A Quadratic Generalization of the Almost Ideal and Translog Demand Systems: An Application to Food Demand in Urban China

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1. Introduction

Selection of the appropriate functional form to use for demand analysis is one of the most crucial issues in empirical studies. In the literature, the almost ideal (AI) demand system of Deaton and Muellbauer (1980) and the translog (TL) model of Christensen et al. (1975) are the two most commonly-used demand specifications. Several new models have been developed during the past three decades, which were modified on the basis of the AI and the TL flexible functional forms. For example, Lewbel (1989) nested the AI and the TL models, which is called either the Lewbel demand system or the AITL model, while Banks et al. (1997) introduced a quadratic version of the AI model (QAI). In addition, the 'translating' procedure, which is interpreted as an introduction of the 'committed quantities' into the original models, was used to modify the basic TL to the generalized TL (GTL) by Pollak and Wales (1980) and the original AI into the generalized AI demand system (GAI) by Bollino (1987), respectively; afterwards, Bollino and Violi (1990) provided a generalized version of the almost ideal and translog demand systems (GAITL) by incorporating the committed quantities into the AITL demand system. Recently, Moro (2003) introduced a quadratic generalization of the Lewbel demand system (QAITL), which nests the QAI and the quadratic TL (QTL) by Beach and Holt (2001) as special cases; unfortunately, the committed quantities are not considered in Moro's model, and moreover, no empirical evidence is provided to support the superiority of his newly developed model.

This paper attempts to provide a small step towards understanding the importance of the choice of functional forms in demand analysis and develops a new demand system which extends Moro's model (2003) by considering the committed quantities as suggested in the literature. This newly developed model is called the "quadratic generalized version of the almost ideal and translog demand systems" (QGAITL).[1] On the basis of Chinese urban household data of four major food items in Jiangsu from the year 2001, empirical evidence is provided and supports this newly developed QGAITL model as superior to all its nested models, including Moro's QAITL model (2003).

The remainder of this paper is planned as follows. Section 2 introduces the new QGAITL model. The data is described in Section 3 and the results are presented and analysed in Section 4. This paper ends with concluding remarks in the last section.

2. The QGAITL demand system

Let u be a given utility value and p be an n- vector of prices. The total expenditure $M = E(u, p)$ is a function of u and p of the form:

$$E(u, p) = c(p) + \exp\{[a(p) + b(p)/((\ln u)^{-1} - g(p))]/d(p)\}, \qquad (1)$$

where $a(p) = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln p_i + (1/2)\cdot \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln p_i \ln p_j$,

$b(p) = \exp(\sum_{i=1}^{n} \beta_i \ln p_i)$,

$c(p) = \sum_{i=1}^{n} p_i \varsigma_i$,

$d(p) = \sum_{i=1}^{n} \alpha_i + \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln p_i$,

$g(p) = \sum_{i=1}^{n} \delta_i \ln p_i$.

Price indices, $a(p)$, $b(p)$, $c(p)$, $d(p)$ and $g(p)$, are functions of parameters in Greek letters and prices in terms of either original or logged prices. In order to satisfy the homogeneity of the expenditure function, demand restrictions on parameters are given by:

$$\sum_{i=1}^{n} \alpha_i = 1,\ \sum_{i=1}^{n} \beta_i = 0,\ \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} = 0,\ \gamma_{ij} = \gamma_{ji}, \text{ and } \sum_{i=1}^{n} \delta_i = 0. \qquad (2)$$

From equation (1), the supernumerary expenditure (Bollino, 1987) can be expressed as

[1] Moro (2003) named his quadratic generalization of Lewbel's demand system as the Q-GAITL; however, for discrimination, his model is called the QAITL and our model, an extension of Moro's model with consideration of committed quantities, is named the QGAITL for consistent abbreviation in the literature.

$M^* = M - c(p)$. Applying Shephard's lemma, the Marshallian demands of the QGAITL model in budget shares (w_i) are given by:

$$w_i = p_i \varsigma_i / M + [M^* / M] \cdot [w_i^*(M^*, p)] \quad (3)$$

where $w_i^*(M^*, p) = \{\alpha_i + \sum_{j=1}^{n} \gamma_{ij} (\ln p_j - \ln M^*)$

$$+ \beta_i [d(p) \ln M^* - a(p)] + [\delta_i / b(p)][d(p) \ln M^* - a(p)]^2 \} / d(p), \quad (4)$$

This newly developed QGAITL nests twelve other models as its special cases, including the quadratic generalized version of the almost ideal (QGAI) and the quadratic generalized version of translog (QGTL) models both of which are also new. Figure 1 shows the relationships among these different demand systems, the testing procedures, and the various parametric restrictions.

3. The data

2001 Chinese urban household data from Jiangsu province are employed and illustrated in this study. The database was obtained from the National Bureau of Statistics (NBS) in China; detailed descriptions of the dataset can be found in Liu (2003). The raw data are composed of quantities (in Kilogram) and expenses (in Yuan) for each consumed commodity in 2001. For simplicity, four major food consumption categories, including grains, pork, fresh vegetables (FV), and fresh fruits (FF), are selected for a total of 774 observations. The price of each category for every household is calculated using unit value, which divides expenditure of the selected food item by its quantity consumed. Most of the households in Jiangsu China consumed these four major food items and hence the zero-consumption problem is not severe in this study; however, for households with no consumption of the selected food items, the unit value cannot be calculated, and therefore, the average price of each food item in every county is used as a "proxy" to each price for the unobservable households.

Meanwhile, in order to compare out-of-sample performance using cross-sectional data, the original dataset is partitioned into two sub-samples by sorting the data from lowest to highest total expenditure of these four food items, and then, every third household was eliminated starting from the household spending the lowest total expenditure of the selected food items (Cranfield et al., 2003). The eliminated households are used for out-of-sample forecasting. In total, 516 households are included in the estimation and 258 for prediction.

The descriptive statistics of the variables in the estimation process are shown in Table 1. The annual average budget share of pork was the highest, accounting for almost 30% of the expenses of the four items, whereas the budget shares of grains and fresh vegetables were weighted approximately even with each slightly over 25% and that of fresh fruits accounting for only 16.4%. The minimum budget shares for all food items were zero, indicating that some households did not spend money on these items. As mentioned earlier, this phenomenon is commonly seen using cross section data which might cause censored-type problems when estimating consumer demands. However, zero consumption for the selected commodities accounted for less than one per cent and thus ignorance of the censoring would not seriously affect econometric results. The total expenditure on these four food items ranged from 50 Yuan to 2,943 Yuan with an average of 770 Yuan per household in 2001. The wide spread in total expenditure may be a result of different consumption patterns, which requires more investigations and is beyond the scope of this study.

4. The empirical results

Selection of an appropriate model among these thirteen alternatives is based on both in-sample evaluation and out-of-sample performance. Based on the assumption of weak separability, all thirteen demand systems of the selected food items are estimated using the full information maximum likelihood (FIML) estimator by dropping the equation of fresh fruits due to singularity.

The estimated parameters of the QGAITL and its twelve nested models are presented in Table 2.[2] The parameter estimates of both the quadratic terms (δ_i) and the committed quantities (ς_i) are worth noting. Models incorporating the quadratic terms, such as the QGAITL, QGAI, QAITL and QAI, produce similar results with a mostly significant positive coefficient for grains but a significantly negative coefficient for fresh vegetables, whereas the QGTL model has reversed signs for grains and fresh vegetables. The estimate of pork from the QGAITL model is also statistically negative. This evidence from the parameter δ_i's supports Banks, Blundell and Lewbel's (1997) finding that the non-linear Engel curve can explain household consumption behavior better than its linear counterpart. As to the committed quantities, estimated parameters of the four food items from all the models, i.e., the QGAITL, QGAI, GAITL, QGTL, GAI and GTL, are inconclusive in signs and insignificantly different from zero. This finding may indicate that households in Jiangsu China may not commit to consumption of certain amounts of each food item. However, from the most restricted LES model, the estimated committed quantities are significantly positive for every food item. This contradiction needs more investigation before a conclusion is made.

Table 3 presents the log-likelihood values (LnL) and likelihood ratio test statistics (LR) for the QGAITL and its nested models. The diagonal elements are the estimated log-likelihood values from the FIML, and the italicized elements under each diagonal element indicate the number of parameters estimated in the model. For example, the QGAITL model consists of 22 parameters to be estimated and its log-likelihood value is 1605.64. The off-diagonal elements are the estimated LR test statistics, and the number of restrictions between the general model and its nested model is in parentheses. In addition, the results of estimation for the QGAITL model

[2] Due to the length limit, some parameter estimates are not shown in Table 2 but are available from the authors upon request.

are presented in Figure 1 with the number of parameters to be estimated shown in parentheses. Obviously, the LR tests are all statistically significant at 5% or better, implying that all the restricted models are rejected against its general counterpart. On the basis of likelihood ratio tests, this study shows that the QAITL model proposed by Mono (2003) is empirically rejected and that the QGAITL is superior to its nested models. Nevertheless, the LR tests confirm several intriguing issues. First, the LR test results of the QAITL against the QGAITL, the QAI against the QGAI, the AITL against the GAITL, the QTL against the QGTL, the AI against the GAI, and the TL against the GTL produce relatively high LR values, which reject the restricted models as preferred ones. This finding indicates the necessity of incorporating committed quantity terms into a demand system. Second, the LR tests show that models with quadratic terms in logged expenditure are superior to its linear models, which is consistent to our previous findings as well as Banks et al. (1997). Lastly, the LR tests strongly support the rejection of the LES models with high LR values, which were also found in Piggott (2003).

Following Cranfield et al. (2003), other criteria are utilized to compare performances among the thirteen models. Based on goodness-of-fit measures and statistical comparison of in-sample residuals, included are: the root mean squared error (RMSE), system-wide RMSE (SRMSE), information inaccuracy (IIA), multivariate Akaike's information criterion (AIC) and the multivariate Schwartz's criterion (SC). The model with the lowest value of each measurement is the preferred model and the results are presented in Table 4. The rank of the performance of the QGAITL model is also revealed in the last row of Table 4. The RMSE results support the QGAITL model as a preferred model with two items, grains and fresh vegetables, being the lowest. The LES model has the lowest RMSE in pork whereas the QGAI is lowest in fresh fruits. In addition, the QGAITL model has the lowest values in SRMSE as well as in IIA. Even though the test statistics of both AIC and SC are not the lowest for the QGAITL model, the differences are quite small between the QGAITL and the preferred models. Hence, according to most of the in-sample evaluations, the QGAITL is superior to its nested models.

Forecasting ability is also important in demand analysis. Table 5 presents the model comparisons based on out-of-sample forecasting and the rank of the QGAITL model among the thirteen alternatives. The QGAITL possesses the lowest RMSE of fresh fruits, the LES has again the lowest RMSE in pork, and the QTL is the lowest in RMSE of grains and fresh vegetables as well as SRMSE and IIA. Therefore, the QTL model proposed by Beach and Holt (2001) is the most preferred model in accordance with forecasting ability. However, the QGAITL model is also appealing based on its rank of second among several forecasting criteria.

Table 6 presents the estimated expenditure and Marshallian price elasticities at sample means of the QGAITL and its nested models. The estimated elasticities at sample means are similar among the QGAITL and its nested models, except for the most restricted LES model. Excluding the LES model, the minimum and the maximum values of each elasticity are also listed in the last two columns of Table 6. Specifically, own-price elasticities are all negative, satisfying the law of demand. The range of the own-price elasticity of pork is between –1.256 and –1.192, indicating that pork is price elastic; whereas grains, fresh vegetables and fresh fruits are all inelastic, with grains' own-price elasticity close to unity. This finding implies that households in Jiangsu China are less sensitive to price changes of both fresh vegetables and fruits. Most of the cross-price elasticities are negative, indicating that the four major food items are mostly complements. The cross-price elasticities between grains and pork are positive, showing they are substitutes; however, the cross-price elasticities of fresh fruits with respect to pork range from –0.011 to 0.015, implying an inconclusive impact of the price changes of pork on the quantities of fresh fruits demanded. Expenditure elasticities of pork and fresh fruits present a distribution from 1.147 to 1.195 for pork and from 0.603 to 0.725 for fresh fruits, respectively;

showing pork to be elastic but fresh fruits inelastic. This finding reveals a strong demand for pork but a relatively weak demand for fresh fruits along with an increase in the expenditures on these four major food items in urban Jiangsu China. However, it is model-dependent to determine whether grains and fresh vegetables are elastic or inelastic since the expenditure elasticities of both grains and fresh vegetables are around unity. Even so, both of their elasticities are higher than that of fresh fruits, which implies households in Jiangsu China would spend more on both grains and fresh vegetables than on fresh fruits. In addition, positive expenditure elasticity of grains reveals that grains are not an inferior good in urban Jiangsu, China. This fresh evidence could undermine the support for Ito, Peterson, and Grant's findings in 1989.

A possible misspecification can be caused by neglecting demographic effects, especially when cross-sectional data is employed. This paper investigates two approaches, including Bollino (1987) and Bollino and Violi (1990). Following Bollino (1987), a simple modification of the QGAITL model is specified and estimated. Assume that each price p_i depends upon a demographic modifying function which is linear in household size hs:

$$p_i^* = (1 + \varphi_i \cdot hs) \cdot p_i , \tag{5}$$

where p_i^* represents a modified price and φ_i indicates parameters in terms of the demographic variable, hs. Hence, the demographic effects can be tested again using the LR tests. The log-likelihood value of the QGAITL model with incorporation of demographic effects is 1628.05. The LR value of the QGAITL against the modified QGAITL model with the demographic effects is 44.81. This value is large enough to reject the original QGAITL model in which the demographic effects are ignored. Additionally, with consideration of household size in all thirteen models, all the LR test statistics of the restricted models against the QGAITL present a relatively large value to reject them, showing strong evidence in support of the superiority of the QGAITL demand system. On the other hand, following Bollino and Violi (1990), an alternative specification is also investigated by assuming that the committed quantity is demographically dependent and to be linear in household size hs, namely:

$$\varsigma_i^* = \varsigma_{0i} + \varsigma_{li} \cdot hs\,, \tag{6}$$

where ς_i^* represents a modified committed quantity for each food category and ς_{0i} and ς_{li} indicate, respectively, intercept and linear parameters in terms of demographic variable, hs. The testing results provide again strong evidence in support of the superiority of the QGAITL model. To sum up, combining both in-sample evaluations and out-of-sample forecasting comparisons, the empirical evidence shows that the quadratic generalized version of the almost ideal and translog demand systems proposed in this paper is superior to its nested models whether demographic effects are considered or not.

5. Concluding remarks

This paper has specified and estimated a quadratic generalized version of the almost ideal and translog demand systems. This new QGAITL model nests twelve other models, including two new ones. Employing 2001 Chinese urban household data from Jiangsu province, all thirteen models are investigated using both in-sample evaluations and out-of-sample forecasting comparisons. Empirical results show that the QGAITL model is superior to its nested models, whether or not demographic effects are incorporated. In urban Jiangsu China, all four major food items satisfy the law of demand and households are sensitive to price changes of pork but are willing to spend more on it. Nevertheless, grains are definitely not an inferior good in urban Jiangsu, China.

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Table 1. Descriptive statistics of the variables used for estimation

Var[1]	Unit	Description	Mean	Stdev[2]	Minimum	Maximum
Dependent variables						
w_1		budget share of grains	0.268	0.107	0.000	0.633
w_2		budget share of pork	0.297	0.110	0.000	0.706
w_3		budget share of fresh vegetables	0.271	0.085	0.000	0.611
w_4		budget share of fresh fruits	0.164	0.112	0.000	1.000
Explanatory variables						
p_1	Yuan/Kg	price of grains	2.464	0.640	1.333	4.944
p_2	Yuan/Kg	price of pork	11.204	1.112	7.917	14.800
p_3	Yuan/Kg	price of fresh vegetables	1.701	0.487	0.614	3.596
p_4	Yuan/Kg	price of fresh fruits	1.782	0.726	0.408	5.000
$\ln p_1$	Yuan/Kg	log price of grains	0.874	0.225	0.287	1.598
$\ln p_2$	Yuan/Kg	log price of pork	2.411	0.101	2.069	2.695
$\ln p_3$	Yuan/Kg	log price of fresh vegetables	0.488	0.302	–0.488	1.280
$\ln p_4$	Yuan/Kg	log price of fresh fruits	0.499	0.401	–0.896	1.609
M	Yuan	total expenditure	770.754	396.507	48.999	2,942.930
lnM	Yuan	log total expenditure	6.526	0.506	3.892	7.987
hs	person	household size	2.901	0.877	1.000	5.000

Note: 1. Var indicates variable.
2. Stdev means standard deviation.

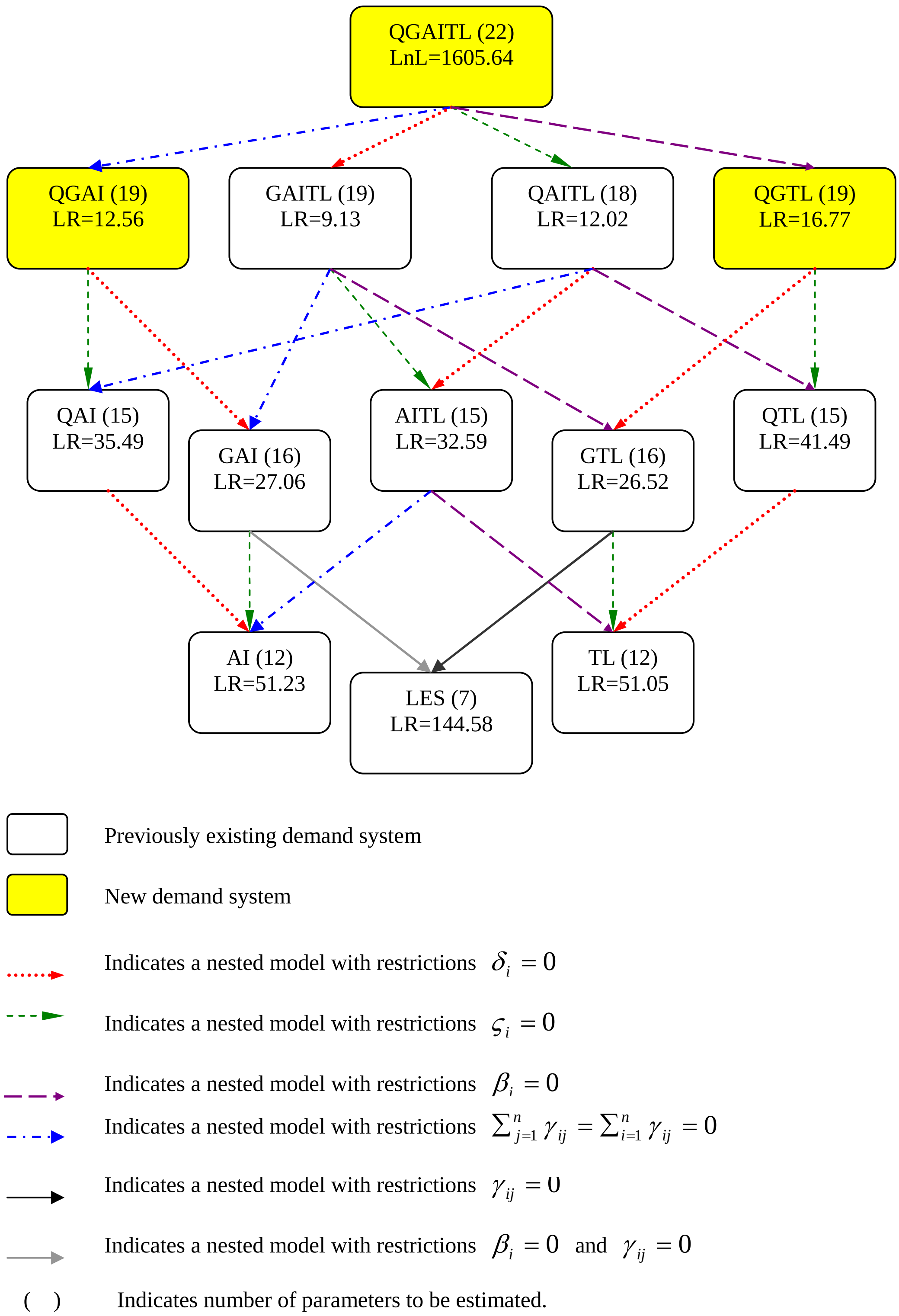

Figure 1. The QGAITL nested models

Table 2. Parameter Estimates of the QGAITL and its nested models

	QGAITL	QGAI	GAITL	QAITL	QGTL	QAI	AITL	QTL	GAI	GTL	AI	TL	LES
δ_1	0.035*	0.033*	–	0.010	–0.020*	0.035*	–	0.010	–	–	–	–	–
	(0.015)	(0.009)		(0.010)	(0.004)	(0.007)		(0.006)					
δ_2	–0.036*	–0.0004	–	–0.004	0.001	–0.010	–	0.006	–	–	–	–	–
	(0.016)	(0.008)		(0.009)	(0.006)	(0.007)		(0.005)					
δ_3	–0.016	–0.032*	–	–0.022*	0.018*	–0.034*	–	–0.018*	–	–	–	–	–
	(0.013)	(0.004)		(0.006)	(0.004)	(0.003)		(0.003)					
ς_1	0.719	–11.135	3.190	–	4.279	–	–	–	–0.509	0.229	–	–	12.250*
	(5.385)	(6.983)	(4.237)		(4.700)				(5.421)	(5.400)			(2.370)
ς_2	0.357	–3.053	–0.040	–	–1.061	–	–	–	–1.450	–1.013	–	–	3.264*
	(1.536)	(3.003)	(2.161)		(2.710)				(2.724)	(2.741)			(0.791)
ς_3	–0.396	–1.100	–1.244	–	–14.786	–	–	–	–10.370	–7.703	–	–	21.748*
	(6.955)	(11.150)	(9.097)		(12.075)				(13.833)	(13.730)			(3.949)
ς_4	7.202	0.464	8.421	–	5.087	–	–	–	4.597	5.495	–	–	16.104*
	(3.876)	(5.829)	(4.515)		(4.508)				(5.675)	(5.706)			(1.492)

Note: Numbers in parentheses are the approximate standard errors and * indicates a coefficient which is statistically significantly different from zero at the 5% level or better.

Table 3. Log-Likelihood values and likelihood ratio tests for the QGAITL and its nested models

	QGAITL	QGAI	GAITL	QAITL	QGTL	QAI	AITL	QTL	GAI	GTL	AI	TL	LES
QGAITL	1605.64 *22*												
QGAI	12.56* *(3)*	1599.36 *19*											
GAITL	9.13* *(3)*		1601.07 *19*										
QAITL	12.02* *(4)*			1599.63 *18*									
QGTL	16.77* *(3)*				1597.25 *19*								
QAI	35.49* *(7)*	22.93* *(4)*		23.48* *(3)*		1587.89 *15*							
AITL	32.59* *(7)*		23.45* *(4)*	20.57* *(3)*			1589.35 *15*						
QTL	41.49* *(7)*			29.47* *(3)*	24.72* *(4)*			1584.90 *15*					
GAI	27.06* *(6)*	14.50* *(3)*	17.93* *(3)*						1592.11 *16*				
GTL	26.52* *(6)*		17.38* *(3)*		9.74* *(3)*					1592.38 *16*			
AI	51.23* *(10)*	38.67* *(7)*	42.09* *(7)*	39.21* *(6)*		15.73* *(3)*	18.64* *(3)*		24.17* *(4)*		1580.03 *12*		
TL	51.05* *(10)*		41.92* *(7)*	39.04* *(6)*	34.28* *(7)*		18.46* *(3)*	9.56* *(3)*		24.54* *(4)*		1580.12 *12*	
LES	144.58* *(15)*	132.02* *(12)*	135.45* *(12)*		127.81* *(12)*				117.52* *(9)*	118.07* *(9)*			1533.35 *7*

Note: The diagonal elements are the estimated log-likelihood values from the FIML, and the italicized elements under each diagonal element indicate the number of parameters in the model. The off-diagonal elements are the estimated LR test statistics, and the number of restrictions between the general model and its nested model is in parentheses. * denotes a significant test statistic at the 5% level or better.

Table 4. Model comparison based on in-sample evaluation criteria[1]

Model	Number of Parameters	RMSE				SRMSE	IIA	AIC	SC
		Grains	Pork	FV	FF				
QGAITL	22	0.0959*	0.1105	0.0753*	0.10117	0.0955*	0.01965*	–14.6088	–14.428
QGAI	19	0.0972	0.1107	0.0755	0.10097*	0.0959	0.01978	–14.6020	–14.446
GAITL	19	0.0962	0.1110	0.0757	0.10118	0.0958	0.01974	–14.6086	–14.452
QAITL	18	0.0960	0.1111	0.0755	0.10194	0.0959	0.01991	–14.6089*	–14.461
QGTL	19	0.0966	0.1109	0.0762	0.10107	0.0960	0.01978	–14.5938	–14.437
QAI	15	0.0969	0.1112	0.0763	0.10272	0.0965	0.02001	–14.5809	–14.457
AITL	15	0.0963	0.1115	0.0765	0.10288	0.0965	0.02019	–14.5865	–14.463
QTL	15	0.0971	0.1119	0.0765	0.10279	0.0968	0.02017	–14.5693	–14.446
GAI	16	0.0974	0.1110	0.0766	0.10100	0.0964	0.01988	–14.5914	–14.460
GTL	16	0.0973	0.1110	0.0767	0.10098	0.0964	0.01987	–14.5925	–14.461
AI	12	0.0971	0.1116	0.0777	0.10286	0.0971	0.02028	–14.5679	–14.469
TL	12	0.0971	0.1115	0.0777	0.10288	0.0971	0.02028	–14.5682	–14.470*
LES	7	0.1065	0.1086*	0.0805	0.10307	0.0995	0.02105	–14.4161	–14.359
Rank of the QGAITL		1	2	1	5	1	1	2	12

Note: * indicates preferred model.

1. The abbreviations are defined as follows: the Root Mean Square Error (RMSE), System-wide RMSE (SRMSE), Information Inaccuracy (IIA), multivariate Akaike's Information Criterion (AIC) and the multivariate Schwartz's Criterion (SC).

Table 5. Model comparison based on out-of-sample forecasting[1]

Model	RMSE				SRMSE	IIA
	Grains	Pork	FV	FF		
QGAITL	0.08559	0.1127	0.0778	0.1008*	0.0941	0.01962
QGAI	0.08680	0.1133	0.0779	0.1009	0.0947	0.01981
GAITL	0.08480	0.1131	0.0782	0.1015	0.0943	0.01973
QAITL	0.08559	0.1128	0.0781	0.1009	0.0943	0.01981
QGTL	0.08483	0.1131	0.0782	0.1010	0.0942	0.01967
QAI	0.08583	0.1132	0.0778	0.1014	0.0944	0.01972
AITL	0.08540	0.1133	0.0794	0.1026	0.0950	0.02015
QTL	0.08460*	0.1134	0.0773*	0.1009	0.0940*	0.01956*
GAI	0.08470	0.1133	0.0780	0.1009	0.0942	0.01962
GTL	0.08461	0.1133	0.0781	0.1009	0.0942	0.01962
AI	0.08478	0.1134	0.0795	0.1023	0.0949	0.01997
TL	0.08473	0.1132	0.0795	0.1019	0.0947	0.01989
LES	0.09391	0.1120*	0.0827	0.1020	0.0976	0.02091
Rank of the QGAITL	9	2	3	1	2	2

Note: * indicates preferred model.

1. The RMSE, SRMSE, and the IIA are defined as in Table 5.

Table 6. Estimated expenditure and price elasticities at sample means

	QGAITL	QGAI	GAITL	QAITL	QGTL	QAI	AITL	QTL	GAI	GTL	AI	TL	LES	Min	Max
Price elasticities															
e_{11}	–0.850	–0.866	–0.905	–0.875	–0.943	–0.945	–0.921	–0.956	–0.989	–0.997	–0.993	–0.995	–0.894	–0.997	–0.850
e_{21}	0.257	0.254	0.302	0.280	0.304	0.296	0.309	0.305	0.303	0.310	0.308	0.308	–0.040	0.254	0.310
e_{31}	–0.315	–0.298	–0.312	–0.313	–0.275	–0.262	–0.309	–0.270	–0.232	–0.231	–0.242	–0.240	–0.039	–0.315	–0.231
e_{41}	–0.189	–0.183	–0.185	–0.192	–0.187	–0.193	–0.178	–0.179	–0.183	–0.184	–0.169	–0.169	–0.035	–0.193	–0.169
e_{12}	0.330	0.334	0.391	0.367	0.382	0.372	0.405	0.387	0.376	0.385	0.393	0.392	–0.049	0.330	0.405
e_{22}	–1.201	–1.192	–1.256	–1.242	–1.238	–1.242	–1.252	–1.243	–1.224	–1.237	–1.248	–1.251	–0.889	–1.256	–1.192
e_{32}	–0.099	–0.118	–0.115	–0.098	–0.122	–0.108	–0.132	–0.119	–0.127	–0.126	–0.120	–0.119	–0.048	–0.132	–0.098
e_{42}	–0.011	–0.003	0.015	0.001	0.009	0.010	0.013	0.007	0.002	0.009	0.006	0.009	–0.043	–0.011	0.015
e_{13}	–0.323	–0.297	–0.315	–0.318	–0.283	–0.278	–0.295	–0.283	–0.249	–0.246	–0.246	–0.244	–0.050	–0.323	–0.244
e_{23}	–0.128	–0.150	–0.143	–0.135	–0.158	–0.153	–0.160	–0.164	–0.167	–0.166	–0.157	–0.156	–0.049	–0.167	–0.128
e_{33}	–0.440	–0.444	–0.429	–0.444	–0.444	–0.472	–0.437	–0.451	–0.464	–0.470	–0.477	–0.483	–0.872	–0.483	–0.429
e_{43}	–0.167	–0.165	–0.172	–0.157	–0.173	–0.144	–0.162	–0.151	–0.180	–0.174	–0.179	–0.175	–0.043	–0.180	–0.144
e_{14}	–0.164	–0.160	–0.163	–0.177	–0.163	–0.187	–0.170	–0.177	–0.160	–0.163	–0.169	–0.170	–0.038	–0.187	–0.160
e_{24}	–0.075	–0.079	–0.066	–0.087	–0.069	–0.090	–0.090	–0.093	–0.071	–0.070	–0.093	–0.092	–0.038	–0.093	–0.066
e_{34}	–0.144	–0.151	–0.149	–0.155	–0.150	–0.150	–0.170	–0.154	–0.151	–0.150	–0.175	–0.173	–0.038	–0.175	–0.144
e_{44}	–0.358	–0.347	–0.368	–0.297	–0.362	–0.283	–0.277	–0.288	–0.360	–0.359	–0.265	–0.271	–0.806	–0.368	–0.265
Expenditure elasticities															
e_{1M}	1.007	0.988	0.991	1.003	1.007	1.038	0.980	1.029	1.021	1.021	1.016	1.016	1.031	0.980	1.038
e_{2M}	1.147	1.167	1.164	1.184	1.161	1.189	1.193	1.195	1.159	1.164	1.190	1.190	1.015	1.147	1.195
e_{3M}	0.998	1.010	1.005	1.010	0.991	0.992	1.048	0.994	0.974	0.976	1.014	1.015	0.996	0.974	1.048
e_{4M}	0.725	0.699	0.709	0.646	0.713	0.610	0.603	0.610	0.721	0.709	0.606	0.605	0.928	0.603	0.725

Note: e_{ij} are the Marshallian price elasticities of demand for the i^{th} good with respect to the j^{th} price, respectively, and e_{iM} are the expenditure elasticities for the i^{th} good where i = 1 for grains, i = 2 for pork, i = 3 for fresh vegetables and i = 4 for fresh fruits, respectively. Following Piggott (2003), the elasticities for the LES model were excluded in ranking the minimum and maximum since this model was comprehensively rejected.