Value of Information and Averting Behavior:
The Case of Toxic Water Contamination

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ABSTRACT: Little theoretical work has been done previously on the welfare valuation of changes in price and quality when consumers are imperfectly informed. The presence of imperfect information is particularly important in the analysis of averting behaviors. We develop a theoretical welfare measure, called *quasi-compensating variation*, as a natural extension of compensating variation (CV). We show that this welfare measure offers not only a money metric of the "value of information", but also a means to appropriately evaluate the welfare effects of various policies when consumers are imperfectly informed of water contamination. With a numerical example and our decomposition results (**Propositions 2 and 3**), we demonstrate that (i) the value of information could potentially account for a large portion of the total welfare gains when regulators simultaneously disseminate accurate information and improve drinking water quality, (ii) the willingness to pay to avoid toxic contamination is strictly larger for imperfectly informed than perfectly informed consumers, and (iii) the distribution of imperfect information among consumers affects the relative performance of the two compelling policy alternatives, "self-protection" and "pollution control".

I. Introduction

In the cases of air-borne and water-borne pollutants, people can adopt averting options, at least to some extent, to protect themselves from their adverse health effects. For example, individuals concerned over the quality of tap drinking water can choose to use water filters and boiled or bottled water to avoid possible intake of microbial and chemical contaminants. A theoretical premise is that, as long as consumers can choose the use of various averting options optimally to adjust the quality of their "personal environment" (Bartik, 1988) for changes in the exogenous water quality, their averting expenditures may be used as a lower-bound estimate for the welfare gain from improving water quality. However, the approach critically relies on a set of underlying information and behavioral factors. In particular, the approach assumes both that all consumers are perfectly informed of the quality (and health effects) of drinking water and that they have access to appropriate averting options. If they cannot use proper averting options, *due to lack of information* or *lack of access (including lack of financial resources)*, then economic losses due to their consequential health damages may be a more appropriate measure of welfare costs. This issue is particularly important in developing countries, where health advisories and drinking water quality standards are unlikely to be well established and disseminated to the public, and where people or government agencies or both have very limited resources.

A rational consumer makes her choices given the information she possesses. In the case of toxic water contamination, the consumer can choose her averting option(s) only suboptimally if she is ignorant or imperfectly informed of the contamination problem. This suboptimal averting behavior results in harmful health effects, which weren't anticipated at the time of her decision (due to imperfect information) and are likely to be realized only after passage of time.[1] Imperfect information, therefore, can lead to significant welfare losses. It seems reasonable to expect, then, that *the consumer's willingness to pay (WTP) for pollution control should be increasing in the degree of imperfect information*. That is, WTP should be larger for those with imperfect information than those with perfect information. On the other hand, prior literature has suggested the problem of *information bias*, which essentially states that the consumer's WTP to avoid toxic pollution is

[1] It is well known that many toxic contaminants such as PCE and TCE may cause cancer in humans only after they are exposed to certain levels over long periods of time whereas every dose or exposure is accompanied with some increased risk of cancer.

likely to be higher for those who are aware of the pollution problem than for those who are not. In fact, one study has empirically shown this information bias effect for both WTP and averting expenditures (Powell, 1991). The story just told suggests (at least conceptually) that one needs to adjust empirical WTP measures for two factors, one for the cost of imperfect information and the other for information bias. However, we do not seem to have a theory to examine these effects. As it turns out, our familiar welfare measures, compensating/equivalent variation (CV/EV) and compensating/equivalent surplus (CS/ES), are not suitable in the presence of imperfect information. We propose alternative welfare measures, called *quasi-compensating variation* (QCV) and *quasi-equivalent variation* (QEV), as a natural extension of CV/EV. We show that these welfare measures offer not only a money metric of the "value of information", but also a means to appropriately evaluate the welfare effects of various policies when consumers are imperfectly informed of water contamination.

To illustrate the problem further, let us consider a simple example provided in Easter and Konishi (2005). Suppose that there is only one toxic pollutant of regulatory concern. We wish to evaluate the welfare costs of this pollutant in drinking water sources. Let E be the consumer's total averting expenditures for this pollutant. Let C_u denote her WTP value for protection against this contaminant. C_u may represent the WTP value to avoid the (short-term and long-term) health risks associated with this contaminant. Suppose that the health risks of this contaminant are well known to regulators, and thus disseminated widely to the public. Suppose further that there are n alternative options for avoiding entirely the health risks from the (potential) contamination, with costs of option j being $C_j, j = 1, ..., n$. Then, her averting expenditure is given by

$$E = \gamma \cdot \min\{C_1, C_2..., C_n, C_u\}$$

where $\gamma = 1$ if she is aware of the contamination risk and 0 otherwise. Note that the vector $\{C_j\}$ could possibly contain zero if no averting option is available to the consumer.[2] "True" welfare costs

[2] By availability we include technological as well as financial feasibility. For example, if a person's income is less than the least expensive averting option, then the vector should contain zero in this formulation.

(or their ideal welfare estimates) of this toxic pollutant must have the property:

$$W = \begin{cases} E & \text{if} \quad \gamma = 1 \\ C_u > 0 & \text{if} \quad \gamma = 0 \text{ or no available option,} \end{cases}$$

whereas *observed* empirical welfare estimates using averting expenditures will result in:

$$\hat{W} = \begin{cases} E & \text{if} \quad \gamma = 1 \\ 0 & \text{if} \quad \gamma = 0 \text{ or no available option.} \end{cases}$$

Thus, we would underestimate the welfare costs for those without accurate information or access to averting options, which can be quite important if a significant portion of the population has either $\gamma = 0$ or no access to avoidance options.[3] In fact, it is widely recognized from the findings of prior avoidance-costs studies (for example, Powell, 1991; Abdalla, Roach, and Epp, 1992; Collins and Steinback, 1993; Kwak and Russell, 1994; and Abrahams, Hubbell, and Jordan, 2000) that (a) even in the United States, a majority of the population may be unaware of certain contamination problems, (b) awareness of the pollution problem is one of the key determinants of averting behaviors, and (c) those informed of the contamination tend to spend more on averting expenditures than do those who are unaware. Despite its importance, such information has not been used effectively in the previous studies. From the point of view of a welfare-maximizing regulator, the estimated percentage of the population with $\gamma = 0$ should be no less important than the estimated averting expenditures for those whose $\gamma = 1$, because the regulator must use a more appropriate estimate of C_u to calculate the welfare costs for those with $\gamma = 0$ and E for those with $\gamma = 1$.[4] What we want is not just the *estimates of WTP for those with* $\gamma = 1$ but the *estimates of aggregate welfare costs* (or equivalently, welfare benefits of reducing toxic water pollution).

A series of questions arise naturally from this example. First, what would be an appropriate measure of welfare costs if the consumer is only partially informed (i.e. if γ is a continuous variable rather than a binary one)? More importantly, what is the size (in money metric) of the welfare cost

[3]Note that, when $\gamma = 1$, $W = \hat{W} = E$ and $E \leq C_u$. Therefore, $\hat{W} \leq C_u = WTP$, which is consistent with the prior theory that averting expenditures is a lower bound estimate for willingness to pay.

[4]We will show that the consumer's willingness to pay to avoid toxic contamination (C_u above) is not a constant and varies with γ (Section IV) and that it is higher for those with $\gamma = 0$ than for those with $\gamma = 1$. Thus, the discrepancy between the observed E and C_u is larger for those with $\gamma = 0$ (Section V).

of being uninformed (or the value of providing accurate information) as a function of information bias? Third, what would be the effects of price changes if the consumer has a more flexible set of options (i.e. if the consumer can choose from a *continuous* set of averting options given the prices)? Lastly, which policy is more welfare-enhancing, to improve water quality or to promote "self-protection" (by providing more accurate information together with lowering the price of averting options) given the distribution of γ? The *valuation* of the welfare effects of imperfect information becomes particularly important when one attempts to answer such policy questions.

The meaning of the term "value of information" needs to be clarified, as it has been widely used in different contexts. The term frequently appears in the quasi-option value literature. The concept of quasi-option value (or Arrow-Fisher-Henry option value) concerns an action with irreversible consequences and focuses explicitly on the intertemporal aspect of decision-making. It is formally the "correction factor" so as to optimally adjust the agent's (sub-optimal) decision due to her failure to recognize the prospect of obtaining full information (about the future benefits). In this connection, quasi-option value is often compared against the expected value of obtaining *perfect* information about the realization of future benefits (See Conrad, 1980 and Hanemann, 1989). Another line of research defines the "value of information" as the increase in *expected utility (or welfare)* by gaining more (refined) information about the distribution of possible outcomes prior to decision-making.[5] For example, Polasky (1992) considered the effects of "exploration" in an exhaustible resource economy. Early exploration can reveal better information about the size of the unproven reserves prior to extraction, leading to higher expected profits (and welfare under some conditions).

The first two lines of research typically consider the cases where the value of acquiring better information is intrinsically positive, at least, to the direct user of that information. It is well known in the industrial organization literature, however, that the value of information can sometimes be negative in multi-agent settings. For example, Mirman, Samuelson, and Schlee (1994) showed that

[5] Formally, Hanemann (1987) defined the expected value of perfect information (EVPI) as the difference in the optimal welfare values between the cases where the agent is allowed to make her decision after the realization of future states and where she makes her decision before the realization. Hanemann's EVPI does not necessarily coincide with the "value of information" defined as the increase in expected utility, because the former compares the values of two intrinsically different decision problems whereas in the latter, the same decision rules can be used both before and after the learning.

more information about demand can hurt duopolists. Schlee (1996) also established the conditions under which the value of public information about product quality is negative to consumers. In these papers, the negative impact of information essentially comes from the interactions among agents — new information changes the agents' motivation and beliefs, leading to their updated decisions, which in turn can negatively affect the other agents. More recently, Morris and Shin (2002) focused on the dual role of public information — of conveying more accurate information as well as creating a focal point for beliefs. They showed that, when public information entails some noise and when private agents have access to independent sources of information, the welfare effects of improved public information is ambiguous.

In contrast to the previous research, the present paper considers a static model with a continuum of price-taking consumers. Moreover, public information does not contain any noise and the consumer's information is represented by a single parameter rather than a distribution. Therefore, the model is admittedly simple in comparison to previous value-of-information studies. Nonetheless, a number of useful results are obtained by focusing on the *valuation* of the welfare gains of obtaining better information about water quality — an area that has not been addressed in prior literature. The main contributions of this paper are three-fold. First, we find that a *money-metric* welfare measure of the value of information can be defined analogously to compensating and equivalent variations. More importantly, this welfare measure can be used to evaluate the welfare gains from a water quality improvement policy of *those who are perfectly informed as well as those who are not*. Second, the welfare measure can be decomposed in such a way as to enable us to analyze the contributions of different factors of a mixed policy (e.g. the policy of simultaneously improving water quality and providing accurate information). Third, with a numerical example, we show that a policy to promote self-protection can yield larger aggregate welfare gains than the policy of improving water quality depending on the distribution of γ. The empirical estimation of its distribution among consumers, then, is important.

The organization of the paper is as follows. In the next section, we review the Courant & Porter (1981) and Bartik (1988) model (henceforth, the CP&B model) and discuss it in relation to our model. In Section III, we fully develop the model and present a series of preparatory results to demonstrate the welfare analysis in the relevant commodity space. We also introduce new welfare measures, called *quasi-compensating* and *quasi-equivalent variation* (QCV/QEV). Section IV discusses the welfare analysis of imperfect information and provides a series of decomposition results (**Lemmas 2, 3** and **Propositions 2, 3**). In Section V, we provide computational results, in which we demonstrate that (i) the value of information could potentially account for a large portion of the total welfare gains when regulators simultaneously disseminate accurate information and improve drinking water quality, (ii) the willingness to pay to avoid toxic contamination is strictly larger for imperfectly informed than perfectly informed consumers, and (iii) the distribution of imperfect information among consumers affects the relative performance of the two compelling policy alternatives, self-protection and pollution control. The last section concludes the paper and discusses key policy implications and potential extensions of our research.

II. Preliminaries

Prior empirical studies frequently cite Courant and Porter (1981) and Bartik (1988) as a theoretical basis of the averting expenditures method. They consider the following model. The quality of one's personal environment or one's health, H, is a function of one's use of averting options x and the ambient quality level Q. H is increasing in each argument. The CP&B model uses an abstract "price" of defensive measures, so that the consumer's defensive expenditure D is a function of H and Q. A notable feature of their model is that the price of the consumer's health, P, has no clear relationship to her choice of averting options and is assumed to be a function *only* of the ambient quality Q where $dP/dQ < 0$. As a result, the price of health is exogenously given by Q. Thus, her defensive expenditure function can be written:

$$D = P(Q)H(x^*(Q), Q) = \hat{D}(Q, H),$$

where $x^*(Q)$ is the optimal choice of averting options given Q. If we assume that ambient quality Q affects utility only through the production of health, the consumer's utility is simply a function of a composite numeraire good z and her health H. Thus, the consumer's problem is written:

$$\max U(z, H) \quad \text{s.t } z + \hat{D}(Q, H) \leq m,$$

where m is income.[6]

In the CP&B model, individuals choose their "health" or quality of personal environment so as to maximize utility, assuming that, given Q, the corresponding optimal use of defensive options $x^*(Q)$ exists. This optimization program by itself is valid, as long as $H(x, Q)$ is a one-to-one function of x given Q. One of the drawbacks of this analytical framework, however, is that it fails to explain a possibly endogenous relationship between the "price" of health and the choice of averting options. In fact, it turns out that in models of this type, including ours, an endogenous relationship arises naturally and the sign of dP/dQ depends on the health production technology H. Attending to this endogeneity is a primary innovation of this paper. Another drawback of their model is that it fails to take into account of the effect of changes in the price of averting options. This price, denoted p to distinguish it from the price of health, has quite different properties from those of P. One of the most important differences is that, even when Q changes, p may not change while P does. p may change via a pricing policy, economic/technological advancement, or (long-run) equilibrium effects of changes in Q. This issue becomes critical when one intends to analyze the welfare effects of a policy that changes p and not Q. For example, regulators may consider providing water filters at low cost to residents of the communities exposed to serious water contamination. Lastly, the CP&B model assumes that $H_x > 0, H_Q > 0, H_{xQ} > 0$, and $H_{xx} = 0$. In the case of drinking water, however, it is more reasonable to assume that $H_{xQ} \leq 0$, because the marginal effect of x is typically lower when the ambient water quality Q is high and vise versa.

III. The Model

To overcome these issues, we employ an alternative formulation that extends the CP&B model. In what follows, we assume for simplicity that there are (i) a composite x of all possible alleviating options $\mathbf{x} = (x_1, ..., x_n)$ and (ii) a composite A of all possible toxic chemicals $\mathbf{A} = (A_1, ..., A_m)$, where the vector $\mathbf{A}$ represents the ambient concentration levels for m toxic or non-toxic pollutants

[6] In Courant & Porter (1981), the consumer's optimization problem is written as:

$$\max_{z,x} U[z, H(x, A)] \quad s.t. \quad z + x = m$$

which seems to be identical to our formulation (**3**) below. However, carefully examining their arguments, one would conclude that x is taken as defensive expenditures (i.e. $x \approx \hat{D}(Q, H)$ above) and does not coincide with our conceptual framework below.

in the ground and surface water bodies. As in Abrahams et al (2000), we introduce a health production function $H = H(x, A)$. It should be understood that A is the ambient level of water pollution, so that health is expected to be monotonically decreasing in A. Furthermore, by this function, we are implicitly assuming that individuals and regulators are perfectly informed about the health effects of using the source water of quality A when an individual adopts a vector of averting behaviors x. For simplicity, we also ignore the joint production of health and utility by water quality.[7] We impose the following regularity conditions on our utility function $U(z, H(x, A))$.

A1 *(i)* $U(\cdot, H)$ *is strictly increasing in* z *for each fixed* H *and* $U(z, \cdot)$ *is strictly increasing in* H *for each fixed* z*; (ii)* $H(\cdot, A)$ *is strictly increasing in* x *for each fixed* A *and* $H(x, \cdot)$ *is strictly decreasing in* A *for each fixed* x*.*

A2 *(i)* $U(\cdot, \cdot)$ *is strictly quasi-concave in* (z, H)*; (ii)* $H(\cdot, A)$ *is strictly concave in* x *for each fixed* A*.*

A3 *(i)* $U(\cdot, \cdot)$ *is continuous in* (z, H)*; (ii)* $H(\cdot, \cdot)$ *is continuous in* (x, A)*.*

A4 *There are upper and lower satiation points for* A*. That is, there exist points* $A_{\max}, A_{\min}$ *such that* $\forall x, \forall A \geq A_{\max}, \quad H(x, A) = 0$ *and* $\forall x, \forall A \leq A_{\min}, \quad H(x, A) = H_{\max}$*. We normalize our space such that* $A_{\min} = 0$*.*

A5 *(i)* $U(\cdot, \cdot)$ *is twice differentiable in* (z, H)*; (ii)* $H(\cdot, \cdot)$ *is twice differentiable in* (x, A)*; and (iii)* $H_{xA} = H_{Ax} = 0$*.*

Therefore, our analysis can be confined to the positive orthant in (z, H)-space. Unless otherwise noted, the domain of H is restricted to $[A_{\min}, A_{\max}]$ in the analysis below. We first develop a series of preliminary results, which support the primary results to come. The consumer's utility-maximization problem is:

$$\max_{z,x} U[z, H(x, A)] \quad s.t. \quad z + p \cdot x \leq m, \tag{1}$$

[7] If the averting options x and the water quality A exhibit joint production of utility and health, then our formulation would be:

$$U = U[z, G(x, A), H(x, A)]$$

where G is a component of utility directly derived from the water consumption x of quality A.

with solutions $z^*(p, m, A)$ and $x^*(p, m, A)$. The "price" of health in the (z, H)-space equals the marginal rate of substitution, evaluated at the optimal x^*:

$$P(A) = \frac{p}{H_x(x^*(p, m, A), A)}. \tag{2}$$

Furthermore, we introduce an *information parameter* to the model. Suppose that we can represent a consumer's "awareness" regarding the quality of drinking water by a parameter $\gamma \in [0, 1]$. It is understood that $\gamma = 1$ means that the consumer is fully informed of the contamination level and $\gamma = 0$ means that she is completely ignorant of the contamination problem. We simply multiply A by this parameter γ.[8] This representation allows us to interpret the parameter γ in a stylized manner. That is, $\gamma = 0$ means that $\gamma A = 0$, so that the consumer perceives the drinking water quality as perfectly safe, whereas $\gamma = 1$ implies $\gamma A = A$, so that she knows its true contamination level A. These polar cases may be understood in the context of a survey questionnaire, in which a researcher phrases her question as "do you know there was a recent contamination problem in your tap water?". We can also think of a case in which the consumer is aware of the contamination problem but may not understand its severity. The consumer with $\gamma \in (0, 1)$, therefore, is understood to be *partially or imperfectly* informed, as she knows that the water quality is somewhat contaminated, yet thinks that it is better than its true quality A. Of course, γ could, at least conceptually, take values greater than 1. We would interpret this to mean that the consumer thinks the water quality is worse than its true quality. Much of the analysis below applies for $\gamma > 1$ without modification. However, unless otherwise noted, we take $\gamma \in [0, 1]$.[9]

[8] We could model this as a more general parameter, in which case we have:

$$H(x, A; \gamma).$$

[9] It may be true that consumers' information structure is more appropriately represented by a *distribution* rather than a *point parameter*. For example, if a researcher phrases a survey question as "how safe do you think the quality of your tap water is?" or "(given a maximum contamination level), what do you think is the contamination level of your tap water?", then the respondent may have a distributional perception about the contamination level. As noted earlier, much of the value-of-information literature focuses on the increase in *expected utility* due to learning relevant information prior to decision-making. Thus, the prior and posterior distributions of possible outcomes is typically the focus of their analysis. Much of the analysis below could be extended to the case where consumer's information is a distribution rather than a point parameter. Given a probability measure μ on its compact support $[0, 1]$ and an underlying utility $u(z, H)$, we define an expected utility as:

$$U[z, H(x, A; \mu)] \stackrel{def}{=} \int u[z, H(x, \gamma A)] \, \mu(d\gamma).$$

Two points must be made clear. First, by "information", we refer to information concerning the water contamination level, but not information about the risk (i.e. probability distribution) of contracting a disease from drinking the contaminated water. Therefore, γ captures neither the consumer's risk beliefs nor risk attitudes. In other words, the consumer (and the regulator) knows with certainty what health effects she would have if she knows the true quality of water. As evidenced by Dickie and Gerking (1996), the consumer's perception about the chance of contracting a disease (i.e. risk belief) may be one of the key determinants of the willingness to pay to avoid the disease. Though the risk-belief aspect may be incorporated into the model, its practical use may be limited. Since the health risk information is often subject to a great deal of scientific uncertainty, it is difficult to evaluate the welfare losses due to having incorrect risk beliefs. Second, in the value-of-information literature, there is typically an explicit account of *timing*. In such a context, our formulation may correspond to the case in which the agent does not know what the *current* state is and makes her decision knowing exactly what would happen in the future if she did. However, the model is static, and we do not consider the timing issue either.

The following preliminary results will be useful in the sequel. All proofs are omitted, except those of **Propositions 1-3**, which appear in the appendix included in the paper.

Result 1 *Define a transformed utility function in (z,x)-space, $\tilde{U}(z,x;A) = U(z,H(x,A))$. Under A2, $\tilde{U}(\cdot,\cdot;A)$ is strictly quasi-concave in (z,x) for each fixed A.*

Result 2 *Let $A_0 \neq A_1$ and let $\bar{v} \in \mathbb{R}$ be fixed. Then, under A1, two indifference curves $I(\bar{v},A_0) = \{(z,x) : \tilde{U}(z,x;A_0) = \bar{v}\}$ and $I(\bar{v},A_1) = \{(z,x) : \tilde{U}(z,x;A_1) = \bar{v}\}$ cannot cross.*

Result 3 *Suppose that A1, A2, and A5 hold and that $U(\cdot,\cdot)$ is homothetic in (z,H). Then the optimal choice of alleviating options $x^*(A)$ is non-decreasing in A.*

Result 4 *Suppose that A2 and A3 hold. Then the optimal choice of alleviating options $x^*(A)$ is continuous in A.*

If the consumer has perfect information, her belief has the property $\mu(1) = 1$. Much of the analysis below could then be extended by applying μ in place of γ. This way of defining information structure makes sense in the current paper, as its objective is to evaluate the welfare losses because of the *choices* made sub-optimally due to imperfect information. In other words, our definition and analysis of quasi-compensating variation apply as long as there are errors in the consumer's choice whether her choice is based on a point parameter or a distribution.

Result 5 *Suppose that A1, A2, and A5 hold and that $U(\cdot,\cdot)$ is homothetic in (z, H). The optimal choice of alleviating options $x^*(\gamma, A)$ is continuous and non-decreasing in γ for each fixed A.*

Results 1 and **2** state that the transformed indifference curves in (z, x)-space are well-behaved. For each fixed utility level, the corresponding level curve will be shifted inwards (i.e. to the south-west in (z, x)-space), as the quality of water improves. Thus, an improvement in ambient water quality shifts the entire family of indifference curves defined in the (z, x)-space. This result is appealing intuitively. As the quality of water improves, the need for self-protection decreases, and therefore the marginal rate of substitution between goods z and x changes. As will be demonstrated with a numerical example later, **Result 3** combined with **Results 1** and **2** implies that the optimal vector (z^*, x^*) moves upwards along the budget line *in the (z, x)-space* as the ambient water quality improves (**Figure 1**). These preliminary results bear upon our main findings, as the (diagrammatic) welfare analyses in the (z, H)-space turn out to be problematic and need to be transferred to the (z, x)-space. The following proposition pins down the problem.

Proposition 1. *Suppose that A1, A2 and A5 hold. Let $x^*(A)$ be the optimal choice of x given A. Then, the "price" of one's personal environment in the (z, H)-space given by*

$$P(A) = \frac{p}{H_x\left(x^*(A)\right)},$$

does not coincide with the "budget line" in the (z, H)-space.

Although the averting expenditure is given simply by a vertical difference $m - z^*$, compensating (equivalent) variation and compensating (equivalent) surplus are not well-defined as a money metric in (z, H)-space. To see why, note that a change in A has two distinct effects. The first effect is an income effect due to increasing the supply of a valuable good H free of charge (i.e. $H(0, A_0) \rightarrow H(0, A_1)$). The second effect is on the implicit price of H. Recall from equation (**1**) that the price of health production is $p/H_x\left(x^*\right)$. Thus, the price of one's health, even when p is fixed, is indeterminate. It is defined only *endogenously* via the optimal choice of x. As we see from **Result 3**, under some regularity assumptions, a decrease in A (weakly) decreases x^*. This has the effect of decreasing the implicit price $p/H_x\left(x^*\right)$. Because the implicit price does not coincide with the budget line, and indeed, is different *before* and *after* the change in A, we have four different "price" vectors that could be used to employ conventional welfare analysis (**Figure 2**). For any of the four conventional welfare measures to work, one needs to be able to pick and *fix* an appropriate price vector *before* and *after* the environmental change. However, none of the four possible prices can be reasonably justified for both before and after the change. Fixing a price before and after the change is justified if optimal choices can arise at that price before and after the change. As long as H_x is monotonic, this cannot happen. Thus, a simple diagrammatic representation is not feasible in the (z, H)-space.[10] Moreover, it may be useful to note that our assumption A5-(iii) is used only to derive the explicit relationship between the budget line and the price (i.e. inequality (**3**) in the proof of **Proposition 1**). A less restrictive assumption, $H_{xA} \geq 0$, would not rescue this problem. The alternative assumption would result in the price of health as $p/H_x(x^*(A), A)$, which is still a function of A via the optimal choice of x. Thus, a change in A necessarily changes the price of health. To overcome this difficulty, we offer an alternative means for diagrammatic analysis of welfare measures. As we now show, transforming a well-defined preference order in the (z, H)-space to that in the (z, x)-space enables us to use CV/EV and CS/ES measures.

Results 1 and **2** above guarantee that, under some reasonable assumptions, preferences in the (z, x)-space are well-behaved. As the transformed utility function $\tilde{U}(z, x; A)$ is quasiconcave for each fixed A, the corresponding indifference curves must show reasonable convexity in the (z, x)-space. Let us consider a water quality improvement from A_0 to $A_1 < A_0$. Given the quasiconcavity of utility, interior solutions would occur at a tangency. We wish to evaluate the welfare changes from $(z^*(A_0), x^*(A_0))$ to $(z^*(A_1), x^*(A_1))$. We can employ a conventional diagrammatic analysis here. The indifference curves exhibit different shapes for A_0 and for A_1, as depicted in **Figure 3**. Let $v_0 = \tilde{U}(z^*(A_0), x^*(A_0); A_0)$ and $v_1 = \tilde{U}(z^*(A_1), x^*(A_1); A_1)$ be the optimal values of utility given A_0 and A_1, respectively. For CV, we evaluate the welfare change at A_1. Thus, we look for the tangency between a decreased budget line and the indifference curve $\tilde{U}(z, x; A_1) = v_0$. Then, CV is simply a vertical difference between the original and the decreased budget lines. Similarly, we look for a tangency point of an increased budget line and an indifference curve $\tilde{U}(z, x; A_0) = v_1$.

[10] Furthermore, as a special case of A2, we could consider the case with $H_x(\cdot) = c =$ constant. In such a case, $P(A) = p/H_x(x^*) =$ constant, so that the willingness-to-pay for a change in A as defined in Courant & Porter (1981) will be zero while CV/EV in the usual sense is nonzero.

The CV and EV are appropriate measures of welfare changes under perfect information, because a consumer can adjust her own consumption levels as environmental changes occur. We can also conduct welfare analysis for changes in the price of averting options. Suppose the price decreases from p_0 to p_1. The price change does not affect the shapes of indifference curves in the (z, x)-space. Moreover, it is clear that, unlike in the (z, H)-space, fixing a price at either p_0 or p_1 before and after the change does not present a problem, because optimal (tangency) solutions arise at each of the prices. Thus, CV/EV for changes in the price of averting options are well-defined in this space.

Introducing imperfect information, however, complicates welfare analysis and prevents us from using conventional measures. CV and EV measures are appropriate measures only when a consumer can adjust her consumption optimally. Under imperfect information, though, she is likely to be "stuck" in the consumption choice that she would not prefer if she had perfect information. Thus, in such a context, it appears more appropriate to use CS/ES measures. However, CS/ES measures will not be appropriate under imperfect information, because her consumption choices, z and x, may be different *before* and *after* the change in environmental quality A. To see this, consider three possible cases. First, the information parameter γ may be the same before and after the change. In this case, the environmental change is represented by $\gamma(A_1 - A_0)$. Note that the consumer's decision is based on a triple $(p, I, \gamma A)$. Thus, it is very likely that she changes her consumption decision as a result of the change in A. Second, the consumer may obtain more accurate information due to the policy change (i.e. $\gamma_0 < \gamma_1 \leq 1$). In such a case, the change is represented by $\gamma_1 A_1 - \gamma_0 A_0$. The last case is that environmental quality does not change, but the consumer obtains more accurate information. The change is then $(\gamma_1 - \gamma_0) A_0$. In all cases, her choices are not completely determined by the environmental change and she adjusts her choices according to the change (sub-optimally unless $\gamma_1 = 1$). Her choices are optimal given the information she has, but would not be optimal if she were perfectly informed. Therefore, CS/ES measures cannot be applied directly. We need to construct alternative welfare measures encompassing all of these cases to deal with the presence of imperfect information.

Definition (Quasi-CV): *Quasi-compensating variation (QCV) is the monetary compensation required to make a person indifferent between the choices made at the new information structure* (γ_1, A_1) *and at the old information structure* (γ_0, A_0), *evaluated by using perfect information* $\gamma = 1$

given the new structure (γ_1, A_1). *Formally, it is the monetary value* QCV *such that:*

$$U\left[z^*(m,p,\gamma_0 A_0), H(x^*(m,p,\gamma_0 A_0), A_0)\right] = U\left[z^*(m-QCV,p,\gamma_1 A_1), H(x^*(m-QCV,p,\gamma_1 A_1), A_1)\right].$$

Note that we cannot use an indirect utility function for this definition, because we are interested in comparing "true" (not "perceived") welfare levels between the choices made at (γ_0, A_0) and (γ_1, A_1). In general, $U\left[z^*(m,p,\gamma A), H(x^*(m,p,\gamma A), A)\right]$ is not equal to $v(m,p,\gamma A) = U[z^*(m,p,\gamma,A),$ $H(x^*(m,p,\gamma,A),\gamma A)]$. The former is the consumer's "realized" welfare level when she chooses a vector $(z^*(m,p,\gamma A), x^*(m,p,\gamma A))$ while the latter is the usual indirect utility function given $(m,p,\gamma A)$ and gives her "perceived" welfare level. In essence, quasi-CV is the money metric of a difference in welfare levels realized at (γ_0, A_0) and at (γ_1, A_1), evaluated at $(\gamma = 1, A_1)$. We are slightly abusing notation here by inserting QCV inside the demands (z^*, x^*) made at (γ_1, A_1). Quasi-equivalent variation can be defined analogously by evaluating the change at $(\gamma = 1, A_0)$. It is very important to recognize that, if $\gamma = 1$ before and after the environmental change, then QCV (QEV) precisely coincides with the usual CV (EV), because in such a case we have:

$$U\left[z^*(m,p,A_0), H(x^*(m,p,A_0), A_0)\right] = U\left[z^*(m-QCV,p,A_1), H(x^*(m-QCV,p,A_1), A_1)\right],$$

or,

$$v(m,p,A_0) = v(m-QCV,p,A_1).$$

In this sense, QCV/QEV are natural extensions of CV/EV for the case of imperfect information.

Analogously, we can define quasi-compensating (QCS) and quasi-equivalent surplus (QES) by fixing the choice of averting options, either before (QES) or after (QCS) the environmental change. An earlier version of the manuscript contained the definition of QCS and QES and welfare analyses using these measures. However, QCS/QES has one undesirable property: CV/EV does not coincide with QCS/QES when consumers are perfectly informed (i.e. $\gamma = 1$). For this reason, we prefer QCV and use it in the subsequent analyses.

IV. The Welfare Analysis

Regulators may be interested in the effects of three distinct policy alternatives. The first scenario is to promote educational programs and strengthen public notification programs concerning the current water quality level, but not change water quality itself (Policy I). For simplicity, we assume that this policy will help all individuals attain $\gamma = 1$. The second alternative is to improve the quality of the drinking water without educational programs (Policy II). The third alternative is to do both (Policy III). Of course, as indicated above, there are several other intermediate cases. For example, even when the regulators intend to achieve $\gamma = 1$ for all individuals, a considerable portion of the population may continue to be badly informed. Welfare-maximizing policies would take into account the idiosyncratic nature of the impacts of educational/informational policies. Though our definition of quasi-compensating variation is useful in analyzing such complications, the current paper will not deal with them explicitly.

The first scenario gives rise to an interesting measure of welfare value — what we call the *welfare measure of the value of information* — as it measures the consumer's willingness to pay to obtain accurate information. It is a measure or money metric of the value of information and differs from the concept of the value of information per se, which is commonly defined simply as the increase in (expected) utility from possessing more (accurate) information a priori. The QCV measure of this value of information can be readily depicted graphically (See **Figure 4**). To see how our diagrammatic analysis works, let us work through **Figure 4**. First, obtain $(z^*(A), x^*(A))$ by solving problem (**1**). This is the "maximal" level of utility she would have achieved if she were perfectly informed. Next, solve the ill-informed consumer's problem:

$$\max_{z,x} U[z, H(x, \gamma A)] \quad s.t. \quad z + p \cdot x \leq m$$

to obtain the "sub-optimal" demand $(z^*(\gamma A), x^*(\gamma A))$. Her "perceived" utility is given by her indirect utility function $v(m, p, \gamma A)$. Now, evaluate this sub-optimal point using her "true" utility function $v(\gamma) = U[z^*(m, p, \gamma A), H(x^*(m, p, \gamma A), A)]$. QCV is the money metric of the difference between the "true" utility and the "maximal" utility when she is allowed to make optimal choices. The following results are immediate:

Lemma 1. *Suppose that A1-A3 hold. The cost of imperfect information* $\gamma \in [0, 1)$*, measured as,*

$$U(z^*(m, p, \gamma A), H(x^*(m, p, \gamma A), A) = U(z^*(m - QCV, p, A), H(x^*(m - QCV, p, A), A)$$

is always nonnegative.

Lemma 2. *Consider a policy represented by the change* (γ, A_0) *to* (γ, A_1) *with* $\gamma \neq 1$ *and* $A_1 < A_0$*. Then, under A1-A3, the quasi-compensating variation of this policy change is given by:*

$$\begin{aligned} QCV &= [\text{cost of imperfect information } \gamma \text{ given } A_0 \text{ (evaluated at } A_1)] \\ &\qquad\qquad - [\text{cost of imperfect information } \gamma \text{ given } A_1 \text{ (evaluated at } A_1)] \\ &= [m - e(p, A_0, \hat{v}_0)] - [m - e(p, A_1, \hat{v}_1)]\,, \end{aligned}$$

where $\hat{v}_0 = \hat{U}(z^*(m, p, \gamma A_0), x^*(m, p, \gamma A_0); A_0)$ *and* $\hat{v}_1 = \hat{U}(z^*(m, p, \gamma A_1), x^*(m, p, \gamma A_1); A_1)$*.*

Lemma 3. *Consider a policy represented by the change from* (γ_0, A_0) *to* (γ_1, A_1) *where* $\gamma_0 < \gamma_1, A_1 < A_0$*, and* $\gamma_1 = 1$*. Then, under A1-A3, the quasi-compensating variation of this policy change can be decomposed into the following:*

$$\begin{aligned} QCV &= [\text{cost of imperfect information } \gamma_0 \text{ given } A_0 \text{ (evaluated at } A_0)] \\ &\qquad\qquad + [\text{welfare gain due to water quality change (given } \hat{v}_0)] \\ &= [m - e(p, A_0, \hat{v}_0)] + [e(p, A_0, \hat{v}_0) - e(p, A_1, \hat{v}_0)]\,, \end{aligned}$$

where $\hat{v}_0 = \hat{U}(z^*(m, p, \gamma_0 A_0), x^*(m, p, \gamma_0 A_0); A_0)$*.*

Lemma 1 states that the QCV value of information (or cost of imperfect information) is always nonnegative. It is, in fact, strictly positive if imperfect information leads to an inefficient level of averting behavior. **Lemma 2** concerns the second policy scenario, in which A changes but γ does not. This scenario does not correct imperfect information, so that consumers are stuck at the suboptimal decisions before and after the change. The lemma says that the welfare gain from this scenario is the *difference* between the costs of imperfect information at the new and old water quality levels (**Figure 5**). Though the difference depicted in the figure appears small, the money metric of the difference may not, as shown in **Section V**.

Lemma 3 lays out a special case of Policy III, in which we suppose that $\gamma_0 < \gamma_1$ and $\gamma_1 = 1$. Intuitively, we anticipate that there will be two sources of welfare improvements: (i) from correcting imperfect information, and (ii) due to a change in water quality. It turns out that our intuitive decomposition of welfare gains is in fact correct and that QCV can be decomposed into two parts. The first component corresponds to the welfare cost of imperfect information. Unlike **Lemma 2**, however, we need to evaluate it at A_0 for this decomposition. The second component, as it turns out, is the saving in expenditures due to the water quality improvement. The decomposition is illustrated in **Figure 6**. It is important to keep in mind that the decomposition is not unique whereas the QCV measure of welfare change due to this policy is. Thus, **Lemma 3** (and therefore **Proposition 2**) states only the existence, not uniqueness, of the decomposition. By combining the proofs of **Lemmas 2** and **3**, we can establish the following proposition for a more general case in which $\gamma_0 < \gamma_1 < 1$ and $A_1 < A_0$.

Proposition 2. *Consider a policy represented by the change from* (γ_0, A_0) *to* (γ_1, A_1) *where* $\gamma_0 < \gamma_1 < 1$ *and* $A_1 < A_0$. *Then, under A1-A3, the quasi-compensating variation of this policy change can be decomposed into the following:*

$$
\begin{aligned}
QCV \;&=\; [\text{welfare gain due to } A_1 \text{ (given } \hat{v}_0)] \\
&\qquad\qquad + [\text{cost of imperfect information } \gamma_0 \text{ given } A_0 \text{ (evaluated at } A_0)] \\
&\qquad\qquad - [\text{cost of imperfect information } \gamma_1 \text{ given } A_1 \text{ (evaluated at } A_1)] \\
&=\; [e(p, A_0, \hat{v}_0) - e(p, A_1, \hat{v}_0)] + [m - e(p, A_0, \hat{v}_0)] - [m - e(p, A_1, \hat{v}_1)]\,,
\end{aligned}
$$

where $\hat{v}_0 = \hat{U}(z^*(m, p, \gamma_0 A_0), x^*(m, p, \gamma_0 A_0); A_0)$ *and* $\hat{v}_1 = \hat{U}(z^*(m, p, \gamma_1 A_1), x^*(m, p, \gamma_1 A_1); A_1)$.

Another interesting policy question may be to ask which policy alternative is more efficient, self-protection or pollution control. By self-protection, we mean the policy of letting consumers protect themselves by simultaneously providing inexpensive filters and more accurate information. To answer such a policy question, we would need both benefit information and cost information. Moreover, the issue becomes more subtle when the government can provide public information only with some noise or error. Thus, the welfare measure of benefits per se does not answer such a question. Nonetheless, the welfare valuation of such a policy does help regulators make a more informed decision. The following decomposition result is obtained in a manner analogous to that of **Lemma 3**.

Proposition 3. *Consider a policy represented by the change from* (γ_0, p_0) *to* (γ_1, p_1) *where* $\gamma_0 < \gamma_1 = 1$ *and* $p_1 < p_0$*. Then, under A1-A3, the quasi-compensating variation of this policy change can be decomposed into the following:*

$$\begin{aligned} QCV &= [\text{cost of imperfect information } \gamma_0 \text{ given } p_1 \text{ (evaluated at } p_1)] \\ &\qquad\qquad\qquad\qquad + [\text{welfare gain due to price change (given } \gamma_0)] \\ &= [m - e(p_1, A, \hat{v}_{p_1})] + [e(p_1, A, \hat{v}_{p_1}) - e(p_1, A, \hat{v}_{p_0})]\,, \end{aligned}$$

where $\hat{v}_{p_0} = \hat{U}(z^*(m, p_0, \gamma_0 A), x^*(m, p_0, \gamma_0 A); A)$ *and* $\hat{v}_{p_1} = \hat{U}(z^*(m, p_1, \gamma_0 A), x^*(m, p_1, \gamma_0 A); A)$.

The second term $e(p_1, A, \hat{v}_{p_1}) - e(p_1, A, \hat{v}_{p_0})$ of the QCV evaluates the welfare value of obtaining the lower price p_1 given γ_0. It is straightforward to verify that this term is always nonnegative. This implies that the higher price of averting options exacerbates the welfare loss due to imperfect information. However, the efficiency of the "self-protection" policy relative to other alternative policies is, in general, indeterminate. It depends not only on the primitives of the model but also on the underlying distribution of γ. This point is addressed in the next section.

V. A Numerical Example.

To illustrate the quantitative impacts of imperfect information, we consider the following numerical example. We use a Cobb-Douglas specification of individual utilities and an additively separable health production function:

$$U(z, H(x, A)) = z^{\eta}(\log(x + 1) + \log(A_{\max} - A))^{(1/2)-\eta}$$

where $A \in [0, A_{\max}]$ and $\eta \in [0, 1/2]$. It is straightforward to verify that this utility function satisfies A1-A3 and A5. Thus, our main results of Sections III and IV can be applied without modification.

For our base case, we set the following numerical values:

$$
\begin{aligned}
p &= 2 \ \ \text{(price of averting options } x\text{)} \\
m &= 10 \ \ \text{(income)} \\
\eta &= 0.25 \ \ \text{(preference parameter)} \\
A_{\max} &= 5 \ \ \text{(worst possible water quality)} \\
A_0 &= 4 \ \ \text{(water quality \textit{before} the policy change)} \\
A_1 &= 0.9 \times 4 \ \ \text{(water quality \textit{after} the policy change)}
\end{aligned}
$$

In the base simulation, we characterize the effects of three alternative policies on individual welfare as a function of parameter γ. As before, Policy I promotes educational programs and public notification systems and is assumed to provide consumer i with parameter γ_{i0} with perfect information: $\gamma_{i1} = 1$. Policy II does not affect her information, but improves the quality of drinking water by 10% from A_0 to A_1. Policy III combines both Policy I and Policy II.

Figure 7 illustrates the relative effectiveness of the three policy alternatives. **Figure 7-(a)** uses the base preference parameter $\eta = 1/4$. The value of information (VOI) or the QCV measure of Policy I increases as γ_0 decreases (i.e. as consumers become more badly informed) for a fixed A_0. This is consistent with our intuition. The consumer's error in choosing defensive options becomes larger and larger as her perception of water quality becomes more and more distant from its true value. In the base simulation, providing more accurate information (in addition to the water quality improvement) can improve welfare gains by up to 30%. More importantly, providing perfect information alone accounts for as much as 53% of her potential welfare gain. This implies that, when regulators cannot afford to implement costly corrective programs (e.g pollution control and water treatment), they could improve the welfare of the uninformed population considerably by providing accurate information as a second-best alternative.

Interestingly, **Figure 7 (a)-(c)** show that the welfare gain from Policy II is also a decreasing function of γ_0. This implies that, even if regulators do not correct for imperfect information, an improvement in water quality can have larger impacts on those who are badly informed of toxic water contamination. The intuition behind this result is that, as water quality improves, the adverse health effect from falsely adopting defensive options is also alleviated. Since this effect is larger for ill-informed consumers, the welfare gain from Policy II is larger for them. We emphasize this result, as it confirms our original point — *the welfare effect of (and therefore the consumer's true willingness to pay for) pollution control changes as information changes and it is larger for those unaware than those fully informed of contamination.* As the empirical estimate of WTP is expected to be an increasing function of γ_0, the disparity between the theoretical measure and the empirical estimate of WTP becomes larger as the consumer becomes more and more badly informed. This disparity must be treated carefully in policy analysis, as its magnitude (in money metric) can be potentially quite large.

Figure 7 also provides another set of results. Though our numerical example above seems to work with different primitive parameters, the relative effectiveness of three policy alternatives seems to be most sensitive to the choice of preference parameter η. The η parameter essentially characterizes the relative importance to the consumer of the two goods, z and H. With the specification above, the smaller the η becomes, the greater weight she places on her health. **Figure 7-(b)** and **7-(c)** illustrate the impact of parameter η. When the consumer has more preference toward her health, the value of information becomes small and therefore Policy I (Policy II) becomes less (more) effective. In contrast, when she places more weight on the numeraire, the value of information becomes very critical, and therefore, Policy I (Policy II) becomes more (less) effective. This has intuitive appeal in that when the consumer places more weight on her health, she consumes more defensive options and protects herself sufficiently even if she believes the water quality is good. Thus, the welfare loss due to imperfect information is smaller. However, when the consumer places more weight on the numeraire, she tends to spend more on it if she believes the water quality is good. Thus, the welfare loss due to imperfect information is larger. The latter case ($\eta = 3/8$) is interesting because the QCV measure of Policy I exceeds that of Policy II for the consumer with $\gamma = 0$.[11]

Table 1 compares the changes in averting expenditures (corresponding to the change in A) with the QCV measures of the welfare changes due to the three policies above, at varying degrees

[11] Though we experimented with various combinations of primitive parameters, only a limited set of parameters produced the result that Policy I outperforms Policy II.

of imperfect information γ. This comparison is interesting because it illustrates the extent of potential bias in welfare valuation that could occur if one uses the change in averting expenditures as an approximation alone. The *full* welfare cost of not implementing the change (from A_0 to A_1) is equivalent to the welfare gain from Policy III (i.e. the direct welfare cost plus the cost that consumers incur due to ignorance). If we measure only the change in averting expenditures, we would report zero welfare cost for a consumer with $\gamma = 0$, while in fact her welfare cost is 2.88 (or 28.8% of her income). As the consumer gains more accurate information, the difference becomes smaller. As prior theory predicts, averting expenditure *does* offer a lower-bound estimate of WTP (and of full welfare costs). However, this result confirms our original point that if consumers are ill-informed, averting expenditures could be too small to be informative.

Lastly, we simulate four reasonable policy scenarios and compute *aggregate* welfare gains from each scenario for an economy with 100 consumers. The first policy scenario (Scenario A) is simply to reduce the toxic contamination level by 10%. This scenario corresponds to Policy II above and is a strategy commonly undertaken in many developed countries. The second scenario not only improves water quality by 10% but also provides accurate information to all consumers. This policy corresponds to Policy III above and is also used commonly, though the merit of such a policy is frequently overlooked in welfare theory. The third policy scenario is to lower the price of averting options by 20%. The last policy scenario is to simultaneously provide accurate information together with the low pricing of averting options. Our main interest here is to compare the aggregate welfare effect of Scenario D with that of Scenario A or B. However, our main focus is not so much on whether or not a certain policy performs better than the other, but rather on how the distribution of information parameter γ affects the relative performance of different policy alternatives. To do this, we generate a random sample of 100 individuals endowed with information parameter γ_i and income m_i. The $\gamma's$ are generated according to a beta distribution with various combinations of parameters (α, β). The beta function was chosen because it allows us to simulate flexible distributions on the unit interval $[0, 1]$. Income parameter $m's$ are generated from the lognormal density with $\mu = 15$ and $\sigma^2 = 0.5$ and assumed to be independent from γ. As we do not have a priori knowledge about the distribution of the preference parameter η, we ignore the variation in η and fix $\eta = 1/4$ for this simulation.

We employed a Monte Carlo simulation with a relatively small number of repetitions, 100, as each repetition involves 8×100 optimization runs for this economy. **Table 2** reports the means and standard deviations of the simulated aggregate welfare gains for each policy scenario with different combinations of initial policy parameters (p_0, A_0). The beta PDF function with $(\alpha, \beta) = (1, 3)$ (or $(3, 1)$) is highly skewed toward 1 (toward 0) whereas $(\alpha, \beta) = (3, 3)$ gives us a symmetric distribution with mean 0.5. The relative performance of Scenario D ("self-protection") against Scenario A ("pollution control") depends on the choice of initial policy parameters. Pollution-control policy always performs better than self-protection policy when $(p_0, A_0) = (2, 4)$ whereas the situation reverses when $(p_0, A_0) = (3, 3)$. More importantly, regardless of (p_0, A_0), the relative performance of self-protection policy appears to improve, as more and more of the population becomes badly informed about the toxic pollution. Using Welch's t-distribution approximation, we can also test the null hypothesis that the aggregate gain from self-protection policy is less than that from pollution-control policy. With $(p_0, A_0) = (3, 3)$, we can reject the null at the 0.01 significance level for every distribution of γ, but the t-statistic clearly increases as a greater share of the population becomes badly informed. For our simulation analysis to be more informative to regulators, we would still need information on the costs of implementing each policy. Nonetheless, the result seems to demonstrate that the distribution of γ is an important policy factor and needs to be empirically estimated for more efficient policy making.

VI. Conclusion

Conventional welfare measures, CV/EV and CS/ES, are not suitable for evaluating the welfare effects of changes in price and quality in the presence of imperfect information. We propose alternative welfare measures, called quasi-compensating variation (QCV) and quasi-equivalent variation (QEV). These welfare measures not only enables us to analyze the welfare effects of changes in information, but also offers a means to appropriately evaluate the welfare effects of various policies when consumers are imperfectly informed. QCV and QEV measures coincide with CV/EV when people are perfectly informed and, therefore, are natural extensions of CV/EV measures. In addition, QCV and QEV offer a money metric of the consumer's welfare loss when she possesses inaccurate perception about the quality of drinking water. It compares the *realized utility* that the consumer attains when she is badly informed with the *maximal utility* that she could have achieved if she were perfectly informed. We have shown numerically that the consumer's true willingness to pay to improve water quality increases, as she becomes more and more badly informed due primarily to the welfare loss incurred from errors in her choice of averting options. This is in a sharp contrast to prior empirical findings that suggested that the consumers' WTP values are higher for those who are aware of the contamination than those who are not. This inverse relationship between the empirical estimate of WTP and the true welfare cost occurs because of two distinct impacts of imperfect information — one from "information bias" in the survey and the other from the additional welfare loss due to errors in averting behaviors. This relationship must be treated more carefully in welfare valuation studies on toxic contamination. For those who are imperfectly informed and are thereby exposed to certain contamination, it may be more appropriate to use their WTP to avoid the adverse health effects from the exposure.

There are several important policy implications from our findings. First, people in developing countries are typically badly informed about the quality of their drinking water and have limited access to defensive options. Even worse, these countries have limited social and economic resources to employ costly policies that would improve the quality of drinking water. Establishing stringent, enforceable ambient and drinking water regulations could mean considerable social opportunity costs in these countries, at least in the short run. In such a case, the best (short-term) strategy may be for regulators to put more efforts into providing (1) consistent and reliable information and (2) inexpensive, privately installable filters for people without public water sources (Easter and Konishi, 2005). Second, even in developed countries where ambient and drinking water regulations are well established, consumers may be still unaware of some toxic chemicals in drinking water. For example, Abdalla, Roach, and Epp (1992) reported that in a survey of households in the borough of Perkasie in Pennsylvania, only 43.2% of survey respondents were aware of the trichloroethylen (TCE) contamination despite the government's mandatory notification of the contamination problem. As shown above, the cost of imperfect information for those unaware may be quite high and require much more attention from regulators. By appropriately targeting an ill-informed subpopulation, regulators may achieve highly cost-effective results. Another unwelcome implication is that our finding will add another reason why the empirical estimate of averting expenditures may not provide a good approximation to the welfare costs of not controlling toxic pollutants. The averting expenditures can be a good (lower bound) approximation for willingness to pay only if all conditions in Bartik (1981) are met *and* if consumers are fully aware of the water quality. If they are unaware, then the cost of imperfect information can be quite high. Thus, the empirical estimate of welfare costs must be adjusted for this factor. We are unaware of previous empirical studies that have dealt with this issue. Lastly, our simulation demonstrates that the distribution of the information parameter γ is an important policy factor that needs to be estimated. A series of econometric questions arise naturally. One of the most challenging questions will be, in our opinion, how to identify this parameter. Further research efforts will be needed on this front.

Appendix.

Proposition 1. *Suppose that A1, A2 and A5 hold. Let* $x^*(A)$ *be the optimal choice of* x *given* A*. Then, the "price" of one's personal environment in the* (z, H)*-space given by:*

$$P(A) = \frac{p}{H_x\left(x^*(A)\right)},$$

does not coincide with the "budget line" in the (z, H)*-space.*

Proof: By A5, the first-order conditions are sufficient for the unique maximum of **(1)**. Under A5, H_x depends only on x, so that the FONC gives:

$$\frac{U_h(z^*, H(x^*, A))}{U_z(z^*, H(x^*, A))} = \frac{p}{H_x\left(x^*\right)}.$$

By the Implicit Function Theorem, the LHS is the negative of the slope of the indifference curve at (z^*, H^*). Thus, we can interpret $p/H_x\left(x^*\right)$ to be the "price" of H. Now, the question is whether or not this coincides with a budget line in the usual manner. Because the consumer obtains the amount $H(0, A)$ free of charge, her "budget line" starts from the point $(m, H(0, A))$ in (z, H)-space. By Walras Law, $z^* = m - px^*$. Therefore, the optimal point in the (z, H)-space is $(m - px^*, H(x^*, A))$. This implies that the slope of the line connecting the $H(0, A)$ and the $H(x^*, A)$ points is:

$$\frac{-px^*}{H(x^*, A) - H(0, A)}.$$

This is the familiar "budget line". For the price and the budget line to coincide with each other, we must have:

$$\frac{x^*}{H(x^*, A) - H(0, A)} = \frac{1}{H_x\left(x^*\right)}.$$

However, this cannot be true, because H is strictly concave in x. In fact, strict concavity implies that for all $x^* > 0$:

$$H_x(x^*) < \frac{H(x^*, A) - H(0, A)}{x^*} \iff -\frac{1}{H_x(x^*)} < -\frac{x^*}{H(x^*, A) - H(0, A)}. \qquad (3)$$

This means that the budget line is not tangent to the indifference curve at the optimum (z^*, H^*). ■

Proposition 2. *Consider a policy represented by the change from (γ_0, A_0) to (γ_1, A_1) where $\gamma_0 < \gamma_1 < 1$ and $A_1 < A_0$. Then, under A1-A3, the quasi-compensating variation of this policy change can be decomposed into the following:*

$$\begin{aligned} QCV &= [\text{welfare gain due to } A_1 \text{ (given } \hat{v}_0)] \\ &\qquad + [\text{cost of imperfect information } \gamma_0 \text{ given } A_0 \text{ (evaluated at } A_0)] \\ &\qquad - [\text{cost of imperfect information } \gamma_1 \text{ given } A_1 \text{ (evaluated at } A_1)] \\ &= [e(p, A_0, \hat{v}_0) - e(p, A_1, \hat{v}_0)] + [m - e(p, A_0, \hat{v}_0)] - [m - e(p, A_1, \hat{v}_1)] \\ &= e(p, A_1, \hat{v}_1) - e(p, A_1, \hat{v}_0), \end{aligned}$$

where $\hat{v}_0 = \hat{U}(z^*(m, p, \gamma_0 A_0), x^*(m, p, \gamma_0 A_0); A_0)$ and $\hat{v}_1 = \hat{U}(z^*(m, p, \gamma_1 A_1), x^*(m, p, \gamma_1 A_1); A_1)$.

Proof: By definition, QCV is the monetary amount given implicitly by:

$$\begin{aligned} &U\left[z^*(m, p, \gamma_0 A_0), H(x^*(m, p, \gamma_0 A_0), A_0)\right] \\ &\qquad = U\left[z^*(m - QCV, p, \gamma_1 A_1), H(x^*(m - QCV, p, \gamma_1 A_1), A_1)\right]. \end{aligned}$$

The LHS is equal to $\hat{v}_0$, and the RHS without the term QCV is simply $\hat{v}_1$. Thus, QCV is a difference in expenditures, evaluated after the change (i.e. at A_1), to obtain two different utility levels, $\hat{v}_0$ and $\hat{v}_1$. Therefore, under A1-A3, we can write:

$$QCV = e(p, A_1, \hat{v}_1) - e(p, A_1, \hat{v}_0).$$

where $e(p, A, v)$ is the expenditure function defined by $e(p, A, v) = \min\{z + px | U(z, H(x, A)) \geq$

$v\}$. The welfare gain due to the change in water quality from A_0 to A_1 controlling for $v = \hat{v}_0$ is the difference in expenditures, between A_0 and A_1, to obtain the same utility level $\hat{v}_0$. Thus, it can be written as:

$$e(p, A_0, \hat{v}_0) - e(p, A_1, \hat{v}_0).$$

Moreover, by **Lemma 1**, the cost of imperfect information γ_i given A_i (evaluated at A_i) is:

$$m - e(p, A_i, \hat{v}_i) \quad \text{for } i = 0, 1.$$

Thus, combining these terms, we obtain exactly the above QCV. ■

Proposition 3. *Consider a policy represented by the change from (γ_0, p_0) to (γ_1, p_1) where $\gamma_0 < \gamma_1 = 1$ and $p_1 < p_0$. Then, under A1-A3, the quasi-compensating variation of this policy change can be decomposed into the following:*

$$\begin{aligned} QCV &= [\text{cost of imperfect information } \gamma_0 \text{ given } p_1 \text{ (evaluated at } p_1)] \\ &\qquad\qquad + [\text{welfare gain due to price change (given } \gamma_0)] \\ &= [m - e(p_1, A, \hat{v}_{p_1})] + [e(p_1, A, \hat{v}_{p_1}) - e(p_1, A, \hat{v}_{p_0})] \\ &= m - e(p_1, A, \hat{v}_{p_0}), \end{aligned}$$

where $\hat{v}_{p_0} = \hat{U}(z^*(m, p_0, \gamma_0 A), x^*(m, p_0, \gamma_0 A); A)$ *and* $\hat{v}_{p_1} = \hat{U}(z^*(m, p_1, \gamma_0 A), x^*(m, p_1, \gamma_0 A); A)$.

Proof: By definition, QCV is the monetary amount given implicitly by:

$$\begin{aligned} &U\left[z^*(m, p_0, \gamma_0 A), H(x^*(m, p_0, \gamma_0 A_0), A)\right] \\ &\qquad = U\left[z^*(m - QCV, p_1, \gamma_1 A), H(x^*(m - QCV, p_1, \gamma_1 A), A)\right]. \end{aligned}$$

where $\gamma_1 = 1$. The LHS equals $\hat{v}_{p_0}$, and the RHS without the term QCV is simply the indirect utility $v(m, p_1, A)$. Thus, the above equality can be reformulated as:

$$\hat{v}_{p_0} = v(m - QCV, p_1, A)$$

Therefore, QCV is a difference in expenditures, evaluated after the change (i.e. at p_1), to obtain two different utility levels. Under A1-A3, the solution to the equation can be obtained explicitly by:

$$\begin{aligned} QCV &= e(p_1, A, v(m, p_1, A)) - e(p_1, A, \hat{v}_{p_0}) \\ &= m - e(p_1, A, \hat{v}_{p_0}) \end{aligned}$$

By **Lemma 1**, the cost of imperfect information given p_1 is:

$$m - e(p_1, A, \hat{v}_{p_1})$$

On the other hand, by the analogue of **Lemma 2**, the welfare gain due to the price change from p_0 to p_1 given γ_0 can be obtained by:

$$e(p_1, A, \hat{v}_{p_1}) - e(p_1, A, \hat{v}_{p_0})$$

Thus, adding up these, we obtain exactly the above QCV. ■

Figure 1: Indifference Curves in the (z, x)-Space.

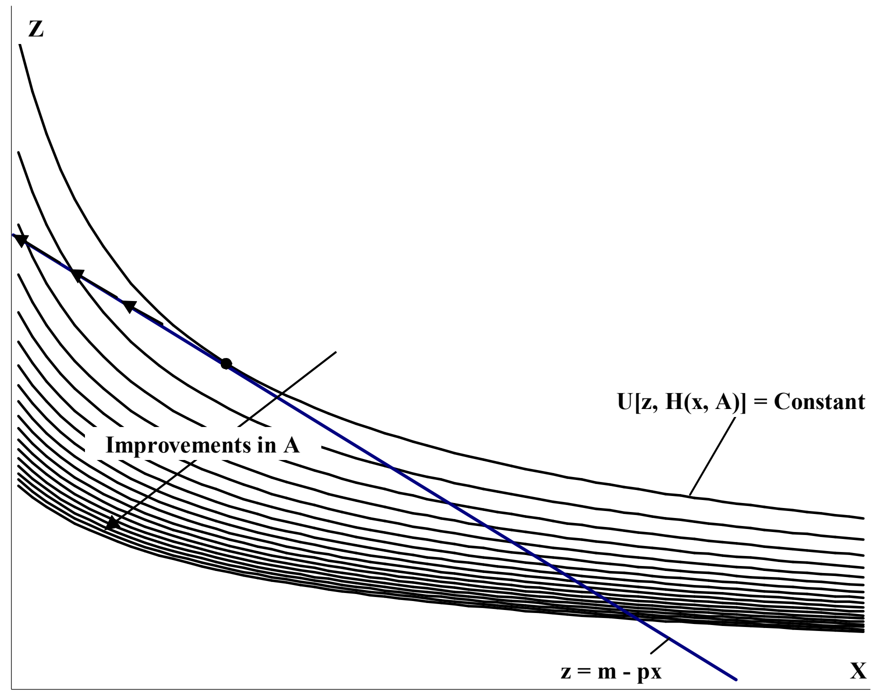


Figure 2: Four "Prices" in the (z, H)-Space.

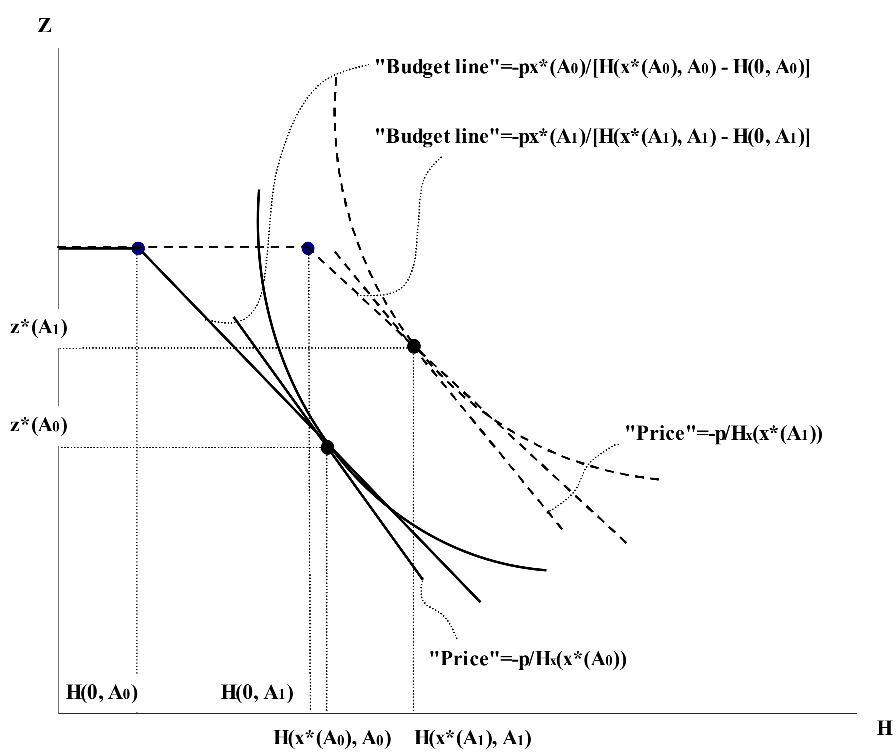


Figure 3: CV and EV Measures of Welfare Changes.

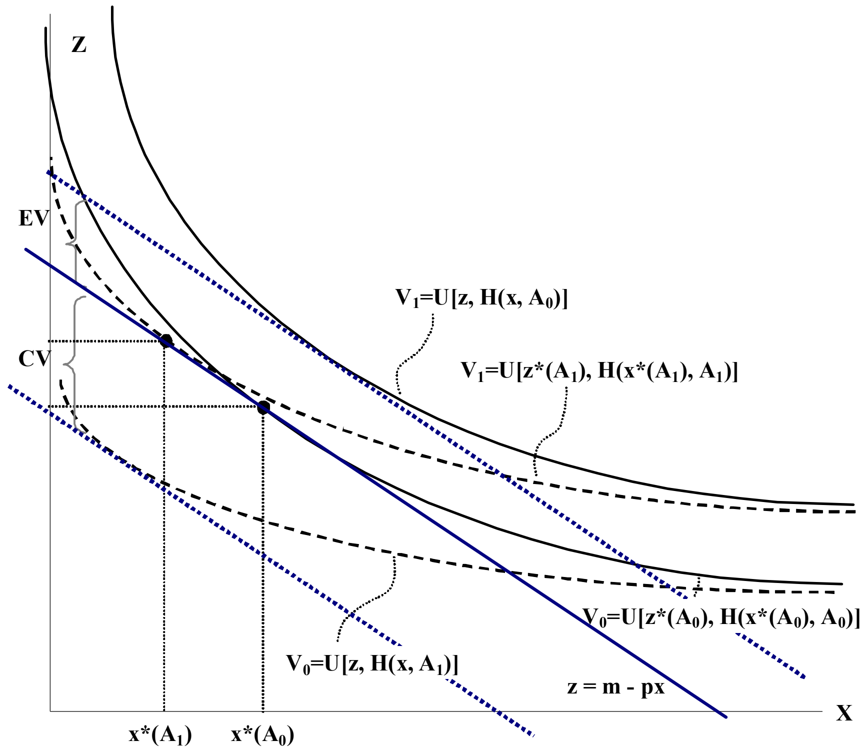


Figure 4: Value of Information, Policy I: $\gamma_0 = \gamma < 1$ and $\gamma_1 = 1$.

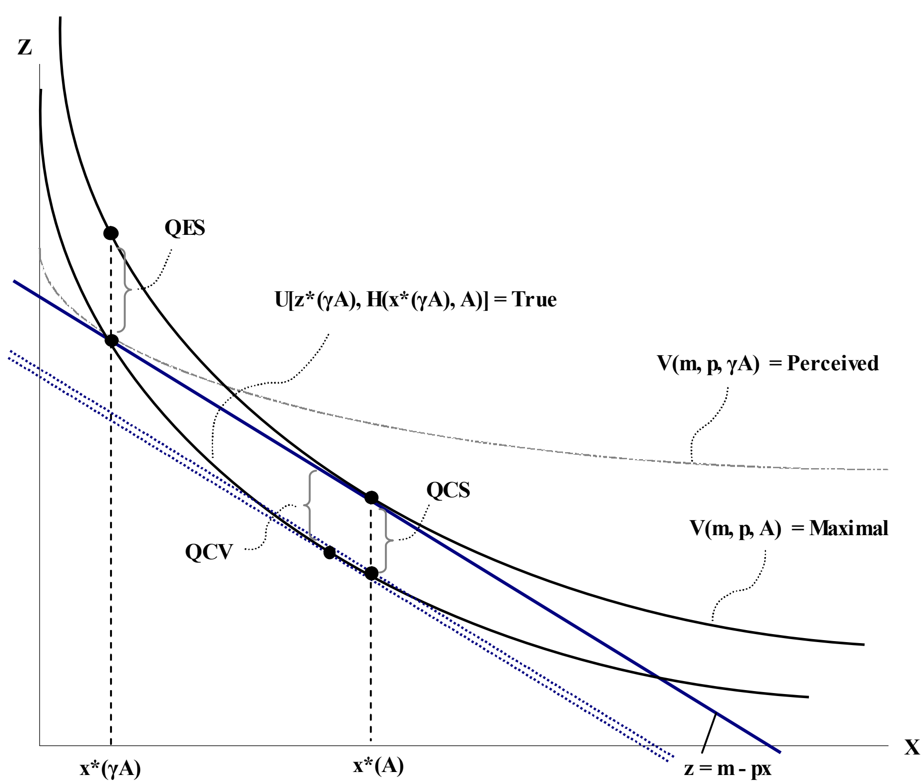


Figure 5: Welfare Gains from Policy II: $\gamma_0 = \gamma_1 = \gamma < 1$ and $A_1 < A_0$.

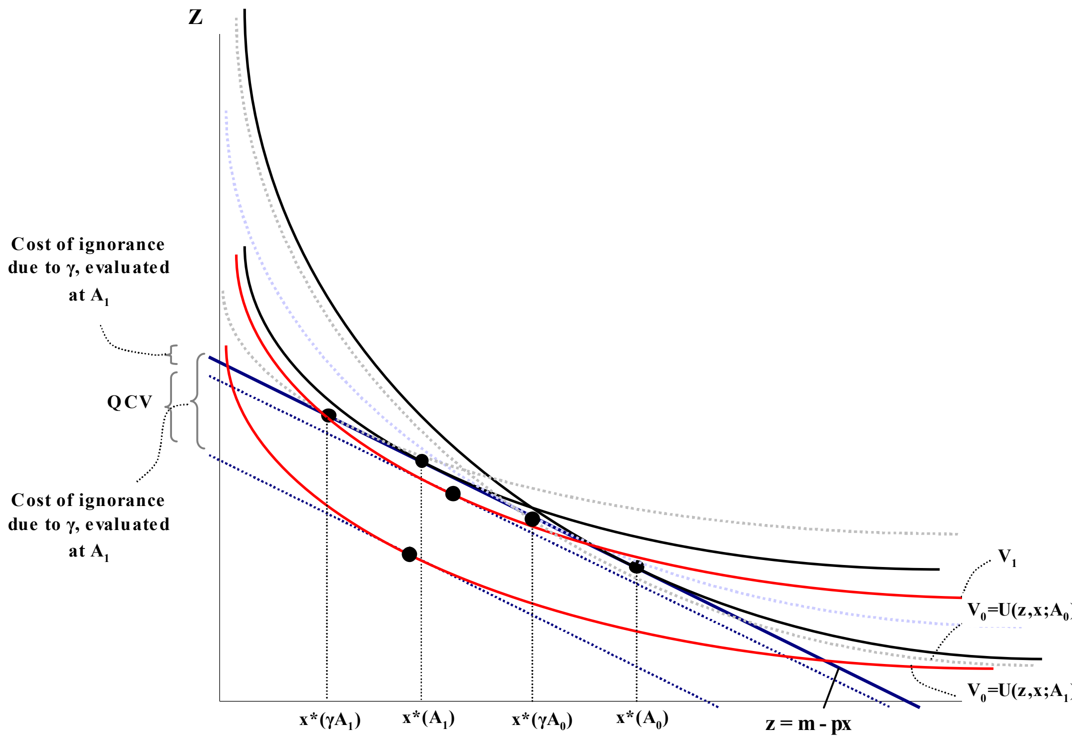


Figure 6: Welfare Gains from Policy III: $\gamma_0 < \gamma_1 = 1$ and $A_1 < A_0$.

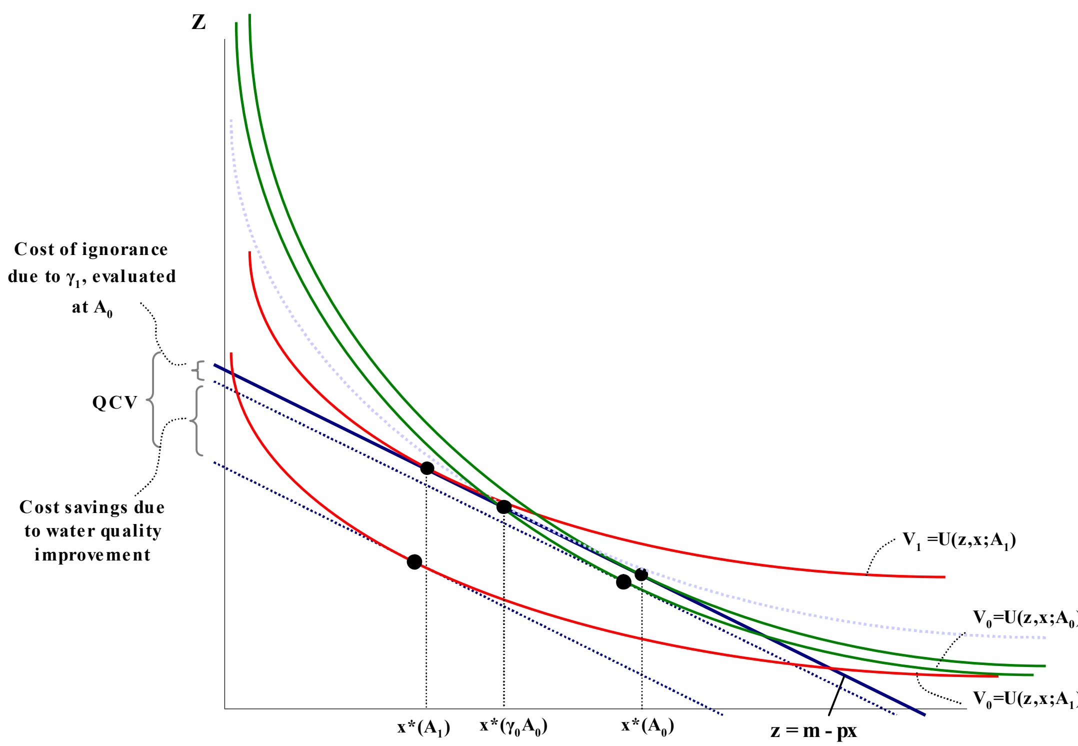
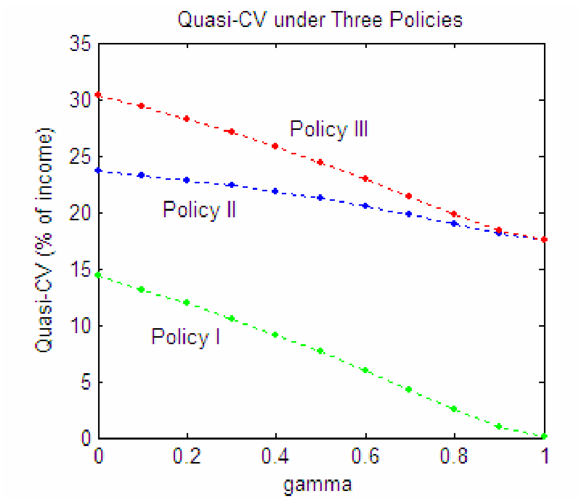
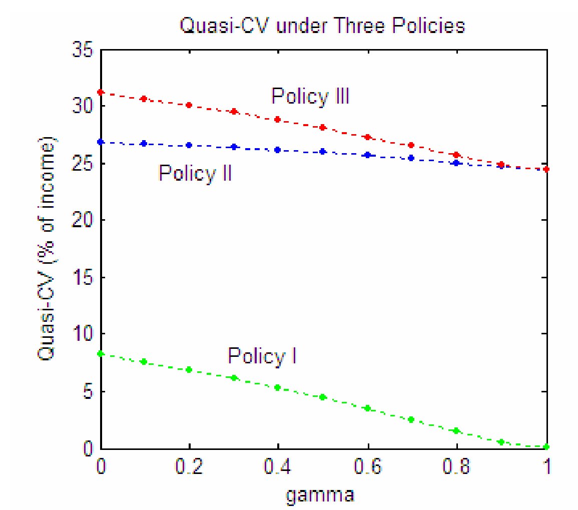


Figure 7: Welfare Gains under Three Policy Alternatives under Different Preference Parameters.

(a) η = 1/4:
Base Case



(b) η = 1/8:
More Weight on Health



(c) η = 3/8:
More Weight on Numeraire

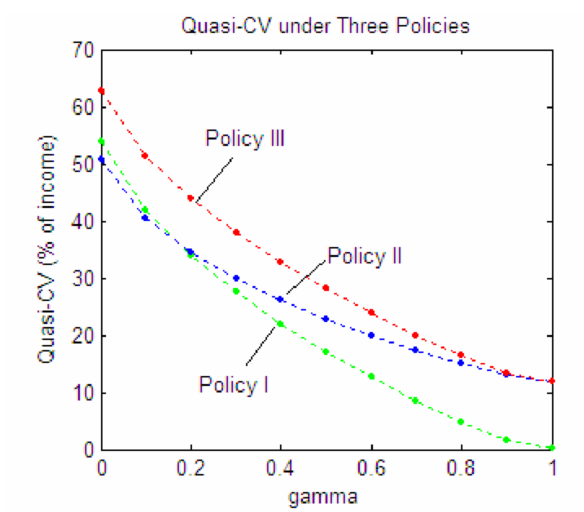


Table 1: Welfare Changes vs. Changes in Averting Expenditures.

	γ					
	0.0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
Chg. in AE	0.001	0.017	0.043	0.093	0.200	0.536
Welfare Changes						
Policy I	1.492	1.235	0.946	0.619	0.261	0.000
Policy II	2.196	2.102	1.991	1.855	1.690	1.549
Policy III	2.880	2.651	2.394	2.102	1.782	1.549
Discrepancy	2.879	2.634	2.350	2.010	1.582	1.013

Note: Changes in averting expenditures are differences in averting expenditures at A_0 and A_1 with γ fixed. Other parameter values are fixed at $p = 2$, $m = 10$, $A_0 = 4$, $A_1 = 3.6$.

Table 2: Summary of Simulated Aggregate Welfare Gains from Four Policy Alternatives.

Policy Scenarios		Distribution of γ				
	α β	**3.0** **1.0**	**3.0** **1.5**	**3.0** **3.0**	**1.5** **3.0**	**1.0** **3.0**
				$p_0 = 2$, $A_0 = 4$		
A. 10% reduction in pollution		304.11 (7.26)	308.20 (7.70)	321.08 (7.14)	329.64 (7.98)	334.40 (8.29)
B. 10% reduction in pollution + full information		318.02 (7.68)	328.90 (8.18)	355.15 (7.50)	377.05 (8.70)	388.36 (8.84)
C. 20% reduction in price		159.87 (4.82)	161.33 (5.11)	165.55 (4.77)	168.02 (5.23)	169.80 (5.47)
D. 20% reduction in price + full information		188.53 (5.62)	199.74 (5.98)	226.35 (5.48)	249.21 (6.37)	260.69 (6.38)
				$p_0 = 2.5$, $A_0 = 3.5$		
A. 10% reduction in pollution		184.60 (4.12)	188.08 (5.63)	192.32 (3.97)	197.21 (4.46)	198.88 (5.19)
B. 10% reduction in pollution + full information		192.76 (4.26)	199.75 (5.63)	211.88 (4.17)	225.39 (4.97)	231.22 (5.74)
C. 20% reduction in price		141.06 (4.37)	142.98 (5.86)	143.88 (4.18)	145.87 (4.58)	146.23 (5.33)
D. 20% reduction in price + full information		153.47 (4.50)	160.51 (5.95)	172.56 (4.40)	186.04 (5.22)	191.82 (6.03)
				$p_0 = 3$, $A_0 = 3$		
A. 10% reduction in pollution		122.91 (3.35)	124.03 (3.42)	126.26 (3.37)	128.65 (3.09)	129.81 (3.35)
B. 10% reduction in pollution + full information		127.44 (3.44)	130.66 (3.58)	137.71 (3.45)	145.52 (3.31)	149.68 (3.58)
C. 20% reduction in price		128.91 (5.15)	129.37 (5.32)	130.01 (5.29)	131.02 (4.78)	131.42 (5.16)
D. 20% reduction in price + full information		134.99 (5.20)	138.20 (5.47)	145.01 (5.34)	152.69 (5.00)	156.72 (5.42)

Note: Distribution of γ is assumed to follow a beta distribution $B(\alpha,\beta)$.

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