Public regulation of R&D racings

Philippe Bontems
University of Toulouse (INRA and IDEI)

James Vercammen
Food and Resource Economics, and Sauder School of Business, University of British Columbia

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Philippe Bontems[†]   James Vercammen[‡]

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Abstract

The purpose of the paper is to theoretically examine the welfare implications of public
sector involvement in agricultural biotechnology R&D. The model assumes that firms
(either a private duopoly consisting of a pair of for-profit firms or a mixed duopoly
consisting of one for-profit firm and one public firm) compete in a winner-take-all patent
race that is subject to R&D spillovers. Unlike previous research, spillovers are explicitly
incorporated into the race, and the size of the prize that accrues to the winner, as well
as the size of the ex post social surplus, is contingent on whether or not the public firm
participates in the stage two product market.

The welfare results concerning the implication of public sector involvement in the R&D
race have several unexpected properties because of the interaction of the three sources of
market failure (underinvestment due to spillovers, overinvestment due to winner-take-all
racing and monopoly pricing in the product market). The main result is that the R&D
subsidy or tax, while effective at improving the efficiency of the R&D outcome, is not
effective at correcting the monopoly pricing market failure in the product market. The
public firm, on the other hand, is able to simultaneously shift the R&D outcome toward
first-best and reduce the expected distortion in the product market. The public firm
invests particularly aggressively when R&D spillovers are high and the deadweight loss
from monopoly pricing in the product market is high.

An important problem with public firm participation in the R&D race is that cost
smoothing inefficiencies arise because the public firm will either invest at a relatively
high level to address the underinvestment externality, or invest at a relatively low level to

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[*]*Corresponding author* : Philippe Bontems, INRA, Université des Sciences Sociales de Toulouse, 21 Allée
de Brienne, 31000 Toulouse, France. Email: bontems@toulouse.inra.fr. Phone: (33) 561 128 522. Fax : (33)
561 128 520.

[†]University of Toulouse (INRA, IDEI).

[‡]Food and Resource Economics, and Sauder School of Business, University of British Columbia

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address the overinvestment externality. Cost smoothing considerations always prevents attainment of first-best when product market externalities are present.

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1   Introduction

The agricultural biotechnology industry is characterized by heavy investment in research and development (R&D) and rapidly increasing concentration in ownership of intellectual property (Pray, Oehmke and Naseem 2005). Competition to secure patents is particularly fierce in the areas of transgenic biotechnology programs to develop herbicide-tolerant and insect-resistant plants (Oehmke 2002). The public sector (i.e., universities and experiment stations) has historically played an important role in agricultural R&D, and continues to be an important contributor in the specific area of agricultural biotechnology R&D (Oehmke 2002; Heisey, King and Rubenstein 2005).

The standard argument for public firm inclusion in agricultural R&D is to correct an R&D underinvestment market failure (Ruttan, 1980; Pinstrup-Andersen 2001). Private sector underinvestment is largely due to monopoly product pricing and R&D spillovers, which arise because of incomplete property rights. Intellectual property right assignments for biotechnology have grown considerably in recent years, so much in fact that policy makers are now concerned with overinvestment and a duplication of effort caused by racing in a "winner-take-all" patenting race (Naseem and Oehmke 2004).

Various papers (e.g., Delbono and Denicolo 1993, Naseem and Oehmke 2004 and Ishibashi and Matsumura 2005) have examined the welfare benefits of replacing a for-profit firm with a public firm in a winner-take-all R&D patent race when there are no R&D spillovers and no product market externalities. As is expected, the public firm raises welfare because it slows down the race and in doing so shifts the equilibrium outcome toward first-best. Although the winner-take-all assumption is extreme and not representative of real-world R&D races (i.e., firms who compete but do not secure the patent will obtain some benefit because future R&D builds on previous R&D), this benchmark case is nevertheless popular to examine because it yields well-defined results.

The purpose of the paper proposed here is to theoretically examine the welfare implica-

tions of public sector involvement in agricultural biotechnology R&D. The model assumes that firms (either a private duopoly consisting of a pair of for-profit firms or a mixed duopoly consisting of one for-profit firm and one public firm) compete in a winner-take-all patent race that is subject to R&D spillovers. Unlike previous research, spillovers are explicitly incorporated into the race, and the size of the prize that accrues to the winner, as well as the size of the ex post social surplus, is contingent on whether or not the public firm participates in the stage two product market. In the private duopoly case, the winner of the patent is allowed to extract monopoly profits from farmers after the innovation has been commercialized. In the mixed duopoly case, monopoly profits are extracted if the winning firm is for-profit, and the technology is distributed using competitive pricing if the winning firm is public.

To establish the robustness of the theoretical results, the equilibrium is examined with and without the assumption that a public regulator uses R&D subsidies or taxes to increase social welfare. Thus, the general game has four stages. In stage one, the regulator announces the R&D subsidy/tax schedule. In stage two, the biotechnology firms involved in the R&D race choose their respective levels of R&D effort. In stage three, the R&D effort levels determine the outcome of the patent race probabilistically. In stage four, the winner distributes the technology to farmers either at a competitive or monopolistic price. The welfare implications of public sector involvement in the R&D race are established by comparing the social welfare for the private duopoly with social welfare for the mixed duopoly.

The welfare results have several unexpected properties because of the interaction of the three sources of market failure (underinvestment due to spillovers, overinvestment due to winner-take-all racing and monopoly pricing in the product market). The main result is that the R&D subsidy or tax, while effective at improving the efficiency of the R&D outcome, is not effective at correcting the monopoly pricing market failure in the product market. The public firm, on the other hand, is able to simultaneously shift the R&D outcome toward first-best and reduce the expected distortion in the product market. The public firm invests particularly aggressively when R&D spillovers are high and the deadweight loss from monopoly pricing

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in the product market is high.

An important problem with public firm participation in the R&D race is that cost smoothing inefficiencies arise because the public firm will either invest at a relatively high level to address the underinvestment externality, or invest at a relatively low level to address the overinvestment externality (R&D expenditures are subject to increasing marginal cost, so equal levels of R&D effort across the two firms maximizes cost-smoothing efficiencies). Cost smoothing considerations always prevents attainment of first-best when product market externalities are present.

The paper is organized as follows. The model is set-up in section 2. We solve the game in section 3. Section 4 is devoted to the exposition of results while we concludes in the last section.

2   The model

2.1   Staging Assumptions

The model has four stages. In stage one, a policy maker maximizes aggregate welfare across all markets by offering a R&D subsidy (e.g., a prize conditioned on successful innovation) or imposing a tax on innovation profits. In stage two, a pair of firms engage in an R&D race to produce a particular type of intellectual property, denoted I. The firms are either a private duopoly consisting of two for-profit firms, or a mixed duopoly consisting of one for-profit firm and one public firm, where the objective of the public firm is to maximize aggregate welfare across all markets. The race is "winner-take-all", so at the end of stage two, either nothing is invented and both firms earn negative profits equal to their R&D costs, or I is created with exclusive control rights belonging to the winning firm, and the losing firm earning negative profits equal to its R&D costs.

If I is created in stage two, this intellectual property (or a derivative technology or product) is marketed in stage three. Successful creation of I in stage two also implies that spinoff innovations are created in stage four. The inventor of I is assumed unable to extract

rents from these spinoffs, and thus the welfare generated by the stage four innovations is an intertemporal spillover from the perspective of decision makers in stages one and two. Unsuccessful creation of I in stage two implies zero welfare gains or losses for all agents in stages three and four.

2.2 Stages Three and Four: Conditional Welfare

Let δ denote the welfare gains arising from the spinoff innovations in stage four conditioned on successful creation of I in stage two. Let Δ^j denote the conditional welfare gain in the stage three market, excluding the profits earned by the successful inventor of I, with $j = f$ if the winning innovator is a for-profit firm and $j = g$ if the winner is a public agency. Associated with this welfare gain is a conditional dead-weight loss of size D^j, $\forall j \in \{f, g\}$, which may emerge in the stage three market due to non-competitive pricing of I. The profits earned by the successful inventor of I (excluding subsidies/taxes established by the regulator in stage one) is denoted π^j. The innovation prize or tax on innovation profits, which is established by the regulator in stage one, is denoted $(\beta - 1)\pi^f$ where $\beta \geq 1$ chosen by the regulator implies a subsidy and $\beta \leq 1$ implies a tax. The values for δ, Δ^j, D^j and π^j are exogenous to the model.

Conditioned on successful creation of I in stage two, aggregate stages two and three welfare (ignore discounting for time) is denoted S, and can be expressed as $S^j = \delta + \Delta^j + \pi^j$, $\forall j \in \{f, g\}$.[1] A key assumption for this analysis is $S^g \geq S^f$. This assumption implies that $(\Delta^g + \pi^g) - (\Delta^f + \pi^f) \geq 0$, which in turn implies that $D^f - D^g \geq C^g - C^f$, where C^j denotes the stage two cost of supplying I by its inventor if I is priced competitively.[2] The $S^g \geq S^f$ assumption implies that if the public firm happens to operate with a cost disadvantage relative to a for-profit firm, then this cost disadvantage is more than offset by

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[1] This measure of social welfare includes a zero net welfare effect when the innovation subsidy/tax, $(\beta - 1)\pi^f$, shifts between general tax revenue and the successful inventor of I.

[2] By definition, $\Delta + \pi$ is equal to the market's willingess to pay for I under competitive pricing less the inventor's cost of supplying I under competitive pricing less the deadweight loss associated with non-competitive pricing. The market's willingness to pay under competitive pricing is independent of pricing, and therefore this term vanishes in the $\Delta + \pi$ comparison.

a dead-weight loss differential for the two firms.[3]

As is shown below, the model can be fully specified in terms of π^f, S^f and S^g. Moreover, the model can be normalized by defining $\alpha^f = S^f/\pi^f$ and $\alpha^g = S^g/\pi^f$. Thus, the stages three and four welfare components of the model can be fully specified with two parameters, α^f and α^g, together with the restriction $\alpha^g \geq \alpha^f \geq 1$. There are also upper bounds on the values of α^f and α^g, but these restrictions are not specified because they do not explicitly enter the analysis.

If a for-profit firm wins the race to create I, then there are four alternative cases that will be considered:

Market Distortion	Spillover		
No	No	$\alpha^g = \alpha^f = 1$	"no externality"
No	Yes	$\alpha^g = \alpha^f > 1$	"pure spillover"
Yes	No	$\alpha^g > \alpha^f > 1$	"market distortion"
Yes	Yes	$\alpha^g > \alpha^f > 1$	"market distortion"

The "no externality" case of $\alpha^g = \alpha^f = 1$ implies a situation of zero intertemporal spillovers in stage four, identical stage three costs for the two types of firms, perfect price discrimination by the for-profit firm and no dead-weight loss associated with non-competitive pricing by the public firm. In this case, aggregate stage two and stage three welfare does not depend on whether I is invented by a for-profit firm or by a public agency. This no externality case was assumed by both Delbono and Denicolo and Ishibashi and Matsumura in their dynamic analysis of an innovation race with a mixed R&D duopoly.

In the "pure spillover" case where $\alpha^g = \alpha^f > 1$, all of the previous conditions apply except now $\delta > 0$, which implies positive stage four spillovers. The specific value of δ does not affect the level of overinvestment in R&D by the two innovating firms in stage two, who are racing in the "winner-take-all" game. However, an increasing value of δ implies that the first-best level of investment is rising. Thus, if δ is increased beyond a sufficiently large value,

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[3]The dead-weight loss for the private firm can vary from zero in the case of perfect price discrimination to a value equal to the standard monopoly dead-weight triangle for the case of uniform pricing. The dead-weight loss created by the public agency's pricing may be positive if the agency is required to use market revenue to cover its fixed costs.

the problem from the regulator's perspective will switch from one of overinvestment by the two firms due to racing to one of underinvestment due to the spillover.

There are two "market distortion" cases described in the previous table, both of which result in $\alpha^g > \alpha^f > 1$. An increase in the spillover parameter δ evenly raises α^g and α^f above 1. On the other hand, a market price distortion created by imperfect price discrimination by the for-profit inventor of I causes both α^g and α^f to rise above 1, and α^g to rise above α^f. Indeed, $\alpha^f - 1$ is a measure of the normalized surplus that could not be captured by the for-profit supplier of I due to imperfect price discrimination, and $\alpha^g - \alpha^f$ is positively related to the size of the dead-weight loss associated with this imperfect price discrimination. In the presence of a given pricing distortion, an increase in the spillover parameter, δ, will raise the values of α^g and α^f while preserving their difference.

3   Solution

3.1   Stage Two R&D Race

Let P_i, $\forall i = 1, 2$, denote the probability that firm i is the successful innovator in the stage two R&D race. This probability of success is a function of the R&D effort of both firms, so $P_i = P_i(x_1, x_2)$ where x_1 and x_2 denote respective levels of R&D effort for the two firms. The stage two cost of R&D is identical for both firms, and is given by the simple quadratic, $.5\phi x^2$.[4] The stage two expected surplus for firm i, denoted U_i, is therefore

$$U_i = \begin{cases} P_i(x_1, x_2)\beta\pi^f - .5\phi x_i^2 & \text{if firm } i \text{ is for-profit} \\ P_i(x_1, x_2)S^g + P_j(x_1, x_2)S^f - .5\phi x_i^2 - .5\phi x_j^2 & \text{if firm } i \text{ is a public agency} \end{cases} \tag{1}$$

Equation (1) shows that a for-profit firm cares only about private expected profits, whereas the public firm cares about expected aggregate welfare for all participants within stages two, three and four. Implicit in the formulation of (1) is the assumption that when neither firm succeeds that occurs with probability $(1 - x_1)(1 - x_j)$, there is no welfare change in stages three and four, and firm j earns negative $.5\phi x_j^2$ in stage two.

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[4]Fixed costs are of interest in this analysis only to the extent that they are such that it is optimal for exactly two firms to engage in the R&D race. It is assumed that the surplus measures derived in stage two include a particular level of fixed costs which achieve this outcome.

To obtained closed form solutions for the problem, assume $P_i = x_i(1 - .5x_j)$. This particular specification emerges if each firm faces probability x of inventing I and probability $1-x$ of not inventing I. In the event that both firms are simultaneously successful in inventing I, then the firm which wins the actual control rights of I is decided by a "flip-of-a-coin".[5] Now substitute $P_i = x_i(1 - .5x_j)$ into equation (1) and divide through by ϕ. Letting $\rho = \pi^f/\phi$, equation (1) can be rewritten as

$$\frac{U_i}{\phi} = \begin{cases} x_i\left(1 - .5x_j\right)\beta\rho - .5x_i^2 & \text{if firm } i \text{ is for-profit} \\ \\ x_i\left(1 - .5x_j\right)\alpha^g\rho + x_j\left(1 - .5x_i\right)\alpha^f\rho - .5x_i^2 - .5x_j^2 & \text{if firm } i \text{ is a public agency} \end{cases}$$
$$(2)$$

It is now possible to proceed with the analysis armed with the result that minimizing the difference between x_1 and x_2 to achieve a particular probability of innovation success is efficient. Returning to (2), the two firms are assumed to choose their respective levels of R&D effort simultaneously. Thus, the Nash assumption is appropriate, and reaction functions can be obtained using the standard procedures of optimization.

3.1.1   The private duopoly case

First consider the case of two for-profit firms, referred to as a private duopoly (variables are superscripted with a "pd"). In this case, the reaction function for firm i can be expressed as $x_i = (1 - .5x_j)\beta\rho$. The equilibrium level of effort, obtained by jointly solving this pair of reaction functions, can be expressed as

$$x_1^{pd} = x_2^{pd} = \frac{2\beta\rho}{2 + \beta\rho}.$$

It is necessary for $0 \leq x_i^{pd} \leq 1$, so restrictions on the various parameters of the model, as well as the stage one control variable, β, are needed. The restrictions for this case and the subsequent cases are imposed gradually as the analysis proceeds.

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[5]The probability that both firms simultaneously invent I is equal to x_1x_2. If firm i has a 50% chance of being first to the patent office in such an event, then the probability of achieving control rights over I is equal to $.5x_1x_2$. Add to this the probability that firm i is successful and firm j is unsuccessful, and the total probability that firm i achieves control rights over I is $.5x_ix_j + x_i(1 - x_j) = x_i(1 - .5x_j)$.

The x_i^{pd} expression can be substituted into the control rights probability function for firm i, $P_i^{pd} = x_i^{pd}\left(1 - .5x_j^{pd}\right)$, to obtain an expression for firm i's equilibrium probability of successfully inventing and controlling I:

$$P_i^{pd} = \frac{4\beta\rho}{(2+\beta\rho)^2}. \tag{3}$$

Let $\widehat{W}^{pd}$ denote the equilibrium level of normalized expected welfare for all market participants in stages two, three and four in the private duopoly case. In the private duopoly case, if the innovation is successful, it will be controlled by a for-profit firm. Consequently, $\widehat{W}^{pd} = \left(P_1^{pd} + P_2^{pd}\right)\beta S^f - .5\phi\left(\left(x_1^{pd}\right)^2 + \left(x_2^{pd}\right)^2\right)$. Dividing through by ϕ and substituting in the above expressions for x_i^{pd} and P_i^{pd} allows an expression for the equilibrium level of $\widehat{W}^{pd}/\phi$, denoted W^{pd}, to be written as

$$W^{pd} = 4\rho^2\left(\frac{2\beta\alpha^f - \beta^2}{(2+\beta\rho)^2}\right). \tag{4}$$

3.1.2 The mixed duopoly case

Next consider the case where the R&D race involves one for-profit firm and one public firm, referred to as a mixed duopoly (variables are superscripted with a "md"). Using (2), the reaction function for the for-profit firm is the same as the previous case: $x_f = (1 - .5x_g)\beta\rho$. The reaction function for the public firm can be expressed as $x_g = (\alpha^g - \overline{\alpha}x_f)\rho$, where $\overline{\alpha} = .5(\alpha^f + \alpha^g)$. Solving these two equations gives expressions for the equilibrium level of effort for these two firms:

$$x_f^{md} = \frac{\beta\rho\left(1 - .5\alpha^g\rho\right)}{1 - .5\beta\overline{\alpha}\rho^2}$$

$$x_g^{md} = \frac{\rho\left(\alpha^g - \overline{\alpha}\beta\rho\right)}{1 - .5\beta\overline{\alpha}\rho^2}$$

Now substitute these expressions into the individual probability functions, $P_i^{md} = x_i^{md}\left(1 - .5x_j^{md}\right)$, to obtain

$$P_f^{md} = \frac{\beta\rho\left(1 - .5\alpha^g\rho\right)^2}{\left(1 - .5\beta\overline{\alpha}\rho^2\right)^2} \tag{5}$$

$$P_g^{md} = \frac{\rho\left(\alpha^g - \overline{\alpha}\beta\rho\right)\left(1 - .5\beta\rho\left(1 + .5\rho\alpha^f\right)\right)}{\left(1 - .5\beta\overline{\alpha}\rho^2\right)^2}$$

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The aggregate welfare function for the mixed duopoly case can be expressed as

$$\widehat{W}^{md} = P_f^{md} S^f + P_g^{md} S^g - .5\phi \left(x_f^{md} \right)^2 - .5\phi \left(x_g^{md} \right)^2 .$$

Divide though by ϕ and substitute in the expressions for x_i^{md} and P_i^{md} to obtain the desired equilibrium expression

$$W^{md} = \frac{\alpha^f N_1 + \alpha^g N_2 - .5 \left(N_3 \right)^2 - .5 \left(N_4 \right)^2}{\rho^{-2} \left(1 - .5\beta \overline{\alpha} \rho^2 \right)^2} \tag{6}$$

where: $N_1 = \beta \left(1 - .5\alpha^g \rho \right)^2$, $N_2 = \left(\alpha^g - \overline{\alpha}\beta\rho \right) \left(1 - .5\beta\rho \left(1 + .5\rho\alpha^f \right) \right)$, $N_3 = \beta \left(1 - .5\alpha^g \rho \right)$ and $N_4 = \alpha^g - \overline{\alpha}\beta\rho$.

3.2 Stage One: Optimal R&D Subsidy/Tax

In stage one, the regulator chooses β to maximize equilibrium profits $\left(W^j \right)^*$ with $j = pd$ for the private duopoly case, and $j = md$ for the mixed duopoly case. Using (4), the welfare maximizing value for β in the private duopoly case is

$$\beta^{pd} = \frac{2\alpha^f}{2 + \rho\alpha^f} . \tag{7}$$

If this expression is substituted into (3), an expression for the policy-optimized probability of success in the private duopoly case can be written as

$$\left(P_i^{pd} \right)^* = \frac{\rho\alpha^f \left(1 + .5\rho\alpha^f \right)}{\left(1 + \rho\alpha^f \right)^2} . \tag{8}$$

Similarly, substitution of (7) into (3) allows an expression for policy-optimized welfare for the private duopoly system as a whole to be written as

$$\left(W^{pd} \right)^* = \frac{\left(\alpha^f \rho \right)^2}{1 + \alpha^f \rho} . \tag{9}$$

The expression for optimal β in the mixed duopoly case is obtained by maximizing W^{md} given by (6):

$$\beta^{md} = \frac{\alpha^f \left(1 - .5\alpha^g \rho \right) - .5 \left(\alpha^g \right)^2 \rho}{1 - .5 \left(1 + \overline{\alpha}\rho \right) \alpha^g \rho} . \tag{10}$$

This expression can be substituted into equations (5) and (6) to obtain expressions for the policy-optimized probability of a successful innovation and the corresponding level of system welfare for the case of mixed duopoly.

3.3 Endogenous Restrictions on ρ

At this point it is useful to digress in order to impose restrictions on ρ, which is a key parameter in the model. As indicated earlier, the welfare measures calculated above include fixed costs. Let F denote the level of fixed cost for one firm, either for-profit or public. In a first-best solution (allowing for spillovers; i.e. $\alpha^f = \alpha^g \geq 1$), the variable efficiency gains from operating two R&D facilities versus one must be greater than F. In the current analysis, the implicit assumption is that the first-best solution entails two R&D facilities incurring fixed costs $2F$ rather than one R&D facility incurring fixed cost F. Suppose that $\alpha^f = \alpha^g = \alpha$ or equivalently $S^g = S^f = S$. As is shown next, this assumption imposes a restriction on the feasible range of values for the normalized market surplus with spillover S/ϕ or equivalently $\alpha\rho$.

Total system surplus plus fixed costs for the two-firm first-best case is given by (9) with $\alpha^f = \alpha$. This is because the optimized policy variable β has eliminated all externalities in the R&D market (i.e., racing and failing to account for spillover). Thus, first-best system welfare, denoted W^{FB}, is given by

$$W^{FB} = \frac{(\alpha\rho)^2}{1 + \alpha\rho}. \tag{11}$$

Total system welfare plus fixed costs for the one-firm first-best case is equal to $.5\,(\alpha\rho)^2$, which can be obtained by optimizing $x\alpha\rho - .5x^2$ with respect to x. Therefore, it is necessary to restrict $\alpha\rho$ to be lower than 1 in order for the probability x to be lower than 1.

The net benefit from operating two R&D facilities versus one in the first best case is hence

$$\text{Net Multi-facility Benefit} = \frac{(\alpha\rho)^2}{1 + \alpha\rho} - \frac{(\alpha\rho)^2}{2} = \frac{(\alpha\rho)^2\,(1 - \alpha\rho)}{2(1 + \alpha\rho)} - F. \tag{12}$$

If F is positive but not excessively large, then it is easy to establish that there exists two critical values for $\alpha\rho$, denoted $(\alpha\rho)^{\min}$, $(\alpha\rho)^{\max} \in (0,1)$, with $(\alpha\rho)^{\min} < (\alpha\rho)^{\max}$, where $(\alpha\rho)^{\min}$ is increasing in F and $(\alpha\rho)^{\max}$ is decreasing in F. For $\alpha\rho \in ((\alpha\rho)^{\min}, (\alpha\rho)^{\max})$, net system first-best welfare is higher with two R&D facilities versus one. A lower bound on $\alpha\rho$

emerges because $\alpha\rho$ is an index of the normalized conditional profitability of the R&D, and so a sufficiently high level of profitability is needed before the fixed costs of the additional R&D operating can be covered. The upper bound on $\alpha\rho$ is more complicated to explain. Notice from (12) that with $F = 0$, net multi-facility benefit is a first increasing then decreasing function, equal to zero both with $\alpha\rho = 0$ and $\alpha\rho = 1$. The maximum value of this benefit occurs with $\alpha\rho = .5\left(\sqrt{5} - 1\right)$. To understand the properties of the net multi-facility benefit function, it is necessary to further digress and examine the issue of cost smoothing versus duplication of effort.

With zero fixed costs, why would a first-best regulator choose interior values for both x_1 and x_2 rather than choosing a corner solution with one x set to zero? To answer this question, it can be seen that a $P = P^0$ indifference curve for the total probability of successful innovation, $P = 1 - (1 - x_i)(1 - x_j) = x_i + x_j - x_i x_j$, is a downward sloping concave function when mapped in x_1 and x_2 space. The concavity of the function reveals the duplication of effort that emerges when two firms rather than one firm engage in identical R&D.[6] However, it can also be seen that indifference curves corresponding to higher value of P^0 are more concave. Thus, the efficiency loss from duplication of effort becomes larger as R&D intensity (i.e., the scale of the project) increases.

A $K = K^0$ indifference curve for the aggregate cost of R&D, $K = .5\phi\left(x_1^2 + x_2^2\right)$, is also a downward sloping concave function when mapped in x_1 and x_2 space. In this case, the concavity of the function implies that efficiency gains arise from cost smoothing (i.e., the cost of achieving a given probability of success is lower with two R&D facilities versus one).[7] In order for the optimal solution to involve positive values for both x_1 and x_2 it is therefore necessary for the $K^0 = .5\phi\left(x_1^2 + x_2^2\right)$ indifference curve to be globally more concave than the $P^0 = x_i + x_j - x_i x_j$ indifference curve. It is easy to show that this condition holds given the

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[6]Specifically, $P = x_i + x_j - x_i x_j$ implies that the total probability of success is less than the sum of the individually probabilities of success if the individual firms were alone in the market. Hence, there is a duplication of effort.

[7]The first-best solution is obtained by first minimizing the cost of achieving a specific probability of success, and then calculating the welfare maximizing probability level using the derived probability cost function.

particular functional forms chosen for this analysis.[8]

The shape of the net multi-facility benefit function given by (12) can now be explained. As $\alpha\rho$ increases, the increasing scale of the project initially increases the net benefit of operating with two R&D facilities, but as $\alpha\rho$ increases toward one, this net benefit eventually decreases and then vanishes. The reason for the decrease is that a higher value for $\alpha\rho$ implies an efficiency loss from duplication of effort that is growing relative to the efficiency gain from cost smoothing. In the extreme case when $\alpha\rho = 1$, the duplication of effort loss is exactly offset by the cost-smoothing gain and the net benefit is equal to zero.

4   Results

4.1   In the absence of externality

We first focus on the "no externality" situation where $\alpha^g = \alpha^f = 1$. Recall that this case is characterized by zero intertemporal spillovers in stage four, identical stage three costs for the two types of firms, perfect price discrimination by the for-profit firm and no dead-weight loss associated with non-competitive pricing by the public firm. We also assume for the moment that no tax/subsidy policy is available, that is $\beta = 1$.

Given the previous results, we obtain that the first best R&D effort is $\rho/(1+\rho)$ for both firms. The equilibrium individual effort for a private duopoly is given by $x_i^{pd} = 2\rho/(2+\rho)$ while for a mixed duopoly we obtain the following allocation:

$$
\begin{aligned}
x_f^{md} &= \frac{\rho(1 - .5\rho)}{1 - .5\rho^2} \\
x_g^{md} &= \frac{\rho(1 - \rho)}{1 - .5\rho^2}.
\end{aligned}
\tag{13}
$$

First, note that there is overinvestment in a private duopoly $(x_i^{pd} > \rho/(1+\rho))$. Second, it can be easily checked that, whatever the value of ρ, a for profit firm exerts more R&D effort in mixed duopoly as $x_f^{md} > x_i^{pd}$ for any $i = 1, 2$. It is also straightforward to remark that

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[8]Suppose the two indifference curves are located on the common point $x_1 = x_2 = x_0$. Hence, $P^0 = 2x_0 - x_0^2$ and $K^0 = .5\left(x_0^2 + x_0^2\right) = x_0^2$. If $x_1 = 0$, then to remain on the K^0 indifference curve it is necessary for $x_2 = 2x_0 - x_0^2$ and to remain on the P^0 indifference curve it is necessary for $x_2 = \sqrt{2}x_0^2$. The K^0 indifference curve is therefore globally more concave than the P^0 indifference curve because $\sqrt{2}x_0^2 < 2x_0 - x_0^2$.

the public firm invests less than the private firm as $x_g^{md} < x_f^{md}$. The positive gap between x_f^{md} and x_g^{md} is also increasing in ρ which suggests that the equilibrium allocation of R&D efforts is more and more asymmetric when the social return of innovation relative to cost is increasing. The public firm even invests less than does a for profit firm in a private duopoly $(x_g^{md} < x_i^{pd}$ for any ρ and any $i = 1, 2)$. Note that x_g^{md} is a concave function with $x_g^{md} = 0$ when $\rho = 1$ or 0.

We are now able to prove the following result regarding the interest of having a mixed duopoly in R&D activities.

Proposition 1 *In the absence of a tax/subsidy policy, the presence of a public firm in a mixed duopoly is welfare improving compared to the situation of a private duopoly. This welfare gain comes at a social cost in terms of non optimal allocation of R&D effort so that a mixed duopoly only achieves a second best optimum, except in the limit case where $\rho = 1$.*

Proof: To prove the result, it suffices to compare the private duopoly welfare and the mixed duopoly welfare in the absence of a tax/subsidy policy (that is when $\beta = 1$). We obtain:

$$\text{Mixed duopoly } W_{\beta=1} - \text{Private duopoly } W_{\beta=1} = \frac{\rho^4 \left(4 + \rho^2 \left(2\rho^2 + \rho - 5\right)\right)}{8(1 + \rho)(2 + \rho)^4(1 - .5\rho^2)}$$

which is positive, increasing and convex for $\rho \in (0, 1)$.

To show that a mixed duopoly only achieves second best optimum, let us compute the difference between first-best welfare and mixed duopoly welfare:

$$\text{First best } W^{FB} \text{ - Mixed duopoly } W_{\beta=1} = \frac{\rho^4(1 - \rho)}{8 \left(1 - .5\rho^2\right)^2 (1 + \rho)}$$

This indicates that welfare for the mixed duopoly equals first best when $\rho = 0$ and $\rho = 1$. This difference is actually first increasing then decreasing in ρ. For all other values of ρ, mixed duopoly welfare is thus strictly less than first-best welfare. ∎

Notice that mixed duopoly achieves first best when $\rho = 1$. The reason for this is that the public firm stops producing at $\rho = 1$ in the mixed duopoly, as indicated by (13). Hence, we

are comparing a single-firm case to a multi-facility first best case. From above we know that in a first-best world, using a single firm versus two firms achieves the same level of welfare when $\rho = 1$. So the mixed duopoly achieves the first-best outcome by shutting down one firm and inducing the private firm to produce at the first-best single firm level. In others words, the efficiency loss of the public firm in terms of misallocation of efforts is increasing in the scale until the difference in efforts becomes sufficiently large. When this difference in efforts is very large, this implies that the public firm is effectively eliminated from participating in R&D and thus cost smoothing is no longer an issue.

Moreover, the inefficiency of the unregulated private duopoly is worse when ρ is larger. This can be seen by establishing that first best welfare and welfare for the private duopoly with $\beta = 1$ are both increasing convex functions, with the former strictly above the latter. If one regulates optimally the R&D activities through a tax/subsidy, the following result can be established.

Proposition 2 *In the absence of externality, regulating a private duopoly through taxation yields first-best optimum.*

Proof: This follows of direct comparison between (9) taken when $\alpha^f = 1$ and the first-best welfare indicated by (11). The optimal R&D tax level is $\beta^{pd} = 2/(2 + \rho) < 1$ for $\rho \in (0, 1)$. Hence, an appropriate positive tax on innovation return allows to eliminate the overinvestment problem in a private duopoly. A direct consequence of Proposition 2 is the following result. ∎

Corollary 3 *Having a public firm in R&D activities without regulating the private firm is not sufficient to obtain first best.*

Obviously, imposing a tax on innovation return for the private firm that competes with a public firm in a mixed duopoly would also restore first best.

4.2 In the presence of spillovers

Here we have $\alpha^g = \alpha^f = \alpha > 1$. It can be shown that most of the results contained in the last subsection are still valid in that case.

First note that in the unregulated private duopoly, R&D efforts and hence profits are independent of the level of spillovers. This is no longer the case when there are spillovers. Indeed, from (7), we deduce the optimal tax/subsidy which is given by

$$\beta^{pd} = \frac{2\alpha}{2 + \rho\alpha}$$

and which allows the regulator to implement the first best effort given by $\alpha\rho/(1 + \rho\alpha)$. More precisely, there exists a unique threshold value $\hat{\alpha}$ such that when $\alpha \leq \hat{\alpha}$ there would be overinvestment due to R&D racing in an unregulated private duopoly and consequently the optimal regulation amounts to tax private firms ($\beta^{pd} < 1$). On the contrary when $\alpha > \hat{\alpha}$, there is underinvestment due to the existence of strong spillovers and the regulator implements a subsidy ($\beta^{pd} > 1$). In the limit case where $\alpha = \hat{\alpha} \equiv 2/(2 - \rho)$ laissez-faire is optimal. Furthermore, for profit firms loose from the introduction of the regulation when spillover are low ($\alpha < \hat{\alpha}$) while they benefit from the introduction of the subsidy when spillovers are high ($\alpha > \hat{\alpha}$). Their profits increase with the level of spillovers, because the tax is less and less important before the threshold value and it becomes eventually a negative tax beyond the threshold value. Proposition 2 can then be extended to the case of pure spillover as follows.

Proposition 4 *In the case of pure spillover, regulating a private duopoly through taxation (when spillovers are low) or subsidy (when spillovers are high) yields first-best optimum.*

Now consider the situation of an unregulated mixed duopoly.

Proposition 5 *The relative value of a mixed duopoly compared to an unregulated duopoly is positive for any value of α. Moreover, it is first decreasing with α up to a threshold value $\hat{\alpha}$ and then increasing. At the threshold value, first best is obtained with a mixed duopoly or an unregulated duopoly as well.*

Actually one can show that when $\alpha < \hat{\alpha}$ (low spillover), the leader in terms of R&D effort in a mixed duopoly is the private firm: The public firm reduces the overinvestment problem by decreasing its effort. Conversely, when $\alpha > \hat{\alpha}$ (high spillover) the problem is now to increase R&D effort and the leader is the public firm.

4.3   In the presence of market distortions and spillovers

The first thing to note is that having a public firm in place is a good thing to diminish the extent of market distortions. This holds for any values of α^f and α^g as soon as $\alpha^f < \alpha^g$. Note that unlike the previous case (pure spillover), regulating the innovation activity of a private firm competing with a public firm in a mixed duopoly would not restore first best, because innovation regulation does not eliminate product market distortions.

Let us consider the situation where the regulatory problem concerns underinvestment (that is when α^f and α^g are large enough). In this case, if there were no product market distortions, the effort of the for profit firm in a mixed duopoly would be higher than the equivalent effort in an unregulated duopoly. If there is sufficiently large market distortions, we would obtain the reverse result as the public firm would induce the for profit firm to decrease its winning probability in order to save for market efficiency losses. The public firm exerts higher effort than the for profit firm and this yields inefficiency in terms of cost smoothing.

[TO BE COMPLETED]

5   Conclusion

[TO BE COMPLETED]

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