizer, but still have access to hybrid maize seed.

These results corroborate those of past studies in many ways. Most studies indicate that in general, ISFM options are more remunerative where purchased fertilizer alone remains unattractive or highly risky, as is the case with the maize-based smallholder farming systems in Malawi. For example, marginal rate of return analysis conducted on baby trials in Malawi also identified maize-pigeon pea intercropping, groundnut-maize intercropping and rotation as being economically attractive to smallholder farmers (Tomlow et al. 2001). In Zimbabwe, Whitebread et al. (2004) reported a 64% higher yield when maize is planted following green manure rotation compared to continuous fertilized maize. Mekuria and Waddington (2002) also reported that ISFM options gave a return to labour of $1.35 per day compared to $0.25 per day when either mineral fertilizers or organic soil fertility management options are used alone in Zimbabwe. In Kenya, Place et al. (2002) reported that the returns to labour from ISFM options ranged from $2.14- $2.68 per day compared to $1.68 per day when only one of the options is used. Economic analysis in central Zambia also indicates that velvet bean and sunhemp green manure followed by maize gives higher rate of returns compared to fertilized maize crop alone (Mwale et al. 2003). Such superior economic performance indicators are also reported by Mekuria and Siziba (2003) in the case of Zimbabwe.

Applying the assumption that all farmers face the same input and maize price ratios, these results imply that on average, use of ISFM in maize production improves profitability compared to use of inorganic fertilizer only. The average profitability indicators also support these results as shown in Table 5. The gross margin per unit of fertilizer and labour is higher when farmers use ISFM. As a result, using average as well as marginal rate of return, the results indicate that it is more profitable for farmers to produce maize under ISFM than using inorganic fertilizer only as shown in Figure 3:

Figure 3: Average Cost of Maize Production
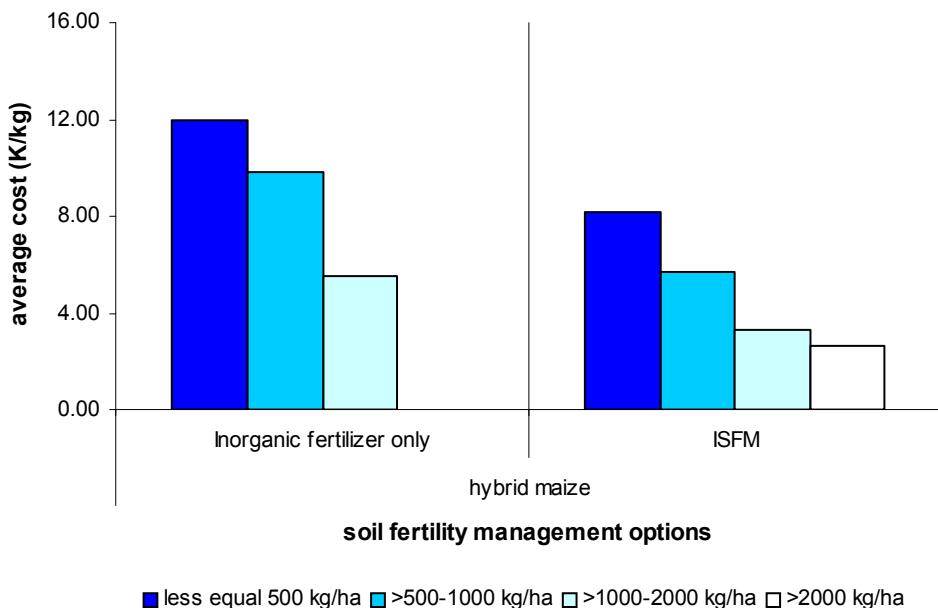
These results agree with those obtained using on-farm trials data which indicate higher yields in green legume rotation systems compared to maize applied with inorganic fertilizer only. *Mucuna* rotation gives the highest optimal yield

compared to maize applied with inorganic fertilizer only. Similarly the optimal yield for groundnut / soybean rotation and maize pigeon pea intercrop is higher than that of maize with inorganic fertilizer only (Kumwenda, 1997; Gilbert, 1998a, b; Sakala et al. 2003).

Table 5: Descriptive Statistics – The Economics of Maize Production

	Hybrid maize	
	Inorganic fertilizer only (N= 110)	Integrated SFM (N=143)
Gross revenue (Kwacha per ha)	9488.80	13124.09
Labour cost (Kwacha per ha)	1816.02	1478.91
Fertilizer cost (Kwacha per ha)	1520.34	1994.42
Gross margin (Kwacha per ha)	6107.44	9650.76
Gross margin per Kg of fertilizer	368.41	530.26
Gross margin per manday	99.91	191.03
Average variable cost per kg of maize	4.80	3.60
Value/Cost ratio (VCR)	2.81	3.78
Marginal Rate of Return (%)	181	278

Note: Hybrid maize includes MH17 and MH18, Kwacha is the local currency, Fertilizers include a combination of 23:21:0+4s and CAN, Integrated soil fertility management (SFM) involves the application of inorganic fertilizers and incorporation of grain legumes i.e. groundnuts (*Arachis hypogea*) or pigeon peas (*Cajanas cajan*) in an intercrop system.

6. Conclusions and Policy Implications

The study clearly shows that maize productivity under ISFM is higher than when farmers use inorganic fertilizer only. Gross margin per unit of inputs is also higher, assuming farmers face the same maize prices and input costs. These results are

likely to be more meaningful among smallholder farmers that can hardly afford optimal levels of inorganic fertilizer, and those in very risky environments. These results in someway also assist to dispel skepticism associated with the benefits of integrated soil fertility management options, especially among farmers who have been crowded out of the agricultural inputs market for reasons of affordability.

In terms of policy implications, ISFM provides scope for improving maize productivity especially where use of inorganic fertilizer is highly unaffordable and risky. Thus there is need for policy interventions to promote smallholder uptake of ISFM options. However, it is important to note that the scope for ISFM to resuscitate the productivity of the maize-based smallholder farmers depends on consistent integration of grain legumes with inorganic fertilizers and access to improved maize varieties. The performance of grain legumes in fixing nitrogen is greatly compromised under low soil fertility conditions. Thus ISFM establishment in smallholder farming systems can be facilitated through cross-compliance interventions through among others, seasonal credit provision to enable farmers to afford inorganic fertilizers and improved maize and legume seeds. Similarly, an improvement in rural output and input markets, including the grain legume market would act as an additional incentive that will motivate farmers to grow grain legumes together with maize. Public extension still remains the main caveat for reaching smallholder farmers with technologies developed by researchers. Where the capacity for public extension is overstretched e.g. due to HIV/AIDS scourge, there is need for policy to create

favourable conditions for the involvement of non-governmental organizations that

have been instrumental in reaching smallholder farmers.

Acknowledgement

References

Ackello-Ogutu, C., Q. Paris and W.A. Williams. 1985. Testing von Liebig Crop Response Function Against Polynomial Specifications. *American Journal of Agricultural Economics*, 67: 873-880.

Angus, J.F., J.W. Bowden and B.A. Keating. 1993. Modeling nutrient response in the field. *Plant Soil* 155 & 156: 57-66.

Belanger, G., J.R. Walsh, J.E. Richards, P.H. Milburn and N. Zaidi. 2000. Comparison of three statistical models describing potato yield response to nitrogen fertilizer. *Agronomy Journal* 92: 902-908.

Benson, T. 1997. *The 1995/96 fertilizer verification trial in Malawi: Economic analysis of results for policy discussion*. Action Group I, Maize Productivity Task Force, Chitedze Research Station, Lilongwe, Malawi.

Benson, T. 1999. *Validating and strengthening the area specific fertilizer recommendations for hybrid maize grown by Malawian smallholders: Maize Commodity Team*. Chitedze Research Station, Lilongwe, Malawi.

Bock, B.R., and F.J. Sikora. 1990. Modified quadratic plateau model for describing plant responses for fertilizer. *Soil Sci. Soc. Am. J.* 54: 1784-1789.

Bullock, D.S. and D.G. Bullock. 1994. Calculation of optimal nitrogen fertilizer rates. *Agronomy Journal* 86: 921-923.

Byerlee, D., Anandajayasekeram, P., Diallo, A., Gelaw, B., Heisey, P.W., Lopez-Pereira, M., Mwangi, W., Smale, M., Tripp, R. and S.Waddington. 1994. *Maize Research in Sub-Saharan Africa: An overview of past impacts and future prospects.* CIMMYT Economics Working Paper 94-03. Mexico, D.F.: CIMMYT.

Chambers, R.G. 1988. *Applied Production Analysis: A Dual Approach.* Cambridge University Press. New York.

Conroy, A., 1997. *Examination of policy options facing Government in the event of a short-fall in maize production.* Discussion paper, Ministry of Finance, Lilongwe, Malawi.

Chirwa, E.W. (1996). An Econometric Analysis of the Determinants of Agricultural Credit Repayment in Malawi. *Agronomy Journal* 1-2: 107-122.

Chirwa, E.W. 2003. *Fertilizer and Hybrid Seeds Adopting Among Smallholder Maize Farmers in Southern Malawi.* Department of Economics. University of Malawi (manuscript).

Diagne, A., and M. Zeller. 2001. *Access to credit and its impacts on welfare in Malawi*. IFPRI Research Report 116. International Food Policy Research Institute, Washington D.C.

Donovan, G., and Casey, F. 1998. *Soil fertility management in Sub-Saharan Africa.* World Bank Technical Paper No. 408. World Bank. Washington, D.C.

Dorward, A. 1999. Farm Size and Productivity in Malawian Smallholder Agriculture. *Journal of Development Studies* 35, 5: 141-161.

Edriss, A., H. Tchale and P. Wobst (2004): The Impact of Labour Market Liberalization on Maize Productivity and Rural Poverty in Malawi. Project paper submitted to the *Journal of Development Studies*. http://www.pasad.uni-bonn.de

Franke, M.D., B.R. Beattie, and M.F. Embelton. 1990. A comparison of alternative crop response models. *American Journal of Agricultural Economics* 72: 597-602.

Frohberg, K. 2001. Comments on the paper 'Review of Agricultural Trade Models: An Assessment of Models with EU Policy Relevance'. Pp. 32-39. In: Frandsen, S.E. and M.J.H. Staehr (Eds.) *Assessment of GTAP Modeling Framework for Policy Analysis from a European Perspective*.

Gilbert, R.A. 1998a. Comparison of best-bet soil fertility interventions: Preliminary results. In: *Annual report of the cereals commodity group for 1997-98*. Ministry of Agriculture and Irrigation, Lilongwe, Malawi.

Gilbert, R.A. 1998b. Undersowing green manures for soil fertility management in the maize-based cropping systems of Malawi. In: Waddington, S., H.K. Murirwa, J.D.T. Kumwenda, D. Hikwa and F. Tagwira (Eds.). *Soil Fertility Research for Maize Based Farming Systems in Malawi and Zimbabwe.* Proceedings of the Soil Fertility Network Results and Planning Workshop, 7-11 July 1997, Mutare, Zimbabwe.

Green, D.A.G and D.H. Ng'ong'ola. 1993. Factors affecting fertilizer adoption in less developed countries: An application of multivariate logistic analysis in Malawi. *Journal of Agricultural Economics* 44: 99-109.

Heisey, P.W.and M. Smale. 1995. *Maize technology in Malawi. A green revolution in the making?* CIMMYT Research Report No.4. Mexico City, Mexico.

Jorgenson, D.W. and Fraumeni, B.M. 1981. Relative Prices and Technical Change. Pp. 17-47. In: E.R. Berndt, E. R., *Modeling and Measuring Natural Resource Substitution*.

Kamanga, B.C., Kanyama-Phiri, G. and S. Minae. 2000. Maize production under tree-based cropping systems in southern Malawi: A Cobb-Douglas approach. *African Crop Sciences Journal,* 8, 4: 429-440.

Keyser, M.A. 1998. *Formulation and Spatial Aggregation of Agricultural Production Relationships within the Land Use Change (LUC) Model.* Interim Report IR-98-092. International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria.

Kumwenda, J.D.T. and R.A. Gilbert, R.A. 1998 Biomass production by legume green manures on exhausted soils in Malawi. A Soil Fertility Network Trial. In: S. Waddington, H.K. Murirwa, J.D.T. Kumwenda, D. Hikwa and F. Tagwira (Eds.). *Soil Fertility Research for Maize Based Farming Systems in Malawi and Zimbabwe.* Proceedings of the Soil Fertility Network Results and Planning Workshop, 7-11 July 1997, Mutare, Zimbabwe.

Kydd, J. 1989. Maize research in Malawi. Lessons from failure. *Journal of International Development* 11: 112-144.

Lau, L.J. 1978. Testing and imposing monotonicity, convexity and quasi-convexity constraints. In: Fuss, M. and McFadden, D. (Eds.): *Production Economics: A Dual Approach to Theory and Applications.* Volume I: The Theory of Production, Amsterdam.

Lau, L.J., 1986. Functional Forms in Econometric Model Building. Pp. 1515-1565. In: Griliches, Z., and Intriligator, M.D. (Eds.), *Handbook of Econometrics*, Volume III.

McFadden, D. 1978. Cost, Revenue and Profit Functions, in: Fuss, M., D. McFadden 1978. *Production Economics: A Dual Approach to Theory and Applications*, Vol. 1: The Theory of Production, Vol. 2: Applications of the Theory of Production; North-Holland, New York.

Minot, N., M. Kherallah and P. Berry. 2000. *Fertilizer market report and determinants of fertilizer use in Benin and Malawi.* MSSD Discussion Paper No. 40. International Food Policy Research Institute, Washington D.C.

Mekuria, M and S. Siziba. 2003. Financial and risk analysis to assess the potential adoption of green manure technology in Zimbabwe and Malawi. In: S.R. Waddington (Ed.) *Grain legumes and green manures for soil fertility in Southern Africa: Taking Stock of Progress.* Soil FertNet, CIMMYT, Harare, Zimbabwe.

Mekuria, M., and S. Waddington. 2002. Initiatives to encourage farmer adoption of soil fertility technologies for maize-based cropping systems in Southern Africa. Pp. 219-234. In: Barrett, C.B., Place, F and Abdillahi, A. (Eds.): *Natural Resources Management in African Agriculture: Understanding and Improving Current Practices.* CABI, Wallington, UK.

Mwale, M., C.Masi, J. Kabongo and L.K. Phiri. 2003. Soil fertility improvement through the use of green manure in central Zambia. In: S.R. Waddington (Ed.) *Grain legumes and green manures for soil fertility in Southern Africa: Taking Stock of Progress.* Soil FertNet, CIMMYT, Harare, Zimbabwe.

Mwangi, W.M. 1997. Low use of fertilizers and low productivity in sub-Saharan Africa. *Nutrient Cycling. Agro-forestry Systems* 47: 135-147.

Nkonya, E. 2001. *Modeling Soil Erosion and Fertility Mining: The case of wheat production in Northern Tanzania*. Unpublished PhD Dissertation. Kansas State University.

Place, F., S. Franzel, J. Dewolf, R. Rommelse, F. Kwesiga, A. Niang and B. Jama. 2002. Agroforestry for soil fertility replenishment: evidence on adoption processes in Kenya and Zambia. Pp.275-286. In: Barrett, C.B., Place, F and Abdillahi, A. (Eds.): *Natural Resources Management in African Agriculture: Understanding and Improving Current Practices.* CABI, Wallington, UK.

Ryan, D.L. and Wales, T.J. 2000. Imposing Local Concavity in the Translog and Generalized Leontief Cost Functions. *Economic Letters* 67: 253 – 260.

Sadoulet E., and A. de Janvry. 1995. *Quantitative Development Policy Analysis.* The John Hopkins University Press. Baltimore and London.

Sakala W.D., Ligowe, I. and Kayira, D. 2003. Mucuna–maize rotations and short fallows to rehabilitate smallholder farms in Malawi. Pp. 161-163. In: Waddington, S.R. 2003. Grain Legumes and Green Manures for Soil Fertility in Southern Africa: Taking Stock of Progress.

Sauer, J., K. Frohberg and H. Hockmann. 2004. *Black-box frontiers and implications for development policy: theoretical considerations.* Development Policy Discussion Paper No. 92. Center for Development Research, Bonn, Germany.

Smale, M., and T. Jayne. 2003. Maize in Eastern and Southern Africa: *Seeds of success in retrospect.* Environment and Production Technology Division. Discussion paper No. 97. International Food Policy Research Institute, Washington D.C.

Twomlow, S.J., J. Rusike and S.S. Snapp. 2001. Biophysical or economic performance-which reflects farmer choice of legume 'best-bets' in Malawi. Pp. 480-486. In: *7th Eastern and Southern Africa Regional Maize Conference.*

Whitbread, A.M., O. Jiri and B. Maasdorp. 2004. The effect of managing improved fallows of *Mucuna pruriens* on maize production and soil carbon and nitrogen dynamics in sub-humid Zimbabwe. *Nutrient Cycling in Agroecosystems* 69: 59-71.

Appendix I: Bordered Hessian Decomposition

$$-(\Delta\Delta') = -\left[\begin{pmatrix}\delta_{11}&0&0&0\\\delta_{21}&\delta_{22}&0&0\\\delta_{31}&\delta_{32}&\delta_{33}&0\\\delta_{41}&\delta_{42}&\delta_{43}&\delta_{44}\end{pmatrix}\begin{pmatrix}\delta_{11}&\delta_{12}&\delta_{13}&\delta_{14}\\0&\delta_{22}&\delta_{23}&\delta_{24}\\0&0&\delta_{33}&\delta_{34}\\0&0&0&\delta_{44}\end{pmatrix}\right]=$$

$$=\begin{pmatrix}-\delta_{11}\delta_{11} & -\delta_{11}\delta_{12} & -\delta_{11}\delta_{13} & -\delta_{11}\delta_{14}\\ -\delta_{11}\delta_{21} & -\delta_{12}\delta_{12}-\delta_{22}\delta_{22} & -\delta_{12}\delta_{13}-\delta_{22}\delta_{23} & -\delta_{21}\delta_{14}-\delta_{22}\delta_{24}\\ -\delta_{11}\delta_{31} & -\delta_{31}\delta_{12}-\delta_{23}\delta_{22} & -\delta_{31}\delta_{13}-\delta_{23}\delta_{23}-\delta_{33}\delta_{33} & -\delta_{31}\delta_{14}-\delta_{23}\delta_{34}-\delta_{33}\delta_{34}\\ -\delta_{11}\delta_{14} & -\delta_{21}\delta_{14}-\delta_{22}\delta_{24} & -\delta_{31}\delta_{14}-\delta_{23}\delta_{34}-\delta_{33}\delta_{34} & -\delta_{14}\delta_{14}-\delta_{24}\delta_{24}-\delta_{34}\delta_{34}-\delta_{44}\delta_{44}\end{pmatrix}$$

[A1]

Appendix II: Regularity Details for the Sample Mean

	LABOUR	*FERTILIZER*	*SEED*
Monotonicity $\left[\partial(q/q')/\partial(x_i/x_i')\right]>0$	0.0316	0.2218	0.2282
Diminishing Marginal Returns $\left[\partial^2(q/q')/\partial(x_i/x_i')^2\right]<0$	-0.0096	-0.0304	-0.0313
	BH₁	*BH₂*	*BH₃*
Quasi-Concavity $(-1)^j D_j \geq 0$	-0.0010	0.0006	-1.3E-05