On the Productive Value of Biodiversity

by

Jean-Paul Chavas[1]

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Abstract: The paper investigates the value of biodiversity as it relates to the productive value of services provided by an ecosystem. It analyzes how the value of an ecosystem can be "greater than the sum of its parts." First, it proposes a general measure of the value of biodiversity. Second, this measure is decomposed into four components, reflecting the role of complementarity, scale, convexity, and catalytic effects. This provides new information on the sources and determinants of biodiversity value. Third, the paper examines the role of uncertainty. In this context, the role of risk and of downside-risk exposure and their effects on the value of biodiversity are explored. This provides useful insights on how management and policy decisions can affect the value of biodiversity.

Keywords: biodiversity, productive value, complementarity, scale, convexity, catalytic effect, uncertainty.

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[1] Professor, University of Wisconsin, Taylor Hall, Madison, WI 53706. Email: jchavas@wisc.edu.

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1. Introduction

Since the development of food production some 10,000 years ago, agriculture has been in the business of using the earth ecosystem to satisfy man's nutritional needs. In this context, agriculture has seen major improvements in technology (e.g., Boserup, 1975, 1981). Over the last century, growth in agricultural productivity has been large enough to allow the earth to feed its growing human population. But this has been associated with significant alterations in the earth environment, which have raised concerns about the sustainability of current agricultural practices. Since the number of cultivated plants remains relatively small (Heiser), the development of agriculture has implied a decline in biodiversity. Previous research has documented that biodiversity is an important component of ecological systems (e.g., Tilman and Downing; Tilman et al.; Heal; Wood and Lenne). There is empirical evidence that a loss of biodiversity can have adverse effects on the functioning of ecosystems (e.g., Laureau and Hector; Naeem et al.; Tilman and Downing; Tillman et al.). Yet, while the term biodiversity has acquired a positive connotation, its measurement and conceptualization remains difficult. A number of indices of biodiversity have proposed, including the Shannon index and the Simpson index (see Hill; Lande; May; Polasky and Solow; Simpson). These indices have been used extensively in the empirical analysis of biodiversity issues (e.g., Heisey et al.; Meng et al.; Priestley and Bayles; Smale et al., 1998, 2002, 2003; Smale; Wood and Lenné). However, there is a debate on which diversity index is most appropriate (see Keylock; Routledge; Tsallis). At this point, it appears that no particular index is always superior. This is made clear when the value of biodiversity is found to depend on the presence and nature of complementary among services provided by an ecological system (e.g., Faith et al.; Justus and Sarkar; Loreau and Hector). Weitzman (1992, 1998) has proposed measuring biodiversity through a diversity function based on a measure of dissimilarity. Weitzman (1992) showed that this diversity function is one half of the Shannon index. However, Brock and Xepapadeas have argued that a more diverse ecosystem can be much more valuable even when the increase in dissimilarity is almost zero. Using Bt-corn for motivation, they show that biodiversity can stimulate productivity by reducing pest populations. This reflects the complexity of ecosystems. It also suggests the need for further research on the characterization and valuation of biodiversity.

The objective of this paper is to develop a general analysis of the productive value of biodiversity. The research focuses on the productive services provided by an ecological system. For example, in the context of agriculture, this would involve environmental goods (including ecological capital and environmental services) used in the production of food. The analysis applies under general conditions, allowing for non-convexity, lack of free-disposal in environmental goods, and dynamics. First, we propose a general measure of the productive value of biodiversity. The measure is designed to answer the question: is the value of an ecosystem "greater than the sum of its parts"? This involves a thought experiment where an ecosystem is split into separate sub-systems, holding technology and resource use constant. The analysis makes use of Luenberger's shortage function (Luenberger, 1995) as a representation of the underlying technology. Our proposed measure of the value of biodiversity has a monetary interpretation that applies even when environmental services are non-market goods. And when positive, the value of biodiversity means that the ecosystem is worth more than "the sum of its parts."

Second, we show that the value of diversity can be decomposed into four additive parts: one associated with complementarity, one with scale effects, one with convexity effects, and one with catalytic effects. Complementarity means that there is positive synergy across sub-systems, where some environmental goods have positive effects on the marginal productivity of others. The role of complementarity has been identified in previous research (e.g., Faith et al.; Justus and Sarkar; Loreau and Hector). Our analysis shows that complementarity is indeed an important component of the value of biodiversity. The scale component establishes linkages between the scale of an ecosystem and its functioning. For example, under increasing returns to scale, we show how an ecosystem can be "too small" to function properly, thus contributing to a positive value of biodiversity. The convexity component reflects the role and nature of the underlying technology. It shows that diminishing marginal productivity contributes to a positive value of biodiversity. This identifies the role of resource scarcity in the valuation of biodiversity. Finally, the catalytic component measures the possible discontinuous effects of environmental goods around 0. This decomposition provides new information on the sources and determinants of biodiversity value.

Third, the analysis is extended to apply under uncertainty. This is relevant since the future productivity of dynamic ecosystems is often imperfectly known. The role of risk and of downside-risk exposure and their effects on the value of biodiversity are explored. This provides useful insights on how management and policy decisions can affect the value of biodiversity.

2. The Productive Value of Environmental Goods

Consider an ecological system as a production process involving m private goods and n environmental goods. Let $z = (z_1, \ldots, z_{m+n}) = (z_a, z_b)$, where $z_a = (z_1, \ldots, z_m) \in \Re^m$ is the quantity of the m private goods, and $z_b = (z_{m+1}, \ldots, z_{m+n}) \in \Re^n$ is the quantity of the n environmental goods. Using the netput notation, quantities are defined to be negative for inputs (i.e., $z_i \leq 0$ when the i-th netput is an input) and positive for outputs (i.e., $z_i \geq 0$ when the i-th netput is an output). The underlying production technology is denoted by the set $Z \subset \Re^{m+n}$, where $z \equiv (z_a, z_b) \in Z$ means that private goods z_a can be feasibly produced in the presence of environmental goods z_b. Throughout, we assume that the set Z is closed, and that it exhibits free disposal with respect to the private goods z_a (where free disposal in z_a means that, for any $z \equiv (z_a, z_b) \in Z$, $z_a' \leq z_a$ implies that $(z_a', z_b) \in Z$). However, we do not assume that the set Z is convex, or that it exhibits free disposal with respect to z_b. Thus, our analysis applies under non-convexity and under scenarios where <u>the environmental goods z_b do not exhibit free-disposal</u>. Finally, when the netputs z take place in different time periods, our approach allows for general dynamics. To illustrate, letting z = (z_{1t}, $z_{1,t-1}$, z_{2t}), the dynamics of the system can be represented by $Z = \{(z_{1t}, z_{1,t-1}, z_2): z_{1t} = h(z_{1,t-1}, z_{2t}), (z_{1t}, z_{2t}) \in Z'\}$, where z_{1t} is a vector of state variables at time t (e.g., physical and ecological capital), z_{2t} is a vector of control variables at time t, $z_{1t} = h(z_{1,t-1}, z_{2t})$ is the state equation describing the evolution of the ecosystem over time, and Z' is the feasible set for (z_{1t}, z_{2t}). In this context, the z's can capture stock effects (e.g., the effects of ecological capital) as well as flow effects (e.g., food production). This means that our analysis applies to a very general technology characterizing the productivity of the ecological system.

This is illustrated in Figure 1, where m = 1, n = 1, and the upper bound of the feasible set Z is given by the line ABCDEFGH. In the region EFG both z_a and z_b are considered as outputs. This would apply to a healthy ecosystem that allows for the production of valuable ecological services as well as private goods. In the region BCDE, the environmental good z_b is an input in the production of z_a. This corresponds to situations where the ecological system is used mainly to produce private goods (e.g., agriculture using the ecological system to produce food). Region GH corresponds to a case of environmental enhancement where the private good z_a is an input into the production of the environmental good z_b (e.g., protecting the ecological system that provides clean water for New York City). Finally, in the region AB, both z_a and z_b are inputs. This would correspond to unproductive ecological systems where the production of private goods becomes impossible (e.g., on Mount Everest). Note that the technology Z in Figure 1 is <u>not</u> convex (e.g., the region CDEF). And it does <u>not</u> exhibit free disposal in the environmental good z_b in the regions ABC and DEF.

We are interested in providing a general representation of the frontier technology given by the boundary of Z. Such a representation is given by the shortage function proposed by

Luenberger. Let $g \in \Re^m_+$ be a reference bundle of private goods satisfying $g \geq 0$, and $g \neq 0$. For a given g, the shortage function S(z, g) evaluated at point $z \equiv (z_a, z_b)$ is defined as

$$S(z, g) = \min_\alpha \{\alpha: (z_a - \alpha\, g, z_b) \in Z\}, \text{ if there is an } \alpha \text{ such that } (z_a - \alpha\, g, z_b) \in Z\},$$
$$= +\infty \text{ otherwise.} \tag{1}$$

The shortage function S(z, g) measures the number of units of the reference bundle g reflecting the distance between point $z \equiv (z_a, z_b)$ and the frontier technology. It has some useful properties (see Luenberger):

1. $z \in Z$ implies $S(z, g) \leq 0$,
2. Under free disposal in z_a, $Z = \{z: S(z, g) \leq 0\}$,
3. Under free disposal in z_a, $S(z_a, z_b, g)$ is non-decreasing in z_a,
4. $S(z_a + \alpha\, g, z_b, g) = \alpha + S(z, g)$, for any α.

Property 1 shows that $S(z, g) \leq 0$ is associated with the feasibility of the netputs $z \equiv (z_a, z_b)$. Under free disposal in z_a, property 2 implies that $S(z, g) \leq 0$ provides a complete characterization of the technology. In this case, $S(z, g) = 0$ if and only if z is on the upper bound of the feasible set Z, with $S(z, g) = 0$ providing a multi-input multi-output functional representation of the underlying frontier technology. Under free disposal in z_a, property 3 states that the shortage function $S(z_a, z_b, g)$ is non-decreasing in the private goods z_a. Note that, in general, $S(z_a, z_b, g)$ can be either increasing or decreasing in the environmental goods z_b. As suggested by property 3, it would be non-decreasing in z_b if the technology exhibited free disposal in z_b. But it would be decreasing in z_b in regions where free disposal in z_b fails to hold. Finally, if $S(z_a, z_b, g)$ is twice differentiable in z, property 4 implies that $\frac{\partial S(z_a, z_b, g)}{\partial z_a}\, g = 1$ and that $\frac{\partial^2 S(z_a, z_b, g)}{\partial (z_a, z_b) \partial z_a}\, g = 0$.

The shortage function is illustrated in Figure 1. Consider evaluating it at point J, where the private good $z_a > 0$ is an output (represented by the distance OK in Figure 1) and the environmental good $z_b < 0$ is an input (where $|z_b|$ is given by the distance OL). Given the reference bundle g (represented by JM in Figure 1), the shortage function S(z, g) evaluated at point J is given by -JN/JM.

As a further illustration, consider the case where g = (1, 0, …, 0). Then $S(z, g) = \min_\alpha \{\alpha: (z_1 - \alpha, z_2, \ldots, z_{m+n}) \in Z\} = z_1 - G(z_c)$ where $z_c = (z_2, \ldots, z_{m+n})$ and $G(z_c) = \max \{z_1: (z_1, z_c) \in Z\}$ is the largest possible z_1 that can be obtained given other netputs z_c. In this case, under differentiability, $\partial S/\partial z_1 = 1$ and $\partial S/\partial z_c = -\partial G/\partial z_c$, implying that $-\partial S/\partial z_c$ can be interpreted as measuring the marginal product of z_c.

For a given $z \equiv (z_a, z_b)$, the shortage function S(z, g) in (1) provides a convenient basis for analyzing the productive value of the environmental goods z_b. To see that, consider a change in environmental goods from z_b^1 to z_b^2. Then, define

$$P(z_a, z_b^1, z_b^2, g) = S(z_a, z_b^1, g) - S(z_a, z_b^2, g). \tag{2}$$

Starting from the point $z \equiv (z_a, z_b^1)$, $P(z_a, z_b^1, z_b^2, g)$ in (2) measures the number of additional units of the reference bundle g that can be obtained from changing environmental goods from z_b^1 to z_b^2. To illustrate, consider the case where z_b are inputs (with $z_b < 0$) and (2) is evaluated under a technology exhibiting free disposal in z_b. As suggested by property 3, $S(z_a, z_b, g)$ would be non-decreasing in z_b. Then, with $z_b < 0$, any increase in the environmental inputs from $|z_b^1|$ to $|z_b^2|$ would mean a <u>decrease</u> in z_b, implying that $P(z_a, z_b^1, z_b^2, g) \geq 0$ in (2). In this case, increasing environmental input z_b can make it possible to produce more of the private goods z_a, with $P(z_a, z_b^1, z_b^2, g) \geq 0$ measuring the additional number of units of the private goods g that can be produced.

To note the role of free disposal for the environmental goods z_b, consider the case of an increase in the environmental input from point J in Figure 1. With $z_b < 0$, increasing the environmental input $|z_b|$ means a decrease in z_b from point J, implying an increase in the shortage function. This reflects the fact that free disposal in z_b does not hold in the region BC of Figure 1, and that the shortage function $S(z_a, z_b, g)$ is now decreasing in z_b in the neighborhood of point J. In this case, any increase in the environmental input $|z_b|$ implies that $P(z_a, z_b^1, z_b^2, g) < 0$ in (2). This illustrates that, without free disposal, increasing environmental input $|z_b|$ can reduce the ability to produce the private goods z_a, with $P(z_a, z_b^1, z_b^2, g) < 0$ measuring the associated reduction in the number of units of g that can be produced.

In the case where the private goods z_a are also market goods with prices p, a monetary evaluation of $P(z_a, z_b^1, z_b^2, g)$ in (2) is

$$V(z_a, z_b^1, z_b^2, p, g) = P(z_a, z_b^1, z_b^2, g)\ (p\ g)$$

$$= [S(z_a, z_b^1, g) - S(z_a, z_b^2, g)]\ (p\ g). \qquad (3)$$

Starting from the point $z \equiv (z_a, z_b^1)$, $V(z_a, z_b^1, z_b^2, p, g)$ in (3) gives a monetary value of the private goods that can be obtained when environmental goods change from z_b^1 to z_b^2. Then, comparing (2) and (3) gives the following result.

Proposition 1: When the reference bundle g is chosen to have unit value (with $p\ g = 1$), $P(z_a, z_b^1, z_b^2, g)$ in (2) gives a monetary value of changes in environmental goods from z_b^1 to z_b^2.

This provides some guidance for choosing the reference bundle g. When g is chosen such that $p\ g = 1$, Proposition 1 shows that $P(z_a, z_b^1, z_b^2, g)$ in (2) measures the monetary value of changes in environmental goods. This measure is attractive on several grounds: it allows the analysis of environmental goods as "non-market goods", (i.e., goods with no observable price); it allows for a general technology underlying the productivity implications of an ecological system; it does not require the technology to be convex; and it does not require that the environmental goods satisfy “free disposal.”

As shown in Proposition 1, equations (2) and (3) provide absolute measures of changes environmental goods. Note that these measures can be easily modified into relative measures. To see that, consider the case where $z_b^1 = 0$ and $z_b^2 = z_b$. Then, equation (2) becomes

$$P(z_a, 0, z_b, g) = S(z_a, 0, g) - S(z_a, z_b, g), \tag{2'}$$

where $P(z_a, 0, z_b, g)$ measures the total value of the environmental goods z_b when p g = 1. In situations where $P(z_a, 0, z_b, g) \neq 0$, a relative measure of changes in environmental goods from z_b^1 to z_b^2 can be written as

$$\begin{aligned} R_1(z_a, z_b^1, z_b^2, g) &\equiv P(z_a, z_b^1, z_b^2, g)/P(z_a, 0, z_b^2, g) \\ &= [S(z_a, z_b^1, g) - S(z_a, z_b^2, g)]/[S(z_a, 0, g) - S(z_a, z_b^2, g)]. \end{aligned} \tag{4}$$

$R_1(z_a, z_b^1, z_b, g)$ in (4) measures the value of the change from z_b^1 to z_b^2 as a proportion of the total value of z_b^2 given in (2'). Finally, note that, in situations where (z_a, z_b^2) is on the upper bound of the feasible set, then $S(z_a, z_b^2, g) = 0$ and equation (4) reduces to

$$R_1(z_a, z_b^1, z_b^2, g) = S(z_a, z_b^1, g)/S(z_a, 0, g), \tag{4'}$$

showing that a ratio of shortage functions provides a simple relative measure of environmental changes.

3. The Value of Biodiversity

Equations (2) and (3) measure the productive value associated with a change in environmental goods. However, it is often of interest to know more about the source of this value. The concerns about biodiversity provide a good example. Indeed, biodiversity issues typically arise when it is believed that the value of an ecosystem is greater than the value of its parts. This suggests the need to evaluate the value of environmental goods both for their "total value" and for the "sum of their parts." To address this issue, consider a thought experiment where the ecological system is split into K separate sub-systems, keeping technology and the total amount of resources constant. The key question is: is the original system more productive than the K sub-systems?

To answer this question, denote by I_b the set of environmental goods in z_b, and consider a partition of the set $I_b = \{I_{b1}, I_{b2}, \ldots, I_{bK}\}$, with $2 \leq K \leq n$. Let $z_{bk} = \{z_j: i \in I_{bk}\}$ denote the environmental goods in the subset I_{bk}, k = 1, …, K, with $z_b = (z_{b1}, \ldots, z_{bK})$. For a given $z \equiv (z_a, z_b) \in Z$, consider K situations where $z^k \equiv (z_a{}^k, z_b{}^k) \neq 0$ for k = 1, …, K, and where $\sum_{k=1}^{K} z^k = z$. Using the shortage function (1), we propose the following measure of <u>diversity</u>

$$D(z, g) = \sum_{k=1}^{K} S(z^k, g) - S(z, g), \tag{5}$$

where $z = \sum_{k=1}^{K} z^k$. Equation (5) compares two situations involving netputs z: one where the netputs z are involved in a single production process; and the other situation where there are K <u>separate</u> production processes, with z_k being the netputs used in the k-th production process. With $z = \sum_{k=1}^{K} z^k$, it follows that, in each situation, the same aggregate amounts of resources are used to produce the same aggregate netputs. In this context, equation (5) provides a measure of the number of units of the reference bundle g that can be saved by producing z jointly (compared to producing the same aggregate netputs z in K separate production processes). Intuitively, $D(z, g) > 0$ if there are productivity gains associated with a joint use of the netputs z. This reflects that $D(z, g) > 0$ corresponds to situations where "the whole is worth more than the sum of the parts." From (5), this would be associated with the <u>subadditivity</u> of the shortage function.

To help further motivate (5), consider the case where p g = 1. Then, use equation (2) to define $P_k = S(z_a, 0, g)/K - S(z^k, g)$ as measuring the value of the environmental goods in z^k, k = 1, ..., K, where $\sum_{k=1}^{K} z_a{}^k = z_a$. Note that $S(z_a, 0, g)$ is divided by K to reflect the fact that the original ecosystem is being evaluated in the context of K separate systems. Then, the value of the "sum of the parts" across the K systems is

$$\sum_{k=1}^{K} P_k = S(z_a, 0, g) - \sum_{k=1}^{K} S(z^k, g),$$

$$= P(z_a, 0, z_b, g) - D(z, g),$$

using (2') and (5). It follows that $D(z, g) = P(z_a, 0, z_b, g) - \sum_{k=1}^{K} P_k$. This shows that the value of diversity $D(z, g)$ in (5) is indeed the difference between the total value of the environmental goods z_b, $P(z_a, 0, z_b, g)$, and the value of the "sum of its parts", $\sum_{k=1}^{K} P_k$.

As indicated in proposition 1, when the reference bundle g is chosen such that p g = 1, then $D(z, g)$ in (5) provides a monetary measure of the value of diversity. As such, equation (5) provides an <u>absolute measure</u> of diversity. However, it can be easily used to obtain a relative measure. In situations where the total value $P(z_a, 0, z_b, g)$ in (2') is non-zero, a <u>relative measure</u> of diversity can be written as

$$R_D(z, g) \equiv D(z, g)/P(z_a, 0, z_b, g)$$

$$= [\textstyle\sum_{k=1}^{K} S(z^k, g) - S(z, g)]/[S(z_a, 0, g) - S(z_a, z_b, g)]. \tag{6}$$

where $z \equiv (z_a, z_b) = \sum_{k=1}^{K} z^k$. $R_D(z, g)$ in (6) measures the value of diversity as a proportion of the total value of z_b given in (2'). In situations where $z \equiv (z_a, z_b)$ is on the upper bound of the feasible set, then $S(z_a, z_b, g) = 0$ and equation (6) reduces to

$$R_D(z, g) = \textstyle\sum_{k=1}^{K} S(z^k, g)/S(z_a, 0, g), \tag{6'}$$

showing that a ratio of shortage functions provides a simple relative measure of diversity.

Note that equation (5) defines diversity in the general context of the netputs z, which include both the private goods z_a and the environmental goods z_b. Given our interest on biodiversity, we want to focus our attention on diversity issues related only to environmental goods. In this context, it will be useful to define $z^k \equiv (z_a^k, z_b^k)$ in (5) in a more specific way. Consider choosing

$$z_a^k = z_a/K, \tag{7a}$$

and

$$z_i^k = \beta\, z_i \text{ if } i \in I_{bk}, \tag{7b}$$

$$= z_i\, (1-\beta)/(K-1) \text{ if } i \in I_b \backslash I_{bk}, \tag{7c}$$

$k = 1, \ldots, K$, for some $\beta \in (1/K, 1]$. First, note that equations (7a)-(7c) always satisfy $z = \sum_{k=1}^{K} z^k$. This guarantees that the same aggregate netputs are involved in both situations. Second, equation (7a) divides the market goods z_a equally among the K production processes. This imposes "no diversity" in the use of the private goods z_a across the K production processes. Third, equations (7b)-(7c) establish the patterns of specialization for the environmental goods z_b. The parameter β in (7b) represents the proportion of the original environmental netputs $\{z_i: i \in I_{bk}\}$ that are produced in the k-th process. And from (7c), $\beta/(K-1)$ represents the proportion of the original netputs $\{z_i: i \in I_b \backslash I_{bk}\}$ produced in the k-th process. When $\beta = 1$, this corresponds to the case of <u>complete specialization</u> where the k-th process relies exclusively on environmental netputs in the subset I_{bk} (with $z_{ik} = z_i$ if $i \in I_{bk}$) with $z_{ik} = 0$ for $i \in I_b \backslash I_{bk}$. In such situations, each of the K process is associated with a <u>complete loss of biodiversity</u> in environmental goods z_b across elements of the partition $I_b = \{I_{b1}, I_{b2}, \ldots, I_{bK}\}$. Alternatively, when $\beta \in (1/K, 1)$, this allows for <u>partial specialization</u>. Then, each of the K process is associated with a <u>partial loss of biodiversity</u> in environmental goods z_b across elements of the partition $I_b = \{I_{b1}, I_{b2}, \ldots, I_{bK}\}$. Thus, the parameter $\beta \in (1/K, 1]$ allows for varying amount of specialization in the environmental netputs among the K production processes. Alternatively stated, it allows for varying amount of loss of biodiversity across the K processes. In general, the degree of specialization in each production process increases with β. This means that the loss in biodiversity in the K processes also increases with β.

With $z^k \equiv (z_a^k, z_b^k)$ given in (7a)-(7c), equation (5) becomes

$$D(z, \beta, g) = \sum_{k=1}^{K} S(z^k, g) - S(z, g), \tag{8}$$

where $\beta \in (1/K, 1]$. Equation (8) provides a measure of the <u>value of biodiversity</u>. It measures the number of units of the reference bundle g that can be saved when the environmental goods z_b are part of a joint production process in the ecological system (compared to the case where the environmental goods z_b are part of K specialized production processes satisfying (7a)-(7c) and producing the same aggregate netputs z). Using arguments similar to the ones presented in Proposition 1 yields the following result.

Proposition 2: When the reference bundle g is chosen to have unit value (with p g = 1), then D(z, β, g) in (8) is a monetary measure of the value of biodiversity.

4. A Decomposition

While equation (8) provides a basis to evaluate the value of biodiversity, it is of interest to identify the sources of this value. In this section, we develop a general decomposition of the benefits associated with biodiversity, thus providing new insights into thier sources.

Below, we consider the case where the shortage function S(z, g) is a smooth function of z, except possibly at z = 0. We allow discontinuity at zero to reflect the possible presence of catalytic effects where productivity can be very different when z moves away from zero. To capture such effects, let $S(z, g) \equiv S_v(z, g) + S_f(z, g)$. This decomposes the shortage function S(z, g) into a "variable function" $S_v(z, g)$ and a "fixed function" $S_f(z, g)$. We assume that the variable shortage function $S_v(z, g)$ is continuous in z. The fixed shortage function $S_f(z, g)$ is defined as a step function satisfying $S_f(0, g) = 0$, with possible discontinuities at z = 0. Thus, $S_f(z, g)$ is constant with respect to z as long as the set of non-zero netputs does not change. The jump-discontinuities of $S_f(z, g)$ (an hence S(z, g)) at z = 0 reflect the possible presence of catalytic effects of z in the production process.

We start from the partition $I_b = \{I_{b1}, \ldots, I_{bK}\}$, where I_{bk} denotes the environmental goods that the k-th ecological process specializes in, k = 1, …, K, with $2 \leq K \leq n$. We use the following notation. Let $z_a = \{z_i: i \in I_a\}$, $z_{bk} = \{z_i: i \in I_{bk}\}$, $z_b = (z_{b1}, \ldots, z_{bK})$, $z_{b\backslash bk} = (z_{b1}, \ldots, z_{b,k-1}, z_{b,k+1}, \ldots,$

z_{bK}), and $z_{b,i:j} = (z_{bi}, z_{b,i+1}, \ldots, z_{b,j-1}, z_{bj})$ for $i < j$. From equations (7), it follows that $z^k = (z_a/K, \beta z_{bk}, (1-\beta) z_{b\backslash bk})$. Our main result is stated next. (See the proof in the Appendix).

<u>Proposition 3</u>: Given $S(z, g) \equiv S_v(z, g) + S_f(z, g)$, assume that $S_v(z, g)$ is continuously differentiable in z_b almost everywhere. Under equations (7), the value of biodiversity $D(z, \beta, g)$ in (8) evaluated at netputs $z = (z_a, z_b)$ can be decomposed as follows

$$D \equiv D_C + D_R + D_V + D_A, \tag{9}$$

where

$$D_C \equiv \sum_{k=1}^{K-1} \Big\{ \int_{z_{bk}(1-\beta)/(K-1)}^{\beta z_{bk}} \frac{\partial S_v}{\partial \gamma}(z_a/K, z_{b,1:k-1}(1-\beta)/(K-1), \gamma, z_{b,k+1:K}(1-\beta)/(K-1), g)\, d\gamma$$

$$- \int_{z_{bk}(1-\beta)/(K-1)}^{\beta z_{bk}} \frac{\partial S_v}{\partial \gamma}(z_a/K, z_{b,1:k-1}(1-\beta)/(K-1), \gamma, \beta z_{b,k+1:K}, g)\, d\gamma \Big\}, \tag{10a}$$

$$D_R \equiv K\, S(z/K, g) - S(z, g), \tag{10b}$$

$$D_V \equiv S(z_a/K, \beta z_b, g) + (K-1)\, S(z_a/K, z_b(1-\beta)/(K-1), g) - K\, S(z/K, g), \tag{10c}$$

and

$$D_A \equiv \sum_{k=1}^{K} S_f(z_a/K, \beta z_{bk}, z_{b\backslash bk}(1-\beta)/(K-1), g)$$

$$- S_f(z_a/K, \beta z_b, g) - (K-1)\, S_f(z_a/K, z_b(1-\beta)/(K-1), g). \tag{10d}$$

Proposition 3 gives a decomposition of the value of biodiversity $D(z, g)$ in (8) into four additive terms: D_C given in (10a), D_R given in (10b), D_V given in (10c), and D_A given in (10d).

The term D_C in (10a) depends on how $z_{b\backslash bk}$ affects the marginal shortage of z_{bk}, $k = 1, \ldots, K$. It reflects the presence of <u>complementarity</u> among environmental netputs in z_b. To see that, consider the case where the shortage function is twice continuously differentiable in z_b. Then, equation (10a) can be written as

$$D_C \equiv -\sum_{k=1}^{K-1} \int_{z_{b,k+1:K}(1-\beta)/(K-1)}^{\beta z_{b,k+1:K}} \int_{z_{bk}(1-\beta)(K-1)}^{\beta z_{bk}} \frac{\partial^2 S_v}{\partial\gamma_1 \partial\gamma_2}(z_a/K, z_{b,1:k-1}(1-\beta)/(K-1), \gamma_1, \gamma_2, g)\, d\gamma_1\, d\gamma_2. \quad (10a')$$

Equation (10a') makes it clear that the sign of D_C depends on the sign of $\partial^2 S/\partial z_{bk}\partial z_{b\backslash bk}$, k = 1, …, K. As discussed above, the marginal shortage can be interpreted as the negative of the marginal product. In this context, define <u>complementarity between z_{bk} and $z_{b\backslash bk}$</u> as any situation where the shortage function satisfies $\partial^2 S/\partial z_{bk}\partial z_{b\backslash bk} < 0$. Indeed, with $\partial S/\partial z_{bk}$ reflecting the negative of the marginal product of z_{bk}, complementarity (with $\partial^2 S/\partial z_{bk}\partial z_{b\backslash bk} < 0$) means that z_{bk} has positive effects on the marginal product of $z_{b\backslash bk}$, implying positive synergies between z_{bk} and $z_{b\backslash bk}$. Then, it is clear from (10a) that $D_C > 0$ if the shortage function exhibits complementarity between z_{bk} and $z_{b\backslash bk}$, k = 1, …, K. Thus, proposition 3 establishes that complementarity among environmental netputs (as reflected by the term D_C) is one of the components of the value of biodiversity. This is consistent with previous literature that has identified complementarity as an important contributing factor to the value of biodiversity (e.g., Faith et al.; Justus and Sarkar; Loreau and Hector).

To interpret the term D_R in (10b), we make use of lemma 1 in the Appendix. Given K ≥ 2, lemma 1 implies that $K\, S(z/K, g) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} S(z, g)$ under $\begin{Bmatrix} \text{decreasing returns to scale (DRTS)} \\ \text{constant returns to scale (CRTS)} \\ \text{increasing returns to scale (IRTS)} \end{Bmatrix}$.

It follows that

$$D_R \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0 \text{ under } \begin{Bmatrix} \text{DRTS} \\ \text{CRTS} \\ \text{IRTS} \end{Bmatrix}. \quad (10b')$$

Equation (10b') implies that D_R vanishes under CRTS, but is positive (negative) under IRTS (DRTS). Thus, the term D_R can be interpreted as capturing <u>scale effects</u> generated as the netput vector z is produced in more specialized ways. Also, equation (10b') shows that $D_R \geq 0$ under non-decreasing returns to scale. Intuitively, more specialized processes involve smaller scales of operation. Under IRTS, such processes (associated with lower biodiversity) would appear less productive (their scale of operation is "too small"), implying that the scale effect contributes positively to the value of biodiversity ($D_R > 0$). Alternatively, under DRTS, the specialized processes would appear more productive (as the scale of operation of the integrated process is "too large"), implying a negative scale effect ($D_R < 0$). Thus, proposition 3 establishes how the scale of ecosystem and the nature of returns to scale for the underlying process (as reflected by the term D_R) can affect the value of biodiversity.

The term D_V in (10c) reflects the effect of convexity. To show it, we make use of lemma 2 in the Appendix. Lemma 2 states that the shortage function S(z, g) is convex in z when the feasible set Z is convex. It follows that, under the convexity of Z, the shortage function satisfies $\sum_{j=1}^{K} \theta_j S(z^j, g) \geq S(\sum_{j=1}^{K} \theta_j z^j, g)$ for $\theta_j \in [0, 1]$ satisfying $\sum_{j=1}^{K} \theta_j = 1$. Choosing $\theta_j = 1/K$, $z^1 = (z_a/K, \beta z_b, g)$ and $z^j = (z_a/K, z_b (1-\beta)/(1-K), g)$ for j = 2, …, K, it follows from (10c) that $D_V \geq 0$. Thus, a convex technology is sufficient to imply that $D_V \geq 0$. Intuitively, a convex technology means diminishing marginal productivity, a standard characterization of resource scarcity. This suggests that the term D_V reflects the role of resource scarcity. In this context, proposition 3 shows that resource scarcity contributes positively to the value of biodiversity.

Finally, the term D_A in (10d) reflects catalytic effects around z = 0. Indeed, in the absence of discontinuity of the shortage function S(z, g), then $S_f(z, g) = 0$ and thus $D_A = 0$ in equation (10d). When S_f is non-zero, note that D_A can be positive, zero, or negative. Since $S_f(z, g)$ is step function with possible discontinuities only around z = 0, $S_f(z, g)$ is a constant as along as the set of non-zero netputs does not change. Then, from equation (10d), $\beta \in (1/K, 1)$ implies $D_A = 0$. Alternatively, the fixed-netput component D_A can be non-zero only when $\beta = 1$. It means that the role of catalytic effects is relevant in the value of biodiversity only when $\beta = 1$, i.e., only under complete loss of biodiversity in environmental goods across elements of the partition $I_b = \{I_{b1}, I_{b2}, \ldots, I_{bK}\}$. In the context where $\beta = 1$, from equation (10d), D_A is positive if and only if

$\sum_{k=1}^{K} S_f(z_a/K, z_{bk}, 0, g) > S_f(z_a/K, z_b, g) + (K\text{-}1)\, S_f(z_a/K, 0, g)$. Then, catalytic effects contribute to the value of biodiversity. This corresponds to situations where a complete loss of biodiversity generates a discontinuous decrease in the productivity of the specialized processes. Thus, Proposition 3 shows how a complete loss of biodiversity can affect the value of biodiversity through the catalytic component D_A.

Proposition 3 provides useful information on conditions contributing to the value of biodiversity. It generates the following result.

<u>Corollary 1</u>: Sufficient conditions for a positive value of biodiversity are:

1) there is complementarity between z_{bk} and $z_{b\backslash bk}$, $k = 1, \ldots, K$, ($D_C > 0$),
2) the technology exhibits non-decreasing returns to scale ($D_R \geq 0$),
3) the technology Z is convex (with $D_V \geq 0$), and
4) $D_A \geq 0$.

Thus, the value of biodiversity can arise from complementarity among environmental goods in z_b ($D_C > 0$), from increasing returns to scale ($D_R > 0$), from a convex technology ($D_V \geq 0$), and/or from catalytic effects (when $D_A \geq 0$). This identifies the role of complementarity as an important contributing factor to the value of biodiversity. However, it also shows that complementarity is in general neither necessary nor sufficient to generate a positive value for biodiversity. For example, under decreasing returns to scale (DRTS), equation (10b') implies that $D_R < 0$. This reflects the fact that, under DRTS, the smaller and more specialized processes require fewer resources to produce the same aggregate outputs. When this scale effect dominates the other components in (9), then $D < 0$, i.e. biodiversity would have a negative value even in the presence of complementarity. Alternatively, B_V can become negative under a non-convex technology. Again if this negative convexity effect dominates the other components in (9), then D

< 0, and biodiversity would have a negative value even in the presence of complementarity. Finally, we have shown that the catalytic effect D_A is present only under complete loss of biodiversity in environmental goods across elements of the partition $I_b = \{I_{b1}, I_{b2}, \ldots, I_{bK}\}$. Scenarios where D_A is positive and large can arise when a complete loss of biodiversity is associated with a large decline in the productivity of the specialized processes. In such cases, the value of biodiversity can be positive even in the absence of complementarity. This illustrates the usefulness of the decomposition provided in Proposition 3.

5. The Case of Uncertainty

Often, the productivity of an ecosystem is not known with certainty. This is particularly relevant when the ecosystem involves significant dynamics. Then, the future productivity of the ecosystem may depend on many factors that are not fully known. In this section, we investigate the case where some elements of the underlying technology are uncertain. This is done by considering that the feasible set Z depends on some factors denoted by the vector v, factors that are not perfectly known at the present time. Then, the underlying technology is represented by $Z(v)$, where $z \in Z(v)$ means that netputs z are feasible given v. For example, v could represent future weather patterns, where good (bad) weather would expand (contract) the feasible set Z. Then, $z \equiv (z_a, z_b) \in Z(v)$ means that private netputs z_a and environmental netputs z_b can be produced under conditions v. Given $Z(v)$, the frontier technology can be represented by the shortage function $S(z, v, g)$ in (1), where feasibility means $S(z, v, g) \leq 0$, and $S(z, v, g) = 0$ means that netputs z are on the frontier technology given v.

The vector v representing uncertainty can be treated as a vector of random variables. Assume that v has a probability distribution reflecting the (possibly subjective) evaluation of the uncertainty. In general, note that this distribution can depend on the netputs z. Then, being a function of v, the shortage function $S(z, v, g)$ is also random and has a probability distribution.

Below, we explore the characterization of the distribution of S(z, v, g) using its moments (e.g., Antle). For that purpose, assume that S(z, v, g) takes the following specification:

$$S(z, v, g) = f_1(z, g) + [f_2(z, g) - f_3(z, g)^{2/3}]^{1/2} e_2(v) + [f_3(z, g)]^{1/3} e_3(v), \quad (11)$$

where the random variables $e_2(v)$ and $e_3(v)$ are independently distributed and satisfy $E[e_2(v)] = E[e_3(v)] = 0$, $E[e_2(v)^2] = E[e_3(v)^2] = 1$, $E[e_2(v)^3] = 0$, $E[e_3(v)^3] = 1$, and $E(\cdot)$ denotes the expectation operator based on the (subjective) probability distribution of v (distribution which can depend on z). This means that the random variables $e_2(v)$ and $e_3(v)$ are normalized (i.e., they are each distributed with mean zero and variance 1). In addition, $e_2(v)$ has zero skewness ($E[e_2(v)^3] = 0$) while the random variable $e_3(v)$ is asymmetrically distributed and has positive skewness ($E[e_3(v)^3] = 1$). It follows from (11) that

$$E[S(z, v, g)] = f_1(z, g), \quad (12a)$$

$$E[(S(z, v, g) - f_1(z, g))^2] = f_2(z, g), \quad (12b)$$

$$E[(S(z, v, g) - f_1(z, g))^3] = f_3(z, g). \quad (12c)$$

This shows that the specification (11) provides a convenient representation of the first three central moments of the distribution of S(z, v, g). Indeed, from (12a), the first moment (the mean) is given by $f_1(z, g)$. From (12b), the second central moment (the variance) is given by $f_2(z, g) \geq 0$. And from (12c), the third central moment (measuring skewness) is given by $f_3(z, g)$. Since the functions $f_1(z, g)$, $f_2(z, g)$ and $f_3(z, g)$ can take any form, this provides a flexible representation of the impacts of netputs z on the distribution of S under uncertainty. In addition, if we treat the distribution of $e_2(v)$ and $e_3(v)$ as given, then the three moments $f_1(z, g)$, $f_2(z, g)$ and $f_3(z, g)$ are sufficient statistics for the distribution of S(z, v, g) in the specification (11).

Note that equation (11) can be interpreted as a standard regression model where $f_1(z, g)$ is the regression line representing mean effects, and $e_1 \equiv \{[f_2(z, g) - f_3(z, g)^{2/3}]^{1/2} e_2(v) + [f_3(z, g)]^{1/3} e_3(v)\}$ is an error term with mean zero, variance $f_2(z, g)$ and skewness $f_3(z, g)$. By considering explicitly skewness effects, it is a generalization of the standard Just-Pope mean-variance specification for a stochastic technology (Just and Pope, 1978, 1979). The effects of netputs z on the variance of S can help determine whether the i-th netput is risk increasing (with $\partial f_2(z, g)/\partial|z_i| > 0$), risk neutral (with $\partial f_2(z, g)/\partial|z_i| = 0$), or risk decreasing (with $\partial f_2(z, g)/\partial|z_i| < 0$), i = 1, …., n+m. However, equation (11) goes beyond the Just-Pope mean-variance specification by allowing for skewness effects. In situations where exposure to downside risk is relevant and skewness effects are important (e.g., the case of catastrophic events), specification (11) provides a framework to analyze such issues. It allows an assessment of the effects of netputs z on the skewness of S. For example, in the case where g = (1, 0, …, 0) and z_1 is an output, then $S(z, v, g) = z_1 - G(z_2, …, z_{m+n}, v)$, where $G(z_2, …, z_{m+n}, v) = \max_{z1} \{z_1: z \in Z(v)\}$ is a stochastic production function. Then, given $E[e_3(v)^3] = 1$ in (11), the i-th netput is said to be downside-risk increasing if $\partial f_3(z, g)/\partial|z_i| > 0$, downside-risk neutral if $\partial f_3(z, g)/\partial|z_i| = 0$, or downside-risk decreasing if $\partial f_3(z, g)/\partial|z_i| < 0$, i = 2, …, m+n. As such, the specification (11) provides a useful basis to investigate how uncertainty affects the productivity of the ecosystem. And the effects of netputs z on the mean, variance and skewness of S show how ecosystem management affects both its average productivity and the associated risk (including downside risk exposure).

Using specification (11) provides additional information on the value of biodiversity. From equation (8) and Proposition 2, the shortage function (11) gives a basis for evaluating the value of biodiversity under uncertainty (given the random variables v)

$$D(z, v, \beta, g) = \sum_{k=1}^{K} S(z^k, v, g) - S(z, v, g), \tag{8'}$$

where $z^k \equiv (z_a^k, z_b^k)$ satisfies equations (7a)-(7c), k = 1, …, K.

Note that Proposition 2 involves the condition: p g = 1. Under uncertainty, this can raise questions in situations where some prices in p may be state-contingent (i.e., if they depend on the random variables v). A simple way of avoiding this issue is to restrict the choice of the bundle g to private goods that are chosen *ex ante*, i.e. before the random variables v become observed. Then, if these goods are market goods, their prices would also be *ex ante* prices with two desirable properties: 1/ they would <u>not</u> be state-contingent (i.e., they would not depend on v); and

2/ they can be taken to be current prices that are readily observable. Thus, provided that one restricts the bundle g to include only private goods that are chosen *ex ante*, Proposition 2 would apply under uncertainty, with D(z, v, β, g) in (8') providing a monetary value of biodiversity (as long as g is chosen such that p g = 1).

To see the usefulness of the specification (11), consider substituting (11) into (8'). This gives the following result.

<u>Proposition 4</u>: Given the specification (11) and assuming that p g = 1, the value of biodiversity D(z, v, β, g) in (8') can be written as

$$\begin{aligned} D(z, v, \beta, g) = {} & \textstyle\sum_{k=1}^{K} f_1(z^k, g) - f_1(z, g) \\ & + \{\textstyle\sum_{k=1}^{K} [f_2(z^k, g) - f_3(z^k, g)^{2/3}]^{1/2} - [f_2(z, g) - f_3(z, g)^{2/3}]^{1/2}\}\, e_2(v), \\ & + \{\textstyle\sum_{k=1}^{K} [f_3(z^k, g)]^{1/3} - [f_3(z, g)]^{1/3}\}\, e_3(v), \end{aligned} \tag{13}$$

implying that

$$D_1(z, \beta, g) \equiv E[D(z, v, \beta, g)] = \textstyle\sum_{k=1}^{K} f_1(z^k, g) - f_1(z, g), \tag{14a}$$

$$\begin{aligned} D_2(z, \beta, g) & \equiv E[(D(z, v, \beta, g) - D_1)^2] \\ & = \{\textstyle\sum_{k=1}^{K} [f_2(z^k, g) - f_3(z^k, g)^{2/3}]^{1/2} - [f_2(z, g) - f_3(z, g)^{2/3}]^{1/2}\}^2 \\ & \quad + \{\textstyle\sum_{k=1}^{K} [f_3(z^k, g)]^{1/3} - [f_3(z, g)]^{1/3}\}^2, \end{aligned} \tag{14b}$$

$$D_3(z, \beta, g) \equiv E[(D(z, v, \beta, g) - D_1)^3] = \{\textstyle\sum_{k=1}^{K} [f_3(z^k, g)]^{1/3} - [f_3(z, g)]^{1/3}\}^3. \tag{14c}$$

Proposition 4 evaluates the effects of uncertainty on the value of biodiversity D. First, equation (13) indicates how the uncertainty v affects D through the two random variables $e_2(v)$ and $e_3(v)$. Second, equations (14a)-(14c) presents the first three central moments of the value of biodiversity. The first moment (i.e., the mean) of D is given by D_1 in equation (14a): it depends on the properties of the mean shortage function $f_1(\cdot, g)$. The second central moment (i.e., the variance) of D is given by D_2 in equation (14b). It depends on the properties of both the variance and the skewness of the shortage function, $f_2(\cdot, g)$ and $f_3(\cdot, g)$. Finally, the third central moment (i.e., the skewness) of D is given by D_3 in equation (14c) and depends on the properties of the skewness function $f_3(\cdot, g)$. Equations (14a)-(14c) provide a basis to evaluate how the netputs $z \equiv (z_a, z_b)$ affect the distribution of the value of biodiversity.

In a way similar to Proposition 3, note that the mean value of diversity $D_1(z, \beta, g) = \sum_{k=1}^{K} f_1(z^k, g) - f_1(z, g)$ in (14a) can be decomposed. Consider the case where the function $f_1(z, g)$ can be written as $f_1(z, g) \equiv f_{1v}(z, g) + f_{1f}(z, g)$. The function $f_{1v}(z, g)$ is a "mean variable-shortage function", assumed to be continuous in z and continuously differentiable in z_b almost everywhere. And the function $f_{1f}(z, g)$ is a "mean fixed-shortage function" reflecting possible catalytic effects: it is a step function which satisfies $f_{1f}(0, g) = 0$ and can exhibit jump discontinuities around z = 0.

Proposition 5: Given p g = 1 and under equations (7), the mean value of biodiversity D_1 in (14a) evaluated at netputs $z = (z_a, z_b)$ can be written as

$$D_1(z, \beta, g) \equiv D_{1C} + D_{1R} + D_{1V} + D_{1A}, \tag{15}$$

where

$$D_{1C} \equiv \sum_{k=1}^{K-1} \{ \int_{z_{bk}(1-\beta)/(K-1)}^{\beta z_{bk}} \frac{\partial f_{1v}}{\partial \gamma} (z_a/K, z_{b,1:k-1} (1-\beta)/(K-1), \gamma, z_{b,k+1:K} (1-\beta)/(K-1), g)\, d\gamma$$

$$- \int_{z_{bk}(1-\beta)/(K-1)}^{\beta z_{bk}} \frac{\partial f_{1v}}{\partial \gamma} (z_a/K, z_{b,1:k-1} (1-\beta)/(K-1), \gamma, \beta z_{b,k+1:K}, g)\, d\gamma \}, \tag{16a}$$

$$D_{1R} \equiv K f_1(z/K, g) - f_1(z, g), \tag{16b}$$

$$D_{1V} \equiv f_1(z_a/K, \beta z_b, g) + (K-1) f_1(z_a/K, z_b (1-\beta)/(K-1), g) - K f_1(z/K, g), \tag{16c}$$

and

$$D_{1A} \equiv \sum_{k=1}^{K} f_{1f}(z_a/K, \beta z_{bk}, z_{b\backslash bk} (1-\beta)/(K-1), g)$$

$$- f_{1f}(z_a/K, \beta z_b, g) - (K-1) f_{1f}(z_a/K, z_b (1-\beta)/(K-1), g). \tag{16d}$$

Proposition 5 gives a decomposition of the mean value of biodiversity $D_1(z, g)$ in (15) into four additive terms: D_{1C} given in (16a), D_{1R} given in (16b), D_{1V} given in (16c), and D_{1A} given in (16d). As discussed in Proposition 3, they reflect respectively complementarity effects, scale effects, convexity effects, and catalytic effects with respect to the mean value of diversity $D_1(z, \beta, g)$. This provides useful insights on the sources and determinants of mean biodiversity value.

Note that the variance and skewness of biodiversity given in equations (14b) and (14c) are more complex than (14a). Yet, they provide a basis for evaluating the effects of netputs z on the distribution of biodiversity value. This would be important in situations where decision makers are risk averse (meaning that a higher variance makes them worse off) and downside-risk averse (meaning that a more negative skewness makes them worse off; see Menezes et al.). Then, the identification of the effects of z_i on D_2 in (14b) and D_3 in (14c) is of special interest. Indeed, assuming differentiability, finding that $\partial D_2/\partial |z_i| < 0$ (> 0) mean that the i-th netput is reducing (increasing) the variance of the value of biodiversity. And finding that $\partial D_3/\partial |z_i| > 0$ would imply that the i-th netput is increasing the skewness of D. This means reducing the probability of a disastrous decline in the value of biodiversity. To the extent that such situations can be associated with improved resilience of the ecological system (see Holling), this can provide useful insights into the economics and management/policy implications of ecosystem resilience. This illustrates how management decisions (involving the choice of netputs z) can affect the mean, variance as well as skewness of the value of biodiversity.

6. Concluding Remarks

We have presented an analysis of the value of biodiversity in an ecosystem. This is particularly relevant in the context of agriculture, where the ecosystem involves environmental goods (including ecological capital and environmental services) being used in the production of food. The analysis applies under general conditions, allowing for non-convexities, lack of free disposal in environmental goods, and dynamics. In this context, we relied on Luenberger's shortage function to provide a measure of the productive value of biodiversity. When positive, this value reflects the fact that an ecosystem is worth more than the "sum of its parts." We showed that this value can be decomposed into four additive components, reflecting complementarity effects, scale effects, convexity effects, and catalytic effects. While the identification of these components indicates that biodiversity value can be complex, our analysis provides new and useful information on its sources and determinants. We also examined the effects of uncertainty on the value of biodiversity. This is relevant since the future productivity of ecosystems is often imperfectly known. In this context, we showed how management decisions can affect the mean, variance as well as skewness of the value of biodiversity. This should help guide empirical research investigating the importance of biodiversity and its implications for ecosystem management.

While our investigation focused on the productive value of biodiversity, we should keep in mind that this value is only a part of the total value of an ecosystem. This indicates the need to place the analysis in the broader context of ecological-economic interactions. This would include the value of biodiversity to consumers. Under uncertainty, this means examining the role of risk preferences and their implications for the design and implementation of risk management schemes. And in a dynamic context, this would include addressing the issue of how new information that becomes available over time is used in ecosystem management. These appear to be good topics for future research.

Appendix

Proof of Proposition 3:

From equation (8), the value of biodiversity is

$$D \equiv \sum_{k=1}^{K} S(z_a/K, \beta\, z_{bk}, z_{b\backslash bk}\,(1\text{-}\beta)/(K\text{-}1), g) - S(z, g) > 0. \tag{A1}$$

Define

$$d_1 \equiv S(z_a/K, \beta\, z_{b1}, z_{b\backslash b1}\,(1\text{-}\beta)/(K\text{-}1), g) + S_v(z_a/K, z_{b1}\,(1\text{-}\beta)/(K\text{-}1), \beta\, z_{b\backslash b1}, g).$$

And letting $z_{b,i:j} = (z_{bi}, z_{b,i+1}, \ldots, z_{b,j-1}, z_{bj})$ for $i < j$, define

$$\begin{aligned} d_k \equiv\ & S(z_a/K, \beta\, z_{bk}, z_{b\backslash bk}\,(1\text{-}\beta)/(K\text{-}1), g) \\ & + S_v(z_a/K, z_{b,1:k}\,(1\text{-}\beta)/(K\text{-}1), \beta\, z_{b,k+1:K}, g) \\ & - S_v(z_a/K, z_{b,1:k-1}\,(1\text{-}\beta)/(K\text{-}1), \beta\, z_{b,k:K}, g), \end{aligned}$$

for $k = 2, \ldots, K\text{-}1$. Using $d_1, \ldots, d_{K-1}$, and given $S(z, g) = S_v(z, g) + S_f(z, g)$, expression (A1) can be alternatively written as

$$D \equiv \sum_{k=1}^{K-1} d_k + S_f(z_a/K, \beta\, z_{b,K}, z_{b\backslash bK}\,(1\text{-}\beta)/(K\text{-}1), g) - S(z, g). \tag{A2}$$

When $S_v(z, g)$ is continuous in z_b everywhere and continuously differentiable in z_b almost everywhere, note that d_1 can be alternatively written as

$$\begin{aligned} d_1 =\ & S(z_a/K, \beta\, z_{b1}, z_{b\backslash b1}\,(1\text{-}\beta)/(K\text{-}1), g) + S_v(z_a/K, z_{b1}\,(1\text{-}\beta)/(K\text{-}1), \beta\, z_{b\backslash b1}, g) \\ & - S_v(z_a/K, \beta\, z_b, g) + S_v(z_a/K, \beta\, z_b, g) \\ & - S_v(z_a/K, z_b\,(1\text{-}\beta)/(K\text{-}1), g) + S_v(z_a/K, z_b\,(1\text{-}\beta)/(K\text{-}1), g), \\ =\ & S_f(z_a/K, \beta\, z_{b1}, z_{b\backslash b1}\,(1\text{-}\beta)/(K\text{-}1), g) \\ & + \int_{z_{b1}(1-\beta)/(K-1)}^{\beta\, z_{b1}} \frac{\partial S_v}{\partial \gamma}(z_a/K, \gamma, z_{b\backslash b1}\,(1\text{-}\beta)/(K\text{-}1), g)\, d\gamma \\ & - \int_{z_{b1}(1-\beta)/(K-1)}^{\beta\, z_{b1}} \frac{\partial S_v}{\partial \gamma}(z_a/K, \gamma, \beta\, z_{b\backslash b1}, g)\, d\gamma \\ & + S_v(z_a/K, \beta\, z_b, g) + S_v(z_a/K, z_b\,(1\text{-}\beta)/(K\text{-}1), g). \end{aligned} \tag{A3}$$

Similarly, d_k can be alternatively written as

$$d_k \equiv S(z_a/K, \beta\, z_{bk}, z_{b\backslash bk}\, (1\text{-}\beta)/(K\text{-}1), g)$$

$$+ S_v(z_a/K, z_{b,1:k}\, (1\text{-}\beta)/(K\text{-}1), \beta\, z_{b,k+1:K}, g)$$

$$- S_v(z_a/K, z_{b,1:k-1}\, (1\text{-}\beta)/(K\text{-}1), \beta\, z_{b,k:K}, g)$$

$$- S_v(z_a/K, z_b\, (1\text{-}\beta)/(K\text{-}1), g) + S_v(z_a/K, z_b\, (1\text{-}\beta)/(K\text{-}1), g),$$

$$= S_f(z_a/K, \beta\, z_{bk}, z_{b\backslash bk}\, (1\text{-}\beta)/(K\text{-}1), g)$$

$$+ \int_{z_{bk}\,(1-\beta)/(K\text{-}1)}^{\beta\, z_{bk}} \frac{\partial S_v}{\partial \gamma}(z_a/K, z_{b,1:k-1}\, (1\text{-}\beta)/(K\text{-}1), \gamma, z_{b,k+1:K}\, (1\text{-}\beta)/(K\text{-}1), g)\, d\gamma$$

$$- \int_{z_{bk}\,(1-\beta)/(K\text{-}1)}^{\beta\, z_{bk}} \frac{\partial S_v}{\partial \gamma}(z_a/K, z_{b,1:k-1}\, (1\text{-}\beta)/(K\text{-}1), \gamma, \beta\, z_{b,k+1:K}, g)\, d\gamma$$

$$+ S_v(z_a/K, z_b\, (1\text{-}\beta)/(K\text{-}1), g), \qquad \text{(A4)}$$

$k = 2, \ldots, K\text{-}1$. Substituting (A3) and (A4) into (A2) yields

$$D \equiv S_v(z_a/K, \beta\, z_b, g) + (K\text{-}1)\, S_v(z_a/K, z_b\, (1\text{-}\beta)/(K\text{-}1), g)$$

$$+ \sum_{k=1}^{K} S_f(z_a/K, \beta\, z_{bk}, z_{b\backslash bk}\, (1\text{-}\beta)/(K\text{-}1), g)$$

$$+ \sum_{k=1}^{K-1} \{ \int_{z_{bk}\,(1-\beta)/(K\text{-}1)}^{\beta\, z_{bk}} \frac{\partial S_v}{\partial \gamma}(z_a/K, z_{b,1:k-1}\, (1\text{-}\beta)/(K\text{-}1), \gamma, z_{b,k+1:K}\, (1\text{-}\beta)/(K\text{-}1), g)\, d\gamma$$

$$- \int_{z_{bk}\,(1-\beta)/(K\text{-}1)}^{\beta\, z_{bk}} \frac{\partial S_v}{\partial \gamma}(z_a/K, z_{b,1:k-1}\, (1\text{-}\beta)/(K\text{-}1), \gamma, \beta\, z_{b,k+1:K}, g)\, d\gamma\}$$

$$- S(z, g). \qquad \text{(A5)}$$

Given $S_v(z, g) = S(z, g) - S_f(z, g)$, it follows that (A5) can be written as

$$D \equiv D_C + D_R + D_V + D_A,$$

where

$$D_C \equiv \sum_{k=1}^{K-1} \{ \int_{z_{bk}\,(1-\beta)/(K\text{-}1)}^{\beta\, z_{bk}} \frac{\partial S_v}{\partial \gamma}(z_a/K, z_{b,1:k-1}\, (1\text{-}\beta)/(K\text{-}1), \gamma, z_{b,k+1:K}\, (1\text{-}\beta)/(K\text{-}1), g)\, d\gamma$$

$$- \int_{z_{bk}\,(1-\beta)/(K\text{-}1)}^{\beta\, z_{bk}} \frac{\partial S_v}{\partial \gamma}(z_a/K, z_{b,1:k-1}\, (1\text{-}\beta)/(K\text{-}1), \gamma, \beta\, z_{b,k+1:K}, g)\, d\gamma\},$$

$$D_R \equiv K\, S(z/K, g) - S(z, g),$$

$$D_V \equiv S(z_a/K, \beta\, z_b, g) + (K\text{-}1)\, S(z_a/K, z_b\, (1\text{-}\beta)/(K\text{-}1), g) - K\, S(z/K, g),$$

and

$$D_A = \sum_{k=1}^{K} S_f(z_a/K, \beta\, z_{bk}, z_{b\backslash bk}\, (1\text{-}\beta)/(K\text{-}1), g)$$

$$- S_f(z_a/K, \beta\, z_b, g) - (K\text{-}1)\, S_f(z_a/K, z_b\, (1\text{-}\beta)/(K\text{-}1), g).$$

<u>Lemma 1</u>: For any $k \in (0, 1)$,

$$S(k\, z, g) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} k\, S(z, g) \text{ under } \begin{Bmatrix} \text{DRTS} \\ \text{CRTS} \\ \text{IRTS} \end{Bmatrix}.$$

<u>Proof</u>: By definition, the technology Z exhibits increasing returns to scale (IRTS), constant returns to scale (CRTS) or decreasing returns to scale (DRTS) if $\alpha\, Z \subset Z$, $\alpha\, Z = Z$, or $\alpha\, Z \supset Z$, respectively, for all $\alpha > 1$. Let $k \in (0, 1)$. Consider the case where there is a γ satisfying $(k\, z_a - \gamma\, g, k\, z_b) \in Z$. Then

$$S(k\, z, g) = \min_\gamma \{\gamma: (k\, z_a - \gamma\, g, k\, z_b) \in Z\},$$

$$= k \min_\delta \{\delta: (z_a - \delta\, g, z_b) \in (1/k)\, Z\}, \text{ where } \delta = \gamma/k,$$

$$\begin{Bmatrix} < \\ = \\ > \end{Bmatrix} k\, S(z, g) \text{ when } (1/k)\, Z \begin{Bmatrix} \supset \\ = \\ \subset \end{Bmatrix} Z, \text{ i.e., under } \begin{Bmatrix} \text{DRTS} \\ \text{CRTS} \\ \text{IRTS} \end{Bmatrix}.$$

<u>Lemma 2</u>: If the set Z is convex, the shortage function S(z, g) is convex in z.

<u>Proof</u>: Consider any two netput vectors $z \in R^{n+m}$ and $z' \in R^{n+m}$. First assume that S(z, g) and S(z', g) are finite. It follows that $(z - S(z, g)\, g) \in Z$ and $(z' - S(z', g)\, g) \in Z$. Let $z'' = \theta\, z + (1\text{-}\theta)\, z'$, for any scalar $\theta \in [0, 1]$. If the set Z is convex, it follows that

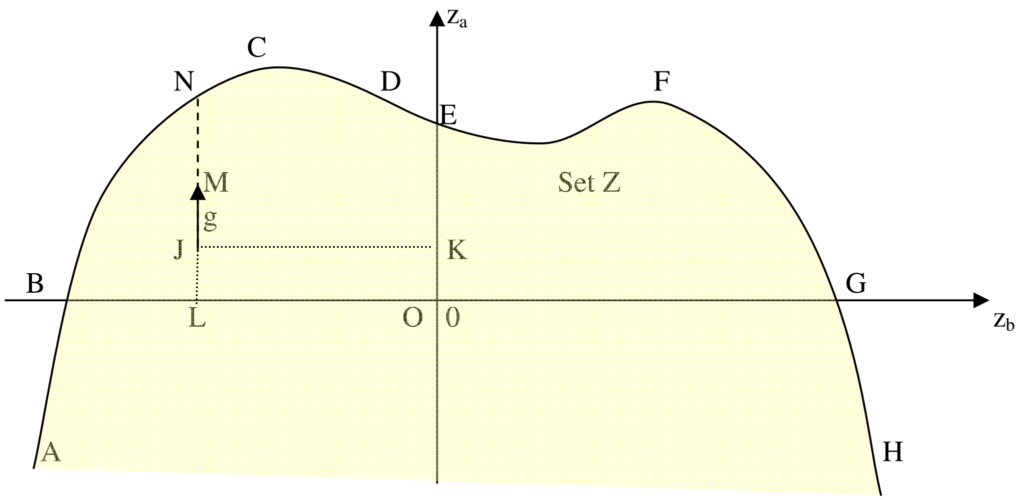
$$[z'' - \theta\, S(z, g)\, g - (1\text{-}\theta)\, S(z', g)\, g] \in Z.$$

The shortage function being defined as a minimum in (1), this yields

$$S(z'', g) = S(\theta z + (1-\theta) z', g) \leq \theta S(z, g) + (1-\theta) S(z', g).$$

Second, consider the case where S(z, g) and/or S(z', g) are infinite. Then, the above inequality always holds. Thus, the function S(z, g) is convex in z.

Figure 1: An illustration of the technology



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