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Estimating Disaggregate Production Functions: An Application to Northern Mexico

by

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Abstract

This paper demonstrates a robust method for achieving disaggregation in the estimation of flexible-form farm-level multi-input production functions using minimally-specified data sets. Since our ultimate goal is to address important questions related to the distributional effects of policy changes, we place emphasis on the ability of the model to reproduce the characteristics of the existing production system and to predict the outcomes of these changes at a high level of disaggregation. Achieving this requires the use of farm-level models that are estimated across a wide spectrum of sizes and types, which is often difficult to do with traditional econometric methods, due to limitations of data. The approach to estimating flexible-form production functions used in this paper overcomes these limitations, and also avoids the problems that frequently hinder the application of budget-based representative farm models to these type of analyses – namely, that of poor calibration to observed behavior. In our estimation procedure, we use a two-stage approach that first generates a set of observation-specific shadow values for incompletely priced inputs, such as irrigation water or family labor, which are used in the second stage, along with the nominal input prices, to produce estimates of crop-specific production functions using Generalized Maximum Entropy (GME) methods. These functions are able to capture the individual heterogeneity of the local production environment, while still allowing the production function to replicate the input usage and outputs produced in the sample data. Since we are able to generate demand, supply, and substitution elasticities, a wide range of policy responses can be modeled.

Our paper demonstrates this methodology through an empirical application to Mexico, drawing from a small set of cross-section data collected in the northern Rio Bravo regions. The estimates show that there is considerable heterogeneity in the behavioral response of farmer households of different sizes, both in terms of the returns to scale, as well as in the elasticities of substitution and derived demands for water. Compared to the aggregate-level estimation, we obtain much more accurate and informative policy response behavior, when shocks are imposed on the model.

I. Introduction

This paper develops a methodology for estimating highly-disaggregated production function models from minimal data sets. Disaggregated models enable the distributional effects of policies to be measured across farm size or location. In addition if, as is common, there is heterogeneity in the sample, spatial differences in policy impacts and input use are also important. Also, with a heterogeneous sample, a disaggregated set of models may predict policy response by farmers more accurately, where aggregation bias exceeds the small sample error in disaggregated models. Throughout the paper, we assume that the sample size is fixed, and strive to maximize the policy information from it. The central empirical question facing a researcher is what level of disaggregation makes the best use of the data set for the purpose in hand. Our primary research focus is on the prediction of policy impacts on farmers in terms of their net income, and use of natural resources in farm production.

A primal approach to production functions has several attractive properties for production models subject to fixed factor constraints. An important characteristic of primary farm survey data is the frequent occurrence of incomplete factor prices due to subsidized inputs, family labor, and government regulation. This absence of market prices for family labor, water, and (often) land makes the traditional dual approach inoperable. In addition, when surveyed, farmers may recall information relating to the primal variables (namely, quantities) more readily and accurately than that which corresponds to the dual (namely, prices and costs). Finally, primal production models are able to directly interact with more detailed models of physical processes, making them more directly applicable to environmentally-related research questions.

Despite the very small sample size, the use of Generalized Maximum Entropy (GME) estimators enables us to estimate all the model parameters, and to evaluate

their fit with three alternative measures: R^2, percent absolute deviation, and normalized entropy. Since we are interested in models that can address policy questions, the emphasis in this paper is on the ability of the model to reproduce the existing production system and to predict the outcomes of policy changes at a highly disaggregated level.

In many developed and developing agricultural economies there is considerable emphasis on the effect of agricultural policies and production on the environment, and conversely, the effect of environmental policies on the agricultural sector. This emphasis may rekindle interest in the use of production function models for many important policy problems. There are several reasons why primal-based production functions are suited to the analysis of agricultural- environmental policy. First, environmental values are measured in terms of the physical outcomes of agricultural activity. Second, some environmental policies are formulated as direct constraints on input use. Third, economic models of agricultural and environmental policy impacts often have to formally interact with process models of the underlying physical systems, thus requiring the specification of economic output in terms of values that are primal in nature.

Several authors have emphasized the need to spatially disaggregate models for environmental policy analysis (Antle and Capalbo, 2001; Just and Antle, 1990). However, such disaggregation is often made difficult either by the limited availability of disaggregate data or, if such data is present, the lack of enough degrees of freedom to identify disaggregate parameters within a classical estimation framework. Generalized Maximum Entropy (GME) estimation techniques (Golan *et al*., 1996(a)) have come into increasing use by researchers who seek to achieve higher levels of disaggregation in the face of these data problems (Lence and Miller, 1998; Lansink *et*

al., 2001; Golan *et al*., 1994, 1996(b)). Given the inherent heterogeneity of soils and other agricultural resources, aggregating across heterogeneous regions leads to aggregation bias. Conversely, ill-conditioned or ill-posed GME estimates may be less precise due to the small samples on which they are based. An additional advantage that speaks in favor of maximum entropy-based alternatives is the ability to formally incorporate additional non-sample data or informative priors into the estimation process, in a Bayesian fashion.

Substitution activity at the intensive and extensive margins is a key focus of agricultural-environmental policy analysis. A basic policy approach is to provide incentives or penalties that lead to input substitution under a given agricultural technology. Such substitutions at the intensive margin can reduce the environmental cost of producing traditional agricultural products or that of jointly producing agricultural and environmental benefits. These policies cannot be evaluated without explicit representation of the agricultural production process. It follows, therefore, that the potential for substitution should be explicitly modeled within a multi-input multi-output production framework.

The disaggregated multi-input, multi-output CES model in this paper has the ability to simulate response at all the three margins of substitution that represent farmer response to changes in price, cost or resource availability. The same approach has been applied to other models based on flexible functional forms, such as, quadratic, square root, generalized Leontieff and trans-log specifications.

This combination of methodology distinguishes our approach with other GME production analyses using in the literature (Zhang and Fan, 2001; Lence and Miller, 1998). The GME estimates given in this paper do, however, converge to consistent estimates when the sample size is increased and have been shown to have the same

asymptotic properties as conventional likelihood estimators (Mittlehammer *et al*., 2000).

GME estimators require the definition of support values for each parameter that are implicit bounded priors on the parameters. Several authors have shown that support values specification can have a strong influence on the resulting estimates. In addition, if the support values are specified in an *ad hoc* manner it is possible that there is no feasible solution to the resulting GME estimation problem. We use values from a calibrated optimization model to ensure that the supports are centered on values that are a feasible solution to the data constraints, and consistent with prior parameter values. Given the support values, we estimate the production function parameters, input shadow values, and returns to scale in a simultaneous GME specification.

This specification of support values differentiates our approach with other GME production analyses using in the literature (Zhang and Fan, 2001; Lence and Miller, 1998), in fact, the empirical GME literature says very little about how a set of feasible and consistent support values are defined for several interdependent parameters. We differ from Heckelei and Wolff (2003) by using calibrated optimization models to define the prior sets of support values, but, like Heckelei and Wolff, we estimate production function parameters, and factor input shadow values, in a simultaneous GME specification.

In addition, we generate the finite sample distribution properties of the resulting GME estimates by applying a bootstrapping procedure (Efron and Tibsharani, 1993). To our knowledge, this is the first time that the bootstrap method has been used to obtain parameter distributions for GME-based estimators. Previous work has tested GME results for sensitivity to their support spaces, or has used Monte Carlo results to

approximate asymptotic parameter distributions. However, since our aim is to use small data samples, bootstrapping seems a natural method to generate the finite sample properties, and can be simply implemented.

The ability to simulate policy alternatives reliably with constrained profit maximization requires a model that satisfies the marginal and total product conditions and has necessary properties of curvature embodied in the second-order profit maximizing conditions. It is our belief that those who use policy models are more interested in reproducing observed behavior and simulating beyond the base scenario, than in testing for the curvature properties of the underlying production function. Within our simulation framework, we can also impose policy restrictions in the form of constraints on the estimated farm production model.

Section II of the paper briefly reviews modeling methods used to estimate the effect on land use of agricultural and environmental policies. Section III develops the production model estimation and bootstrap procedure within the GME framework. Section IV contains an empirical application to a data set from a primary survey of 27 farms in the Rio Bravo region of Northern Mexico. The randomly selected farm sample contains a very wide range of farm sizes. The central point is whether the production parameters of different farm sizes vary sufficiently to make disaggregated models, better estimates of policy response than estimates based on the whole sample. Essentially we are testing whether disaggregated policy models are better predictors of farmer behavior despite the minimal data sets used by the GME estimators. Conclusions are drawn in Section V

II. Methods for Modeling Disaggregated Agricultural Production.

The approach that we use in this paper addresses the shortcomings of representative farmer models enumerated by Antle and Capalbo (2001), when they cite the limited range of response in the typical representative farm model. The disaggregated production models capture the individual heterogeneity of the local production environment, whether it is in terms of land quality or farm-size specific effects, and allows the estimated production functions to replicate the differences in input usage and outputs.

Love (1999) made the point that the level of disaggregation matters in terms of the degree of firm-level heterogeneity and other localized idiosyncrasies that get averaged out of the sample. This affects the likelihood of observing positive results for tests of neo-classical behavior, such as cost minimization or profit maximization. In our approach, we impose curvature conditions on the estimated production function, since we are aiming for models that reproduce observed behavior in a theoretically-consistent manner through simulation, rather than simply testing the hypothesis of its presence or absence. The relative stability we observe within cropping systems, despite the presence of substantial yield and price fluctuations is informal empirical evidence that farmers act as if their profit functions are convex in crop allocation. The gradual adjustment of agricultural systems to changes in relative crop profitability suggests that farmers adjust by progressive changes over time, along all the margins of substitution, rather than going from one corner solution to the next.

Zhang and Fan (2001) conclude that the behavioral assumptions of profit maximization are too strong for the example to which they applied a GME production function estimation. While their level of aggregation was severe, they made the case

for using GME on the basis of its ability to incorporate non-sample information and to deal with imperfectly observed activity-specific inputs. Within our framework, we are able to implement more flexible functional forms for production than that used by Zhang and Fan, as well as avoid imposing constant returns to scale, as a result of our higher level of disaggregation.

Just *et al* (1983), stated in their classic production paper that their:

"Methodology is based on the following assumptions that seem to characterize most agricultural production:

(a)Allocated inputs. Most agricultural inputs are allocated by farmers to specific production activities..

(b)*Physical constraints*. Physical constraints limit the total quantity of some inputs that a farmer can use in a given period of time …

(c) *Output determination*. Output combinations are determined uniquely by the allocation of inputs to various production activities aside from random, uncontrollable forces."

Just *et al*'s specification admits jointness in multioutput production only through the common restrictions on allocatable inputs. The specification in this paper has constraints on the land available, but also allows for jointness between crops in a region as reflected by the deviations of crop value marginal products from the opportunity cost of restricted land inputs.

The current range of approaches to agricultural production modeling and the associated analysis of environmental impacts, seems to fall into three groups, namely, disaggregated calibrated or constrained programming models (McCarl, 2000 ; Alig *et*

al., 1998; CVPM[1], 1997; CAPRI[2], 2000) disaggregated logistic land use models (Wu and Babcock, 1999) , and aggregate econometric land use models (Mendelsohn *et al*., 1994, Antle and Valdivia, 2006).

III Using Generalized Maximum Entropy to Estimate Production Functions

The nature of the data set defines the estimation method to be used. For disaggregated policy models, the available data usually takes the form of a cross-sectional survey sample taken over each disaggregated region. A reassuring characteristic of generalized maximum entropy (GME) estimators is that while they can estimate consistent parameter values from ill-conditioned or ill-posed problems, their large sample estimates enjoy the usual classical properties (Mittlehammer *et al*, 2000). The GME estimation approach advanced in this paper is completely in accord with classical econometric estimators for large sample problems and uses a standard bootstrap approach to estimate GME parameter distributions. The novelty of the paper lies in the idea that the modeler does not have to accept the stricture of conventional degrees of freedom, but may specify a complex model at the level of disaggregation that is thought to minimize the effect of estimation errors and aggregation bias on the model outcome. The modeler can specify flexible multi-input production functions for any number of observations and calibrate closely to the base conditions. Essentially we show that a minimal level of data that would, in the past, have restricted the modeler to a simple linear programming model, can now be calibrated and reconstructed as a set of multi-input CES production functions.

[1] Central Valley Production Model , used in the 1997 Programmatic Environmental Impact Statement of the Central Valley Project Improvement Act (see references).

[2] Common Agricultural Policy Regional Impact (http://www.agp.uni-bonn.de/agpo/rsrch/capri/)

The first order conditions for optimal allocation have to incorporate the shadow value of any constraints on inputs. Since the allocatable inputs are restricted in quantity, and rotational interdependencies can exist between crops, we use a model modified to conform to the principles of Positive Mathematical Programming – or PMP (Howitt 1995), on each data sample so as to obtain a numerical value for a prior value for the shadow price that may exist in addition to the allocatable input cash price.

Before the GME reconstruction program is solved, support values have to be defined for each parameter and error term. To ensure that the set of support values spans the feasible solution set, we define the support values as the product of a set of five weights and functions of the average Leontieff yield over the data set, and for a particular crop and input combination. The support values for the error terms are defined by positive and negative weights that multiply the left-hand side values of the equation.

The non-constant returns to scale CES production function is defined as:

$$(2) \qquad y_i = \alpha_i \left(\sum_j \beta_{ij} x_{ij}^{\gamma_i}\right)^{\frac{rts_i}{\gamma_i}}$$

Where rts_i is the returns to scale parameter for crop i, and $\gamma_i = \frac{\sigma_i - 1}{\sigma_i}$ where σ_i is the elasticity of substitution.

The GME reconstruction problem becomes:

(3)

$$Max \quad \Sigma_{i,p} prts_{i,p} - \ln prts_{i,p} + \Sigma_{i,p} psub_{i,p} - \ln psub_{i,p} + \Sigma_{j,p} plam_{j,p} - \ln plam_{j,p}$$
$$+ \Sigma_{i,j,p} pb_{i,j,p} - \ln pb_{i,j,p} + \Sigma_{n,i,j,p} pe1_{n,i,j,p} - \ln pe1_{n,i,j,p} + \Sigma_{n,i,p} pe2_{n,i,p} - \ln pe2_{n,i,p}$$

$$Subject \ \ to:$$

$$\frac{c_{i,j} + lam_j}{price_i} = \gamma_i \, \beta_{ij} \, x_{n,i,j}^{(\gamma_i - 1)} \quad \frac{rts_i}{\gamma_i} \alpha_i \left[\sum_j \beta_{ij} \, x_{n,i,j}^{\gamma_i} \right]^{\left(\frac{rts_i}{\gamma_i}\right) - 1} + \Sigma_p (pe1_{n,i,j,p} \ *ze1_{n,i,j,p}) \quad \forall \, i, j$$

$$tprod_{n,i} = \alpha_i \left[\sum_j \beta_{ij} \, x_{n,i,j}^{\gamma_i} \right]^{\frac{rts_i}{\gamma_i}} + \Sigma_p (pe2_{n,i,p} \ *ze2_{n,i,p}) \qquad \forall \, i$$

`Equation (3) is subject to the usual constraints on the discrete probability functions, and the product of the probabilities and support parameters that are needed to derive the estimated coefficients for returns to scale (rts_i), elasticity of substitution (σ_i), the shadow value of allocatable inputs (lam_j), and the CES share parameters (β_{ij}). The CES scale parameter is directly estimated without support values.

The objective function is the usual sum of the entropy measures for the parameter probabilities. Following the normal GME procedure, the entropy of the error term probabilities is also maximized. The first data based equations in (3) are the first order conditions that set the cost ratio equal to the marginal physical product. If some inputs are restricted, the input cost in the first order equation includes the estimated shadow values as well as the nominal input price.

The second data based equations in (3) fit the production function to the observations on total production. While it is not usual practice to include both the marginal and total products as estimating equations in econometric models, we think that the information in the total product constraint is particularly important for two reasons. First, information on crop yields and areas is likely to be the most precisely

known by farmers. While farmers are often doubtful and reluctant about stating their costs of production to farm surveyors, they are more mindful of their yields and use them as a primary metric of production performance. Second, while the marginal conditions are essential for behavioral analysis, policy models also have to accurately fit the total product to be convincing to policy makers and correctly estimate the total impact on the environment and the regional economy of policy changes. Fitting the model to the integral as well as the marginal conditions should improve the policy precision of the model.

Due to the separability assumption on the production functions, the estimation problem can be solved rapidly by looping through individual production functions, since the linkage between the production of different crops is defined by the shadow values and allocatable input constraints.

We note that the supply functions, derived input demands, their associated elasticities, and the elasticities of substitution are obtainable from a data set of any size from one observation upwards. Clearly the reliance on the support space values and the micro theory structural assumptions is much greater for minimal data sets. However the approach does enable a formal approach to disaggregation of production estimates, since the specification of the problem is identical for all sizes of data sets.

A problem which commonly hinders the widespread adoption of GME and entropy-based methods by practitioners of more conventional methods, can be encapsulated in the following question – *while I might accept that entropy maximization results in an efficient distribution of the parameter, how do I know that the expected value of the parameter is a reliable point estimate?*. In short, the potential user is understandably asking for the variance of the coefficient's estimate. To date the response from ME advocates is to reassure the potential user that the

asymptotic properties of the estimate have asymptotic consistency. Such a response, which is based purely on asymptotically valid arguments, do not offer much reassurance of an estimator whose primary use and comparative advantage lies in small–sample estimates. It follows that there is a need to generate GME parameter error bounds using the small data sets in which GME excels. Using a bootstrap-based method (Efron and Tibsharani, 1993) with the GME estimation, we are able to generate variances for all the production function parameters and corresponding pseudo t values. This will enable the analyst to have a formal measure of precision for each parameter, while drawing upon a method with more refined asymptotic properties (Cameron and Trivedi, 2005). In addition, having calculated the variance of a set of critical policy parameters such as the disaggregated elasticities of substitution and returns to scale, we can then apply statistical tests for significant differences between the parameters —which reflect, implicitly, the value-added of disaggregating by farm size.

IV. The Empirical Reconstruction of Regional Crop Production in Rio Bravo.

Data Restrictions

Ideally, production models are reconstructed from a consistent time series of regional data, which includes all the crop inputs and outputs and their associated prices. Unfortunately, such rich, consistent data sets are rarely available. In some cases, comprehensive cross-section survey data is available, but it is rarely collected for more than one year. The empirical example in this paper is a small cross-section farm survey collected by FAO enumerators for a sub-sample of 27 farms in the Rio Bravo region of Mexico in 2005, which is representative of the kind of primary

production data sets that are commonly collected in developing and developed countries.

Production Function Specification

Within a farm size, we assume that the production of different crops is connected by the restrictions on the total land, water available. Labor is treated as a normal variable input, as the proportions of family and wage labor varied widely across the sample. The CES production function is written as:

$$(4) \qquad y_i = \alpha_i \left(\beta_{i,land} x_{i,land}^{\gamma_i} + \beta_{i,water} x_{i,water}^{\gamma_i} + \beta_{i,labor} x_{i,labor}^{\gamma_i}\right)^{\frac{rts_i}{\gamma_i}}$$

where y_i is the farm output of a given crop and $x_{i,j}$ is the quantity of land, water or labor allocated to crop production for that farm size class.

The policy simulation problem defined over n farms and i crops in each farm size class for a single year is :

$$(5) \qquad \begin{aligned} &\text{Max } \Sigma_n \ \Sigma_i \ p_i \ \alpha_i \left[\sum_j \beta_{ij} \, x_{n,i,j}^{\gamma_i}\right]^{\frac{rts_i}{\gamma_i}} - \sum_j \omega_{i,j} \, x_{i,j} \\ &\text{subject to} \quad \Sigma_n \ x_{1ni} \le X_1 \ (\text{Land}) \\ &\qquad\qquad\quad \Sigma_n \ x_{2ni} \le X_2 \ (\text{Water}) \end{aligned}$$

where the total annual quantities of irrigated land and water (X_1 and X_2) are limited for each farm. By changing the RHS quantity of water available on the constraint, we can generate a derived demand function for each farm class.

Estimation Results

Estimation of the full set of parameters for the production function with three inputs requires that each regional crop be parameterized in terms of six parameters, three for the share coefficients, a scale parameter, the returns to scale parameter, and the elasticity of substitution. In addition, two resource constraint shadow values (on land and water) are estimated for each farm size group. The 27 observations can be disaggregated into three size classes based on their production of the dominant crops, sorghum and maize. The sample statistics are shown in Table 1. The small farm group has 12 farms surveyed, the medium sized group has 6 farms, and the large farm group has 9 farms in it. With six parameters per crop production function, all farm groups have small or minimal degrees of freedom, in fact, allowing for the estimation of shadow values, the medium farm group has a small negative degrees of freedom. This extreme case provides a severe test of the disaggregated GME approach.

Table 1. Cultivated land (in ha) and average water used (in m^3/ha) for selected crops by farm size

Farm size / Crop	Small		Medium		Large		Summary	
	Cultivated land (ha)	water used (m^3/ha)*	Cultivated land (ha)	water used (m^3/ha)*	Cultivated land (ha)	water used (m^3/ha)*	Total Cultivated land (ha)	Average Water Used
Alfalfa	1.5	23,000	10.0	16,000	129.0	18,558	140.5	19,186
Wheat			19.4	5,000	77.0	5,000	96.4	5,000
Maize	3.0	8,000	50.1	5,325	1,358.3	5,236	1,411.4	6,187
Cotton			34.0	8,000	290.0	8,138	324.0	8,069
Melon	10.0	17,000			180.0	2,600	190.0	9,800
Sweet Potato					20.0	7,000	20.0	7,000
Beans	0.5	5,000					0.5	5,000
Sorghum	15.0	7,600	83.0	4,172	2,198.0	2,023	2,296.0	4,598
Average	30.0	12,120	196.5	7,699	4,252.3	6,936	4,478.8	8,105

(*) Average of water used per hectare

The data for this study was collected by an FAO (2005) survey of 45 farms in the Rio Bravo region during 2005. The number of farms surveyed, by state are: Chihuahua 12, Coahuila 8, Nuevo Leon 4 and Tamaulipas 21. Farm-level data on inputs usage, outputs and costs and farm characteristics were used. Total revenues took into account government support programs. An equivalent crop price was calculated on a per hectare basis. Three very large farms were removed from the sample as atypical, in addition, farms that grew no maize or sorghum were omitted from the estimation, reducing the estimated data base to 27 observations.

Five out of twelve irrigation districts in the Rio Bravo region are represented in the sample. In addition, irrigation units in Delicias, Chihuahua and in the Bajo Rio Bravo were included in the surveys. Eight crops were selected for this analysis namely: alfalfa, wheat, maize, cotton, melon, sweet potato, beans and sorghum.

Table 2. Returns to Scale

	Field	Forage	Maize	Sorghum	Wheat
All Farms	0.369,	0.431,	0.658,	0.67	0.402
Small Farms	0.385,	0.444,	0.411,	0.615	
Medium Farms			0.511,	0.437	
Large Farms			0.387,	0.39	

Table 3. Elasticity of Substitution

	Field	Forage	Maize	Sorghum	Wheat
All Farms	0.721,	0.729,	0.397,	0.761	0.713
Small Farms	0.720,	0.726,	0.709,	0.702	
Medium Farms			0.699,	0.697	
Large Farms			0.714,	0.718	

Tables 2 and 3 show considerable variation in the returns to scale and elasticities of substitution both between farm size groups and crops. For example, sorghum and wheat have higher substitution elasticities than the other dominant crop, maize. As expected, the returns to scale decrease as farm size increases for both maize and

sorghum. The differences in these two parameter values across farm size groups will be reflected in the response to input price or quantity changes. The intensive margin of adjustment is determined by the elasticity of substitution, while changes at the extensive margin are determined by the curvature of the production function summarized by the decreasing returns to scale parameter. Intuitively one would expect less ability to respond by crop mix or land area changes on small farms.

Measures of Goodness of Fit.

Tables 4 and 5 show the goodness of fit of the model in two ways, the R^2 values for crop production, and the percent absolute deviation (PAD) of the in-sample predictions. The R^2 values range from 0.77 to 0.15 and seem consistent with results from estimates based on larger samples of cross-section survey data. Likewise, the PAD measure shows reasonable prediction errors. Another measure of the overall information content of the GME estimates is the normalized entropy measure (Golan *et al* (1996). The normalized entropy values for the different samples are used to calculate the information index (Soofi, 1992), which measures the reduction in uncertainty attributable to the GME estimates. The information indices (1 - normalized entropy) for all sample sizes show significant reductions in uncertainty. The index values are: All farm sample, 0.830, Large farm sample, 0.769, Medium farm sample, 0.709, and Small farm sample 0.768.

Table 4. R^2 of Farm Production

	Field	Forage	Maize	Sorghum	Wheat
All Farms	0.375,	0.369,	0.269,	0.319	0.528
Small Farms	0.374,	0.393,	0.299,	0.142	
Medium Farms			0.696,	0.263	
Large Farms			0.190,	0.290	

Table 5. Percent Absolute Deviation of Farm Production

	Field	Forage	Maize	Sorghum	Wheat
All Farms	3.68	6.55	40.0	40.87	1.50
Small Farms	5.549	15.102	24,495	37.518	
Medium Farms			16.797	37.749	
Large Farms			9.319	12.712	

The estimation of shadow values for the fixed, but allocatable inputs land, and water, are a very important component in the estimation of farmer response to changes in the cost of allocatable inputs in developing economies. The results in Table 6 show that the shadow values of land exceed the nominal costs in all farm size groups, and for water, the shadow value is equal to or greater than the total input cost. Clearly, for this sample, any estimation based only on the nominal input costs will be very biased, and policy responses will be similarly distorted.

Table 6. Input Shadow Values

	Land		Water	
	Shadow value	Nominal Cost	Shadow value	Nominal Cost
Small Farms	959.82	762	255.59	222.02
Medium Farms	1947.57,	637.1	855.28	185.85
Large Farms	1208.32,	977.27	223.56	223.06

Calculating GME Parameter Distributions Using a Bootstrap

Bootstrap methods have been used for the past twenty years to approximate the distribution of a statistic by systematically resampling the original sample data. The GME bootstrap uses a uniform random distribution to select observations from the original sample of "n" observations with replacement. Having generated the bootstrap observations, the GME program described above calculates the GME estimates of the production function coefficients $rts_{i,B}$, where there are "i" crops. We calculate the

bootstrapped returns to scale $rts_{j,B}$ and run the bootstrap loop for 500 (B) iterations. The estimated asymptotic variance for a given GME parameter estimate, for instance the returns to scale for the i th crop $\hat{rts}_j$, can be estimated from the B bootstrapped estimates $\hat{rts}_{j,B}$ as:

$$(6) \qquad Var\,\hat{rts}_j = \frac{1}{B}\sum_{b=1}^{B}\left[\hat{rts}_{j,b} - \hat{rts}_j\right]\left[\hat{rts}_{j,b} - \hat{rts}_j\right]'$$

For simplicity of presentation we restrict the tables to one crop and three production function parameters. Sorghum is selected since it is the crop grown most widely in the random sample. Differences in the production functions are tested using the returns to scale parameter (RTS), the elasticity of substitution (Sub) and the CES scale parameter. From theory, one would expect that the RTS will decrease as farms size increases, the elasticity of substitution that measures the intensive margin of adjustment has no theoretical reason to differ with farm size for the same crop, and the scale parameter is expected to differ with farm size. Table 7 shows the mean and variance of the three parameters by farm size

Table 7. Sorghum Production Parameters by Farm Size

	Small Farm		Medium Farm		Large Farm	
	Mean	Variance	Mean	Variance	Mean	Variance
RTS	0.615	0.02 **	0.437	0.017	0.39	0.056 *
Substitution	0.615	0.263	0.688	0.019 **	0.717	0.158 *
Scale	8.552	251.25	48.445	256863.53	125.5	28102.5

** significant at 1%, * significant at 5%.

The results in Table 7 show that, as expected, the returns to scale decrease with larger farms, the elasticity of substitution shows no trend, and the scale parameter increases. To formally evaluate whether there are significant differences in these three parameters between the farm sizes, we used the bootstrap results to generate pair-wise tests. The results are shown in Table 8 below.

Table 8. t values for differences in Sorghum Production Parameters

	Small- Medium	Small- Large	Medium- Large
RTS	2.578 **	2.721 **	0.44
Substitution	-0.338	-0.494	-0.170
Scale	-0.276	-2.423 **	-0.423

Table 8 supports the expected production function properties, in that the returns to scale in the small farm group are significantly larger than both the medium and large farm group. The increase in RTS between the medium and large farm group is not significant. As expected, the scale parameter shows an increase between each group, but because of the imprecision in the bootstrap results for the medium farm group, the only significant difference is between the small and large farm scale parameters.
The results in tables 7 and 8 show that the combination of bootstrapping and GME enables formal tests of the disaggregated estimates, and in this case, justifies the disaggregation by farm size.

Simulating Differences in Water Policy Response Functions

The estimated production functions for different farm size samples are used in equation (5) to simulate the production response for each farm in the size group. The

interval elasticity of demand for water is calculated by decreasing the total available quantity of water to each farm in 10% increments and measuring the change in the shadow value. Due to the sample variation in the estimates we do not expect that all farms in a given sample will have binding water constraints when simulated using the estimated production function coefficients for that sample. Production functions and demands were estimated for the aggregate farm sample, and the small, medium and large farm samples, as defined in the previous section. Each model was parameterized over a 50% reduction in the water available. Interval elasticities over a 10% change were calculated for each farm in the group that had non-zero shadow values on water in the range. The interval elasticities showed a remarkable consistency over the different farm size groups. The water demand elasticity for small farms is -0.645, for medium farms -0.755, for large farms -0.691, and for the aggregated sample – 0.678. These elasticity values are consistent with the majority of empirical analyses..

Despite this similarity in the interval elasticities, the derived demand functions for different farm size groups differ greatly. To test the policy value of disaggregating demand estimation by farm size, a demand function was obtained by regression on the water quantities and shadow values generated for each farm in the sample when parameterized by water reductions. Table 9 shows the fits and parameter values.

Table 9. Inverse Water Demand Functions

Farm Size	Demand Equation	R^2
Small	P = 618.65 - 97.63 Ln(Q)	0.78
Medium	P = 3024.2 – 440.54 Ln(Q)	0.74
Large	P = 1290.4 – 127.69 Ln(Q)	0.33
Aggregate	P = 792.61 – 117.37 Ln(Q)	0.75

To compare the aggregate and disaggregate water demand functions, the disaggregated and aggregated estimated functions are plotted over the same range of potential water reductions. The functions can be thought as measuring the impact of a water tax policy or the cost of a quantitative reallocation.

Figures 1- 3 show the functions. In figure 1 for small farms the aggregate function is the closest approximation in that the difference is a constant over-valuation of water which would introduce a constant distortion into policies.

Figure 2 that compares the aggregate and medium farm functions shows very large under-valuations over most of the quantity range. The demands coincide at large quantities, but differ in value by a factor of four at very low quantities. Thus the stronger the policy, the greater the under-valuation of policy-induced response on the part of the farmer.

Figure 1. Water Demand- Small farm

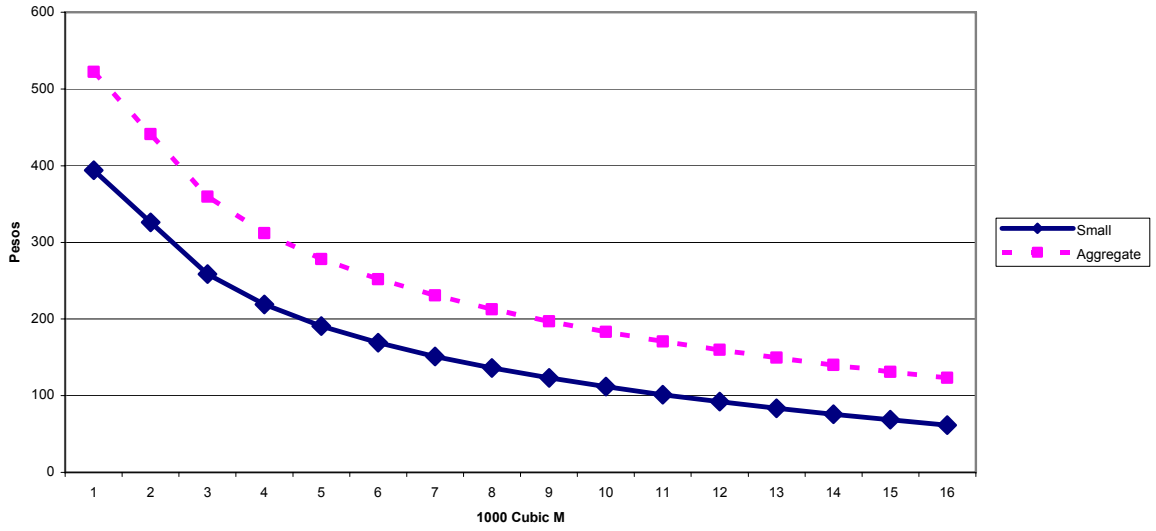


Figure 2. Medium Farm Water Demands

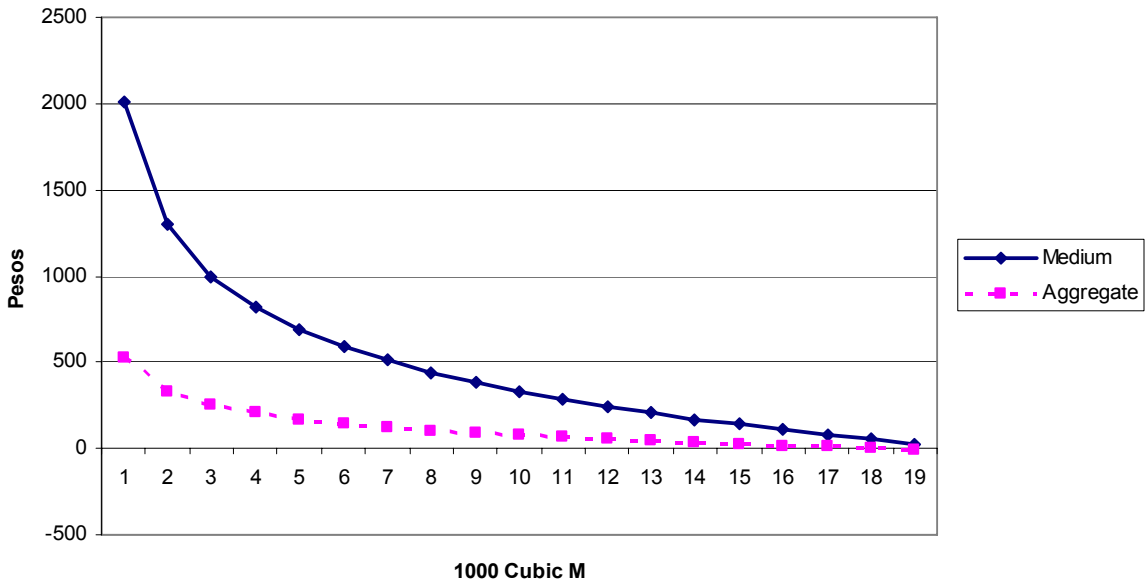
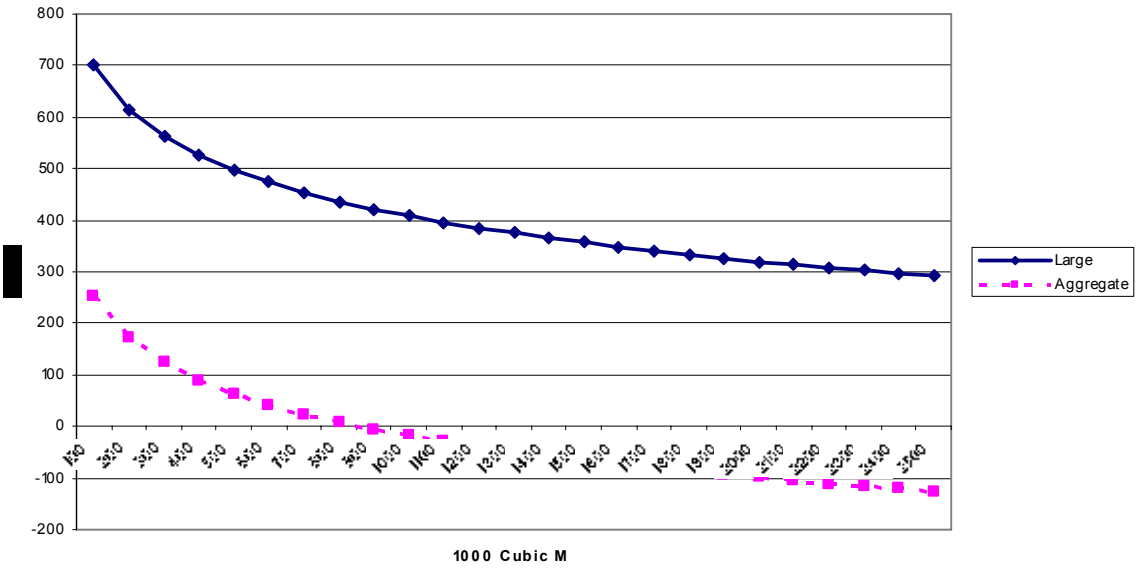


Figure 3. compares the functions for large farms. Due to bias toward small farms in the aggregate set of farms with binding water constraints, the aggregate function under-values the large farm data so badly that it is unusable for policy analysis.

Figure 3. Large Farm Water Demands



The results in figures 1-3 clearly show that despite similarity in the interval elasticities, the water demand function estimated using the aggregate data set is unusable for the large farm group, and has the expected upward and downward bias in the small and medium farm groups respectively. For this empirical example, the estimation of policy models disaggregated by farm size clearly gains more in the reduction of aggregation bias that it loses from small sample imprecision.

V. Conclusions

This paper shows that by using a GME approach, it is possible to reconstruct flexible form production function models from a data set of modest size. A researcher can empirically reconstruct a similar theoretically-consistent flexible form production model using data that ranges from minimal (or even negative) degrees of freedom to full econometric data sets with standard degrees of freedom. The convergence of GME estimates to conventional estimates as the sample size increases means that as the data set is expanded there is movement along the continuum between optimization-based and pure econometric models.

The disaggregated production models yield all the comparative static properties and parameters of large sample models. The effect of any constraints on inputs is directly incorporated in the estimates through the simultaneous estimation of the shadow values of the allocatable resources. An advantage of modeling production functions is that they are readily understood by physical scientists, who can add valuable, physically-based information, in the form of prior support values or constraints.

In this example the aggregation bias in the aggregate model swamped any gains in reducing the small sample error. The disaggregated model yielded greater precision

in its regional response. The gain from disaggregation of production models is an empirical result that needs substantially more testing before one can conclude that it is a commonly-shared phenomenon. In this example, the empirical results show that the disaggregated estimates have similar strong explanatory power as the aggregate sample, as measured by R^2, absolute deviation and the entropy information index. Despite the similar measures of elasticity, the disaggregated samples show a wide variation in the derived demand for water that directly influences policy response. The use of disaggregated estimates is clearly supported by the results.

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