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**Modeling the Effects of Mitigating Apple Maggot Spread into Apple  
Production Region of Washington State\***

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# Modeling the Effects of Mitigating Apple Maggot Spread into Apple Production Region of Washington State

Zishun Zhao, Thomas Wahl, and Thomas Marsh

## **Introduction:**

Biological invasions are occurring worldwide with an increasing pace.

Increased trade between countries and advances in transportation technologies, which allow for fast transoceanic delivery, have contributed to the risk of introduction of invasive species. Imports of goods such as fresh produce, animals, and timber and its derivatives (such as solid wood packaging materials) represent important pathways for the introduction of IS. Increase in the rate of population growth and movement of people, as well as alteration of the environment, have been associated with an increase in the rate of introductions and risk associated with “biotic invaders” in the last 40 years (Pimentel et al, 2000).

Even though only 16% of the non-indigenous species (NIS) are considered to have a high impact as pests (OTA, 1993), they could cause great harm to a country’s economy, especially in the agricultural sector. There are several examples of the devastating effects of invasive species. Grape Phylloxera (*Daktulosphaira vitifoliae*) is an aphid native to North America that between 1865 and 1875 caused the destruction of a large portion of the wine grape industry in Europe. The damage caused by this insect completely modified growing techniques and wine grape distribution around the world. This case has also an historical importance; the destructive effects of this pest brought several European countries together to the “International Convention on Measures to be taken against *Phylloxera vastatrix*” signed in 1878, the first international agreement on the spread of plant pests (Ebbel, 2003). Other more recent examples of important IS include Mediterranean fruit fly or Medfly (*Ceratitidis capitata*), gypsy moth (*Lymantria dispar* L.), and citrus canker. considered one of the world’s most destructive agricultural pests. Rough assessments

of the economic losses due to invasive species in the U.S. range from tens to hundreds of billions of dollars per year. Pimentel et al., (2000) estimated that NIS in the U.S. cause significant environmental damage and economic losses of approximately \$137 billion per year, \$16 billion in the agricultural sector.

Government agencies concerned with invasive species usually have two approaches to managing them: A) decisions are taken to stop potential invasive species before they enter the country (*ex-ante*) or B) decisions are made as to how to control invasive species after they have arrived (*ex-post*) (Maguire, 2001). In 1999, the United States (U.S.) spent an estimated \$590 million to prevent and control IS (Mumford, 2002). Preventative measures range from inspection and quarantine to completely block the inflow of pathways and mitigation strategies range from slowing the spread of an IS to complete eradication. To select the optimal alternative policy and to allocate limited resource among different locations and on different time, policymakers need comprehensive economic analysis that fully considers the characteristics of a biological invasion and their impacts on ecological services.

In this paper, we develop a simulation framework for analyzing economic impacts of an IS on perennial fruit production and consumption that can be used to evaluate IS management alternatives. Fruit production heavily depends on ecological services provided by the native habitat and hence particularly vulnerable to biological invasions as shown in some of the examples. As the fourth largest fruit producer in the world, the US fruit industry generated \$12.6 billion in revenue in the year 2000 (Fruit and Tree Nut Yearbook 2002). The US also imported 13 billion pounds of fresh fruit and exported 5.7 billion pounds in 2000. An IS outbreak in the fruit production sector could cause great economic consequences. The long life cycle of perennial fruit trees makes fruit supply irresponsive to market price in the short term. Sudden shocks to the production system can cause wide fluctuations in fruit markets. The long life cycle

also carries along the effects through years like the baby boom effects observed in human society. To fully evaluate the potential economic impacts of an IS, we propose a dynamic simulation framework for perennial fruit trees that integrate population dynamics of fruit trees and dissemination dynamics of the IS. An implementation for apple production with apple maggot invasion is used to illustrate the use of this model in evaluating policy alternatives.

### **Conceptual Framework**

Conceptually, a fruit producer maximizes his expected profit subject to the constraint of population dynamics. Yields and production costs are determined by the production system and production environment. In the context of a biological invasion, a producer's production environment is determined by whether or not his orchard is infested with the particular pest or disease under consideration. The number of orchards infested is determined by the dynamic process of dissemination as well as mitigation strategies that can modify the process. Fruit products are sold on domestic and foreign markets to realize profit. The domestic producers also compete with imported fruits on domestic markets. It is important to include international trade in the analysis because as important pathways for spreading IS they are subject to changes when IS outbreaks occur. These components of the conceptual model are explained in detail in the following sessions.

### ***Population Mechanics and Production***

The process of perennial fruit production consists of a productive population that evolves according to its biological features and the producer's decision to adjust the population. There is a long lag between investment decision and revenue generation due to the time required for a tree (or group of trees) to reach its productive stage. Most fruit species will require a few years before they can start efficiently producing fruit. This time period will vary depending on several factors such as

species, rootstocks, density, climate condition, etc. Thus, keeping track of the total area of trees of different ages is essential to generate the correct total supply. We differentiate the stock of productive planting area by tree age. Each age group evolves according to the following equations:

$$(1.a) \quad K_{t+1}^{j+1} = K_t^j - RM_t^j$$

$$(1.b) \quad K_t^0 = NP_t$$

where  $K_t^j$  is the total area of age  $j$  trees at time  $t$ ,  $RM_t^j$  is the area to be removed from the stock of age  $j$  trees at time  $t$ , and  $NP_t$  is the area of newly planted trees. Any planting area that is not chosen to be removed during the current period progresses into the stock of the next age group.

We assume that fruit is the only product of the industry. Each year's fruit production is given by

$$(2) \quad FP_t = \sum_{j=0}^u y^j K_t^j$$

where  $FP$  is the total fruit production,  $y^j$  is the yield per acre of age  $j$  trees and  $u$  is the upper bound on the productive age. While we, for simplicity, treat the yields and the corresponding input requirements as exogenously given, they could be the optimal yields generated by a separate simulation model such as FRUPRO (Winter 1986), which relates various inputs to the yields of a particular orchard (with trees of the same age). In our model, we only consider the input requirements and the related yield changes introduced by a particular invasive species. The total production supplies are for both export and domestic markets. The domestic supply is then given by:

$$(3) \quad SD_t = FP_t + M_t - E_t$$

where  $SD_t$  is the total domestic supply and  $E_t$  and  $M_t$  are exports and imports respectively.

***Inventory Update Policy:***

For our analysis we assume that the fruit grower has one single objective: to maximize the total present value of all future profits. Under the grower's control are the addition to and subtraction from the productive stock of trees (orchard); effective management of production inputs, such as tree density at establishment of the orchard, fertilizer, pesticides, etc; and the selective administration of labor for the different orchard operations such as pruning, fruit thinning and harvest. We also assume that the grower's only source of revenue is from fruit sales. When the orchard management system is exogenously given or pre-determined, the representative producer's problem with regard to a particular block of land with or without trees is essentially an investment decision. If the total expected and discounted present value of cash flow of the best alternative management system is higher than zero, then it's profitable to plant an additional block of trees or keep the trees on that block. We assume that the industry as a whole is facing diminishing marginal returns and increasing marginal costs because as more trees are planted less productive land and/or land that has better alternative use (opportunity cost) has to be used. The total area of trees is then determined by the marginal producer whose expected net profit is exactly zero. To accommodate various boundary conditions, we model the inventory update problem as a mixed complementary problem.

Fruit producers make their decision base on their expected profit. The only source of revenue is from selling fruit. Let  $E_t(P_{t+l})$  be the price expectation of time  $t+l$  based on information available at time  $t$ , the total expected revenue is

$\sum_{i=0}^{\infty} \left[ b^i \left( E_t(P_{t+i}) \sum_{j=0}^u y^j K_{t+i}^j \right) \right]$ . Total expected cost consists of capital cost, labor cost,

material cost, planting cost, and removal cost. For simplicity, we use three cost terms including planting cost for new trees  $PC$ , maintenance cost  $MC$ , and removal cost  $RC$ . The planting cost includes preparation of land, planting, and purchasing necessary equipment. We assume that the planting cost is increasing in acreage of new plantings, given by

$$(4) \quad PC_t = pc(K_t^0), pc' > 0$$

It is also assumed that the maintenance cost is increasing in total acreage to accommodate the diminishing marginal return and increasing marginal cost, expressed as

$$(5) \quad MC_t^j = mc_j \left( \sum_{j=0}^u K_t^j \right), mc_j' > 0, 1 \leq j \leq u.$$

Removal cost  $RC$  is assumed to be fixed for all time. The total cost over the planning

horizon is  $\sum_{i=0}^{\infty} \left[ b^i \left( PC_{t+i} NP_{t+i} + \sum_{j=1}^u (MC_{t+i}^j K_{t+i}^j + RC \cdot RM_{t+i}^j) \right) \right]$ .

The representative producer's problem is to

$$\begin{aligned} & \underset{NP_{t+i}, RM_{t+i}^j}{MAX} \left[ \sum_{i=0}^{\infty} b^i \left( E_t(P_{t+i}) \sum_{j=0}^u (y^j K_{t+i}^j) - \left( PC_{t+i} NP_{t+i} + \sum_{j=1}^u (MC_{t+i}^j K_{t+i}^j + RC \cdot RM_{t+i}^j) \right) \right) \right] \\ (6) \quad & \text{s.t.} \quad (a) K_{t+1}^{j+1} = K_t^j - RM_t^j \\ & \quad (b) K_t^0 = NP_t \\ & \quad (c) NP_t \geq 0 \\ & \quad (d) 0 \leq RM_t^j \leq K_t^j \\ & \quad (e) RM_t^u = K_t^u \end{aligned}$$

Under the assumption of perfect competitiveness, i.e. the representative producer takes price and cost as given, the constrained optimization problem can be solved to yield the following Kuhn-Tucker conditions



$$(7.a) \quad \sum_{l=1}^{u-j} b^l (E_t(P_{t+l})y^{j+l}) - \sum_{l=1}^{u-j} b^l MC_t^{j+l} - b^{u-j} RC \geq -RC \quad \perp \quad RM_t^j \geq 0 \quad \forall j \geq 1$$

$$(7.b) \quad \sum_{l=1}^u b^l (E_t(P_{t+l})y^l) - \sum_{l=1}^u b^l MC_t^l - b^u RC \leq PC \quad \perp \quad NP_t \geq 0$$

For ease of discussion, let  $PVI_t^j = \sum_{l=1}^{u-j} b^l (E_t(P_{t+l})y^{j+l})$  denote the present value of all

cash inflow generated by an acre of fruit trees and  $PVO_t^j = \sum_{l=1}^{u-j} b^l MC_t^{j+l} + b^{u-j} RC$

denote the present value of all cash outflow for the same acre, the above optimality conditions can be expressed as

$$(8.a) \quad PVI_t^j - PVO_t^j \geq -RC \quad \perp \quad RM_t^j \geq 0 \quad \forall j \geq 1$$

$$(8.b) \quad PVI_t^0 - PVO_t^0 \leq PC \quad \perp \quad NP_t \geq 0$$

It is clear that the optimality conditions are essentially a set of investment decisions.

The first condition says that if leaving an acre of trees on the block of land is more profitable than removing it now, then none of the age  $j$  trees should be removed; on the other hand if some but not all of the trees are removed, then it must be true that leaving the acre of trees as they are is as profitable as removing them now; if all the trees are removed, keeping the trees must be less or equally profitable as removing them. The second condition deals with new plantings. If a positive amount is planted, then it must be true that the expected profit from planting new trees is zero; if the profit from planting new trees is negative, then no new trees would be planted. The set of optimality conditions can be formulated as a complementary programming problem and solved using GAMS.

### ***Markets and Market-clearing Prices:***

Fruit markets are where the producers obtain information to form their expectations, and where expected production profits can be realized. To capture the

potential impact of an IS outbreak on market environment in a broad spectrum, both domestic and international markets are included. Domestic demand for fruit is defined using inverse demand relationships. Let  $D_t$  be the demand for meat,  $PMeat_t$  be the price, and  $IN_t$  be the income. Domestic demand for fruit in price dependent form can be expressed as:

$$(9) \quad P_t = d(D_t, IN_t).$$

Assuming that the exchange rate is fixed over time, the export demand for fruit is a function of the domestic price plus tariff or the tariff equivalent of trade barriers

$$(10) \quad E_t = ed(P_t + TF_t)$$

The import demand for foreign fruit products, assuming that the imported fruit and domestically produced fruit are homogeneous, is also a function of the domestic price

$$(11) \quad M_t = md(P_t).$$

Assuming a perfectly competitive market, the equilibrium price is given by solving the market-clearing condition (Varian, 1992).

$$(12) \quad FP_t + M_t = D_t + E_t.$$

In general, both imports and exports can be segmented into countries or trade regions to better accommodate alternative trade policies and bilateral agreements.

In summary, equations (1)-(12) completely describe a partial equilibrium system for fruit production and consumption. The bio-economic model was kept as general as possible so that it could be adapted to model different species of perennial fruit production in an open economy. With given yields, costs, demand equations and starting inventories for a specific fruit species and corresponding production system, an simulation model can be implemented to simulate production and consumption responses to various shocks. Coupled with an IS dissemination

mechanism, the simulation model can be used to evaluate the potential economic impact of an IS outbreak and analyze the benefit of alternative mitigation policies.

### **IS Introduction and Dissemination**

Upon the establishment of an invasive pest species, production is differentiated into infested and non-infested according to the change in production environment. Production in the infested area is assumed to be more costly to reflect the need for controlling the pest. As the pest spreads, inventory of trees makes the transition from non-infested to infested status. Production cost and yield are modified accordingly. Transition from non-infested to infested is dictated by the speed of spread.

To model the spread of an invasive pest species, we choose to use the population front advance model proposed by Sharov and Liebhold (1998). In this model, the population front of a pest species, or the boundary between infested and non-infested area, advances linearly at a constant speed. The speed at which population front advances is governed by

$$(13) \frac{cn_0V}{r^2} \left[ \exp\left(\frac{r}{V}\right) - \frac{r}{V} - 1 \right] = K$$

where  $c$  is the rate at which new colonization is established,  $K$  is a colony's carry capacity,  $n_0$  is the initial number of individuals in a colony,  $r$  is the intrinsic growth rate, and  $V$  is the relative speed of population front advance<sup>1</sup>. This model provides a linkage between mitigation effort and the spread speed. The population spread can be slowed/stopped through reducing/preventing the establishment of new colonies. Although analytical solution is not possible due to the complexity of this model, comparative static analysis can be made.

### **Implementation in Apple Production and Economic Evaluation of Apple Maggot Spread**

Apple maggot is a major apple pest that's native to North America. Untreated orchards that are infested with apple maggot could lose 30-70% of total production. Historically, the pest has been affecting apple production in the east coast. It was introduced to the Portland area in 1979 and began to spread to California, Washington, and Idaho. Over the years, apple maggot becomes established in all counties west of the Cascades in Washington where 2% of the state's apple production is located. In recent years, trappings of apple maggot flies have been found in Yakima and Kittitas, two of the major apple production counties. In 2004, part of Kittitas and part of Yakima were quarantined for apple maggot. Now that the natural barrier of Cascades has been penetrated, the major apple production area, which accounts for 98% of Washington's apple production, is under great threat.

The spread of apple maggot could make serious economic impact to the region. Washington State is the number 1 apple producing state in the U.S., accounting for 65 to 75 percent of all apples sold in the fresh market (Fruit and Tree Nuts Outlook, 2005). Establishment of apple maggot will raise the production cost. It is stipulated that 3 insecticide sprays need to be applied to control it, raising cost per acre by \$30-50 (Reissig 1988). The spread of apple maggot will also affect the export markets. For example, Canada requires all apple shipped to British Columbia either be certified from an apple maggot free area or undergo costly cold treatment. Loss of apple maggot free status will significantly increase exporting cost. The magnitude of this impact can be reflected in the experience with Mexico. Mexico requires all apple imports from the U.S. undergo cold treatment to prevent the introduction of apple maggot, which is estimated to be equivalent to 20-30% tariff (Krissoff et al. 1997).

### ***Implementation and Calibration***

As indicated by the different production cost that can arise upon the establishment of an invasive pest, the production is differentiated by regions

according to apple maggot status. There are a total of three regions in the implementation—Washington infested, Washington non-infested, and the rest of U.S. where the rest of U.S. is considered to be infested by apple maggot. A separate set of equations (1)-(8) is specified for each region. The transition from Washington Infested to Washington Non-infested is determined by the speed of population advance. Each region has its own set of inventories, production cost, and yields. The also assume that all products from the three regions are homogeneous for domestic production so that one domestic demand is sufficient. The total export of apple to Canada is segmented into Washington export and the rest of U.S. export to accommodate the different treatments Canada imposed according to apple maggot status

The model was calibrated to the base year 2002. Demand (domestic and foreign) elasticities are estimated using data from various sources. Production costs are derived from various publications of orchard budget forms. It is assumed that a 1% increase in total production will increase the maintenance cost by 1%, representing supply elasticity of 1 in all input factors. The planting cost is also assumed to increase by 1% when new planting is increased by 1%. Producers are assumed to form naïve expectations on prices. Initial values of inventories are extrapolated from acreage of bearing and nonbearing data published in 2002 agricultural census. The speed of population spread of apple maggot is calculated through dividing the total land area infested in Washington by 24 years (assuming liner population advance at constant speed). It is also assumed that further population advance is in a linear pattern and apple orchards are equally disperse throughout the apple maggot free area. When an acre of apple trees becomes infested, the production cost is increased by \$45 (assuming 3 sprays are needed for economic viable

production and each spray costs \$15/acre). Cost of exporting to Canada is increased by 30% if the exported product is from apple maggot infested area.

Lack of age specific inventory data is the major limitation in our study. Although the time path of equilibrium price, quantity, and welfare measures are sensitive to initial age structure of the inventories, the total welfare measure in the simulated scenarios is not sensitive.

### ***Simulation Scenarios and Results***

A base scenario, where apple maggot spread resumes its historical speed, is first simulated for comparison with policy scenarios. Under the assumption in this scenario, all apple production in Washington State will be infested in 34 years. A total of 8 policy scenarios, using linear reductions in spread speed to represent increasing effort in mitigation of apple maggot, are simulated.

Figure 1. Domestic apple price response to apple maggot spread in Washington State

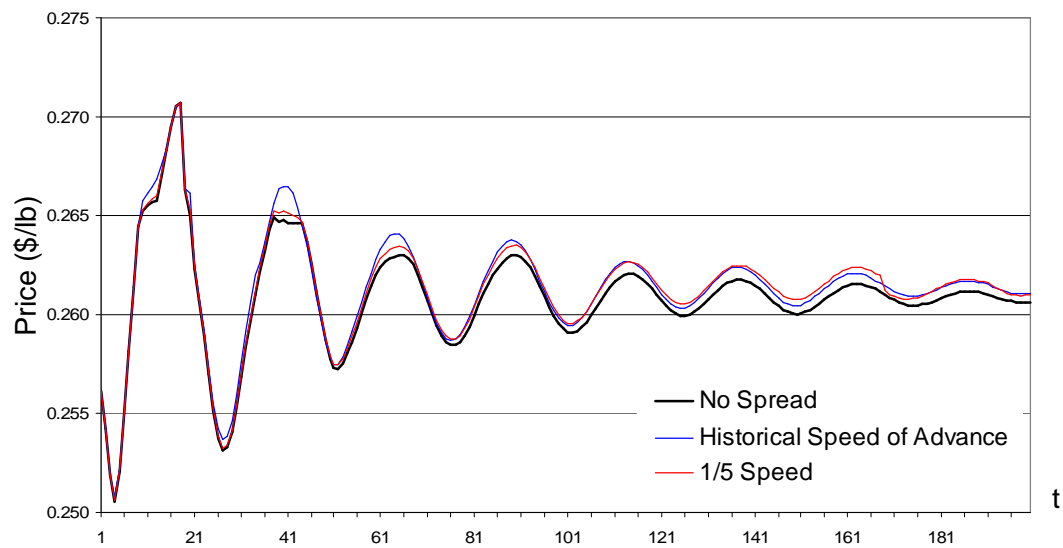


Figure 1 shows the domestic apple price responses for three different scenarios: no spread, spread at historical speed, and spread at 1/5 of historical speed. In all of the three scenarios, equilibrium price displays a cyclical pattern and converge to a long-run equilibrium. When apple maggot is allowed to spread, the long run equilibrium

price is slightly higher than that from the no-spread scenario, reflecting the higher average production cost.

Figure 2. Welfare changes when apple maggot spread at historical speed

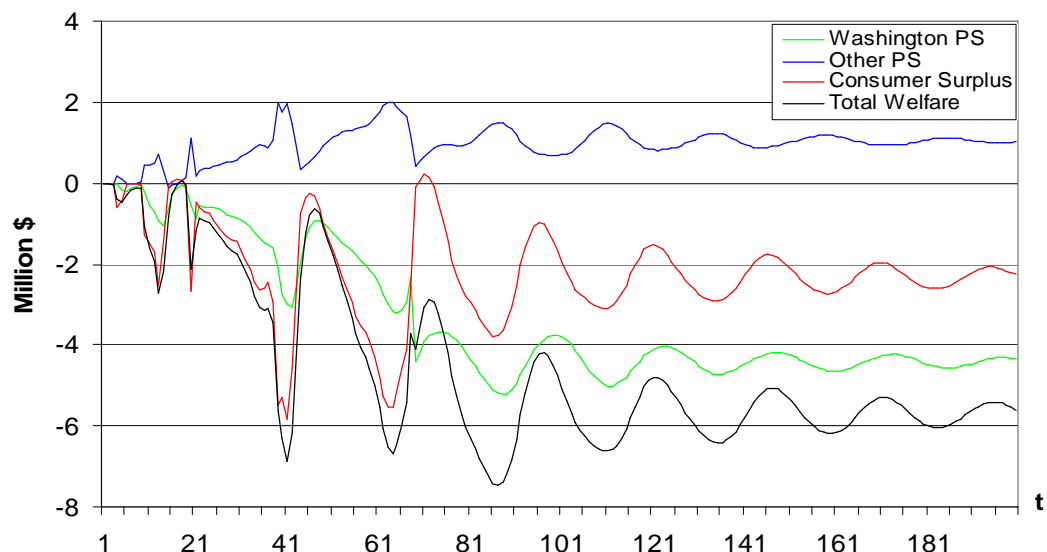


Figure 2 shows the time path of various welfare changes when apple maggot is allowed to spread at its historical speed. As apple maggot spread to affect more production area in Washington State, apple producers in Washington are worse off. Producers in the rest of U.S. are better off because they become relatively more competitive than before. And the consumers are worse off due to the higher apple price. The apple industry as a whole suffers annual loss of \$4 to 8 million.

Total present valued welfare changes for the base scenario and alternative policy scenarios are listed in table 1. As the speed of population spread reduces, total welfare loss decreases linearly. Thus the benefit from slowing the spread is increasing linearly. And the break-even annual spending on mitigation effort increases linearly.

Table 1. Welfare results

Speed of Spread	Total Welfare Loss	Benefit of Control	Break-even Annual Spending
Historical Speed V	-14.76	-	-
0.9 V	-13.47	1.30	0.13
0.8 V	-12.00	2.76	0.28
0.7 V	-10.58	4.18	0.42
0.6 V	-9.11	5.66	0.56

0.5 V	-7.61	7.15	0.71
0.4 V	-6.10	8.67	0.86
0.3 V	-4.58	10.19	1.01
0.2 V	-3.05	11.71	1.16

However, the marginal benefit of 10% speed reduction is maintained at around a constant level of \$1.52 million. The optimal mitigation level is achieved when the present-valued marginal cost of 10% speed reduction is equal to \$1.52 million.

### **Discussion and Conclusion**

The characteristics of perennial fruit production are primarily determined by the long production cycle. The long life cycle of fruit trees causes a significant delay between production decisions and actually changes in fruit supply, leaving the short- to medium-term fruit supply irresponsive to changes in fruit price and production costs. On the other hand, any production decision made is likely to be carried in the tree inventories and hence influences the fruit supply for a long time. Thus, shocks to prices, costs, and initial inventories are very likely to create long term fluctuations in fruit supply as can be reflected from the simulation of apple production. Welfare analysis that does not take into account the dynamic effects cannot capture the full potential of IS outbreak and relevant mitigation strategy.

The partial equilibrium bio-economic model we developed integrates dynamics of perennial fruit tree inventories with dynamics of invasive pest spread, which allows us to capture the dynamics of economic effects that could be caused by an outbreak of an invasive species. The equilibrium time path of prices, producer surplus, and consumer surplus can be simulated as demonstrated in the apple maggot outbreak simulations. When used in evaluating alternative mitigation strategies, these are all relevant information that can help to determine the socially optimal IS policy. In addition, the dynamic welfare distribution effects among producers from different



regions and consumers can help to determine potential financial sources for such policies and to construct government payment programs for welfare redistribution.

The use of the framework to analyze IS outbreaks was demonstrated in the simulations of apple maggot spread in Washington apple industry. While the dynamics of prices and welfare measures should be received with caution due to the fact that we had to extrapolate demographic data, the total welfare measures were shown to be not sensitive to demographic distribution of initial inventories. We arrived at the conclusion that the optimal mitigation level is achieved when the present-valued total marginal cost of 10% reduction in spread speed is equal to \$1.52 million.

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<sup>1</sup> For details of derivation and explanation, refer to Sharov, A. A. and A. M. Liebhold. 1998. "Model of Slowing the Spread of Gypsy Moth (Lepidoptera: Lymantriidae) with a Barrier Zone." *Ecological Applications*, 8(4), 1998, pp. 1170-1179.