Estimating Economies of Scope Using Profit Function: A Dual Approach of the Normalized Quadratic Profit Function

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Abstract

Theoretical relationships between the parameters of the normalized quadratic cost and profit functions were derived. The cost function was recovered from an estimated profit function, and thus economies of scope (EOS) were calculated using a profit function. An empirical example showed that the parameters in the true cost function could be precisely recovered using the estimated profit function. Estimating economies of scope measures using profit functions have several merits over that using cost function, which include easier imposition of curvature, avoiding calculating EOS off the production frontier and capturing the inefficiency in the allocation of output quantities. The EOS calculated using a profit function was based on the Baumol et al.'s concept of EOS and can be easily compared with the EOS that was computed using cost function.

Keywords: duality, economies of scope, normalized quadratic cost and profit function

Introduction

Economies of scope exist if the costs of producing several products together are lower than that of producing these products separately. It measures the percentage of cost savings in producing the outputs jointly in one firm rather than producing the products individually in different firms. The sources of economies of scope lie in the complimentary property among inputs: the marginal cost of producing one product decreases as the output of the other product increases.

Since Baumol, Panzar and Willig's work in the 1980s, economies of scope have become an important concept in measuring economic efficiency in a multiproduct framework. Basically, there are two approaches in estimating economies of scope. One approach is nonparametric analysis. In the nonparametric approach (Färe), linear programming is used to calculate the minimum multiproduct cost with all/individual outputs, and then the cost of producing multiproducts jointly and the cost of producing these products individually are compared. The advantage of nonparametric method is that it doesn't require specific functional forms, which avoids the problem of distorting the technology by imposing a functional relationship on the cost function. However, this method does not necessarily allow setting the output equal to zero leading to some possible approximation error (Coffey and Featherstone).

The second approach involves directly estimating a specified cost function, and comparing the cost of producing multiproducts jointly and the sum of the cost of producing all the products individually. This approach is widely used in studying economies of scope of firms in various industries, such as agricultural and financial industries. The normalized quadratic functional form is widely used to approximate the cost function in the study of economies of scope[1] (Featherstone and Moss, Fernandez-Cornejo et al, Jin et al and Cohn et al). The main problem with the parametric approach is that the data used to estimate cost functions are not always on the efficient frontier. Because scope economies are defined only on the efficiency frontier, testing economies of scope by "using data off the frontier could confound scope economies with X-efficiencies." (Berger et al 1993a). In addition, imposing curvature in a profit function is easier than that in a

1: Another widely used functional form is the translog function. However, the translog function form is multiplicative in output and the cost will be zero when one or more outputs are zero in the cost function. Taking the log of a zero output causes problems. Most research replaces the zero output with 10 percent of the mean output but the effect on the measurement of the scope economies of replacing the zero output with a small portion of mean output is still unclear.

cost function. Normally, the concavity in outputs and the convexity in inputs have been imposed for the two sub-matrices of the Hessian matrix, and off diagonal sub-matrices are not considered. Using the profit function makes it easier to impose curvature on the off diagonal sub-matrices (Marsh and Featherstone). Berger et al (1993b) also pointed out that measuring scope economies from cost function didn't consider whether the output bundle is optimal, thus lacking the consideration of the revenue effects on the efficiency measure. Therefore, they suggested that more research should concentrate on estimating economies of scope from the profit function, which includes both revenue and cost sides of production. In accordance with their argument, Berger et al (1993b) provided a new concept of optimal scope economies, which determines "whether a firm facing a given set of prices and other exogenous factors should optimally produce the entire array of products or specialize in some of them." With an unrestricted profit function, the optimal quantities of outputs can be derived using Hotelling's lemma $\frac{\partial \Pi}{\partial p_i} = y_i$. If the optimal quantities of all outputs are determined to be positive at given exogenous prices, optimal scope economies exist at that point. The new concept of economies of scope, however, loses its connection with the classic definition of economies of scope, in that while it can be determined whether scope economics exist, the magnitude of scope measures cannot be determined.

Following Berger et al's suggestion, this paper provides a novel way to estimate economies of scope using a profit function. Different from Berger et al's (1993b) approach, we use the classic concept of scope economies that was first provided by Baumol et al.

Duality and Recovering Cost Function from Unrestricted Profit Function

To determine economies of scope (EOS), the cost of producing multiproducts jointly and the sum of the cost to produce these products individually need to be compared. Economics of scope measures how much savings could occur if the products are produced jointly rather than producing them separately. Specifically, EOS is:

$EOS = \frac{\sum_i C(Y_i) - C(Y)}{C(Y)}$ where $C(Y_i)$ is the cost of producing only Y_i by a individual firm, and $C(Y)$ is the cost of producing all outputs by a single multiproduct firm. If EOS is positive, economies of scope exist and firms can be more cost efficient by diversification in production.

According to duality theory, a profit maximizing firm must also minimize cost, and the unrestricted profit function from profit maximization problem contains the same information as the cost function from cost minimization problem (Mas-Colell et al.). Theoretically, it's possible to link the parameters of the profit function to the parameters in the cost function. Lau (1976) proved that under perfect competition, a restricted profit (cost) function or production function can be recovered from an unrestricted profit function and vice versa. Lusk et al. test the relationship between the parameters of production function, unrestricted profit function and restricted profit function empirically. Therefore, estimating the economies of scope using the profit function is to obtain a cost function from a profit function, and then use the concept of EOS determined by the cost function to calculate economies of scope.

We use the normalized quadratic functional form in this paper, because it is self-dual in cost and profit functions and the links between these two functions do not depend on particular data points (Lusk et al.).

It seems plausible to be able to recover the cost function from the profit function. However, the unrestricted profit function is calculated from the difference between the maximized revenue and the minimized cost, and calculating revenue and cost functions both involve the first order derivative of the corresponding objective functions. Obtaining the cost function from a profit function involves the opposite process, which is integration. Compared to taking the derivative of a function, integrating a function is relatively more difficult and in some cases may be intractable. To begin the derivation, we begin with a cost function and use the maximization process to calculate the unrestricted profit function theoretically. If the parameters of profit function (Y) can be expressed using the parameters of the cost function (X), such as $Y = f(X)$, an inverse relationship can be obtained, which expresses the parameters of cost function using the parameters from the profit function : $X = f^{-1}(Y)$. In the case that the theoretical inverse relationships cannot be obtained when there are highly nonlinear relationships between the parameters of those two functions, the parameters of cost function can be recovered from the profit function empirically using an algorithm to $\underset{X}{Min} Y - f(X)$. As long as a cost function can be expressed using parameters from a profit function, economies of scope can be calculated using the parameters from unrestricted the profit function.

Theoretical Relationship between Cost and Unrestricted Profit Functions

Suppose that we have a normalized quadratic cost function C(W,Y)=min wx where W is a vector of normalized input prices and Y is a vector of output quantities. C(W,Y) has following properties:

1. C(W,Y) is continuous in (W,Y) and differentiable in W and Y

2. C(W,Y) is linear homogenous and concave in W

3. C(W,Y) is convex in Y

The normalized cost function with n+1 inputs and m outputs is:

$$(1)\quad C(W,Y)=b_0+\underset{1*n}{B}*\underset{n*1}{W}+\underset{1*m}{A}*\underset{m*1}{Y}+0.5*\underset{1*n}{W}^{'}*\underset{n*n}{BB}*\underset{n*1}{W}+0.5*\underset{1*m}{Y}^{'}*\underset{m*m}{CC}*\underset{m*1}{Y}+\underset{1*n}{W}^{'}*\underset{n*m}{AA}*\underset{m*1}{Y}$$

where C(W,Y) is the cost, and W is a vector of input prices, both are normalized on the n+1 input price, which implies that the cost function satisfies the homogeneity condition. Formally,

$B=[\mathrm{b}_1\ \mathrm{b}_2......\mathrm{b}_\mathrm{n}]$

$W=[\mathrm{w}_1\ \mathrm{w}_2......\mathrm{w}_\mathrm{n}]^{'}$

$A=[\mathrm{a}_1\ \mathrm{a}_2......\mathrm{a}_\mathrm{m}]$

$Y=[\mathrm{y}_1\ \mathrm{y}_2......\mathrm{y}_\mathrm{m}]^{'}$

$$BB=\begin{bmatrix} b_{11} & b_{12} & ...... & b_{1n} \\ b_{21} & b_{22} & ...... & b_{2n} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ b_{n1} & b_{n2} & ...... & b_{nn} \end{bmatrix}$$ and $b_{ij}=b_{ji}$ to satisfy symmetry condition in input prices

$$CC=\begin{bmatrix} c_{11} & c_{12} & ...... & c_{1m} \\ c_{21} & c_{22} & ...... & c_{2m} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ c_{m1} & c_{m2} & ...... & c_{mm} \end{bmatrix}$$ and $c_{ij}=c_{ji}$ to satisfy symmetry condition in output quantities

$$AA = \begin{bmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1m} \\ a_{21} & a_{22} & \ldots\ldots & a_{2m} \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ a_{n1} & a_{n2} & \ldots\ldots & a_{nm} \end{bmatrix}$$

Curvature conditions required that cost function is concave in input prices W and convex in output quantities Y. The Hessian matrix of the normalized quadratic cost function on input prices and output quantities are just BB and CC respectively. The curvature and symmetry conditions together imply that the matrix BB and CC are negative semi-definite symmetric matrices and positive semi-definite symmetric matrices, respectively. Empirically, Cholesky decomposition can be used to impose curvature globally on the cost function in estimating this function.

Assume both input and output markets are perfectly competitive, the unrestricted profit function can be obtained as a result of following maximization problem

$\Pi = \max \mathrm{P*Y - C(W, Y)}$ where P is a vector of exogenous output prices, $P = [\mathrm{p_1\ p_2 \ldots\ldots p_m}]$.

The first order conditions of profit maximization allow us to determine the optimal output $\frac{\partial \Pi}{\partial Y} = P' - \frac{\partial C(W,Y)}{\partial Y} = 0$ by solving a set of equations. $P' = \frac{\partial C(W,Y)}{\partial Y}$ is the general condition under perfect competition that the output prices equal the corresponding marginal cost of that output. For the normalized quadratic cost function (1), the first order conditions are:

$P' = A' + CC * Y + AA' * W$ and the optimal output quantities are determined by solving for *Y:*

(2) $Y^* = CC^{-1} * (P' - A' - AA' * W)$ [2]

Plug Y^* into the original cost function (1) to solve for the cost function at the optimal output

2: The inverse of CC matrix requires that the production function is strictly convex. It also indicates that there is single point in Y corresponding to one set of prices, making Hotelling's lemma feasible. Throughout this paper, we assume that the production function is strictly convex. If the assumption is relaxed, the inverse of CC may not be found. However, generalized inverse can be used instead to determine inverse matrix of CC to solve for possible solutions of Y. This is beyond the study of this paper, for more details of the theory of generalize inverse, see Ben-Israel.

quantities:

$$
(3)\quad \begin{aligned}
C(W,Y^*) &= b_0 + \underset{1*n}{B}*\underset{n*1}{W} + \underset{1*m}{A}*CC^{-1}*(P' - \underset{m*1}{A'} - AA'*W) \\
&+ 0.5*\underset{1*n}{W'}*\underset{n*n}{BB}*\underset{n*1}{W} + 0.5*[CC^{-1}*(P' - \underset{1*m}{A'} - AA'*W)]'*\underset{m*m}{CC}*[CC^{-1}*(P' - \underset{m*1}{A'} - AA'*W)] \\
&+ \underset{1*n}{W'}*\underset{n*m}{AA}*CC^{-1}*(P' - \underset{m*1}{A'} - AA'*W)
\end{aligned}
$$

Expanding the function via multiplication, gives:

$$
(4)\quad \begin{aligned}
C(W,Y^*) &= b_0 + B*W + (A*CC^{-1}*P' - A*CC^{-1}*A' - A*CC^{-1}*AA'*W) \\
&+ 0.5*W'*BB*W + 0.5*[(P' - A' - AA'*W)'*(CC^{-1})']*(P' - A' - AA'*W) \\
&+ (W'*AA*CC^{-1}*P' - W'*AA*CC^{-1}*A' - W'*AA*CC^{-1}*AA'*W)
\end{aligned}
$$

Because CC and BB are symmetric matrices, $(CC^{-1})'$ is equal to CC^{-1} and $(BB)'$ is equal to BB. By further expanding equation (4) , the cost function is:

$$
(5)\quad \begin{aligned}
&C(W,Y^*) = b_0 - A*CC^{-1}*A' + 0.5*A*CC^{-1}*A' \\
&+ B*W - A*CC^{-1}*AA'*W + 0.5*W'*AA*CC^{-1}*A' + 0.5*A*CC^{-1}*AA'*W - W'*AA*CC^{-1}*A' \\
&+ A*CC^{-1}*P' - 0.5*A*CC^{-1}*P' - 0.5*P*CC^{-1}*A' \\
&+ 0.5*W'*BB*W + 0.5*W'*AA*CC^{-1}*AA'*W - W'*AA*CC^{-1}*AA'*W \\
&+ 0.5*P*CC^{-1}*P' \\
&- 0.5*W'*AA*CC^{-1}*P' - 0.5*P*CC^{-1}*AA'*W + W'*AA*CC^{-1}*P'
\end{aligned}
$$

Since each term in equation (5) is a scalar, we can transpose any item in the above equation. This allows us to combine $A*CC^{-1}*A$ and $0.5*A*CC^{-1}*A'$ in line 1, cancel out $0.5*W'*AA*CC^{-1}*A'$, $0.5*A*CC^{-1}*AA'*W$ and $W'*AA*CC^{-1}*A'$ in line 2, cancel out third line, combine $0.5*W'*AA*CC^{-1}*AA'*W$ and $W'*AA*CC^{-1}*AA'*W$ in line four, and cancel out the last line of equation (5). As a result, we can find a simplified cost function, which is

(6) $$C(W,Y^*) = b_0 - 0.5*A*CC^{-1}*A' + (B - A*CC^{-1}*AA')*W$$
$$+ 0.5*W'*(BB - AA*CC^{-1}*AA')*W + 0.5*P*CC^{-1}*P'$$

Now, plug the optimal output Y^* (equation (2)) and the above cost function into the profit function $\Pi = P*Y^* - C(W,Y^*)$, the unrestricted profit function is

(7) $$\Pi = P*CC^{-1}*(P' - A' - AA'*W) - [b_0 - 0.5*A*CC^{-1}*A' + (B - A*CC^{-1}*AA')*W$$
$$+ 0.5*W'*(BB - AA*CC^{-1}*AA')*W + 0.5*P*CC^{-1}*P']$$

By simplifying equation (7), we obtain

(8) $$\Pi = -b_0 + 0.5*A*CC^{-1}*A' - A*CC^{-1}*P' + (A*CC^{-1}*AA' - B)*W$$
$$+ 0.5*P*CC^{-1}*P' + 0.5*W'*(AA*CC^{-1}*AA' - BB)*W - W'*CC^{-1}*AA'*P'$$

Equation (8) is the unrestricted profit function expressed by the parameters of a given cost function, input and output prices. It is also a normalized quadratic function. The next step is to find the inverse relationships, to recover the cost function from the unrestricted profit function.

At this point, if the unrestricted normalized quadratic profit function is used as the form

(9) $$\Pi = pb_0 + \underset{1*m}{P}*\underset{m*1}{PA} + \underset{1*n}{PB}*\underset{n*1}{W} + 0.5*\underset{1*m}{P}*\underset{m*m}{PCC}*\underset{m*1}{P'} + 0.5*\underset{1*n}{W'}*\underset{n*n}{PBB}*\underset{n*1}{W} + \underset{1*m}{P}*\underset{m*n}{PAA}*\underset{n*1}{W}$$

the profit function can be expressed by the corresponding cost function, and the parameters in these two functions have the following relationships:

(10) $pb_0 = -b_0 + 0.5*A*CC^{-1}*A'$, $PA = -A*CC^{-1}$, $PB = A*CC^{-1}*AA' - B$

(11) $PCC = CC^{-1}$, $PAA = -AA*CC^{-1}$ and $PBB = AA*CC^{-1}*AA' - BB$

With the explicit relationships between the parameters from the cost and unrestricted profit functions, we can recover the parameters in the profit function from the cost function.

However, recovering the parameters in the cost function using the profit function may not be as straightforward as recovering the parameters in the profit function using cost function. Two cases exist. In the first case, the relationships between the parameters from the cost and profit functions are highly nonlinear, and it may be intractable to find the inverse function given parameters in profit function as a function of parameters in cost function. That is, it's difficult to express the parameters in the cost function explicitly with the parameters in profit function. Empirically, we can use EXCEL or GAMS to recover the unknown parameters in the cost function using parameters from the profit function by the explicit relationships that we have derived in (10) and (11), i.e. parameters in profit function as functions of parameters in cost function. In the second case, the relationships are not highly nonlinear, and it is easy to solve the inverse relationships between the parameters of cost and profit functions, i.e., express the parameters of a cost function as a function of the parameters from the profit function. In this case, the cost function can be recovered directly from the profit function using those relationships.

In our case, it is tractable to solve the inverse relationships between the parameters from the profit function and cost functions. With the relationships in (10) and (11), the parameters in the cost function can be expressed using those in the profit function.

(12) $b_0 = -pb_0 + 0.5 * PA * PCC^{-1} * PA'$, $A = -PA * PCC^{-1}$, $B = PA * PCC^{-1} * PAA' - PB$

(13) $CC = PCC^{-1}$, $AA = -PAA * PCC^{-1}$ and $BB = PAA * PCC^{-1} * PAA' - PBB$

The results in (12) and (13) imply that for a normalized quadratic profit function, the underlying cost function can be recovered using linear algebra computations.

An Empirical Example: A Case of Three Inputs and Two Outputs

Suppose that the normalized quadratic cost/profit function has three inputs (w_1 w_2 w_3) and two outputs (y_1 y_2), and the input prices w_1, w_2 and cost C(W,Y) are normalized on the third input price w_3.

The cost function is

$$(14)\quad \begin{aligned} C(W,Y) &= b_0 + \begin{bmatrix} b1 & b2 \end{bmatrix} * \begin{bmatrix} w1 \\ w2 \end{bmatrix} + \begin{bmatrix} a1 & a2 \end{bmatrix} * \begin{bmatrix} y1 \\ y2 \end{bmatrix} + 0.5 * \begin{bmatrix} w1 & w2 \end{bmatrix} * \begin{bmatrix} b11 & b12 \\ b12 & b22 \end{bmatrix} * \begin{bmatrix} w1 \\ w2 \end{bmatrix} \\ &+ 0.5 * \begin{bmatrix} y1 & y2 \end{bmatrix} * \begin{bmatrix} c11 & c12 \\ c12 & c22 \end{bmatrix} * \begin{bmatrix} y1 \\ y2 \end{bmatrix} + \begin{bmatrix} w1 & w2 \end{bmatrix} * \begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix} * \begin{bmatrix} y1 \\ y2 \end{bmatrix} \end{aligned} .$$

Economies of scope can be expressed as

$$(15)\quad EOS = \frac{C(W,y_1) + C(W,y_2) - C(W,Y)}{C(W,Y)} = \frac{b_0 + b_1 * w_1 + b_2 * w_2 + 0.5 * b_{11} * w_1^2 + b_{12} * w_1 * w_2 + 0.5 * b_{22} * w_2^2 - c_{12} * y_1 * y_2}{C(W,Y)} .$$

The unrestricted profit function corresponding with the cost function is

$$(16)\quad \begin{aligned} \Pi &= pb_0 + \begin{bmatrix} pb1 & pb2 \end{bmatrix} * \begin{bmatrix} w1 \\ w2 \end{bmatrix} + \begin{bmatrix} pa1 & pa2 \end{bmatrix} * \begin{bmatrix} p1 \\ p2 \end{bmatrix} + 0.5 * \begin{bmatrix} w1 & w2 \end{bmatrix} * \begin{bmatrix} pb11 & pb12 \\ pb12 & pb22 \end{bmatrix} * \begin{bmatrix} w1 \\ w2 \end{bmatrix} \\ &+ 0.5 * \begin{bmatrix} y1 & y2 \end{bmatrix} * \begin{bmatrix} pc11 & pc12 \\ pc12 & pc22 \end{bmatrix} * \begin{bmatrix} p1 \\ p2 \end{bmatrix} + \begin{bmatrix} w1 & w2 \end{bmatrix} * \begin{bmatrix} pa11 & pa12 \\ pa21 & pa22 \end{bmatrix} * \begin{bmatrix} p1 \\ p2 \end{bmatrix} \end{aligned} .$$

Following the theoretical results from (12) and (13), the parameters in the cost function (14) can be calculated from the unrestricted profit function (16).

Also, note that the optimal output y1 and y2 in the equation of EOS can be calculated using Hotelling's lemma, where $\frac{\partial \Pi}{\partial p_i} = y_i$. Thus, economies of scope at the optimal quantity can be calculated using the recovered cost function and the optimal output supplies.

In this paper, 500 data points for each of the three input prices and two output prices were generated using Monte-Carlo procedure. The parameters of the normalized cost function (true cost function) were specified so that the cost function satisfied homogeneity, symmetry, and curvature conditions. Homogeneity was satisfied by normalizing all prices and cost by the third input price, symmetry was imposed by letting the Hessian matrices of input prices and output quantities being symmetric, and curvature was imposed using Cholesky decomposition, which means the Hessian matrix on input side can be expressed as $BB = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = -\begin{bmatrix} d_{11} & 0 \\ d_{21} & d_{22} \end{bmatrix} * \begin{bmatrix} d_{11} & 0 \\ d_{21} & d_{22} \end{bmatrix}'$ and Hessian matrix on the output side can be expressed as $CC = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} e_{11} & 0 \\ e_{21} & e_{22} \end{bmatrix} * \begin{bmatrix} e_{11} & 0 \\ e_{21} & e_{22} \end{bmatrix}'$. With input and output prices, as well as the parameters in cost function, the optimal output quantities y^* were calculated using equation (2). The minimized costs $C(W, Y^*)$ were determined using the input prices and optimal output quantities. Maximized profits were the differences between maximized revenue and the minimized cost $\Pi = P * Y^* - C(W, Y^*)$. All the costs and profits were calculated at each of the 500 data points. The unrestricted normalized quadratic profit function was estimated with profit on the left hand side and input and output prices on the right hand side. At this point, we estimate the unrestricted profit function and our objective is to recover the cost function using the relationships between the parameters of cost and profit functions.

In table 1, the first column are the estimated coefficients of the profit function using the generated data, the second column are the recovered parameters of cost function using the estimated coefficients from profit function, and the third column are the assumed true parameters of the cost function that were used to generate the data set. The results in table 1 show that all the recovered parameters in the AA matrix are exactly same as parameters of the true cost function in data generating process, which verified our method.

Using Hotelling's lemma, the optimal output quantities were obtained at each of the 500 data points. The mean cost was also calculated using (14) with recovered parameters of cost function, mean input prices and mean output quantities obtained by Hotelling's lemma. Then EOS at mean optimal output quantities and input prices were calculated using equation (15), which were 0.044, implying relatively weak economies of scope.

Conclusions and Discussions

In this paper, the theoretical relationships between the normalized quadratic cost and unrestricted profit functions were derived. Using the theoretical relationships, the parameters in the cost function can be recovered from the unrestricted profit function that is estimated, which enables the calculation of economies of scope using the profit function. We numerically analyzed the economies of scope based on the normalized profit function which had three inputs and two outputs, using Monte-Carlo data. The recovered parameters for the cost function are identical to the true parameters. Measuring economies of scope using the profit function has a few merits that lack in methods using the cost function. First, imposing curvature on a profit function is easier than imposing curvature on a cost function (Marsh and Featherstone). Second, EOS calculated from the profit function is always on the production frontier, which avoids the problem that EOS from the cost function are not necessarily on the production frontier, violating the condition that EOS required (Berger et al 1993a). Third, EOS calculated from cost functions may incorporate output inefficiencies (Berger et al 1993b). While the economies of scope calculated from profit function can be a result from both the cost saving process on the input side and the optimal allocation of output supply in response to exogenous output prices. In addition, using Berger et al’s (1993b) method to test optimal economies of scope, an unrestricted profit function was estimated, and then the minimum optimal output quantities corresponding to every price was acquired using Hotelling’s lemma. If the minimum optimal output quantities was statistically significant and greater than zero, then optimal scope economies exist. This method does not use the quantitative measurement of EOS defined by Baumol et al, and lacks the ability of comparing the specifically defined EOS with Baumol et al's definition. In this paper, since the basic concept of economies of scope was used, the difference in EOS from both methods (using cost or profit function) can be compared.

We realize that using the profit function to estimate economies of scope is not as straight forward as using the cost function, and the process of recovering the parameter in the cost function adds additional calculations. A normalized cost/profit function with three inputs and two outputs involves 15 unknown parameters. In a production process with n inputs and m outputs, adding one more dimension on the input (output) side will add 2(n+1)+m (2(m+1)+m) parameters. If the relationships between parameters from cost and profit functions can be expressed using linear algebra as with the normalized quadratic cost and profit function, the process should not be a big problem. Without measurement error in prices and quantities, the recovered cost function is identical to the true cost function used to generate the data. However, in an empirical study, data quality are always problem which involves measurement error in prices and quantities that were used to estimate profit or cost functions. Lusk et al's study showed that only under certain restricted conditions, the estimated parameters from production function, unrestricted profit function and restricted profit function satisfied the Hessian Identity relationships derived by Lau. As a result, more research should be conducted to explore how measurement errors in prices and output quantities affect EOS calculated from both profit and cost functions.

References:

Baumol, William J., John C. Panzar and Robert D. Willig. Contestable Markets and Industry Structure. New York: Harcourt Brace Jovanovich, Inc., 1982.

Ben-Israel, Adi and Thomas N. E. Greville. Generalized Inverses: Theory and Applications. New York: A Wiley-Interscience Publication, 1974.

Berger, Allen N., Diana Hancock and David B. Humphrey. "Bank Efficiency Derived from the Profit Functions." *Journal of Banking and Finance*.17 (1993b): 317-347.

Berger, Allen N., William C. Hunter and Stephen G. Timme. "The Efficiency of Financial Institutions: A Review and Preview of Research Past, Present, and Future." *Journal of Banking and Finance*. 17 (1993a): 221-249.

Coffey, Brian K. and Allen Featherstone. "Nonparametric Estimation of Multiproduct and Product-Specific Economies of Scale." Selected Paper of SAEA Annual Meetings, Feb. 2004, Tulsa, OK.

Cohn, Elchanan, Sherrie L. W. Rhine, Maria C. Santos. "Institutions of Higher Education as Multi-Product Firms: Economies of Scale and Scope." *The Review of Economics and Statistics* 71 (May. 1989) 284-290.

Färe, Rolf. "Addition and Efficiency." *Quarterly Journal of Economics*. 101 (Nov. 1986): 861-866.

Featherstone, Allen M., and Charles B. Moss. "Measuring Economies of Scale and Scope in Agricultural Banking." *American Journal of Agricultural Economics*. 76 (Aug. 1994): 655-661.

Fernandez-Cornejo, J., C. M. Gempesaw Π and J. G. Elterich, S. E. Stefanou. "Dynamic Measure of Scope and Scale Economies: An Application to German Agriculture." *American Journal of Agricultural Economics.* 74 (May. 1992): 329-342

Jin,Songqing, Scott Rozelle, Julian Alston and Jikun Huang. "Economies of Scale and Scope and the Economic Efficiency of China's Agricultural Research System." *International Economic Review*. 46 (Aug. 2005): 1033-57.

Lusk, Jayson L., Allen M. Featherstone, Thomas L. Mash and Abdullahi O. Abdulkadri. " The Empirical Properties of Duality Theory." *The Australian Journal of Agricultural and Resource Economics*. 46 (2002) 45-68

Lau, L. J. "A Characterization of the Normalized Restricted Profit Function." *Journal of Economic Theory*. 12 (1976): 131-63.

Marsh, T. L and A. M. Featherstone. "Testing and Imposing Curvature Restrictions on Restricted Profit Functions" Working Paper, Kansas State University, 2004

Mas-Colell, Andreu, Michael D. Whinston and Jerry R. Green. Microeconomic Theory. New York: Oxford University Press, 1995

Table 1
Estimated Profit Function and the Comparison of the True and Recovered Cost Functions

Estimated Parameters in Profit Function		Recovered Parameters in Cost function		True Parameters in Cost Function	
Pb0	-29.340	b0	30.000	b0	30.000
Pb1	-9.912	b1	10.000	b1	10.000
Pb2	-34.901	b2	35.000	b2	35.000
Pa1	-0.147	a1	0.600	a1	0.600
Pa2	-0.616	a2	2.000	a2	2.000
Pb11	0.101	b11	-0.090	b11	-0.090
Pb12	0.080	b12	-0.075	b12	-0.075
Pb22	0.749	b22	-0.740	b22	-0.740
Pc11	0.250	c11	4.000	c11	4.000
Pc12	-0.001	c12	0.010	c12	0.010
Pc22	0.309	c22	3.241	c22	3.240
Pa11	-0.045	a11	0.180	a11	0.180
Pa12	-0.031	a12	0.100	a12	0.100
Pa21	-0.032	a21	0.130	a21	0.130
Pa22	-0.040	a22	0.130	a22	0.130