What Happens when Peter can't Pay Paul: Risk Management at Futures Exchange Clearinghouses

by

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Abstract

We model a futures exchange's clearinghouse as a "bank" holding a portfolio of credit lines available to its clearing members and collateralized with clearing margins or, equivalently, a portfolio of short European put basket options. Consequently, the "bank"model measures the clearinghouse's risk exposure as the sum of the payoff functions of these put options, emphasizing the portfolio diversification and the option-like payoffs. The model is used to assess exchange's clearinghouse's liquidity and credit risk exposure. The model provides exchange clearinghouses and government regulators with a theoretical framework of risk management that systematically integrates clearing margin requirements, credit lines and economic capital.

Keywords: credit risk, liquidity risk, loss distribution, margin requirements, credit lines, economic capital, portfolio diversification, basket option.

JEL Classification: C110, G110, D810.

I. Introduction

Clearinghouses play a central role in settlement and clearing of exchange-traded futures and futures options. A crucial part of the role of clearinghouses is to act as an official "party to every trade," substituting itself as a seller to every buyer and a buyer to every seller. As a result, a clearinghouse is exposed to liquidity and credit risk should one of the clearing members default or fail. Clearinghouse' failure are rare but have been exemplified by the failures or near failures at Paris in 1973, at Kuala Lumpur in 1983 and at Hong Kong in 1987. Stresses and strains at exchanges' clearing operation, even rumors about them, could ripple across the whole financial system, potentially destabilizing the system given the central role exchanges play in the financial system. See Bernanke (1990) and Bates and Craine (1999) for analysis of clearing and settlement at futures exchanges during the 1987 market crash. To safeguard smooth and stable operations, clearinghouses (and government regulators) manage the liquidity and credit risk using an array of tools and mechanisms, which include "mark-to-market", margin requirements, credit lines and economic capital.

Margin requirement is a clearinghouse's first line of defense against possible default by its clearing members. Most previous academic studies estimate margin requirements based on statistical analysis of price changes to meet a coverage probability, the probability that the margin collected sufficient to cover the losses arising from actual price change in the market (e.g., Gay et al., 1986). Various conditional volatility models (Fenn and Kupiec, 1993; Kupiec, 1994; Gemmill, 1994; Kupiec and White, 1996; Kupiec, 1998; Booth and Broussard, 1997) have been used to estimate margin levels in order to accommodate volatility clustering observed in futures and option prices. Similarly, Extreme Value Theory (EVT) has also been applied to estimation of margin levels in order to account for non-normality in futures and option prices (Dewachter and Gielens, 1999; Longin, 1999, 2000; Cotter, 2001). Recently, Day and Lewis (2004) and Day and Lewis (1997) consider futures margins as risk premiums for barrier options and thus evaluate margin requirements based on option pricing theory. Nevertheless, all these previous studies focus on a single exchange-traded product and therefore ignore the portfolio diversification effect. That is the effect due to a clearing member holds long/short positions in multiple products and that the price changes of these products are correlated.

In industry practice, the margin requirements, particularly at clearing members' level, are set by the SPAN (Standard Portfolio Analysis of Risk) system, which is now the official margin setting mechanism of nearly every registered futures exchange and clearing organization in the United States, and many global entities. The SPAN system considers a portfolios of instruments with a single underlying instrument and set margin requirements based on 16 different scenarios of underlying market price and volatility changes of the underlying instrument (Chicago Mercantile Exchange, 2004), and hence the system partially accommodates the portfolio diversification effect. Nevertheless it is still not a true "global" portfolio margining system because it does not account for all the cross-product correlations (Kupiec and White, 1996). Another problem with the system is that margin requirements are assessed based on scenario analysis rather than Monte Carlo simulation, as a result, the system does not capture the whole spectrum of possible value changes of the portfolio under consideration.

While in industry practice exchange clearinghouses manage liquidity and credit risk exposure using an array of tools and mechanisms, including market-to-market, margin requirement, credit lines and economic capital, most previous studies as well as the SPAN system focus on margin requirements and few considers these tools jointedly. Margin requirements are a clearinghouse's first line of defense against its clearing members' possible default, but a clearinghouse also arranges lines of credit agreements with domestic and international banks and collects capital contribution from its clearing members. The clearinghouse could borrow up to the full amount of its credit lines on short notice should a clearing member delays or defaults on its margin variation payment. As the last resort, a clearinghouse may use its economic capital to defuse the possibility of an exchange default. Margin requirements and economic capital pose substantial costs to clearinghouses and clearing firms. Rosenzweig (2003) reports that in 2003 clearing members contribute to four largest futures clearinghouses in the US about *textdollar*875 million in economic capital and many time lager in magnitude in margin requirements. Shanker and Balakrishnan (2005) try to link clearing margins, economic capital and price limits together in a univariate product setting, Nonetheless, they employ a single product approach and thus ignore the portfolio diversification effect identified above.

In sum, previous academic literatures and the SPAN system do not fully account for the portfolio diversification effect and the interlink among different risk management tools. In this study, drawing on an analogy between a clearinghouse and a financial intermediary, such a bank or an insurance firm, we model a futures exchange's clearinghouse as a "bank" holding a portfolio of credit lines available to its clearing members and collateralized with clearing margins. The privilege clearing firms have at a clearinghouse is treated as the privilege of one-day credit lines available to the clearing members via margin accounts and collateralized with clearing margins, whereas the clearinghouse is treated as holding a portfolio of these short-term (one-day) credit lines, or equivalently, a portfolio of short European put basket options. Consequently, we use the "bank" model to measure the clearinghouse's liquidity and credit risk exposure as the sum of the payoff functions of these put options, emphasizing portfolio diversification and option-like payoffs. The model provides exchange clearinghouses and government regulators with a unified framework of risk management that systematically integrates clearing margin requirements, credit lines and economic capital.

The proposed model is closely related to three streams of ideas in the current literature. One is the analogy between a clearinghouse and a financial intermediary (e.g., Bernanke, 1990). The role of an exchange clearinghouse can be divided into two parts: one is to register, match and clear transactions, the operational function of the clearing and settlement operation; the other is to bear liquidity and credit risk should members delay and/or default on payments, the financial function of the clearing and settlement operation. The second stream is the option approach to margin requirements evaluation. Day and Lewis (1997, 2004) suggest that the profit/loss of clearing firms and clearinghouses can be characterized as the payoff functions to (barrier) options and margin requirements can be considered as risk premiums for those options. The third stream is portfolio credit risk models. A portfolio approach to market risk exemplified in various VaR (Value-at-Risk) models is well known and accepted in academics and industry and the approach is making inroads in analysis of credit risk. Various portfolio credit risk models have been developed to analyze the risk of credit-sensitive portfolios, such as portfolios of bank loans, various mortgage-backed securities (MBS), Asset-Backed-Securites (ABS) and Collateralized-Debt-Obligations (CDO). See survey studies (Crouhy et al., 2000; Gordy, 2000), industry reports (Gupton et al., 1997; Kealhofer and Bohn, 2001) and textbooks (Crouhy et al., 2001; Bluhm et al., 2002; Duffie and Singleton, 2003; Lando, 2004). Given the similarity between an exchange clearinghouse and a bank, we apply the concepts and techniques developed in these models to analyzing the risk exposure of clearinghouses.

The basic idea of the model can be illustrated with a thought experiment of an exchange clearinghouse with two products and two clearing members. Assume that the price changes of these two products are perfectly positively correlated and have the same volatility, i.e., $\sigma_1^2 = \sigma_2^2$ and $\rho_{1,2} = 1$, and that member A holds W_1 and W_2 in these two products and $W_1 = W_2$, consequently member B holds $-W_1$ and $-W_2$ in those two products due to the zero-summation of clearing members' positions. Now consider the the profit/loss of clearing members and the risk exposure of the clearinghouse one business day later. Regardless how the price of the products move (up or down), one member will make a profit in both positions of the two products while the other will lose the same amount of money in these positions. Even though the sum of clearing members' profits and losses is zero, the risk exposure of the clearinghouse is not because the clearinghouse has an option-like payoff function, i.e., it suffers loss from member default but does not reap profit from member gains. Assume member *A* and *B* swap their positions in one product and consider the profit/loss and risk exposure again. The same thought experiment can be conducted with correlation being perfectly negative. Logically, one will observe opposite results to what that above. But the point is that the risk exposure of an exchange clearinghouse depends not only on the price dynamics of exchange-traded products but also on the exact position holdings of its members in these products and that the clearinghouse has an option-like payoff function.

The rest of paper is organized as follows. In the next section, we develop the portfolio model for clearinghouse risk management that integrates margin requirements, credit lines and economic capital. In section three, we illustrate how to implement the model using a Monte Carlo simulation. In the last section, we draw conclusions.

II. Model

To begin, we make some simplifying assumptions for ease of exposition and explicitness of the model when these assumptions do not impair the features of the model that are emphasized. We consider only four risk management tools, namely, market-to-market, margin requirements, credit lines and economic capital. We limit the scope of exchange-traded products to futures contracts and hence exclude futures contracts due to their non-linearity in pricing. For simplicity, we assume that N exchanged-traded products follow a multivariate Geometric Brownian motion specified as

$$\frac{dP_{n,t}}{P_{n,t}} = \mu_n dt + \sigma_n dW_{n,t}, \text{ where } P_{n,0} > 0, \text{ and } n = 1, 2, 3, \ldots, N. \tag{1}$$

where μ_n and σ_n denote, respectively, the drift and volatility parameters of the Geometric Brownian motion. $P_{n,t}$ is the price of the n-th product at time t. $(\mathbf{W}_1, \mathbf{W}_2, \ldots, \mathbf{W}_N,)$ is an N-dimensional standard Brownian motion with correlation ρ_{n_1,n_2} between $\mathbf{W}_{n_1}$ and $\mathbf{W}_{n_2}$. In other words, $(W_{1,t_1}, W_{2,t_2}, \ldots, W_{N,t_N}) \sim \mathbb{N}(0, \mathbf{\Sigma})$ with covariance matrix specified as

$$\mathbf{\Sigma} = \begin{pmatrix} t_1 & \rho_{1,2}\sqrt{t_1 t_2} & \cdots & \rho_{1,N}\sqrt{t_1 t_N} \\ \rho_{1,2}\sqrt{t_1 t_2} & t_2 & \cdots & \rho_{2,N}\sqrt{t_2 t_N} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1,N}\sqrt{t_1 t_N} & \cdots & \cdots & t_N \end{pmatrix} \tag{2}$$

According to Itô's Lemma, we have the prices of the products at time t as

$$P_{n,t} = P_{n,0} \cdot \exp\left\{(\mu_n - \frac{1}{2}\sigma_n)t + \sigma_n \sqrt{t} W_{n,t}\right\}, \tag{3}$$

We also assume that the clearinghouse clears and settles the N exchange-traded products for I clearing members, each of which maintains two separate accounts: one house (proprietary) account and one customers' account, according to the rule of Segregation of Customer Funds Regulations (Chicago Mercantile Exchange, 2004). Member i at time t holds a portfolio denoted by $\mathbf{W}_{i,t}^{(1)}$ at its house account and a portfolio denoted by $\mathbf{W}_{i,t}^{(1)}$ at its customers' account, respectively.

With the Geometric Brownian motion and members's portfolios specified, we compute the profit/loss of clearing member i from the portfolios at its house and customers' accounts from time

0 to t:

$$\text{(4)} \qquad \begin{aligned} \Delta V_{i,t}^{(1)} &= \Delta \mathbf{P}' \cdot \mathbf{W}_{i,0}^{(1)} = (\mathbf{P}_t - \mathbf{P}_0)' \cdot \mathbf{W}_{i,0}^{(1)} \\ \Delta V_{i,t}^{(2)} &= \Delta \mathbf{P}' \cdot \mathbf{W}_{i,0}^{(2)} = (\mathbf{P}_t - \mathbf{P}_0)' \cdot \mathbf{W}_{i,0}^{(2)} \end{aligned}$$

where $\mathbf{P}_t$ denotes a column vector of $(P_{1,t}, P_{2,t}, \ldots, P_{N,t})$ and $\Delta V_{i,t}^{(1)}$ and $\Delta V_{i,t}^{(2)}$ are, respectively, the profit/loss of member i's from the portfolios at its house and customers' accounts. Since linear combination of lognormal distributions is not a lognormal distribution, the profits/losss of these portfolios do not follow a lognormal distribution, therefore in the next section we estimate the changes in portfolio values using Monte-Carlo simulation.

Assume the clearinghouse at time 0 collects margins equal to $M_{i,0}^{(1)}$ and $M_{i,0}^{(2)}$ as collateral from clearing member i for the portfolios at its house and customers' accounts, respectively. Previous studies assess margin requirements based on the probability distribution of a single product as a time. We argue that it is more appropriate to assign clearing margin levels based on the probability distributions of the portfolios of the clearing members at their house and customers' accounts. The new approach takes into account the portfolio diversification effect aforementioned. Mathematically, the clearing margins collected from member i at time 0 for the portfolios at its house and customers' accounts are

$$\text{(5)} \qquad \begin{aligned} M_{i,t}^{(1)} &= -\, q(\alpha_1, \Delta V_{i,t}^{1}) \\ M_{i,t}^{(2)} &= -\, q(\alpha_2, \Delta V_{i,t}^{2}) \end{aligned}$$

where $q(\cdot)$ denotes a quantile function of portfolio profit/loss and $1-\alpha_1$ and $1-\alpha_2$ are the coverage levels for member i's house and customers' accounts, respectively. For example, the clearinghouse may assign margin levels across all its clearing members so that these margins can cover up to

90% of potential loss of the clearing members' portfolios over one business day. In practice, clearinghouses may fine-tune the benchmark coverage probabilities based on the creditworthiness of clearing members, clearing members' customers and market condition under consideration.

The approach to margin levels above can be characterized as a"VaR" (Value-at-Risk) approach. Compared with the univariate approach used in the previous academic studies, the new approach captures the portfolio diversification effect. Compared withe the SPAN system used in industry practice, the new approach captures the portfolio diversification effect more thoroughly because it is "global", i.e., it considers all the correlations among the exchange-traded products and it is based on Monte-Carlo simulation rather than scenario analysis.

With the profit/loss of the clearing members' portfolios measured, we calculate the risk exposure of the clearinghouse at time t:

$$
\begin{aligned}
X_{1,t} &= \sum_{i=1}^{I} -\min\left\{\Delta V_{i,t}^{(1)} + M_{i,0}^{(1)} + \Delta V_{i,t}^{(2)} + M_{i,0}^{(2)}, 0\right\} \\
&= \sum_{i=1}^{I} -\min\left\{\left(V_{i,t}^{(1)} + V_{i,t}^{(2)}\right) - \left(V_{i,0}^{(1)} + V_{i,0}^{(2)} - M_{i,0}^{(1)} - M_{i,0}^{(2)}\right), 0\right\} \\
&= \sum_{i=1}^{I} \max\left\{\left(V_{i,0}^{(1)} + V_{i,0}^{(2)} - M_{i,0}^{(1)} - M_{i,0}^{(2)}\right) - \left(V_{i,t}^{(1)} + V_{i,t}^{(2)}\right), 0\right\}.
\end{aligned}
\tag{6}
$$

Here implicity, we assume that the values of the margins do not change during the modeling horizon from 0 to time t, which is usually one business day due to mark-to-market. Given the high credit rating of performance bonds and short modeling horizon, the assumption is reasonable. The equation tells us that the clearinghouse will be exposed to risk, liquidity and/or credit, at time t, whenever at one of its member suffers an aggregate loss at its house and customers' accounts exceeding the sum of its margin deposits for those two accounts. The magnitude of the clearinghouse' risk exposure is measured by the sum of potential losses from clearing members' portfolios. The equation also tell us that the clearinghouse's risk exposure can be measured as the sum of payoff functions of I short European put basket options. See the third line of the equation, where the strike price is $\left(V_{i,0}^{(1)} + V_{i,0}^{(2)} - M_{i,0}^{(1)} - M_{i,0}^{(2)}\right)$. The clearinghouse will have risk exposure from member i whenever the aggregate value of member i's portfolio drops below the strike price. The equation confirms one result from the thought experiment, that is, although profits/losses from clearing members' portfolios sum to zero, the risk exposure of the clearinghouse is not due to its option-like payoff functions. One possible direction of research is to evaluate margin requirements as premiums of short European put basket options based on option pricing theory. It will be an improvement over Day and Lewis (2004) because it takes into account of the portfolio diversification effect. However, in this study, we focus on the risk exposure and management at clearinghouse level instead of appropriateness of margin requirements.

$X_{1,t}$ above measures the probability and magnitude of a clearinghouse's risk exposure due to its members' portfolio losses exceeding the their margin collateral. In practice, a clearinghouse can ask its clearing members which lose money to deposit more money in their accounts, i.e., to meet margin calls. Consequently, clearing embers have to maintain some credit lines either by their own cash equivalent assets or credit lines agreements with banks. We assume that at time 0 the clearinghouse mandates clearing member i to maintain credit lines equal to $L_{i,0}^{(1)}$ and $L_{i,0}^{(2)}$ for its own house and customers' accounts, respectively, besides the margins already deposited. We assume that the clearinghouse uses a similar "VaR" approach to assess credit line requirement. The credit line requirements for clearing member i' house and customers' accounts are

$$(7) \qquad \begin{aligned} L_{i,t}^{(1)} &= -q(\alpha_2, \Delta V_{i,t}^{1}) + q(\alpha_1, \Delta V_{i,t}^{1}) \\ L_{i,t}^{(2)} &= -q(\alpha_2, \Delta V_{i,t}^{2}) + q(\alpha_1, \Delta V_{i,t}^{2}) \end{aligned}$$

The margin requirements and credit lines together can protect the clearinghouse from its members' potential portfolio loss up to a $1 - \alpha_2$ probability level.

Similar to the computation of $X_{1,t}$, with margin requirements and credit lines considered altogether, we can calculate the liquidity risk exposure of the clearinghouse at time t:

$$
\begin{aligned}
X_{2,t} &= \sum_{i=1}^{I} - \min\left\{\left(\Delta V_{i,t}^{(1)} + M_{i,0}^{(1)} + L_{i,0}^{(1)}\right) + \left(\Delta V_{i,t}^{(2)} + M_{i,0}^{(2)} + L_{i,0}^{(2)}\right), 0\right\} \\
&= \sum_{i=1}^{I} \max\left\{\left(V_{i,0}^{(1)} - M_{i,0}^{(1)} - L_{i,0}^{(1)} + V_{i,0}^{(2)} - M_{i,0}^{(2)} - L_{i,0}^{(2)}\right) - \left(V_{i,t}^{(1)} + V_{i,t}^{(2)}\right), 0\right\}.
\end{aligned} \tag{8}
$$

Here again we implicitly assume that the value of the credit lines do not change over the modeling horizon. The same justification is applied. The new measure gauges the probability and magnitude that at least one of clearing members depletes its margin deposits and credit lines to meet margin calls. Once again, the clearinghouse' risk exposure can be interpreted as the sum of I short European put basket options. Here the difference is that the strike price, $\left(V_{i,0}^{(1)} - M_{i,0}^{(1)} - L_{i,0}^{(1)} + V_{i,0}^{(2)} - M_{i,0}^{(2)} - L_{i,0}^{(2)}\right)$, is lower than it is in $X_{1,t}$.

The measure assesses the liquidity gap that the clearinghouse may have to fill in should clearing members exhaust their cash equivalent liquid assets to meet margin calls. Part of the gap may be filled by the final customers of the clearing members because the clearing members are holding positions on their behalf. In this study, we do not intend to assess the liquidity and credit worthiness from these customers. The reasons are two-fold. First, the clearinghouse does not look to non-member customers for performance or attempt to evaluate their creditworthiness or market qualifications, hence the clearinghouse looks solely to the clearing member carrying and guaranteeing the account to secure all payments and margin obligations. Second, the clearinghouse only observes the aggregate positions of the portfolio at a clearing member's customers' account, it does not know the positions and creditworthiness of final customers of each clearing member. The probability distribution of $X_{2,t}$ can help a clearinghouse determine the appropriate size of short-term credit lines it should maintain to meet liquidity shortage. For example, the clearinghouse can arrange credit lines so that the aggregate size of credit lines can cover the risk exposure $X_{2,t}$ up to a desired δ_1 probability level, the rest of the liquidity exposure is left to be covered by the final customers who can draw liquidity from the whole banking system.

Our next step is to measure the credit risk exposure of the clearinghouse. Here we assume that clearing member i has a net capital equal to $C_{i,0}^{(1)}$ besides its margin already deposited and that it will default on its house account when the portfolio loss at its house account exceeds the sum of its margin deposit on the house account and its net capital, i.e., $\Delta V_{i,t}^{(1)} + M_{i,0}^{(1)} + C_{i,0}^{(1)} < 0$. We assume that the clearinghouse mandates that clearing member i must maintain a net capital level at least up to

$$C_{i,t}^{(1)} = -q(\alpha_3, \Delta V_{i,t}^{1}) + q(\alpha_1, \Delta V_{i,t}^{1}). \tag{9}$$

The default at members' customers' accounts is more difficult to model due to the lack of information concerning final customers as discussed above. Therefore instead of taking a "structural" approach as to the default of house accounts above, we use a "reduced-form" approach to modeling default at customers' accounts. We assume the default probability of the portfolio at clearing member i's customers' account is

$$Pr^{(2)}_{i,t} = \left\{ \begin{array}{lll} 0 & \text{when} & \Delta V^2_{i,t} \geq q(\alpha_1, \Delta V^2_{i,t}) \\ \beta_1 & \text{when} & q(\alpha_2, \Delta V^2_{i,t}) < \Delta V^2_{i,t} \leq q(\alpha_1, \Delta V^2_{i,t}) \\ \beta_2 & \text{when} & q(\alpha_3, \Delta V^2_{i,t}) < \Delta V^2_{i,t} \leq q(\alpha_2, \Delta V^2_{i,t}) \\ \beta_3 & \text{when} & -\infty < \Delta V^2_{i,t} \leq q(\alpha_3, \Delta V^2_{i,t}) \end{array} \right\} \tag{10}$$

where $\beta_1 < \beta_2 < \beta_3$ and $\alpha_3 < \alpha_2 < \alpha_1$. In the example of the next section, we calibrate the default function as

$$Pr^{(2)}_{i,t} = \left\{ \begin{array}{lll} 0 & \text{when} & \Delta V^2_{i,t} \geq q(10\%, \Delta V^2_{i,t}) \\ 1\% & \text{when} & q(5\%, \Delta V^2_{i,t}) < \Delta V^2_{i,t} \leq q(10\%, \Delta V^2_{i,t}) \\ 5\% & \text{when} & q(1\%, \Delta V^2_{i,t}) < \Delta V^2_{i,t} \leq q(5\%, \Delta V^2_{i,t}) \\ 25\% & \text{when} & -\infty < \Delta V^2_{i,t} \leq q(1\%, \Delta V^2_{i,t}) \end{array} \right\} \tag{11}$$

The calibration means that there is no default risk when the portfolio loss is covered by the margin deposit, however as the portfolio loss increasingly exceeds the margin deposit, final customers as a whole are more likely to default on their contractual obligations.

The slicing of the probability distribution of the portfolio value into different credit regimes is similar to the credit migration method used in the CreditMetrics (Gupton et al., 1997, p.37, chart 3.3) and (Crouhy et al., 2000, p.75, figure 8). Other "reduced-form" approaches to portfolio credit risk probably can be adopted to evaluate the credit risk of the portfolios at customers' accounts. An alternative is to use a Bernoulli Mixture model as in the CreditRisk+ model (Gordy, 2000; Crouhy et al., 2000) to model default as exogenous events and hence default does not depend on the possible portfolio value changes.

Similar to the computation of $X_{1,t}$ and $X_{2,t}$, we can now compute the credit risk exposure of the clearinghouse at time t

$$X_{3,t} = \sum_{i=1}^{I} -\left[\min\left\{\left(\Delta V_{i,t}^{(1)} + M_{i,0}^{(1)} + C_{i,0}^{(1)}\right), 0\right\} - \min\left\{\Delta V_{i,t}^{(2)} + M_{i,0}^{(2)}, 0\right\} \times Pr_{i,t}^{(2)}\right]$$
$$(12) \qquad = \sum_{i=1}^{I} \left[\max\left\{\left(V_{i,0}^{(1)} - M_{i,0}^{(1)} - C_{i,0}^{(1)}\right) - V_{i,t}^{(1)}, 0\right\} + \max\left\{\left(V_{i,0}^{(2)} - M_{2,0}^{(1)}\right) - V_{i,t}^{(2)}, 0\right\} \times Pr_{i,t}^{(2)}\right].$$

The measure can help the clearinghouse assess the appropriateness of its current economic capital and calculate appropriate economic capital level given a certain targeted protection threshold, δ_2. The economic capital of a clearinghouse serves much as the economic capital of a bank, cushioning the possible credit loss of clearing members' portfolios when the members default on their obligations. The probability, δ_2, should be very small given the importance of the clearinghouse operation and the sensitivity of its operation to its creditworthiness (and perhaps reputation and trustworthiness). It should be much smaller than the shorthold set for liquidity risk exposure, δ_1.

The model developed in this section emphasizes the portfolio diversification effect and option-like payoffs when measuring liquidity and credit risk exposure of a futures exchange's clearinghouse. The model provide a unified theoretical framework for risk management that systematically integrates margin requirements, credit lines and economic capital. In the next section, we illustrate how to use the new model to measure liquidity and credit risk exposure of a clearinghouse and how to assess appropriate clearing margin requirements, credit lines and economic capital.

III. Example

In this section, we illustrate measure of risk exposure and assessment of appropriate margin requirements, credit lines and economic capital of a clearinghouse using Monte Carlo simulation. We assume that a hypothetical clearinghouse settles and clears 10 exchange-traded futures contracts for 10 clearing members. For simplicity, we assume these 10 products are futures contracts for Live Cattle, Lean Hogs, British Pound, Euro FX, Swiss Franc, Japanese Yen, Eurodollar, LIBOR, NASDAQ-100, and S&P 500, which are traded at the CME (Chicago Mercantile Exchange). These products cover major categories of futures contracts traded at the CME: commodity, foreign exchange, equity and interest rates, and are the most popular products in each category. Again for simplicity, only contracts maturing in December 2005 are considered. We analyze the risk exposure of the hypothetical clearinghouse from 10/31/2005 ("today")to 11/01/2005 ("tomorrow"). The parameters of the multivariate Geometric Brownian motion 1 (the expectation vector and covariance matrix) are estimated based on the daily settlement prices of December 2005 contracts for these 10 products from 03/18/2005 to 10/31/2005. The specifications, settlement price and open interests on 10/31/2005 of these 10 futures contracts are reported in table 1 and the estimated means and standard deviations of log settlement price differences of the 10 futures contracts in table 2 and the estimated correlation matrix in table 3

To implement the model numerically, we also need to know the position holding of clearing members at their house and customers' accounts. For simplicity, we assume the sizes of 10 members are proportional to $6:4:2:2:1:1:1:1:1:1$ and the size of house and customers' accounts of any clearing member is proportional to $1:50$. Based on these ratios, we split the open interest of each contract among clearing members' house and customers' accounts using Monte-Carlo simulation. The details of the splitting algorithm is available from the authors upon request.

In the splitting, we ensure that the positions in any contacts across different accounts sum up to zero, and that the total of long (short) position in any contract across different accounts equals the open interest of that contract. An example of simulated position holdings at members' house and customers' accounts are illustrated in figure 1.

Based on the estimated multivariate Geometric Brownian motion, we simulate the settlement prices of the 10 futures contracts on 11/01/2005. With "tomorrow's" prices simulated and clearing members' position holdings specified, we can compute the probability distributions of portfolio value at clearing members' house and customers' accounts on 11/01/2005. Since the distributions can not be solved analytically, we simulate the distribution numerically using Monte Carlo simulation. For the sake of the space, here we only present the probability distributions of member *i*'s portfolios in figure 2. The upper subplot show the probability distribution of portfolio value at clearing member *i*'s house account on 11/01/2005 and the lower subplot that at its customers' account. Based on these kind of distributions, the clearinghouse collect margins from its clearing members to cover up 90% of potential loss at members' house and customers' accounts over the one-business-day modeling horizon. Also the clearinghouse request clearing members maintain additional credit lines, which together with margin requirements can cover up to 95% of the potential portfolio loss at these accounts.

With the probability distributions of portfolio values at clearing members' accounts estimated, we then compute the probability distribution of the risk exposure of the clearinghouse. The simulated probability distribution of the clearinghouse' risk exposure is illustrated in figure 6. The figure shows that given a coverage ratio of 90% (i.e. $\alpha = 10\%$) for margin requirement, there is a 77.09% of probability that the clearinghouse won't be exposed to any risk whatsoever because the aggregate portfolio loss does not exceed the collected margins for each member. However, there is a 22.91% of probability that at lease one member suffers portfolio losses exceeding margins already deposited. In this case, the clearinghouse has exposure to liquidity and/or credit risk. The figure shows that the clearinghouse would have a quite small risk exposure given there is any exposure, but occasionally, the clearinghouse could have substantial risk exposure as indicated by the right tail of the probability distribution of $X_{1,t}$. Depending on whether the clearing members, whose portfolio losses exceed the corresponding margin deposits, pay the margin variations on time and pay them eventually, the risk exposure can be translated in to liquidity risk exposure or credit risk exposure. These two issues are analyzed as follows. Note that the estimated probability distribution of $X_{1,t}$ depend not only on the price dynamics of futures contracts but also on the position holding of clearing members' portfolios at their house and customers' accounts.

The clearinghouse' liquidity risk exposure is assessed using equation 8. The simulated distribution of $X_{2,t}$ is illustrated in figure 8. The figure shows that given a coverage ratio of 95% (i.e., $\alpha_2 = 5\%$) for margin requirement and credit line combined, there is a 88.35% of probability that the clearinghouse would not face any liquidity risk. This means that additional credit line requirements for clearing members provides the clearinghouse a higher level of protection. Nonetheless, there is still a 11.65% of probability that the clearinghouse would have a liquidity risk exposure. The figure further shows the probability and magnitude of the liquidity shortage that the clearinghouse potentially has to fill in given there is a liquidity risk exposure. In case of a temporary liquidity shortage, the clearinghouse has to step in to provide the additional liquidity to facilitate smooth settlement and clearing of exchange trades. Based on the probability distribution of $X_{2,t}$, the clearinghouse can assess the total amount of credit lines it has to secure in order to grantee the smooth clearing and settlement operation. In the example presented in figure 4, it is estimated that the clearinghouse has to secure a total amount of credit lines equal USD 1.95×10^{11} to cover up to 99% of possible liquidity shortage over the one-business-day modeling horizon. The estimated aggregate credit line should provide protection at a level higher level 99% because at least part of the remaining 1% of possible liquidity shortage can be filled in by clearing members' final customers. But the exact protection level is difficult to assess because it is difficult to assess the capacity of financial customers' in providing liquidity, which may eventually depend on how much liquidity available in the whole financial market.

Similarly, the clearinghouse's credit risk exposure is assessed using equation 12. The simulated distribution of $X_{3,t}$ is illustrated in figure 12. The figure shows that given the net capital of the clearing members (equation 9,where $\alpha_3 = 1\%$) and default probability function for customers' accounts defined in equation 11, there is a 94.745% of probability that the clearinghouse would face any credit risk. Nonetheless, there is a 5.255% of probability that the clearinghouse would be exposed to credit risk. The figure shows the probability distribution of the clearinghouse' credit risk exposure given there is credit risk exposure. The probability distribution can help the clearinghouse assess the economic capital it has to acquire in order to cushion potential credit risk and loss up to a target protection level. In the example presented in figure 4, it is estimated that the clearinghouse has to acquire USD 2.70×10^{11} to cover up to 99.9% of possible credit risk exposure over the one-business-day modeling horizon.

IV. Conclusions

Clearinghouses play a central role in settlement and clearing of exchange-traded futures and futures options and is exposed to liquidity and credit risk due to its intermediary role in clearing and settlement operation. Clearinghouses manage the liquidity and credit risk using an array of tools and mechanisms, which include "mark-to-market", margin requirements, credit lines and economic capital. Margin requirement is a clearinghouse's first line of defense against possible default by its clearing members. Most previous academic studies assess margin requirements assessed based on statistical analysis of price changes to meet a coverage probability. These studies often focus on a single exchange-traded product and hence ignore portfolio diversification effect. In industry practice, margin requirements are set by the SPAN (Standard Portfolio Analysis of Risk) system, which partly takes into account of the portfolio diversification effect but it does not account for all the cross-product correlations. While margin requirement is the first line of defense, a clearinghouse also arranges lines of credit agreement with banks to meet possible liquidity shortage and collects capital contribution from its members to defuse the possibility of an exchange default. Although credit lines and economic capital pose substantial cost to clearinghouses and clearing firms, few studies consider these two risk management tools, particularly jointly with margin requirements. In sum, previous academic literatures and the SPAN system do not fully account for the portfolio diversification effect and the interlink among different risk management tools.

In this study, drawing on an analogy between a clearinghouse and a financial intermediary, such as a bank or an insurance firm, we model a futures exchange's clearinghouse as a "bank" holding a portfolio of credit lines available to its clearing members and collateralized with clearing margins. The privilege clearing firms have at a clearinghouse is treated as the privilege of one-day credit lines available to the clearing members via margin accounts and collateralized with clearing margins, whereas the clearinghouse is treated as holding a portfolio of these short-term (one-day) credit lines, or equivalently, a portfolio of short European put basket options. Consequently, we use the "bank" model to measure the clearinghouse's liquidity and credit risk exposure as the sum of the payoff functions of these put options, emphasizing the portfolio diversification and the option-like payoffs. The model provides exchange clearinghouses and government regulators with a unified framework of risk management that systematically integrates clearing margin requirements, credit lines and economic capital.

We then implement the model to measure the liquidity and credit risk exposure of a hypothetical clearinghouse and assess the appropriate clearing margin requirements, credit lines and economic capital for the clearinghouse. The implementation is carried out using Monte Carlo simulation. The numerical results of the example depend on assumptions made in the example concerning price dynamics and position holdings of clearing members, but the underlying methods have broader implications. The numerical results show that given a certain coverage ratio for margin requirements, we can compute the probability distribution of the clearinghouse's risk exposure. With further assumptions made concerning clearinghouse's credit lines and economic capital, we can compute the probability distributions of the clearinghouse' liquidity and credit risk exposure. Based on these probability distributions, we can compute the appropriate aggregate credit line and economic capital that the clearinghouse should secure to safeguard smooth clearing and settlement operations. The example demonstrate the effect of portfolio diversification and option-like payoff of a clearinghouse.

As the next step, we plan to test our model using data from the CFTCs large-trader reporting system. As a part of its market surveillance program, CFTC (Commodity Futures Trading Commission) collects market data and position information from exchanges, clearing members, futures commission merchants (FCMs), foreign brokers, and traders. We intend to collect data for futures and option contracts traded at CBOT and CME for 2005 and test the risk exposure of the clearinghouse affiliated with the CME.[1] In particular, we intend to compare the margin levels set by the SPAN system, the univariate approach and the new portfolio approach, and to assess the coverage probability of the CME's clearinghouse current credit lines and economic capital offers.

In further research, one should relax some of the model's underlying assumptions. For example, one can allow fat-tails and conditional/stochastic volatilities in price dynamics, perhaps even

[1] Futures and futures option contracts traded at CBOT are cleared and settled via the CME's clearinghouse due to CME/CBOT Common Clearing Link.

incorporate jumps into price evolution. In another direction, one should relax linearity assumption and consider option contracts. Other risk management tools used by clearinghouses, such as price limits, position limits, may also be incorporated into the model. One may also use the model to conduct stress test scenario analysis. Finally, we may extend the modeling horizon beyond one business day.

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Figure 1: The position holdings of portfolios at clearing members' house and customers' accounts.

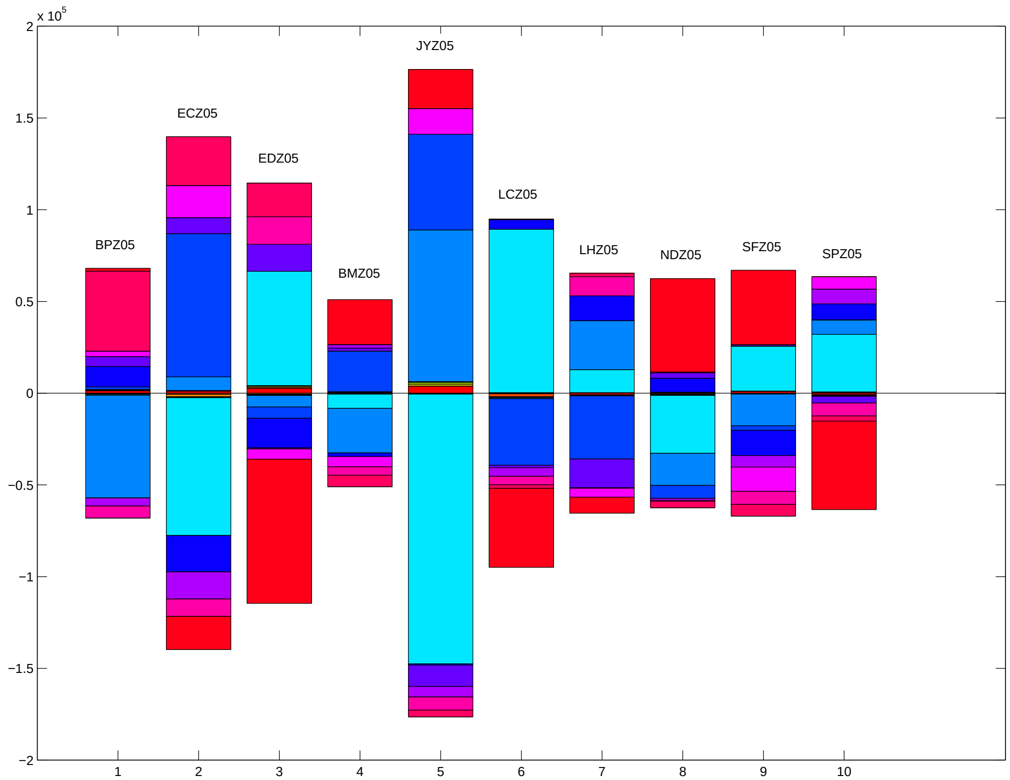

Figure 2: The probability distribution of portfolio values: the house and customers' account of clearing member 1' at time t.

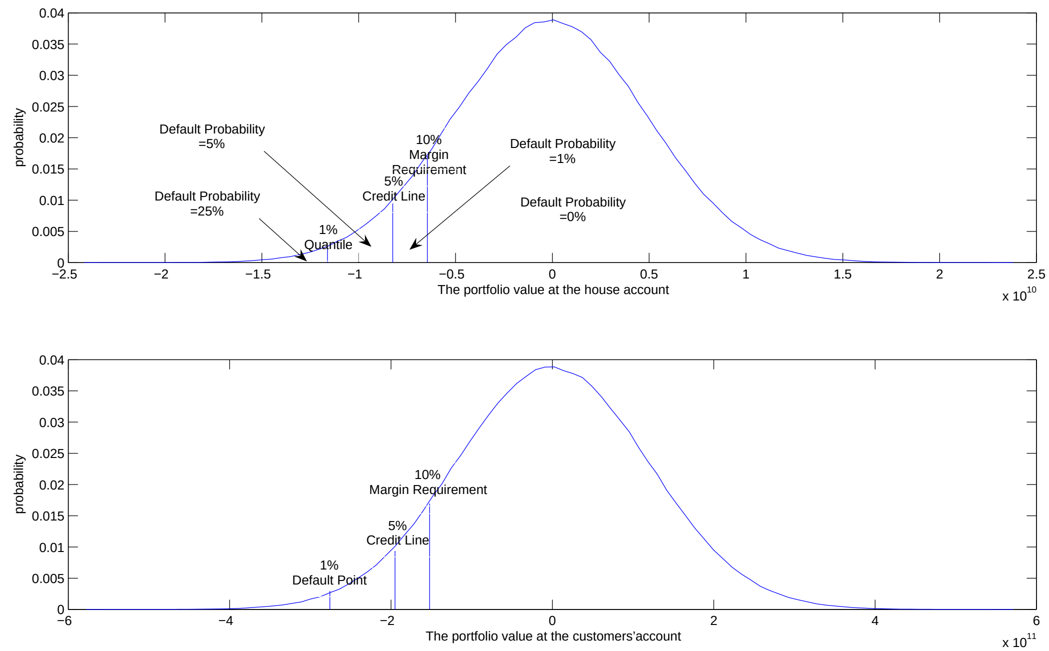

Figure 3: The probability distribution of the clearinghouse' risk exposure in exceeding of margins.

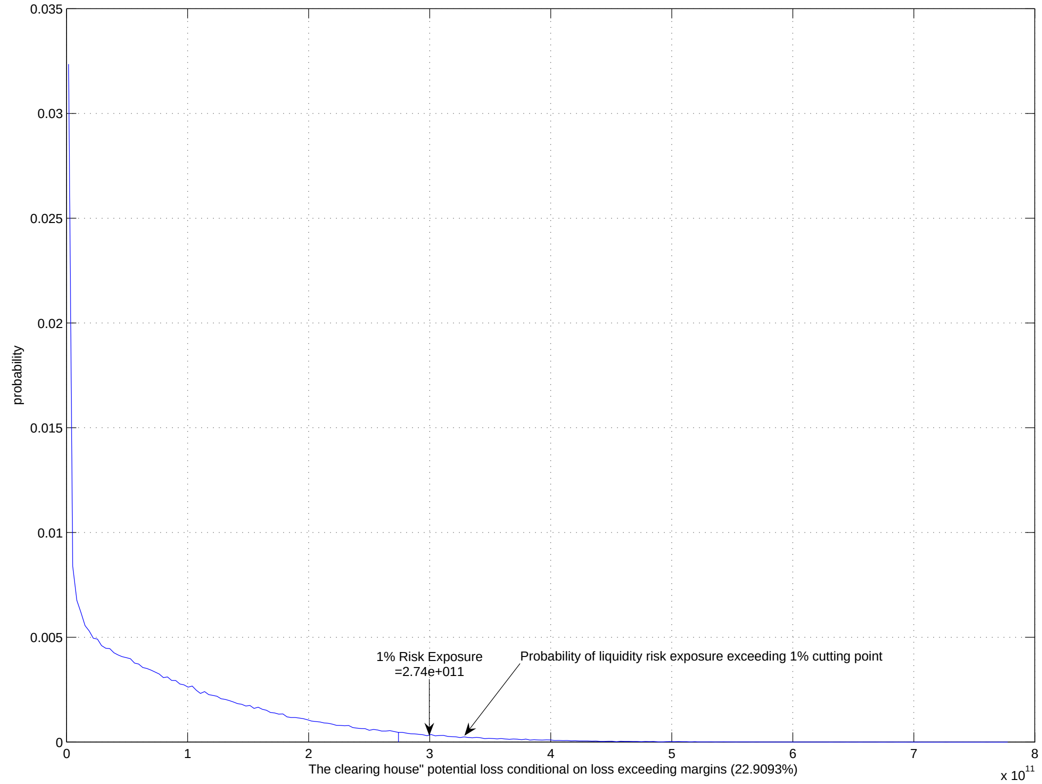

Figure 4: The probability distribution of the clearinghouse' liquidity risk exposure in exceeding of margins and additional credit lines.

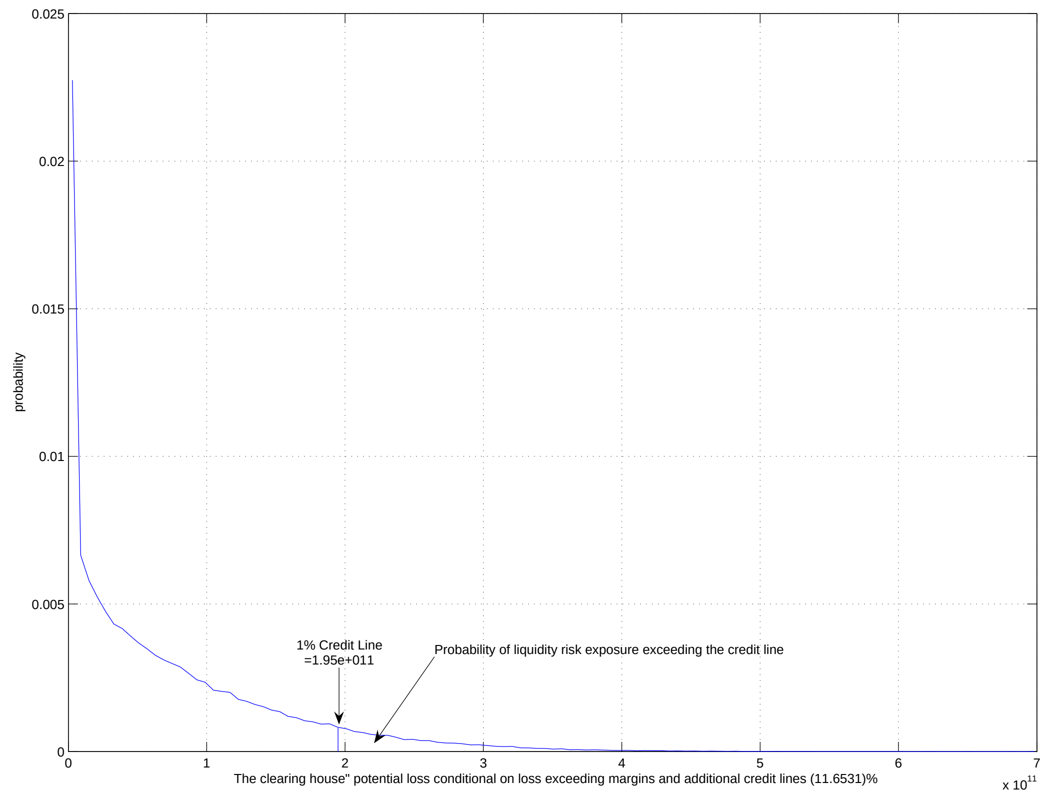

Figure 5: The probability distribution of the clearinghouse' credit risk exposure in exceeding of margins and net capital.

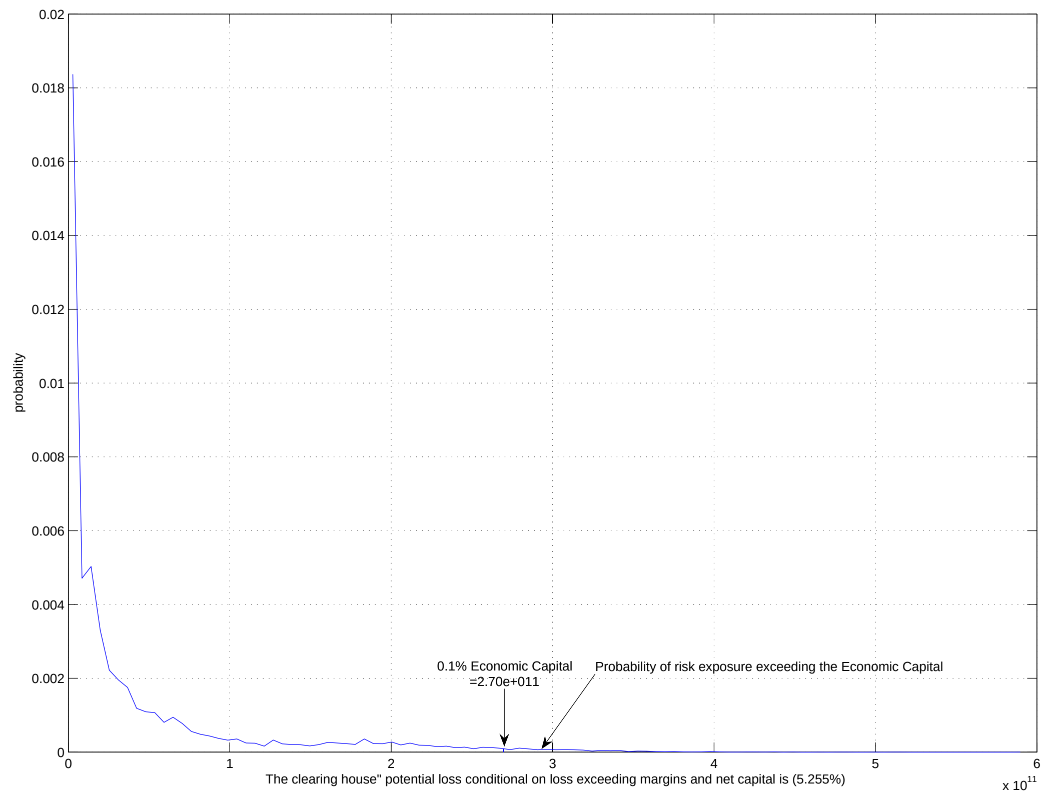

Table 1: **The specifications of the 10 CME products and their open interests and settlements on 10/31/2005.**

Contract	Product	Category	Trade Unit	Settlement	Open Interest
BPZ05	British Pound	foreign exchange	62,500 pounds sterling	1.7693	68048
ECZ05	Euro FX	foreign exchange	125,000 Euro	1.2023	139784
EDZ05	Eurodollar	interest rate	$1,000,000 with a three-month maturity	95.52	1145602
EMZ05	LIBOR	interest rate	$3,000,000 with a one-month maturity	95.625	5098
JYZ05	Japanese Yen	foreign exchange	12,500,000 Japanese yen	0.8642	176432
LCZ05	Live Cattle	commodity	40,000 pounds	90.92	94922
LHZ05	Lean Hogs	commodity	40,000 pounds	61.67	65414
NDZ05	NASDAQ-100	equity	$100 times the NASDAQ-100 Index	1586.5	62440
SFZ05	Swiss Franc	foreign exchange	125,000 Swiss francs	0.7796	67036
SPZ05	S&P 500	equity	$250 times the S&P 500 Stock Price Index	1209.8	634365

Table 2: The means and standard deviations of log settlement differences of the 10 CME products from 03/18/2005 to 10/31/2005.

Contract	Mean	Standard Deviation
BPZ05	-0.00044	0.00536
ECZ05	-0.00071	0.00575
EDZ05	-0.00002	0.00048
EMZ05	-0.00002	0.00045
JYZ05	-0.00080	0.00494
LCZ05	0.00047	0.00609
LHZ05	0.00012	0.01162
NDZ05	0.00031	0.00793
SFZ05	-0.00074	0.00602
SPZ05	0.00004	0.00690

Table 3: The correlation matrix of log settlement differences of the 10 CME products from 03/18/2005 to 10/31/2005.

Contract	BPZ05	ECZ05	EDZ05	EMZ05	JYZ05	LCZ05	LHZ05	NDZ05	SFZ05	SPZ05
BPZ05	1.000	0.776	0.281	0.308	0.670	-0.207	-0.032	-0.102	0.820	-0.035
ECZ05	0.776	1.000	0.214	0.234	0.588	-0.263	-0.079	-0.040	0.960	0.034
EDZ05	0.281	0.214	1.000	0.971	0.187	-0.144	-0.101	-0.162	0.292	-0.119
EMZ05	0.308	0.234	0.971	1.000	0.206	-0.127	-0.091	-0.183	0.303	-0.141
JYZ05	0.670	0.588	0.187	0.206	1.000	-0.108	0.011	0.002	0.630	0.124
LCZ05	-0.207	-0.263	-0.144	-0.127	-0.108	1.000	0.304	-0.026	-0.272	-0.041
LHZ05	-0.032	-0.079	-0.101	-0.091	0.011	0.304	1.000	0.089	-0.071	0.039
NDZ05	-0.102	-0.040	-0.162	-0.183	0.002	-0.026	0.089	1.000	-0.074	0.869
SFZ05	0.820	0.960	0.292	0.303	0.630	-0.272	-0.071	-0.074	1.000	0.011
SPZ05	-0.035	0.034	-0.119	-0.141	0.124	-0.041	0.039	0.869	0.011	1.000