THE ROLE OF YIELD-PRICE CORRELATION IN SETTING OPTIMAL N APPLICATION RATES FOR CORN PRODUCTION

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Abstract: Recent natural gas price increases have resulted in acutely higher fertilizer prices. Profit and environmental concerns have increased interest in identifying optimal nitrogen application rates. Previous estimates of the profit maximizing nitrogen rate (PMNR) have been estimated assuming that price is fixed and exogenous of yield. We construct a model in which price is correlated to yield shocks, and estimate the PMNR. We find that incorporating correlation information systematically affects the PMNR, however, these effects are small.

1   Introduction

Since the dawn of the 'green revolution' in the 1950s in the US, when mechanization and chemical applications greatly accelerated crop production efficiency, farmers have sought to optimize input application levels. For the production of corn, the primary chemical input cost is that of nitrogen fertilizer. For many years, experts recommended N application rates based on the yield potential of the soil with the intent of maximizing yield. Common recommendations took the form of 1 pound of nitrogen per bushel per acre of expected yield. While it has long been recognized that these application rates were likely higher than necessary, the relative cost of nitrogen to corn was quite low, and so the (private) economic consequences of over-application were slight.

Over the past decade, natural gas prices have steadily climbed, as the electrical industry has increasingly built natural gas-fired plants to expand generation capacity. While the average price of natural gas during the early 1990s hovered around $2.00 per million British Thermal Units (mmBTU). Since 2002, prices have averaged over $4.00/mmBTU, and in the Fall of 2005, prices briefly reached $15.00/mmBTU. Because natural gas is the primary feedstock for nitrogen fertilizer, (see Abram and Forster (2005)) the sharply higher natural gas prices have pushed nitrogen fertilizer prices higher, as well.

Higher nitrogen prices have greatly increased the value of improved nitrogen application rates, and the realization that the proper objective of setting optimal rates should not be yield, but instead, profit. The typical approach used to determine the profit-maximizing nitrogen rate (PMNR)[1] is to use yield data gathered from university yield plots which have received various nitrogen application rates over multiple years. Based upon the empirical distribution of yields conditioned upon the application rate, an optimal application rate is determined for prices of nitrogen and corn, which are assumed to be fixed an exogenous.

However, because corn has only one production cycle in the United States in any given year, and the United States is a dominant corn producer, the national average price of corn in the United States is negatively correlated to the realized national yield. Because of this negative correlation, high yields are relatively less valuable than low yields compared to the zero correlation case. Further, the strength of the correlation between prices and yields varies based upon location. The average yield in Iowa is significantly more negatively correlated to national price than the average corn yield in Ohio.

This paper posits a joint distribution for price and local yield, in which the mean of the average yield distribution is conditioned upon the nitrogen application rate, using models previously published in the crop sciences literature. Using this model, PMNRs are determined for Darke County, Ohio and Kossuth County, Iowa. In the case of Kossuth County, Iowa, for which the correlation between yield shocks and price is strongly negative, the use of correlated prices and yields results in the PMNR of five lbs per acre at a nitrogen cost of $0.60/lb. For Darke County, OH, where the correlation between yield shocks and price is actually positive, the PMNR is over seven lbs higher when correlation is considered.

The remainder of the paper is structured as follows; the next section introduces a conceptual model for the interplay of price and yield. The third section discusses the data available for research and the resulting empirical model. The fourth section presents and discusses the

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[1]In the agronomy literature, the modal nomenclature for the profit-maximizing nitrogen rate is 'EONR' or economically optimal nitrogen rate. However, these models are only privately optimal, if externalities such as N runoff exist, they are not socially optimal. Hence the use of the PMNR nomenclature here.

estimation results, and the fifth section contains concluding remarks.

2 Data

Ideally, a study such as this would be conducted with a panel of data consisting of replicated yield plots in a given county, administered different levels of N, with precisely measured yields, over 20 or 30 years. Likewise, county average cash corn and nitrogen prices would be used to compute revenue net of fertilizer, and therefore PMNR. In actuality, properly administered yield plot data is difficult to obtain for a given county over multiple decades. This study uses Ohio Yield Plot data obtained from multiple plots in Northwest Ohio from 1998 through 2005. These plots were treated with 0 to 200lbs of N per acre. This data is used to estimate the nitrogen response model in the next section. Because similar data from Iowa has so far been unavailable, this study proceeds under the assumption that the response curves and distribution of yields in Ohio and Iowa are identical. At which time Iowa data can be utilized, this study will incorporate it.

To estimate the distribution of prices and the correlation between prices and yield, USDA-NASS county yields for 1970-2005 for Darke County, Ohio and Kossuth County, Iowa are used. These counties were chosen because they have the greatest number of corn acres in their respective states. USDA-NASS state average prices are used to represent the prices received by producers.

3 Modeling Yields and Prices

For the ith farm, period t corn yield is denoted $y_{it}(\omega_{it} \mid \nu)$, where ω_{it} is a random variable that represents the weather conditions and ν is the nitrogen applied in pounds per acre. Yield is increasing and concave in both ν and ω. The weather shocks, ω_{it} are correlated, with the degree of correlation between any two locations inversely related to the distance

that separates them. At the aggregate level, national yield, $Y_t(\Omega) = \sum_i y_{it}(\omega_{it} \mid \nu)$ is a function of $\Omega = \{y_{it} \, \forall i\}$.

The correlation between prices and yields is estimated with the model

$$
\begin{align}
P_t &= \alpha_0 + e_{1t} \tag{1} \\
y_t &= \beta_0 + \beta_1 t + e_{2t} \tag{2}
\end{align}
$$

where P_t is the state average price (as reported by NASS) in year t, and y_t is the natural logarithm of the county yield in year t, and

$$
\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} \sim D(M, \Sigma) \tag{3}
$$

where D is a bivariate distribution of mean M and variance/covariance matrix Σ.

The relationship of inputs and yield is well-researched, but there is little definitive evidence to indicate one particular model is superior to all others. Nelson and Preckel (1989) suggest the use of a beta distribution with parameters conditioned on yield-influencing information, such as soil type and fertilizer application rates. This conditional beta distribution implies that there exists some maximal level of yield attainable for any given set of conditioning information. This representation is consistent with Mitscherlich and Spillman models of yield. (See Dillon and Anderson (1990)) As presented by Nelson and Preckel (1989), the probability distribution function of the condtional beta is

$$
p(y) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \times \frac{y^{\alpha-1}(\tilde{y} - y)^{(\beta-1)}}{\tilde{y}^{(\alpha+\beta-1)}} \quad 0 \le y \le \tilde{y} \tag{4}
$$

where α, β and $\tilde{y}$ are parameters and $\Gamma(x)$ is the gamma function.

In the spirit of Nelson and Preckel (1989), the parameters of the beta distribution, α and β are conditioned on the N application rate; $\alpha(N) = \gamma_0 + \gamma_1 N + \gamma_2 N^2$ and $\beta(N) = \delta_0 + \delta_1 N + \delta_2 N^2$. The maximum attainable yield, $\tilde{y}$, must be specified exogenously,
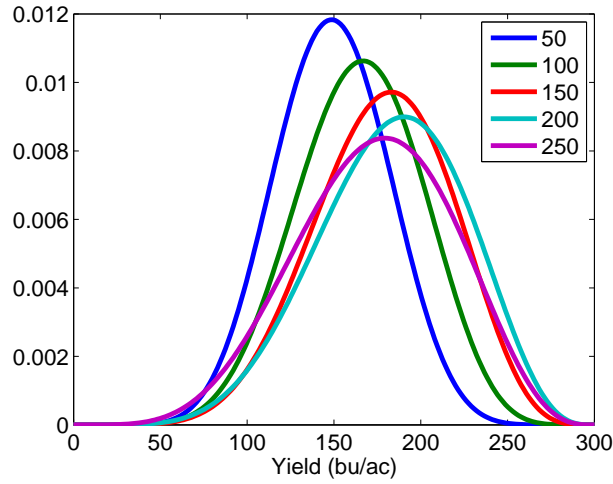
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Figure 1: Distribution of Yields Conditioned on Nitrogen Application Rate.

Table 1: Third and Fourth Moments of Price and Yield Data

	Skewness		Kurtosis	
Series	Iowa	Ohio	Iowa	Ohio
Price	-0.1283	-0.1693	3.0710	3.1341
Yield	-2.1453	-1.6161	9.3436	5.6456

and is chosen to be 300 bushels/acre for this study. Figure 3 plots the estimated distributions of yield for various levels of nitrogen.

The choice of distribution for prices is greatly simplified by the higher moments of the residual of (1) as reported in table 1. While the skewness and kurtosis of yields are both far from values associated with the normal distribution, the skewness and kurtosis of price residuals are very similar to those of the normal distribution.

Copulas are used when a multivariate distribution with arbitrary marginal distributions need to be modeled. A copula distribution is simply a function that maps the probabilities obtained from the cumulative distribution functions of the marginal distributions into a multivariate distribution with some specified dependence parameter. A number of different functions can serve as the copula function, see Cherubini et al. (2004) for a comprehensive discussion. Copula functions differ in their ability to represent dependence among series. For
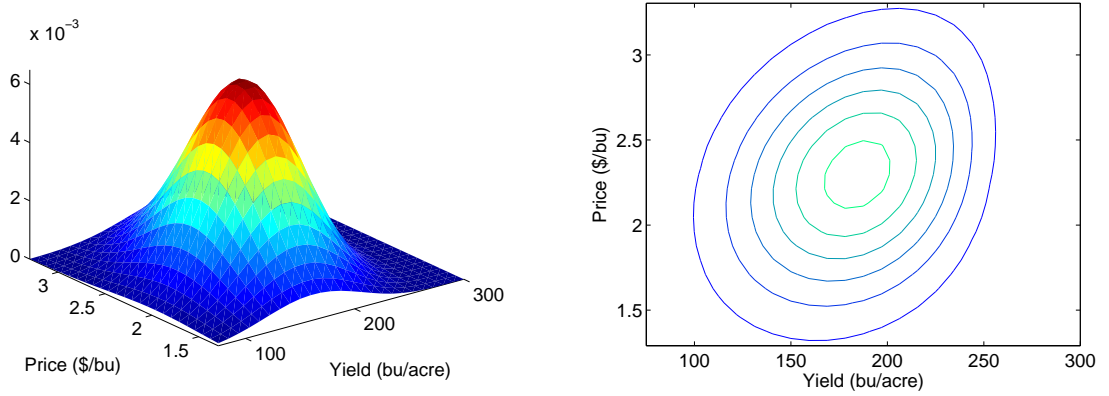
Figure 2: Surface and Topographical Plots of the Joint Distribution of Price and Yield for Darke County, Ohio.

simplicity, this paper uses the so-called Gaussian copula. In the Gaussian copula, for a given observation, the cumulative probability for each marginal is computed. These cumulative probabilities are mapped to corresponding z-scores for a standard normal distribution. These z-scores, along with the correlation parameter, are used to compute the desired statistic for the multivariate distribution using the appropriate multivariate normal function. For the price and yield series modeled here, the linear correlations are 0.2513 for Darke County, Ohio and -0.2712 for Kossuth County, Iowa.

The resulting multivariate distributions of prices and yields are depicted in figures 2 and 3. The topographical plot of figure 2 makes the positive correlation between prices and yields quite apparent. In contrast, the negative correlation in 3 is just as apparent.

In order to estimate the optimal level of nitrogen application, the expected revenue net of nitrogen cost, $\pi = p \cdot y - n * p_N$, is calculated. The product of price and revenue is computed by numerical integration of the bivariate copula distribution. In order to reduce the influence of the extreme tails of the distribution, the range of prices is limited to \$2.00[2] through \$3.50 and the range of yields is limited to 50 bushels through 300 bushels. A golden search algorithm is used to find the PMNR.

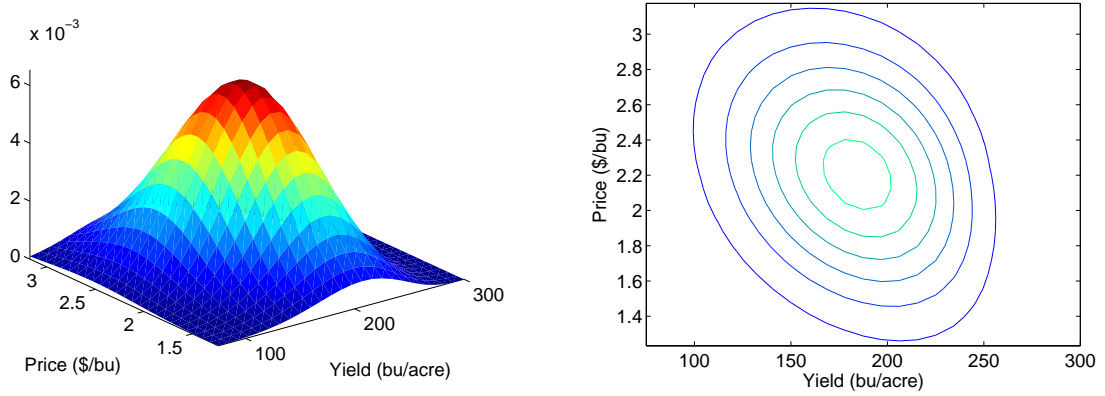[2]The lower bound of \$2.00 is chosen to reflect the presence of loan deficiency payments.

Figure 3: Surface and Topographical Plots of the Joint Distribution of Price and Yield for Kossuth County, Iowa.

4 Estimation Results

Table 4 reports the PMNR, expected yield at PMNR and the profit net of nitrogen costs at PMNR for Ohio and Iowa farms at various N prices. At $0.10/lb., there is no meaningful difference between the use of a fixed price of $2.25 and the stochastic price model for either Ohio or Iowa. At $0.20/lb., the stochastic model implies that Ohio producers should apply approximately 1 additional lb., and Iowa producers should apply approximately 0.5 additional lbs. of N compared to the static case. At $0.30/lb., which is approximately the peak fertilizer price following the Hurricane Katrina-induced run-up in natural gas prices, the difference in PMNR between the static and stochastic cases is only 1.5lbs for Ohio and 1lb for Iowa. It is not until prices rise above $0.50/lb that the differences become relatively large. At $0.60/lb, Ohio producers would reduce application by approximately 7lbs and Iowa producers would reduce production by approximately 5lbs.

5 Conclusion

The identification of the optimal rate of fertilizer application has always been important to producer profitability, but with the increasing prices observed in recent years, accurate

| | | Price=$2.25 | | | Stochastic Price | |
N Cost	PMNR	$E(\pi|\text{PMNR})$	$E(Y|\text{PMNR})$	PMNR	$E(\pi|\text{PMNR})$	$E(Y|\text{PMNR})$
Ohio						
0.10	185.20	386.15	179.85	185.74	403.89	179.87
0.20	175.16	368.12	179.18	176.14	385.78	179.26
0.30	163.89	351.15	177.92	165.46	368.69	178.12
0.40	150.69	335.41	175.86	153.11	352.75	176.28
0.50	134.15	321.13	172.53	138.00	338.16	173.37
0.60	109.90	308.82	166.56	117.24	325.33	168.48
Iowa						
0.10	185.21	386.16	179.86	185.30	397.56	179.86
0.20	175.17	368.13	179.19	175.63	379.51	179.23
0.30	163.90	351.17	177.93	164.82	362.48	178.05
0.40	150.71	335.42	175.87	152.18	346.61	176.12
0.50	134.19	321.13	172.55	136.68	332.13	173.09
0.60	109.96	308.82	166.58	114.93	319.47	167.88

estimation of the PMNR is even more important. Traditional methods of estimation do not account for the (usually negative) correlation of prices and yields. This study utilizes a copula distribution that permits arbitrary marginal distributions to be combined into a joint distribution with specified correlation.

The result is that as nitrogen prices increase, the PMNR increases as well for the Iowa and Ohio examples examined here. While this finding is not surprising for Ohio, in which yields and prices are negatively correlated, it is contrary to the intuitive conclusion for Iowa production, and deserves more scrutiny.

This study remains incomplete. Three primary short-comings must be addressed to substantially increase its applicability. First, second moment estimates of the PMNR need to be computed in order to assess the statistical significance of the findings. Second, better price and yield data need to be collected. Ideally, a long field-level yield series would be combined with posted county prices from the county in which the field is located. Finally, it would be useful to have additional data to validate the form of the nitrogen response model.
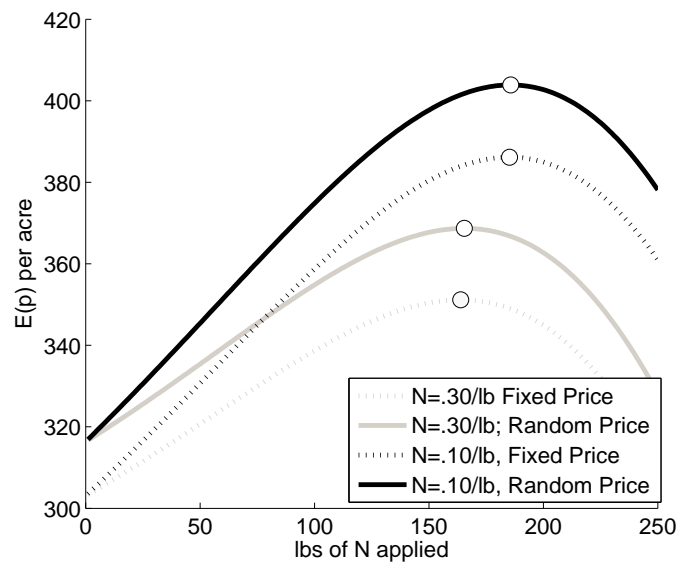
Expected revenue per acre vs nitrogen application rate, Ohio.
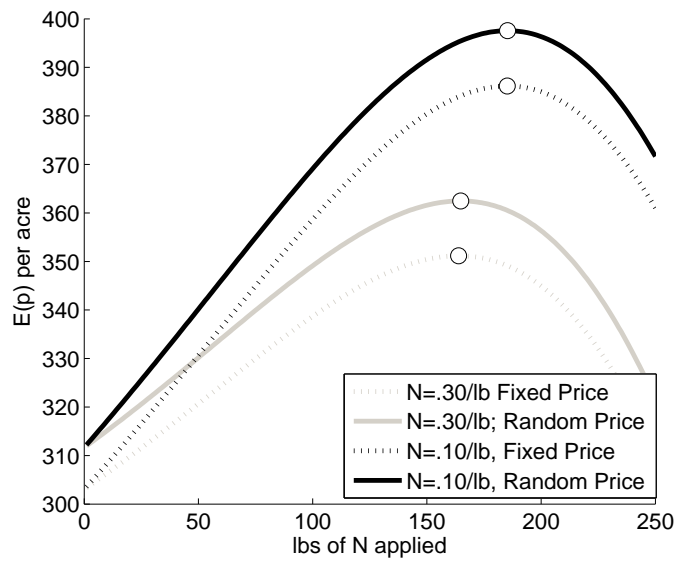
Expected revenue per acre vs nitrogen application rate, Iowa.

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