A Cost Approach to Economic Analysis under Production Uncertainty

by

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Abstract: This paper explores the economics of input decision by a firm facing production uncertainty. It relies on a state-contingent approach to production uncertainty. First, the paper develops a methodology to specify and estimate cost-minimizing input decisions under a state-contingent technology. Second, the analysis is applied to time series data on US agriculture. It finds strong empirical evidence that, in the analysis of input choices, expected output alone does not provide an appropriate representation of production uncertainty. The results provide empirical support for an output-cubical technology. This indicates that an ex post analysis of stochastic technology (as commonly found in previous research) appears appropriate. The analysis also provides evidence that the cost of facing adverse weather conditions has declined in US agriculture over the last few decades.

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1. Introduction

Much research has been done on the microeconomics of uncertainty. Under incomplete risk markets, the effects of uncertainty on economic decisions have typically been investigated under the expected utility model (e.g., Arrow; Pratt). When applied to firm behavior, Sandmo and others have shown the adverse effects of uncertainty under risk aversion. This has stressed the joint importance of risk assessment and risk preferences. Risk assessment is typically presented in the context of probability assessments. And risk preferences are typically evaluated in the context of the expected utility model (e.g., Arrow; Pratt). However, psychologists have documented the presence of systematic bias in probability assessment (e.g., Camerer). And there is evidence that the expected utility model fails to provide an accurate representation of individual risk preferences (e.g., Machina). This raises two questions. First, is a probability assessment always required? Second, are there situations where analyzing firm behavior does not require knowing the decision maker's risk preferences? The objective of this paper is to explore these issues by analyzing firm decisions under production uncertainty.

This paper explores the economics of input decisions made by a firm facing production uncertainty. The issue of investigating cost-minimizing input choices under production uncertainty has been analyzed by Pope and Chavas, Chambers and Quiggin, and others. Under risk aversion, Pope and Chavas have argued that, under risk aversion, expected output alone does not provide an appropriate characterization of cost minimization. Chambers and Quiggin have argued that, standard cost minimization still applies under a state-contingent approach, irrespective of risk preferences. However, so far, the state-contingent approach has not been used empirically. The current challenge is to make it empirically tractable.

This paper proposes a methodology to specify and estimate standard cost-minimizing input choices under production uncertainty and a state-contingent technology. The approach has several attractive characteristics. First, under a state-contingent approach, it does not require a priori risk assessments. This can be seen as an advantage when probability assessments are problematic and impede empirical economic analysis. Second, as argued by Chambers and Quiggin, the analysis applies irrespective of risk preferences. To the extent that assessing risk preferences is often difficult, this broadens the scope of applications of the methodology. Third, the approach provides a basis for investigating the nature of the state-contingent technology. In particular, it allows the empirical analysis of substitution possibilities across states of nature. As noted by Chambers and Quiggin, previous research has commonly assumed an "output-cubical technology", where there is no possibility of substitution among state-contingent outputs. Our approach provides a basis for testing this hypothesis.

The usefulness of the proposed methodology is illustrated in an econometric application to US agriculture. We find strong evidence that, in the analysis of input choices, expected output alone does not provide an appropriate representation of production uncertainty. The results indicate empirical support for an output-cubical technology. This indicates that an ex post analysis of stochastic technology (as commonly found in previous research) appears appropriate. The analysis also provides evidence that the cost of facing production risk has declined in US agriculture over the last few decades.

The paper is organized as follows. The basic model of the state-contingent approach to cost-minimizing input choice under production uncertainty is presented in section 2. Section 3 investigates how the substitution among state-contingent outputs can be estimated from the cost function. Section 4 discusses the measurement of stochastic outputs, with the aim of closing the gap between theory and empirical work. Section 5 proposes a parametric specification, with an econometric application to US agriculture presented in section 6. Section 7 discusses the empirical results. Finally, section 8 presents concluding remarks.

2. The model

Consider a firm making decisions under production uncertainty. The uncertainty is represented by S mutually exclusive states of nature. The firm chooses n inputs $\mathbf{x} = (x_1, \ldots, x_n) \in R^n$ to produce outputs $\mathbf{y} = (y_{11}, \ldots, y_{m1}; \ldots ; y_{1s}, \ldots, y_{ms}) \in R^{mS}$, where y_{is} is the quantity of the i-th output produced under the s-th state of nature, i = 1, …, m, s = 1, …, S. Under technology t, the stochastic production technology is represented by the possibility set $F(t) \subset R_+^n \times R_+^{mS}$, where $(\mathbf{x}, \mathbf{y}) \in F(t)$ means that outputs y can be produced using inputs x. The set F(t) provides a general ex ante representation of the production technology under production uncertainty. Throughout, we assume that, for each y, the input requirement set $G(\mathbf{y}, t) = \{\mathbf{x}: (\mathbf{x}, \mathbf{y}) \in F(t)\} \subset R_+^n$ is closed and convex.

In general, production decisions depend on the nature of risk preferences of the decision maker. However, production uncertainty is typically associated with lags in the production process. In this context, input decisions are made first before the state of nature and the possible output realizations become known. Denote by $\mathbf{w} = (w_1, \ldots, w_n) \in R_{++}^n$ the vector of prices for $\mathbf{x}$. Assume that input prices $\mathbf{w}$ are known at the time when inputs $\mathbf{x}$ are chosen. Also assume that the decision maker exhibits preferences that are non-satiated in income. Then, inputs **x** are chosen in a way consistent with the cost minimization problem

$$C(\mathbf{w}, \mathbf{y}, t) = \text{Min}_{\mathbf{x}} \{\mathbf{w} \cdot \mathbf{x}: \mathbf{x} \in G(\mathbf{y}, t)\}. \tag{1}$$

Indeed, if input choices do not minimize cost, then under income non-satiation, choosing x according to (1) would improve the welfare of the decision maker. Thus, cost minimizing behavior (as given in (1)) represents economic rationality for the firm irrespective of the nature of risk preferences of the decision maker. Below, we will use expression (1) as a general representation of input choice under production uncertainty.

Denote $\mathbf{x}^c(\mathbf{w}, \mathbf{y}, t) = \text{argmin}_{\mathbf{x}} \{\mathbf{w} \cdot \mathbf{x}: \mathbf{x} \in G(\mathbf{y}, t)\}$. In general, the cost function $C(\mathbf{w}, \mathbf{y}, t) = \mathbf{w} \cdot \mathbf{x}^c(\mathbf{w}, \mathbf{y}, t)$ is positively linearly homogeneous and concave in **w**. And in the case where $C(\mathbf{w}, \mathbf{y}, t)$ is differentiable in **w**, it satisfies Shephard's lemma:

$$\mathbf{x}^c(\mathbf{w}, \mathbf{y}, t) = \partial C(\mathbf{w}, \mathbf{y}, t)/\partial \mathbf{w}. \tag{2}$$

Equation (2) provides a convenient framework to investigate economic behavior under uncertainty. Throughout the paper, we will rely on (2) as a representation of economic rationality for input decisions under production uncertainty. Also, we will use (2) as a means of obtaining information about the nature of the underlying production technology. From duality, it is well known that the cost function $C(\mathbf{w}, \mathbf{y}, t)$ in (1) provides a convenient framework to investigate the nature of substitution among inputs. In particular, the Allen elasticity of substitution between inputs i and j is given by $\sigma_{ij} = \frac{\partial^2 C}{\partial w_i \partial w_j} \frac{C}{(\partial C/\partial w_i)(\partial C/\partial w_j)}$, or using Shephard's lemma (2), $\sigma_{ij} = \frac{\partial x_i^c}{\partial w_j} \frac{C}{x_i^c x_j^c}$ (e.g., Chambers).

Below, we will be particularly interested in exploring the nature of substitution across states. This is at the heart of a debate about whether an ex post production function provides an appropriate representation of the ex ante technology under production uncertainty. This question has been raised by Chambers and Quiggin, who have shown that an ex post production function is appropriate if and only if the ex ante production technology is "output cubical" across states, i.e., with no possibility of substitution across states. Following Powell and Gruen, this can be conveniently characterized by the Allen elasticity of transformation applied across states. In this context, the technology is "output cubical" is the Allen elasticity of transformation between any y_{is} and $y_{is'}$ is zero for all $s \neq s'$ and for all $i = 1, \ldots, m$. But how can we recover the Allen elasticities of transformation between outputs from the cost function (1)? This question is addressed in the next section.

3. Elasticities of transformation and duality

Powell and Gruen define elasticities of transformation between outputs. Such elasticities provide useful information about the possibility of substitution among outputs. While Powell and Gruen present Allen elasticities of transformation using the production function, this section uses duality to explore how to obtain elasticities of transformation from the cost function $C(\mathbf{w}, \mathbf{y}, t)$ in (1).

To develop the relevant duality results, let $\mathbf{g} \in R^n_+$ be some reference input bundle satisfying $\mathbf{g} \neq \mathbf{0}$. Given the input requirement set $G(\mathbf{y}, t)$, following Luenberger and Chambers et al., define the directional distance function

$$D(\mathbf{x}, \mathbf{y}, \mathbf{g}, t) = \max_\beta \{\beta: (\mathbf{x} - \beta\, \mathbf{g}) \in G(\mathbf{y}, t)\} \text{ if there is a } \beta \text{ such that } (\mathbf{x} - \beta\, \mathbf{g}) \in G(\mathbf{y}, t)$$
$$= -\infty \text{ otherwise.}$$

The function $D(\mathbf{x}, \mathbf{y}, \mathbf{g}, t)$ measures the distance between point $(\mathbf{x}, \mathbf{y})$ and the boundary of the feasible set, expressed in units of the reference bundle $\mathbf{g}$. Under free input disposability (where $\mathbf{x} \in G(\mathbf{y}, t)$ implies that $\mathbf{x}' \in G(\mathbf{y}, t)$ for all $\mathbf{x}' \geq \mathbf{x}$), $\mathbf{x} \in G(\mathbf{y}, t)$ is equivalent to $D(\mathbf{x}, \mathbf{y}, \mathbf{g}) \geq 0$. In this case, the directional distance function $D(\mathbf{x}, \mathbf{y}, \mathbf{g}, t)$ provides a complete representation of the technology, where $D(\mathbf{x}, \mathbf{y}, \mathbf{g}) = 0$ is an implicit multi-output production function representing the boundary of the feasible region. Below, we will assume that $D^*(\mathbf{x}, \mathbf{y}, \mathbf{g}, t)$ is twice continuously differentiable in $(\mathbf{x}, \mathbf{y})$. Also, we will make use of the "normalized" distance function $D^*(\mathbf{x}, \mathbf{y}, \mathbf{w}, t) \equiv [\mathbf{w}\, \mathbf{g}]\, D(\mathbf{x}, \mathbf{y}, \mathbf{g}, t)$.

Using $D^*(\mathbf{x}, \mathbf{y}, \mathbf{w}, t) = 0$ as a multi-output production function and following Allen, and Powell and Gruen, the elasticity of transformation between any two outputs y_i and y_j can be defined as $\tau_{ij} = - \frac{\sum_{k=1}^{m} (\partial D^*/\partial y_k)\, y_k}{y_i y_j} \frac{K^c_{ij}}{\det(K)}$, where $K = \begin{bmatrix} \partial^2 D^*/\partial y^2 & (\partial D^*/\partial y)^T \\ \partial D^*/\partial y & 0 \end{bmatrix}$ is the bordered Hessian of $D^*(\mathbf{x}, \mathbf{y}, \mathbf{g}, t)$ with respect to $\mathbf{y}$, and K^c_{ij} is the (i, j)-th cofactor of K. Outputs i and j are said to be substitutes (complements) if $\tau_{ij} < 0$ (> 0).[1] And in the two output case $(m = 2)$, we have $\tau_{12} \in [-\infty, 0)$, where $\tau_{12} \to 0$ corresponds to fixed output-proportions (Powell and Gruen). In the general case, τ_{ij} measures the responsiveness of output-mix ratio to changes in the corresponding marginal rate of substitution.

Our main result is stated next (see the proof in the Appendix).

<u>Proposition 1</u>: Assume that $G(\mathbf{y}, t)$ is a convex set and that free input disposability holds. Then, the Allen elasticity of transformation between outputs i and j is given by

$$\tau_{ij} = \frac{\sum_{k=1}^{m} (\partial C/\partial y_k)\, y_k}{y_i y_j} \frac{H^c_{ij}}{\det(H)}, \tag{3}$$

where $H = \begin{bmatrix} -\frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial y \partial w}\left(\frac{\partial^2 C}{\partial w^2}\right)^{+} \frac{\partial^2 C}{\partial w \partial y} & -(\partial C/\partial y)^{T} \\ -\partial C/\partial y & 0 \end{bmatrix}$, H^{c}_{ij} is the (i, j)-th cofactor of H,

and $\left(\frac{\partial^2 C}{\partial w^2}\right)^{+}$ denotes the generalized inverse of $\left(\frac{\partial^2 C}{\partial w^2}\right)$.

Equation (3) gives an evaluation of the Allen elasticity of transformation among outputs from the cost function. In the presence of state-contingent outputs, this provides a basis for investigating the possibility of substitution across states (e.g., whether or not the state-contingent technology is "cubical"). See below.

4. Measuring stochastic outputs

Consider a situation involving T observations on the firm. It will be convenient to think that different observations correspond to different time periods. In this context, we assume that each observation on the firm can be associated with a different technology, where “t” represents both time and a “technology index”, t = 1, …, T. It follows that the input requirement set G(**y**, t) allows for possible technological change across observations. The t-th observation consists in observing inputs $\mathbf{x}_t = (x_{1t}, \ldots, x_{nt})$, input prices $\mathbf{w}_t = (w_{1t}, \ldots, w_{nt})$, and outputs $(y_{1t}, \ldots, y_{mt})$. Under production uncertainty, for each t, the ex post outputs realization $(y_{1t}, \ldots, y_{mt})$ is only one of the many possible realizations of outputs. The output realizations that are possible ex ante are $\mathbf{y}_t = (y_{1t1}, \ldots, y_{mt1}; \ldots; y_{1tS}, \ldots, y_{mtS})$, where y_{its} is the quantity of the the i-th output produced at time t under the s-th state of nature. The problem is that, for each t, only one of the S possible output realizations is typically observed. With ex ante outputs being incompletely observed, this means that neither the cost function $C(\mathbf{w}_t, \mathbf{y}_t, t)$ nor the input demand functions $\mathbf{x}^c(\mathbf{w}_t, \mathbf{y}_t, t)$ are empirically tractable. In order to make $C(\mathbf{w}_t, \mathbf{y}_t, t)$ and $\mathbf{x}^c(\mathbf{w}_t, \mathbf{y}_t, t)$ empirically tractable, it is necessary to impose some structure on the problem. Here, we propose a method to generate all possible outputs y based on the T observations of the firm.

First, we know that the ex post outputs realization $(y_{1t}, \ldots, y_{mt})$ is one of the possible ex ante realizations $\mathbf{y}_t = (y_{1t1}, \ldots, y_{mt1}; \ldots; y_{1tS}, \ldots, y_{mtS})$ at time t. In this context, one option is to estimate the ex post technology relating realized outputs $(y_{1t}, \ldots, y_{mt})$ to input use, conditional on the particular state of nature obtained under the t-th observation, $t = 1, \ldots, T$. To make this approach empirically tractable, stationary assumptions are needed to establish how the states of nature affect outputs across observations. This is typically done by treating the states as random variables, and making stationary assumptions on the probability distribution generating these random variables. For example, in the single output case $(m = 1)$, assuming that the states are independently distributed across observations, the regression of output on input use provides a framework to estimate an ex post production function, where the presence of heteroscedasticity can reflect the effects of input use on the variability of output (e.g., Antle; Just and Pope, 1978). This approach is convenient and has been commonly used in the analysis of stochastic technology. Its main limitations are three: first, by embedding the factors determining the state of nature into a single scalar-valued random variable and then embedding this variable in a technology, it imposes separability of the stochastic factors determining the state of nature (in an agricultural example, these would typically be viewed as random inputs such as weather and pest infestations) on the underlying technology; second, while it works well in a single output case, it can only be applied in a multioutput setting under the restrictive assumption on the technology of input nonjointness; and third, and perhaps most importantly, it focuses exclusively on the observed outputs. As such, the approach neglects the potential outputs that could have been obtained had nature selected different states. Is this neglect important for economic analysis? Chambers and Quiggin have showed that this neglect is acceptable under an "output-cubical technology" exhibiting no possibility of output substitution across states. In this case, the ex ante technology can be expressed entirely in terms of the ex post technologies across states (see Chambers and Quiggin, p. 53-55). This suggests that, in the absence of output substitution across states, an ex post analysis of stochastic technology is appropriate. However, one should keep in mind that this does not imply ex post cost minimization. Indeed, since inputs are chosen before the state of nature is known, their choice must be feasible ex ante, i.e. for all possible states of nature (and not just the particular state of nature that was observed). This means that, under an output cubical technology, ex post cost functions are a lower bound on the ex ante cost function $C(\mathbf{w}_t, \mathbf{y}_t, t)$ (Chambers and Quiggin, p. 134-135).

But what if the stochastic technology is not "output-cubical"? Then, there are possibilities of output substitution across states. In this case, as argued by Chambers and Quiggin, an ex post analysis of stochastic technology is inappropriate. It would neglect the effects of input choices on the distribution of outputs across states. For example, labor use can contribute to conserving water and affect the drought-resistance of a crop. Then, important output trade-offs exist across states of nature. Capturing these trade-offs require an ex-ante representation of the technology. This raises the important question: how to do this empirically?

A natural place to start is to explore whether the output observations $(y_{1t}, \ldots, y_{mt})$, $t = 1, \ldots, T$, can be used to recover the ex ante technology? This is a difficult problem. The reason is that outputs depend on inputs, on the state of nature, as well as on the underlying technology. We have an identification problem. Under production uncertainty, we cannot estimate the ex ante technology without observing all possible outputs (meaning outputs under all possible states, and not just for the realized state). And without knowing the underlying technology, we do not know what outputs could have been under different states of nature (at least when the technology is not output-cubical). Thus, under general production uncertainty, knowing the actual outputs (y_{1t}, …, y_{mt}) does not provide enough information to know the distribution of all possible outputs or the underlying ex ante technology. In an attempt to resolve this issue, we need to impose some a priori structure on the process generating the states of nature. Below, we propose a general methodology to recover possible ex ante outputs using actual outputs (y_{1t}, …, y_{mt}). We know that (y_{1t}, …, y_{mt}) is one of the possible outputs for the t-th observation., Recall that y_{ist} denotes the quantity of the i-th output produced under the s-th state of nature at time t. For the i-th output, assume the existence of positive numbers μ_{is} and σ_{is}, i = 1, …, m, s = 1, …, S. for each i, define a random variable e_i for which the s-th realization is given by $e_{is} \equiv (y_{is}/\mu_{is})^{1/\sigma_{is}}$, s = 1, …, S. It follows that the ex ante outputs can be written as

$$y_{ist} = \mu_{it}\ e_{is}^{\sigma_{it}}, \tag{4}$$

Equation (4) defines the variable $e_{is} \equiv (y_{is}/\mu_{it})^{1/\sigma_{it}}$ as measuring the relative changes in the i-th output across states of nature. Thinking of (y_{i1t}, …, y_{iSt}) as a random variable that can take different values across states, this simply defines e_i as a new random variable obtained from a deterministic transformation of the original one. This imposes no a priori restriction on the nature of production uncertainty. Indeed, for each t, it allows for an arbitrary distribution of the effects of production uncertainty on outputs. In addition, note that the term σ_{it} can be interpreted as a "spread parameter", allowing the spread of the distribution of the i-th output across states to vary across observations t. However, equation (4) does impose a stationarity restriction. It assumes that, except for the spread effects captured by σ_{it}, the relative effects of production uncertainty on each output are similar across observations t.

Next, assume that there exists auxiliary variables z_{it} with the following property. When s is the state occurring under the t-th observation, z_{it} satisfies

$$z_{it} = k_{it}\ e_{it}^{\sigma_{it}}, \qquad (5)$$

i = 1, …, m, and t = 1, …, T. This establishes the variables z as proxy variables for the measurement of production uncertainty. Indeed, by definition of z_{it}, for the t-th observation, the states of nature have the same relative effects on the i-th output as they have on z_{it}. Below, we will discuss which variables appear to be good candidates for z. Assume that k_{it} and σ_{it} can be consistently estimated. Assuming that all variables are positive, equation (5) can be written as $\ln(z_{it}) = \ln(k_{it}) + \sigma_{it} \ln(e_{is})$. This can be treated as a standard econometric model with $\ln(z_{it})$ as the dependent variable, $\ln(k_{it})$ as the regression line, $\sigma_{it} \ln(e_{it})$ as the error term, and σ_{it} as capturing possible heteroscedasticity. In the case where $\ln(e_{it})$ has mean zero and variance 1, then $\ln(k_{it})$ can be interpreted as the expected value of $\ln(z_{it})$, and σ_{it} as the standard deviation of the error term for the i-th output and the t-th observation. As shown by Antle, after choosing a parametric specification for k_{it} and σ_{it}, a moment-based approach can be used to obtain consistent estimate of the parameters. See below.

When s is the state occurring under the t-th observation, it follows from equation (5) that $e_{it} = (z_{it}/k_{it})^{1/\sigma_{it}}$. This generates $((z_{i1}/k_{i1})^{1/\sigma_{it}}, \ldots, (z_{iT}/k_{iT})^{1/\sigma_{iT}})$ as estimates of T realized values of the random variable e_i. For the t-th observation and from equation (4), this can be used to obtain the simulated state-contingent outputs at time t

$$\mathbf{y}_t^e = \{y_{irt}:\ y_{irt} = y_{it}\ (z_{ir}/k_{ir})^{\sigma_{it}/\sigma_{ir}}/(z_{it}/k_{it});\ r = 1, \ldots, T;\ i = 1, \ldots, m\}. \qquad (6)$$

t = 1, …, T. Again, note that the term σ_{it}/σ_{ir} in (6) allows for the spread of the distribution of the i-th output across states to vary across observations. We want to stress here that μ_{it} in (4) does not play any role in the evaluation of simulated outputs $\mathbf{y}_t^e$ in (6). To the extent that the μ_{it}'s are expected to reflect the underlying technology and the associated economic tradeoffs, this means that our proposed scheme for evaluating ex ante outputs can be applied independently of the nature of the technology. Of course, the validity of the approach relies crucially on the validity of the stationary assumption (4) and of equation (5).

5. Parametric specification

In general, consistent estimates of k_{jt} and σ_{it} can be used to generate simulated state-contingent outputs $\mathbf{y}_t^e$ from equation (6). In turn, this can be used to obtain consistent estimate of the cost function $C(\mathbf{w}, \mathbf{y}, t)$ and of cost minimizing behavior $\mathbf{x}^c(\mathbf{w}, \mathbf{y}, t)$. This section discusses specification issues raised in this approach. When using the state contingent outputs $\mathbf{y}_t^e$, the problem becomes one of specifying and estimating $C(\mathbf{w}_t, \mathbf{y}_t^e, t)$ and of cost minimizing behavior $\mathbf{x}^c(\mathbf{w}_t, \mathbf{y}_t^e, t)$ based on a sample of T observations. In this context, the state contingent outputs $\mathbf{y}_t^e = \{y_{irt}: y_{irt} = y_{it}\ (z_{ir}/k_{ir})^{\sigma_{it}/\sigma_{ir}}/(z_{it}/k_{it});\ r = 1, \ldots, T;\ i = 1, \ldots, m\}$ include mT variables at each time period t. Even when m = 1, including such a large number of explanatory variables is problematic. Typically, many of the elements of $\mathbf{y}_t^e$ will tend to be correlated in the sample, creating serious multicollinearity problems. This makes it very difficult to estimate $C(\mathbf{w}_t, \mathbf{y}_t^e, t)$ and $\mathbf{x}^c(\mathbf{w}_t, \mathbf{y}_t^e, t)$ directly. And the collinearity problems would become even more severe when m > 1. This suggests a need to develop an econometric approach that can avoid such problems. The solution is to propose a "parsimonious" parametric specification of $C(\mathbf{w}_t, \mathbf{y}_t^e, t)$ and $\mathbf{x}^c(\mathbf{w}_t, \mathbf{y}_t^e, t)$ that would not involve "too many" parameters while still allowing the estimation of substitution possibilities across states.

This can be done in two possible ways. A first approach to a parsimonious parametric specification can be obtained by representing the distribution of the e's by a few parameters. If the distribution belongs to a specific parametric form, then the associated parameters are sufficient statistics and provide all the relevant information. Alternatively, the first few central moments of the distribution can be used (assuming that they exist). In this context, one issue is: how many moments are needed to represent the distribution? If the decision maker is risk neutral, we know that only the first moment is relevant in the decision making process. This is the assumption made by Pope and Just (1996) and Moschini in their analysis of production uncertainty. However, if the decision maker is not risk neutral (e.g., under risk aversion), then the first moment is not sufficient to characterize production decisions under risk. Then, at minimum, the first two moments (and possibly higher moments) are needed. This issue will be investigated empirically below.

A second approach to a parsimonious parametric specification involves working with a coarsened partition of the state space. To see that, for the i-th output and the t-th observation, define (K_i-1) values b_{ikt} satisfying $b_{i1t} < b_{i2t} < \ldots < b_{i,K_i-1,t}$. For each i and t, this establishes K_i intervals, $V_{i1t} = [-\infty, b_{i1t}]$, $V_{ikt} = (b_{i,k-1,t}, b_{ikt}]$, $k = 2, \ldots, K_i-1$, and $V_{i,K_i,t} = (b_{i,K_i,t}, +\infty]$, $i = 1, \ldots, m$, $t = 1, \ldots, T$. Assume that the partitions are chosen such that there is at least one observation y_{irt} satisfying $y_{irt} \in V_{ikt}$ for each i, k and t. Define the indicator variables

$$I_{ikrt} = 1 \text{ if } y_{irt} \in V_{ikt},$$
$$= 0 \text{ otherwise.}$$

Let $y_{ikt} = (\sum_{r=1}^{T} I_{ikrt}\, y_{irt})/(\sum_{r=1}^{T} I_{ikrt})$ denote the conditional mean of y_{irt} in the k-th partition related to the i-th output at time t. Define $\mathbf{y}_t^K = \{y_{ikt}: k = 1, \ldots K_i-1; i = 1, \ldots, m\}$. Next, consider specifying the cost function as $C(\mathbf{w}_t, \mathbf{y}_t^K, t)$ and the input demand functions as $\mathbf{x}^c(\mathbf{w}_t, \mathbf{y}_t^K, t)$. The choice of the state partition provides some flexibility for capturing the economic tradeoffs between outputs across states. At one extreme, for each i and k, the finest partition would be obtained if $K_i = T$, generating a single observation at time t in each element of the partition. This would be very flexible. However, as noted above, under production uncertainty, it is not practical (it involves too many parameters to estimate).

At the other extreme, the coarsest partition would be obtained if $K_i = 1$ for each i. This would help reduce greatly the number of parameters to estimate. However, this appears too restrictive for at least three reasons. First, it would amount to replacing the distribution of each y_{irt} across states by its corresponding unconditional means $(\sum_{r=1}^{T} y_{irt})/T$. Since the mean is in general not a sufficient statistic for most distributions, this would likely involve important loss of information. Second, if the decision maker is risk neutral, then it could be argued that the mean is the only relevant variable that would influence the decision making process (as assumed by Pope and Just (1996) and Moschini). However, this would not apply under risk aversion. To the extent that there is considerable evidence that most decision makers are risk averse, this would fail to capture the effects of risk and risk aversion on production behavior. Third, using unconditional means as representations of production uncertainty would make it impossible to estimate econometrically the elasticity of substitution across states. We are interested here in estimating such elasticities. This alone would rule out the use of unconditional means $(\sum_{r=1}^{T} y_{irt})/T$ in the representation of output uncertainty.

If either $K_i = 1$ and $K_i = T$ appears undesirable, this suggests that a reasonable choice of partitions would satisfy $1 < K_i < T$. In general, this choice involves tradeoffs between providing a flexible representation of the underlying technology (with flexibility improving as the K_i's increase) and parsimony and ease of estimation (where estimating the model becomes easier as the K_i's decrease). The approach is illustrated below in an empirical application.

6. An Econometric Application

Consider the case of where the state space is partitioned to give $\mathbf{y}_t^K = \{y_{ikt}: y_{ikt} = (\sum_{r=1}^{T} I_{ikrt}\, y_{irt})/(\sum_{r=1}^{T} I_{ikrt});\ k = 1, \ldots, K_i;\ i = 1, \ldots, m\}$. We focus our discussion on the case of the generalized Leontief cost function (see Diewert; Lopez)

$$C(\mathbf{w}_t, \mathbf{y}_t^K, t) = h(\mathbf{y}_t^K, t)\ [\sum_i \sum_j \alpha_{ij}\, w_{it}^{1/2}\, w_{jt}^{1/2}] + \sum_j w_{jt}\, g_j(\mathbf{y}_t^K, t) \quad (7)$$

where $\alpha_{ij} = \alpha_{ji}$ for all $i \neq j$, $\mathbf{y}_t^K = \{y_{ikt}: y_{ikt} = (\sum_{r=1}^{T} I_{ikrt}\, y_{irt})/(\sum_{r=1}^{T} I_{ikrt});\ k = 1, \ldots, K_i;\ i = 1, \ldots, m\}$, and $h(\mathbf{y}_t^K, t)$ and $g_j(\mathbf{y}_t^K, t)$ take some parametric form (see below). Diewert has shown that this specification is flexible in the sense that it does not impose a priori restrictions on the possibilities of substitution among inputs. It includes as a special case a Leontief technology when $\alpha_{ij} = 0$ for all $i \neq j$, and a homothetic technology when $g_j(\mathbf{y}_t^K, t) = 0$, $j = 1, \ldots, n$ (Shephard). The possibilities of substitution among outputs are captured by the functions $h(\mathbf{y}_t^K, t)$ and $g_j(\mathbf{y}_t^K, t)$. Under production uncertainty, this involves possible substitution both among the m different outputs as well as across states of nature.

Then, we consider the following specification for $h(\cdot)$:

$$h(\cdot) = \sum_i \sum_k \beta_{ik}\, y_{ikt} + \sum_{i,i'} \sum_{k \neq k'} \beta_{ii',kk'}\, y_{ikt}\, y_{i'k't}], \quad (8)$$

subject to the normalization rule $\beta_{i1} = 1$, with $\beta_{ii',kk'} = \beta_{i'i,k'k}$ for all $i \neq i'$ and $k \neq k'$. Note that the parameters $\beta_{ii',kk'}$ in (8) capture the possibilities of substitution among different outputs (for $i \neq i'$) as well as different states (for $k \neq k'$). We consider the following specification for $g_j(\cdot)$:

$$g_i(\cdot) = \gamma_{0i} + \gamma_{ti}\, t,\ i = 1, .., n. \quad (9)$$

Finally, using Shephard's lemma (2), under the specifications (7), (8) and (9), the cost minimizing input demand functions under production uncertainty take the form

$$x_{it}^{c}(\mathbf{w}_t, \mathbf{y}_t^K, t) = h(\mathbf{y}_t^K, t)\,[\textstyle\sum_j \alpha_{ij}\,(w_{jt}/w_{it})^{1/2}] + g_i(\mathbf{y}_t^K, t), \qquad (10)$$

$i = 1, \ldots, n$, $t = 1, \ldots, T$.

7. Econometric Results

In this section, the above model is applied to US agriculture. Annual data on US agriculture were obtained from the US Department of Agriculture. They include price and quantity data for four inputs (labor, capital, material and land) and one aggregate output for the period 1949 to 1999 (see Ball et al.). Thus, by working with an aggregate output, the analysis presented below focuses on the single output case, with $m = 1$.

The evaluation of production uncertainty requires an empirical basis to estimate equation (5). We use a crop yield index as the auxiliary variable z capturing production uncertainty. This seems reasonable: once acreage decisions are made, production uncertainty manifest itself entirely through yield effects. As a result, yield fluctuations are due in large part to unpredictable weather effects and pest damages. First, we measure z as a yield index, calculated from annual data on "yield per acre planted" for the major US crops (corn, wheat and soybeans). Second, we ran a regression $\ln(z_t) = \ln(k_t) + \sigma_t \ln(e_t)$, with the regression line $\ln(k_t)$ including selected explanatory variables. The explanatory variables are a time trend (to capture technological progress over time) and relative prices for inputs and outputs (to capture the effects of changing market conditions on yield). After controlling for technological change and price effects, the error term of the regression is interpreted as reflecting production uncertainty. We investigated whether the variance of the error term changed over time. Using a Lagrange multiplier test (based on squared residuals regressed on squared fitted values), we failed to find statistical evidence of heteroscedasticity (the p-value for the test was 0.67). As a result, we proceeded with assuming that the variance σ_t^2 was constant over time. With a constant variance σ^2, we obtained consistent estimates of $e_t = \exp[(\ln(z_t) - \ln(k_t))/\sigma]$, t = 1, …, T. Under a stationarity assumption (as discussed above), we used these estimates to generate the state-contingent outputs in equation (3).

Next, we used the specification of cost-minimizing input demands given in (10). The specification was estimated for K = 2. This can be interpreted as considering two states of nature, e.g., "bad weather" (k = 1) and "good weather" (k = 2). While this is a rather coarse representation of the state space, it will be convenient for the investigation reported in this paper.[2]

We first estimated equations (10) with four inputs: labor, capital, material, and land. However, the estimates showed that the cost function was not concave in capital price (i.e., the demand for capital was found to upward sloping, which is inconsistent with cost minimization). We interpreted this as indirect evidence that the demand for capital may involve significant dynamics that are not captured in (10). This suggested the need either to address dynamics explicitly, or alternatively to conduct the analysis conditional on capital. To the extent that the dynamics of capital can be complex, we opted for the second option. As a result, the empirical analysis presented below focuses on the demand for three inputs, labor, material and land, taking capital as given. In this specification, we introduced the effects of capital on the demand for other inputs by letting the γ_{0i} in (9) to vary with capital, with $\gamma_{0i} = \gamma_{ai} + \gamma_{bi}$ Capital. Associating i = 1 with labor, i = 2 with material, and i = 3 with land, equation (10) was estimated by maximum likelihood. The resulting parameter estimates are presented in Table 1. To take into consideration possible heteroscedasticity, the standard errors in Table 1 are White-corrected robust standard errors.

Table 1 shows that the model provides a good fit to the data. The R-square varies between 0.934 for material to 0.991 for labor. Most parameters are statistically different from zero at the 5 percent significance level. With $\beta_1 = 1$ by normalization, note that the estimate of β_2 (0.9855) is not statistically different from 1. Also, the coefficient β_{12} is found to be negative and statistically significant. The null hypothesis that $\beta_{12} = 0$ is strongly rejected at the 1 percent significance level. This indicates the presence of significant interactions across states of nature. Note that a cost specification that would depend only on expected output would be obtained as a special case with $\beta_2 = 1$ and $\beta_{12} = 0$. Using a Wald test, this hypothesis is strongly rejected at the 1 percent level. This indicates that, under uncertainty, a cost specification that would depend only on expected output is inappropriate.[3] As discussed above, this has at least two implications. First, if decision makers are risk averse under incomplete markets, then focusing on expected output alone fails to capture the role of risk management in input choice. Second, even if firm managers are risk neutral, our result indicates that focusing narrowly on expected output is not enough to provide a complete characterization of the stochastic technology and its implications for production behavior. On the one hand, it should not be a surprise to find out that the mean of a distribution is in general not a sufficient statistics for representing the whole distribution. On the other hand, our empirical findings show that this lack of sufficiency is empirically relevant when characterizing cost minimizing behavior.

The parameter estimates for the α's reported in Table 1 indicate that price effects are statistically significant. These price effects are found to be consistent with production theory: the cost function is concave in input prices. Evaluated at sample means, the price elasticities of input demands are reported in Table 2. As expected, input demands are downward sloping. The price elasticities of land are found to small. This is consistent with land being close to being a fixed factor in agriculture. The price elasticities of labor and material are larger but remain inelastic, with an own-price elasticity of -0.387 for labor and -0.299 for material. The cross-price elasticity between labor and material is positive, indicating that they are substitute inputs. The parameter estimates for the γ's indicate that technological progress has been biased against labor (with $\gamma_{t1} < 0$ corresponding labor-saving technical change)) and in favor of material (with $\gamma_{t3} > 0$ identifying material-using technical change).

Using equation (3) and the parameter estimates reported in Table 1, the elasticity of transformation between states was estimated. Evaluated at sample means, the elasticity of transformation was calculated to be $\tau_{12} = -0.001$. This is very close to zero. Recall that $\tau_{12} = 0$ corresponds to an output-cubical technology with zero possibility for substitution between states. This indicates that the possibility of output substitution between states is extremely limited. We interpret this as empirical evidence in favor of an output-cubical technology. In other words, our analysis supports the validity of the ex post analyses of stochastic technology commonly found in previous research (e.g., Antle; Just and Pope, 1978).

Finally, the parameter estimates were used to evaluate the marginal cost of outputs $MC_{kt} = \partial C/\partial y_{kt}$ for state k at time t. Recall that k = 1 corresponds to "bad weather" while k = 2 corresponds to "good weather". Figure 1 reports the evolution of the relative marginal cost MC_{1t}/MC_{2t} over the period 1970-1999. It shows that the marginal cost of production tends to higher under "bad weather" (compared to "good weather"). It also shows two interesting characteristics. First, the relative marginal cost MC_{1t}/MC_{2t} has been declining over the last few decades. It means that the marginal cost of production under adverse weather conditions is not as high as it used to be. Second, the variability in the relative marginal cost MC_{1t}/ MC_{2t} has declined over time. In particular, the relative marginal cost is much more stable in the 1990's than it was in the 1970's. These findings reflect the nature of the underlying technology under production uncertainty. They hold irrespective of risk preferences. In this context, this provides evidence that the cost of facing production risk has declined in US agriculture over the last few decades.

8. Concluding Remarks

This paper investigated production uncertainty when input decisions are made before uncertain outputs are known. Using a state-contingent approach, we developed a methodology to specify ands estimate cost-minimizing input choices. The proposed approach exhibits at least two attractive characteristics. First, it does not require a probability assessment of the unknown outputs. This can be useful when such probability assessments are difficult to make. Second, it does not depend on the risk preferences of the decision maker. Given the current controversies about the validity of the expected utility model, this provides a framework to conduct economic analysis while avoiding such controversies. In addition, this appears useful when one realizes that risk preferences can be somewhat difficult to assess empirically and that they typically vary across individuals.

In this context, the challenge was to develop a methodology that is empirically tractable. The main issue arises from the fact that, at each time period, only one of the many possible states is typically observable. Our methodology proposes to measure all possible states, relying on auxiliary variables that can be used to simulate these states under stationarity conditions. This provides a framework to conduct econometric analysis of cost-minimizing behavior under a general state-contingent technology. The empirical tractability of the approach was illustrated in an empirical application to US agriculture.

The application demonstrates that an econometric analysis of state-contingent technology is possible and useful. Two important results were obtained. First, we found strong evidence that restricting the analysis of input choice to include only expected output is not appropriate. This reflects the fact, under risk aversion, the role of risk management in input choice can be important. More generally, this stresses the point that, for a general stochastic technology, mean output is not a sufficient statistic for the distribution of outputs. Second, we found econometric evidence that the possibility of substitution between states is very limited. We interpret this as evidence in favor of an "output-cubical" technology. This indicates that an ex post analysis of stochastic technology (as commonly found in previous research) appears appropriate. Finally, our analysis provides evidence that the cost of facing adverse weather conditions has declined in US agriculture over the last few decades.

Although our proposed approach is empirically tractable, it is also the subject of limitations. First, our measurement of state-contingent outputs requires stationarity assumptions. It would be useful for future research to explore whether our stationarity assumption could be relaxed. Second, our empirical analysis has neglected econometric issues related to simultaneity bias and measurement errors. Further research on these issues is needed. Finally, our econometric estimation was limited to two states of nature. This clearly appears restrictive. In principle, our methodology can handle any number of states. However, collinearity problems are likely to arise when the number is states is large (due to the associated increase in the number of parameters to estimate). By reducing the econometrician's ability to obtain reliable parameter estimates, collinearity problems remain a challenge for future econometric use of the state-contingent approach.

Table 1: Parameter estimates

Parameter	Estimate	Standard error	P-value
β_2	0.9855	0.3824	0.013
β_{12}	-0.0071	0.0012	0.000
α_{11}	-0.5172	0.1248	0.000
α_{22}	-0.5887	0.1173	0.000
α_{33}	-0.0172	0.0070	0.018
α_{12}	0.4689	0.1694	0.008
α_{13}	-0.0089	0.0114	0.441
α_{23}	0.0286	0.0211	0.183
γ_{a1}	163.0669	21.3335	0.000
γ_{a2}	68.0519	9.0431	0.000
γ_{a3}	86.9531	1.1305	0.000
γ_{t1}	-1.1908	0.2480	0.000
γ_{t2}	0.2214	0.2288	0.338
γ_{t3}	-0.4277	0.0186	0.000
γ_{b1}	-1.3516	0.2354	0.000
γ_{b2}	1.2207	0.4069	0.004
γ_{b3}	-0.1078	0.0305	0.001

Note: Log Likelihood = -349.8972
Number of Observations = 51
R-square = 0.992 for labor, 0.934 for material, and 0.985 for land.

Table 2: Price Elasticities

Price Elasticities	Price of Labor	Price of Material	Price of Land
Quantity of Labor	-0.387	0.392	-0.015
Quantity of Material	0.260	-0.299	0.039
Quantity of Land	-0.003	0.013	-0.010

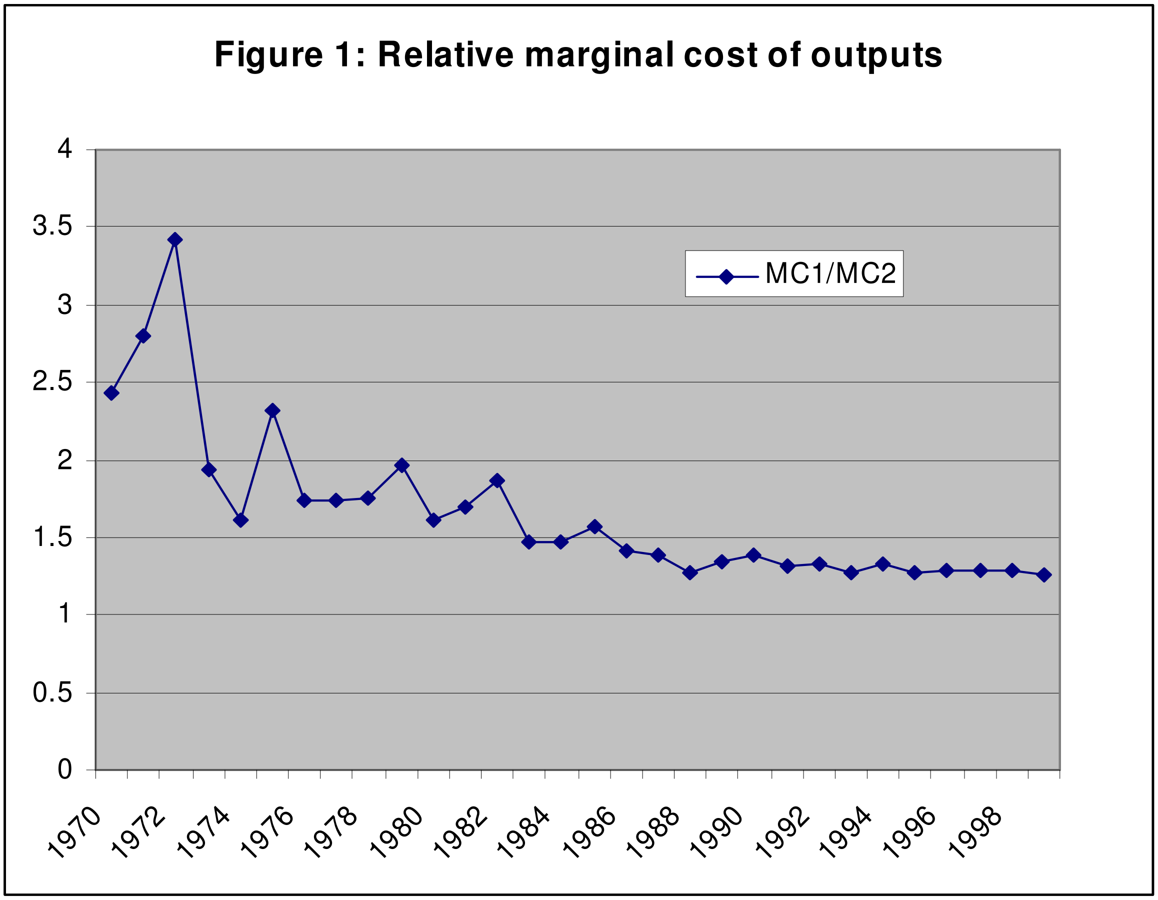
Figure 1: Relative marginal cost of outputs
MC1/MC2
4
3.5
3
2.5
2
1.5
1
0.5
0
1970
1972
1974
1976
1978
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998

Appendix

Proof of Proposition 1: Under free input disposability and the convexity of the set G(**y**, t), the cost function C(**w**, **y**, t) in (1) and the distance functions D(**x**, **y**, **g**, t) satisfy the following duality relationships (see Luenberger; Chambers et al.)

$$C(\mathbf{w}, \mathbf{y}, t) = \mathbf{w}\,\mathbf{x}^c(\mathbf{w}, \mathbf{y}, t) = \inf_{\mathbf{x}\geq \mathbf{0}} \{\mathbf{w}\,\mathbf{x} - D(\mathbf{x}, \mathbf{y}, \mathbf{g}, t)\,\mathbf{w}\,\mathbf{g}\}, \quad \text{(A1)}$$

and

$$D(\mathbf{x}, \mathbf{y}, \mathbf{g}, t) = \inf_{\mathbf{w}\geq \mathbf{0}} \{[\mathbf{w}\,\mathbf{x} - C(\mathbf{w}, \mathbf{y}, t)]/(\mathbf{w}\,\mathbf{g})\}, \quad \text{(A2)}$$

which has $\mathbf{w}^c(\mathbf{x}, \mathbf{y}, g, t)$ for solution. Given $D^*(\mathbf{x}, \mathbf{y}, \mathbf{w}, t) \equiv [\mathbf{w}\,\mathbf{g}]\,D(\mathbf{x}, \mathbf{y}, \mathbf{g}, t)$ and under differentiability, the envelope theorem applied to (A1) and (A2) yields that $\frac{\partial C}{\partial w} = \mathbf{x}^c$ (Shephard's lemma) and $\frac{\partial D^*}{\partial x} = \mathbf{w}^c$. It follows that $\frac{\partial C}{\partial w}(\mathbf{w}, \mathbf{y}, t) = \frac{\partial C}{\partial w}[\frac{\partial D^*}{\partial x}(\frac{\partial C}{\partial w}, \mathbf{y}, \mathbf{w}, t), \mathbf{y}, t]$.

Differentiating with respect to **w** yields

$$\frac{\partial^2 C}{\partial w^2} = \frac{\partial^2 C}{\partial w^2}\,\frac{\partial^2 D^*}{\partial x^2}\,\frac{\partial^2 C}{\partial w^2}. \quad \text{(A3)}$$

Similarly differentiating $\frac{\partial D^*}{\partial x}(\mathbf{x}, \mathbf{y}, \mathbf{w}, t) = \frac{\partial D^*}{\partial x}[\frac{\partial C}{\partial w}(\frac{\partial D^*}{\partial x}, \mathbf{y}, t), \mathbf{y}, \mathbf{w}, t]$ with respect to **x** yields

$$\frac{\partial^2 D^*}{\partial x^2} = \frac{\partial^2 D^*}{\partial x^2}\,\frac{\partial^2 C}{\partial w^2}\,\frac{\partial^2 D^*}{\partial x^2}. \quad \text{(A4)}$$

Equations (A3) and (A4) establish that $\frac{\partial^2 C}{\partial w^2}$ and $\frac{\partial^2 D^*}{\partial x^2}$ are generalized inverses of each other, with $\left(\frac{\partial^2 C}{\partial w^2}\right)^+ = \frac{\partial^2 D^*}{\partial x^2}$ (where the superscript "+" denotes the generalized inverse). In addition, applying the envelope theorem to (A1) gives

$$\frac{\partial C}{\partial y}(\mathbf{w}, \mathbf{y}, t) = -\frac{\partial D^*}{\partial y}(\mathbf{x}^c, \mathbf{y}, \mathbf{w}, t). \quad \text{(A5)}$$

Differentiating $\frac{\partial D^*}{\partial y}(\mathbf{x}, \mathbf{y}, \mathbf{w}, t) = -\frac{\partial C}{\partial y}[\frac{\partial D^*}{\partial x}(\mathbf{x}, \mathbf{y}, \mathbf{w}, t)\mathbf{,}\ \mathbf{y,}\ t]$ with respect to **x** and **y** gives

$$\frac{\partial^2 D^*}{\partial y \partial x} = -\frac{\partial^2 C}{\partial y \partial w}\frac{\partial^2 D^*}{\partial x^2}, \tag{A6}$$

$$\frac{\partial^2 D^*}{\partial y^2} = -\frac{\partial^2 C}{\partial y^2} - \frac{\partial^2 C}{\partial y \partial w}\frac{\partial^2 D^*}{\partial x \partial y}. \tag{A7}$$

It follows that

$$\frac{\partial^2 D^*}{\partial y^2} = -\frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial y \partial w}\frac{\partial^2 D^*}{\partial x^2}\frac{\partial^2 C}{\partial w \partial y}. \tag{A8}$$

Substituting (A5) and (A8) into the definition $\tau_{ij} = -\frac{\sum_{k=1}^{m}(\partial D^*/\partial y_k)\, y_k}{y_i y_j}\frac{K_{ij}^c}{\det(K)}$ (where K = $\begin{bmatrix} \partial^2 D^*/\partial y^2 & (\partial D^*/\partial y)^T \\ \partial D^*/\partial y & 0 \end{bmatrix}$ and K_{ij}^c is the (i, j)-th cofactor of K) yields the desired results.

Footnotes

[1] The Allen elasticity of transformation can also be defined from the revenue function R(p, x, t) = $p\,y^*(p, x, t) = \max_y \{p\,y: (-x, y) \in F\}$, where $p > 0$ is the vector of output prices and $y^*(p, x, t)$ are the revenue maximizing output supplies. Then, the Allen elasticity of transformation between outputs i and j is given by $\tau_{ij} = \frac{\partial^2 R}{\partial p_i \partial p_j} \frac{R}{(\partial R/\partial p_i)(\partial R/\partial p_j)}$, or using the envelope theorem, $\tau_{ij} =$

$$\frac{\partial y_i^*}{\partial p_j} \frac{R}{y_i^* y_j^*}.$$

[2] Some experimentation with finer representations of the state space indicated that collinearity problems can arise rather quickly. These problems should be kept in mind. As collinearity reduces our ability to obtain reliable parameter estimates, it places some limits on how many states can be realistically analyzed econometrically using a state-contingent approach.

[3] We also investigated this same hypothesis using a moment-based approach, where the cost function C(·) was specified to depend on both the mean and the variance of output (the variance being evaluated using our state contingent approach). The null hypothesis that the variance effect was zero was also strongly rejected at the 1 percent significance level. Again, this provides evidence that expected output alone does not provide an appropriate representation of production uncertainty under cost minimizing behavior.