Estimating a Demand Function
for Poultry Litter

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Abstract

Excess poultry litter could be a sustainable source of crop nutrients outside of nutrient-saturated regions if crop farmers are willing to utilize it. Using nearly 150 observations of actual poultry litter purchases in Oklahoma, Arkansas, and Missouri we estimate a demand function for poultry litter produced in northwest Arkansas.

Keywords: poultry litter, factor demand estimation, econometrics, northwest Arkansas

This paper is a draft and includes preliminary results. Please contact the author for an updated version of this paper.

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Introduction

In production theory, the solution to the optimization problem faced by the producer yields the factor demand functions (depending on the problem, other type of functions can also be derived). Factor demand functions, also referred to as input demand functions, are derived taking into account a production function, which we can also designate as a technology, and can be classified into three types: (1) constant cost factor demands, (2) conditional factor demands, and (3) general case factor demands. The first type can be obtained when we assume that the producer maximizes output subject to costs. The resulting constant cost factor demand is a function of input prices and total cost, as indicated by the name; this is similar to the ordinary demands obtained in utility/consumer theory. The second type is obtained when we minimize cost subject to a certain output level; thus the resulting constant output (or conditional) factor demands depend on the input prices and the level of output. These are similar to the compensated demand functions in consumer theory. Finally, the third type is obtained from the profit maximization problem, which is an unconstrained problem. These are the general demand functions and they depend on the factor prices as well as the output prices.

The theory is well established and can be consulted in any microeconomics book (for example see Varian). However, the empirical estimation of such functions is sometimes problematic. In order to qualify as a demand function, a function must conform to certain restrictions on the parameters that result from the assumed production function (Beattie and Taylor). Because many times the production function is discontinuous or very nonlinear, it is also common for the mathematical derivation of the demand function to be untreatable. Other

times the demand function, which is nonlinear in the parameters, may be rather difficult to estimate because of the nonlinearity itself (ibid.) or because of the quality and availability of data.

For the purpose of this paper we will estimate the general demand functions of poultry litter. First we will develop the theoretical background for a function to qualify as a demand function. We will do this assuming two different technologies: a Cobb-Douglas production function and a quadratic production function. For the mathematical derivation we will borrow extensively on Beattie and Taylor. We will spend some time addressing estimation procedures and previous work done in the field. We will then describe the idiosyncrasies of poultry litter as an input and how the data set came to be. The results of our estimation will follow along with some discussion of estimation difficulties. Finally we will provide a brief summary and some concluding remarks.

Theoretical Framework

We assume the producer faces an optimization problem such that profit is maximized with respect to the production factor amounts. In the case of a one output, two factors of production we write it as

$$(1) \qquad \max_{x_1, x_2} \pi = py - r_1 x_1 - r_2 x_2,$$

where π is profit, p is output price, r_1 and r_2 are the costs of the production factors x_1 and x_2, and y is crop yield such that $y = f(x_1, x_2)$ and this relationship is ideally concave. The first production technology we will consider is that illustrated by the Cobb-Douglas production function. This is a popular function because the mathematical derivation is rather straight forward. However many authors, such as Beattie and Taylor, caution that it may not represent

reality all that well as it assumes complementary production factors. The general form of the Cobb-Douglas production function is described in equation 2,

(2) $y = Ax_1^{b_1} x_2^{b_2}$,

where A, b_1, and b_2 are parameters. For strict concavity to occur the following restrictions apply:

$0 < b_1 < 1$, $0 < b_2 < 1$, $0 < b_1 + b_2 < 1$ and $A > 0$.

The second production function we will consider is the quadratic which takes the form

(3) $y = c_0 + c_1 x_1 + c_2 x_2 + 0.5 d_1 x_1^2 + 0.5 d_2 x_2^2 + d_3 x_1 x_2$,

where the linear parameters are $c_0, ..., c_2$ and the nonlinear parameters are $0.5 d_1, 0.5 d_2$, and d_3; the parameters of the quadratic terms are halved for ease of computation as is done in Beattie and Taylor. The appeal of the quadratic function is that it is a second-order Taylor series approximation to any nonlinear function and it can be used for factors that are competitive, complementary or independent. For this production function to be strictly concave, thus ensuring a global optimum, $d_1 d_2 > d_3^2$, $d_1 < 0$, $d_2 < 0$, $c_1 > 0$, and $c_2 > 0$. If $d_3 < 0$, the factors are competitive; if $d_3 = 0$, the factors are independent and if $d_3 > 0$, the factors are complementary.

Estimating a Factor Demand Function for the Cobb-Douglas Technology

The mathematical derivation of the factor demand function assuming the Cobb-Douglas technology is a trivial problem commonly used in academic examples. From the first order conditions of the optimization problem we can derive the expansion path that is

(4) $x_2 = \left(r_1 b_2 / r_2 b_1 \right) x_1$.

After introducing the expansion path in each of the first order conditions and solving for each of the factor amounts, we obtain the factor demands that are functions of the factor prices as well as the output price, as can be seen in equations 5 and 6:

(5) $x_1(r_1,r_2,p) = (Ap)^{\frac{1}{1-b_1-b_2}} (b_1/r_1)^{\frac{1-b_2}{1-b_1-b_2}} (b_2/r_2)^{\frac{b_2}{1-b_1-b_2}}$

(6) $x_2(r_1,r_2,p) = (Ap)^{\frac{1}{1-b_1-b_2}} (b_1/r_1)^{\frac{b_1}{1-b_1-b_2}} (b_2/r_2)^{\frac{1-b_1}{1-b_1-b_2}}$.

The above factor demands can be easily made linear by taking a double-log transformation, which yields in the case of $x_1(r_1,r_2,p)$

(7) $\ln x_1(r_1,r_2,p) = \alpha_0 + \alpha_1 \ln p + \alpha_2 \ln r_1 + \alpha_3 \ln r_2$

where $\alpha_0 = \left[\ln A + (1-b_2)\ln b_1 + b_2 \ln b_2\right]/(1-b_1-b_2)$, $\alpha_1 = 1/(1-b_1-b_2)$,

$\alpha_2 = (b_2-1)/(1-b_1-b_2)$, and $\alpha_3 = (-b_2)/(1-b_1-b_2)$. We could similarly rewrite the expression for $\ln x_2(r_1,r_2,p)$ as

(8) $\ln x_2(r_1,r_2,p) = \gamma_0 + \gamma_1 \ln p + \gamma_2 \ln r_1 + \gamma_3 \ln r_2$

where $\gamma_0 = \left[\ln A + (1-b_1)\ln b_1 + b_1 \ln b_2\right]/(1-b_1-b_2)$, $\gamma_1 = 1/(1-b_1-b_2)$,

$\gamma_2 = (b_1-1)/(1-b_1-b_2)$, and $\gamma_3 = (-b_1)/(1-b_1-b_2)$. The restrictions on the parameters in terms of the αs and γs can be derived from the restrictions on the parameters A and bs and are $\alpha_1 > 1$,

$\alpha_2 < -1$, $\alpha_3 < -1$, $\gamma_1 > 1$, $\gamma_2 < -1$, $\gamma_3 < -1$, $\alpha_2 + \alpha_3 = \gamma_2 + \gamma_3 = -\alpha_1 = -\gamma_1$, $-1 < \alpha_3/\alpha_1 < 0$,

$-1 < \gamma_3/\gamma_1 < 0$, and finally $-1 < (\alpha_3 + \gamma_3)/\alpha_1 < 0$. The intercept parameters, α_0 and γ_0, are not restricted because if $0 < A \leq 1 \Leftrightarrow (\alpha_0 < 0 \wedge \gamma_0 < 0)$ and if $A > 1 \Leftrightarrow (\alpha_0 \in \mathbb{R} \wedge \gamma_0 \in \mathbb{R})$.

Estimating a Factor Demand Function for the Quadratic Technology

The derivation of the factor demand using a quadratic production function is more cumbersome than the previous derivation but following the guidelines in Beattie and Taylor it can be easily achieved using matrix algebra. Using matrices the profit function for this technology can be written as

(9) $\pi = p\left(c_0 + \mathbf{C'X} + 0.5\mathbf{X'DX}\right) - \mathbf{r'X}$

where $\mathbf{C'} = \begin{bmatrix} c_1 & c_2 \end{bmatrix}$, $\mathbf{X'} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} d_1 & d_3 \\ d_3 & d_2 \end{bmatrix}$, and $\mathbf{r'} = \begin{bmatrix} r_1 & r_2 \end{bmatrix}$. The first order conditions of the optimization problem as described in equation 1 can be written as

(10) $\mathbf{C} + \mathbf{DX} = (1/p)\mathbf{r}$

Solving for the *x*s, we obtain the factor demand equations that are

(11) $\mathbf{X}^* = \mathbf{D}^{-1}\left[(1/p)\mathbf{r} - \mathbf{C}\right]$

and that equivalently can be written algebraically as

(12) $x_1\left(r_1, r_2, p\right) = \dfrac{c_2 d_3 - c_1 d_2}{d_1 d_2 - d_3^2} + \dfrac{d_2}{d_1 d_2 - d_3^2}\left(\dfrac{r_1}{p}\right) - \dfrac{d_3}{d_1 d_2 - d_3^2}\left(\dfrac{r_2}{p}\right)$

and

(13) $x_2\left(r_1, r_2, p\right) = \dfrac{c_1 d_3 - c_2 d_1}{d_1 d_2 - d_3^2} - \dfrac{d_3}{d_1 d_2 - d_3^2}\left(\dfrac{r_1}{p}\right) + \dfrac{d_1}{d_1 d_2 - d_3^2}\left(\dfrac{r_2}{p}\right).$

Since these two functions are linear in the normalized factor prices, one could simply write them as

(14) $x_1\left(r_1, r_2, p\right) = \beta_0 + \beta_1\left(\dfrac{r_1}{p}\right) + \beta_2\left(\dfrac{r_2}{p}\right)$

and

$$(15) \quad x_2 (r_1, r_2, p) = \delta_0 + \delta_1 \left(\frac{r_1}{p} \right) + \delta_2 \left(\frac{r_2}{p} \right).$$

The restrictions on the parameters are that $\beta_1 < 0$, $\delta_2 < 0$, and $\beta_2 = \delta_1$ always; if the factors are competitive, $\beta_0 < 0$, $\beta_2 > 0$, $\delta_0 \in \mathbb{R}$ and $\delta_1 > 0$; if the factors are complementary, $\beta_0 \in \mathbb{R}$, $\beta_2 < 0$, $\delta_0 > 0$ and $\delta_1 < 0$; and finally if the factors are independent then $\beta_0 > 0$, $\beta_2 = 0$, $\delta_0 > 0$, and $\delta_1 = 0$.

Previous Work

Lavoisier's Law of Conservation of Mass simply states that nothing is created or destroyed, instead everything is transformed (the one exception to this law occurs in nuclear reactions). The problem of excess poultry litter nutrients in specific locations (the same could be said of the commercial production of meat of other animal species), particularly phosphorus, is not the result of newly produced nutrients—it is the result of a process through which large amounts of feed, basically nutrients, are transported into regions of concentrated poultry production and then transformed mainly into poultry meat and animal manure. While the nutrients in the meat get redistributed around the world, most of the nutrients in the litter remain in or near the production site. The solution to the problem must include a geographical redistribution of the poultry litter, as defended by Gollehon et al. If this redistribution occurs by utilizing the litter to fertilize crops, then the demand for fertilizers and natural gas could be reduced and soil quality could be improved beyond merely the replenishing of nutrients, such as through added organic matter. Skeptics say, among other things, that litter use is too time-consuming, nutrient level is uncertain, and farmers do not have necessary equipment to handle litter.

There has been little research done on actual demand data. Most studies that address the demand for poultry litter are based on crop needs according to US Census data or other sources. For example, Jones and D'Souza use poultry litter demand as one of the components in their model that optimizes poultry litter trading among watersheds in West Virginia. Because actual demand data was nonexistent, they used a proxy variable obtained by computing farm nutrient requirements in terms of nitrogen and phosphorus. Govidansamy and Cochran derive a demand for litter that is based on crop price, yield responses and litter transportation costs; their optimal demand reflects ideal behavior by producers but it is not based on actual demand data. Feinerman, Bosch and Pease also derive a manure demand function analytically assuming a Von Liebig production function but do not estimate its parameters from actual data. Lichtenberg and Parker looked into the economic value of poultry litter under six different alternative uses: land application as a crop fertilizer, compost, pelletization, electric power generation, cogeneration of steam and electric power, and forest fertilization. Other studies have conducted surveys that directly elicit from potential litter producers how much litter they are willing to demand. Lynch and Tjaden's study looked into forest landowners' willingness to use poultry litter as a fertilizer under different incentives. Carreira, Goodwin, and Hamm conducted a survey of potential poultry users and asked about their willingness to purchase poultry litter at different price levels.

Background and Data Sources

Benton and Washington counties, in northwest Arkansas, produce 20% of the broilers and other chicken meat sold in Arkansas, the second-ranked state in the U.S. in production, after Georgia (NASS/USDA). Recent water quality concerns in the region have triggered lawsuits (see City of Tulsa v. Tyson Foods, Inc. which was vacated due to a settlement; another pending

lawsuit was initiated by the Oklahoma Attorney General in June 2005) that may threaten local land-application of litter due to limited nutrient removal from local land by crops and pasture. A non-profit organization (Eucha/Spavinaw BMPs, Inc.) was created as a result of the settlement to facilitate the movement of excess poultry litter from the nutrient surplus area of the Eucha/Spavinaw Watershed to other locations where it can be land-applied in a sustainable manner that is advantageous to the land and crops. Another non-profit organization (BMPs, Inc.) was created to address the nutrient surplus issues of the Illinois River Watershed.

The convenience and ready availability of commercial fertilizers and their known concentrated nutrient content are some of the reasons why crop producers are using relatively little animal manures to fertilize their land. Manure, such as poultry litter, is commonly referred to as animal waste, a derogative term that incorrectly conveys the idea that this byproduct has no value. Increasing prices of natural gas, a typical input in nitrogenous commercial fertilizers, could make poultry litter more attractive to crop farmers. In Northwest Arkansas, poultry growers relied on it for over 40 years as a source of nutrients and organic matter for their pasture lands so that cattle could be produced on otherwise poor land. However, as mentioned above, recent concerns may threaten local land-application of most litter produced in Northwest Arkansas. It is unlikely that the excess-nutrient situation will be repeated elsewhere because we currently have sufficient knowledge (Sharpley et al.) of the idiosyncrasies of phosphorus movement in the soil.

The current tournament-contract structure pervasive in the American poultry industry is the result of over 40-years of vertical integration, specialization, and geographical concentration. The ultimate results of this process benefited the American consumer through lower poultry prices and greater product variety and quality. In crude terms, the industry is composed of

integrators who provide chicks, feed, and medication, and contractors who provide labor, housing, and operating inputs. The broiler industry, worth over $20.4 billion (NASS/USDA), faces two major threats: foreign competition and environmental concerns regarding poultry litter management. Brazil is the number one exporter of poultry in the world, a lead that traditionally had been held by the U.S. Brazil's comparative advantage relies in lower production costs in areas such as labor and feed.

If the litter problem is not solved in a manner advantageous to the region, it is not farfetched to imagine a relocation of much of the Northwest Arkansas poultry industry, possibly abroad, considering that the US poultry industry has been losing competitiveness to the Brazilian industry. Transporting the litter to alternative locations where it can be applied to crops with better nutrient removal rates could be a solution to the problem (Gollehon et al), but litter adoption by crop farmers is not widespread because of litter's bad reputation.

The data contain information on 219 actual poultry litter purchases which occurred between March of 2005 and February of 2006 and result from the work of BMP, Inc. and Eucha/Spavinaw BMP, Inc. The variables recorded include city and state of source and destination of poultry litter (Arkansas, Oklahoma, and Missouri), tonnage purchased, price per ton, type of land where litter was applied (cut land or land already in production), crop where litter was applied, irrigated or non-irrigated land, number of acres where litter was applied, application rate of litter, type of application (surface of incorporation), whether or not additional commercial fertilizer was applied. About 148 respondents mentioned forage or pasture as the destination of the litter; other crops mentioned included corn, rice, wheat, soybeans, and cotton. For the purpose of this paper, we will focus on estimating the demand for litter to be used in the fertilization of pasture land.

We obtained monthly prices from data published by NASS-USDA; specifically we collected the price on pasture (hay), cattle, corn, rice, soybean, wheat, and chemical fertilizer (we computed the price for fertilizer from the mixed fertilizer composite index). Whenever possible we used prices for the south central United States; if these were unavailable then the national price was used. Because pasture land is often used to produce cattle, we obtained the prices for cattle as well as the price for hay to be used as alternative output prices in the estimation.

Preliminary Results

The preliminary parameter estimates of the demand function regressions are presented in Appendix I. The results of the final models will be presented at the conference. The parameter estimates of the demand function assuming a Cobb-Douglas production technology are presented in Table 1. This is a double-log model where the natural log of the amount of litter purchased is regressed against the natural logs of cattle beef price,[1] poultry litter price, and chemical fertilizer price. For the estimation of this model we used restricted least squares because, as we had seen earlier, $\alpha_2 + \alpha_3 = -\alpha_1$. The estimate of the Lagrangean associated with the restriction is not statistically significant, which indicates that we fail to reject the restricted model. There is some other evidence supporting this model as the R^2 is 0.2035 and the F-test of model significance has a p-value of less than 0.0001. The signs of the parameters are in accordance to the theoretical framework developed previously but a joint F-test does not support the hypothesis that $\alpha_2 < -1$ and $\alpha_3 < -1$ (p-value less than 0.0001). The latter conclusion indicates that at least one of the parameters violates the assumption; in this case the parameter estimate for α_2, corresponding to the own price elasticity, is not statistically significant. We conducted a White's test of

heteroskedasticity and obtained a test statistic of 2.12, which is less than the critical Chi-square value of 12.59 thus we failed to reject homoskedasticity.

The basic assumption for a Cobb-Douglas technology is that the two inputs are complementary. In the case of fertilizer and manure, it can be argued that the relationship between the two can be complementary or competitive. The nutrients in poultry litter do not provide exactly that nutrients required by the plant, thus many times either some nutrients must be oversupplied if the producer only relies on the manure and not enough soil nutrients are available or some chemical fertilizer must be applied to supply the remaining nutrients required by the plant. From the data collected by BMPs, Inc and Eucha-Spavinaw BMPs, Inc, at least two producers indicated that chemical fertilizer was purchased in addition to the litter. Our model results indicate that the data show some support in favor of a demand function derived from a Cobb-Douglas production technology.

The parameter estimates of the demand function assuming a quadratic function technology are presented in Table 2. The function is more flexible when it comes to the relationship between inputs but as we will see, the statistical fit of the data was very poor and basic assumptions were not upheld. The dependent variables in this model were the ratio of the input prices to the output price. The R^2 of the regression is 0.0802 and the F-test of model significance fails to reject the hypothesis that the dependent variables have no explanatory power. The parameter estimate for the own price is not statistically significant and violates the assumption that $\beta_1 < 0$, undermining a basic requirement for a demand function. Thus the data show no evidence that the demand for poultry manure is derived from a quadratic production technology.

Conclusions

In this paper we attempted to estimate a poultry litter demand function based on actual transaction data. We investigated two different production technologies that yielded two different functional forms for the demand curves. In the Cobb-Douglas technology we used a restricted double-log model and obtained parameter estimates that closely conform to those expected. In the quadratic technology the parameter estimates violated basic principles of demand theory. Thus we conclude that there is some statistical evidence to support that the demand for poultry litter follows a functional form derived from a Cobb-Douglas technology.

Notes:

1. We also used hay prices instead of cattle beef prices as one of the explanatory variables but found that the resulting model violated more restrictions than the one presented here, particularly the sign for α_2 was positive.

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Appendix I. Tables

Table 1. Regression Parameters Estimated in SAS Proc Reg for the Double-Log Restricted Model of Poultry Litter Demand Assuming a Cobb-Douglas Production Function ($R^2 = 0.2035$)

Variable	Parameter	Estimate	Standard Error	P-value
Intercept	α_0	18.0276	2.6102	<0.0001
ln P	α_1	10.2716	1.7263	<0.0001
ln r_1	α_2	-0.0690	0.0446	0.8774
ln r_2	α_3	-10.2027	1.8150	<0.0001
Restriction: $\alpha_2 + \alpha_3 = -\alpha_1$	Lagrangean	-0.3078	0.2529	0.2248

Note: (i) The dependent variable was the natural log of the amount of poultry litter purchased (tons), P is the price of cattle beef ($/cwt), r_1 is the price of poultry litter ($/ton) and r_2 is the price of chemical fertilizer ($/ton).

(ii) The White test for heteroskedasticity has a Chi-square test value of 2.12 smaller than the critical value of 12.56.

Table 2. Regression Parameters Estimated in SAS Proc Reg for the Model of Poultry Litter

Demand Assuming a Quadratic Production Function ($R^2 = 0.0802$)

Variable	Parameter	Estimate	Standard Error	P-value
Intercept	β_0	8939.6083	2390.4499	0.0003
r_1/P	β_1	2009.7131	3171.0647	0.5273
r_2/P	β_2	-2460.8998	707.5829	0.0007

Notes: The dependent variable was the quantity of poultry litter purchased (tons), P is the price

of cattle beef ($/cwt), r_1 is the price of poultry litter ($/ton) and r_2 is the price of chemical

fertilizer ($/ton).