Count Data Models of Prescribed Fire Escapes

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Abstract

We specify several count data models, parameterizing the probability densities in terms of their means for easier comparison between models. In addition, we derived a correction of these probability densities for differences in sample sizes, which is a contribution to the count data literature as far as we are aware. We then empirically implement these models using data from a mail survey of firms using prescribed fire to estimate the expected number of escapes from prescribed burns. We find that the not correcting for sample size differences can lead to erroneous conclusions concerning the statistical significance of variables.

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An estimated 190 million acres of federal forest land and rangeland in the U.S. face high risk of catastrophic fire due to hazardous level of fuels accumulation. In addition, more than 107 million acres of non-federal lands are classified in the Historical Fire Regime Condition Class 3 (the most altered from natural fire frequency), indicating the amount of privately owned land also in critical need of hazardous fuels reduction. This status is largely due to decades of fire suppression that has created hazardous accumulation of fuels and increases the spread of wildfires that threaten life, property, and ecology.

Prescribed fire is a primary tool to reduce these hazardous fuel accumulations. Fire causes change that is biologically necessary to maintain many healthy ecosystems. Resource managers have learned to use fire to cause changes in plant and animal communities to meet their objectives. Prescribed burning is fire applied in a knowledgeable manner on a specific land area under selected weather conditions to accomplish predetermined, well-defined management objectives. Prescribed fire is more affordable with much less risk to the habitat and destruction of site and soil quality than chemical or mechanical methods. Hence, the Healthy Forests Restoration Act of 2003 mandates the use of prescribed burning and thinning on federal lands to reduce fuel loads that promote catastrophic fires. No comparable program exists for private lands.

According to a recent study, the fear of liability is as the most significant barrier to the application of prescribed fire by private landowners. Risk and liability concerns have decreased private consultants' and contractors' willingness to conduct prescribed burns. Liability insurance covering prescribed fire is not readily available to the private sector and liability concern has forced many small private businesses to discontinue their prescribed fire services. This problem limits additional resources for prescribed burning on federal lands and diminishes prescribed burning on private lands. The primary risk for prescribed fire is that the prescribed fire will escape from its intended boundaries and cause property damage or personal injury before the escaped fire is extinguished.

This paper has two goals. The first goal is to identify characteristics and practices of prescribed burners that make prescribed fire escapes less likely. The intent is to make recommendations that will make prescribed burns safer and less damaging in order to encourage prescribed burning. The second goal is to use the collected data to evaluate the empirical performance of a variety of count data models for estimating prescribed fire escapes. Since the number of prescribed burns varies for each prescribed burner in a year, we extend count data models to correct for sample size differences.

This study is part of a larger research effort to develop data and an analysis to facilitate the development of a privately provided liability insurance policy for prescribed burners. The current low level of private prescribed fire activity is partially due to the difficulty, expense, and/or inability of obtaining liability coverage for contractors desiring to conduct prescribed burns for private landowners. Hence, the analysis is focused on identifying for insurance purposes what characteristics make escapes more likely and estimating the magnitude of these effects.

What follows is first a presentation of various types of count data models, including Poisson, negative binomial models, and other alternatives to these models, and then a description of two types of hurdle models. Next, the likelihood functions for these count data models are reported, and then the data and estimation described. Finally, estimation results are reported and we summarize our empirical findings.

Poisson Models

We begin with a general model and derive the simpler (and more commonly used) models as restrictions on the parameters. The non-negative integer y_i is the number of escapes for prescribed burner i and has probability density function $f(y_i)$, which we express in terms of the mean μ and other parameters determining the variance. These parameters can either be estimated directly or as functions of a vector of regressors x_i, e.g., $\mu = \exp(x_i'\beta)$. In the model exposition that follows, we suppress the subscript i.

The hybrid generalized Poisson distribution can be expressed as

(1) $$f(y) = \left(\frac{\mu}{1+\alpha\mu^{\theta+1}}\right)^{y}\left(1+\alpha\mu^{\theta}y\right)^{y-1}\exp\left[\frac{-\mu(1+\alpha\mu^{\theta}y)}{1+\alpha\mu^{\theta+1}}\right]/\,y!.$$

The mean is μ and the variance is $\mu(1 + \alpha\mu^{\theta+1})^2$. Restrictions on the parameters α or θ give several other Poisson models.

When $\theta = -1$, the distribution becomes the generalized Poisson distribution

(2) $$f(y) = \frac{\mu}{(1+\alpha)^{y}}\left(\mu+\alpha y\right)^{y-1}\exp\left[\frac{-(\mu+\alpha y)}{1+\alpha}\right]/\,y!,$$

where $\alpha \geq \max(-\frac{1}{2},-\frac{1}{4}\mu)$ and $f(y)$ is defined to equal zero when $\alpha < 0$ and y exceeds the largest positive integer m satisfying $m\alpha + \mu > 0$. Again the mean is μ, while the variance is $\mu(1 + \alpha)^2$.

When $\theta = 0$, the distribution becomes the restricted generalized Poisson distribution, since, relative to the generalized Poisson distribution, it has been restricted so that its variance is not proportional to its mean (and the mean-variance ratio constant). The density function is

(3) $$f(y) = \left(\frac{\mu}{1+\alpha\mu}\right)^{y}\left(1+\alpha y\right)^{y-1}\exp\left[\frac{-\mu(1+\alpha y)}{1+\alpha\mu}\right]/\,y!.$$

Again the mean is μ, while the variance is $\mu(1 + \alpha\mu)^2$.

Finally, when $\alpha = 0$, the distribution becomes the Poisson distribution:

(4) $f(y) = \mu^y \exp(-\mu) / y!$.

Again, the mean is μ, while the variance is also μ.

The primary motivation for developing the more general Poisson models reported in equations (1)-(3) is to avoid equality of the mean and variance ("equi-dispersion") as imposed by the Poisson model in equation (4). The more general distributions permit the variance to be less than or to exceed the mean (under dispersion or over dispersion) and allow testing to determine which assumption the data support.

Another extension is to adjust the Poisson model to correct for differences in sample size (Maddala, p. 53ff). The sum of independent Poisson random variables has a Poisson distribution with mean equal to the sum of the means (Evans, Hastings, and Peacock). In the context of prescribed burning, let n_i be the number of prescribed burns conducted in a year by prescribed burner i. Again, suppressing the subscript i for model exposition, for the Poisson distribution, the probability density function with different sample sizes is

(5) $f(y) = (n\mu)^y \exp(-n\mu) / y!$.

The mean and variance are $n\mu$, where now μ is the mean rate of escapes per prescribed burn, while $n\mu$ is the mean number of escapes per year (Maddala, p. 53).

The sum of independent generalized Poisson random variables has a generalized Poisson distribution (Consul, p. 15) with a mean equal to the sum of the individual means and the variance equal to the sum of the individual variances. It can be shown that in terms of equation (2), the parameter μ is replaced by the $n\mu$, while the parameter α does not change, where now μ is the mean rate of escapes per prescribed burn and $n\mu$ is the mean number of escapes per year.

Hence, the generalized Poisson probability density function with different sample sizes can be expressed as

$$(6) \qquad f(y) = \frac{n\mu}{(1+\alpha)^y}\left(n\mu + \alpha y\right)^{y-1} \exp\left[\frac{-(n\mu+\alpha y)}{1+\alpha}\right] / \, y!,$$

which has mean $n\mu$ and variance $n\mu(1 + \alpha)^2$.

For the restricted generalized Poisson distribution, it can be shown that in terms of equation (3), the parameter μ is replaced by $n\mu$ and the parameter α by α/n. Hence, after simplification, the restricted generalized Poisson probability density function with different sample sizes can be expressed as

$$(7) \qquad f(y) = \left(\frac{\mu}{1+\alpha\mu}\right)^y n\left(n + \alpha y\right)^{y-1} \exp\left[\frac{-\mu(n+\alpha y)}{1+\alpha\mu}\right] / \, y!.$$

which has mean $n\mu$ and variance $n\mu(1 + \alpha\mu)^2$.

For the hybrid generalized Poisson distribution, it can be shown that in terms of equation (1), the parameter μ is replaced by $n\mu$, the parameter α by $\alpha/n^{\theta+1}$, and the parameter θ does not change. Hence, after simplification, the hybrid generalized Poisson probability density function with different sample sizes can be expressed as

$$(8) \qquad f(y) = \left(\frac{\mu}{1+\alpha\mu^{\theta+1}}\right)^y n\left(n + \alpha\mu^{\theta} y\right)^{y-1} \exp\left[\frac{-\mu(n+\alpha\mu^{\theta} y)}{1+\alpha\mu^{\theta+1}}\right] / \, y!.$$

which has mean $n\mu$ and variance $n\mu(1 + \alpha\mu^{\theta+1})^2$. Note that equation (8) can be used to obtain the generalized Poisson and restricted generalized Poisson density functions with different sample sizes by restricting the parameter θ to equal –1 and 0 respectively, and the Poisson density with different sample sizes by restricting α to equal 0, just as these densities were derived from equation (1).

Negative Binomial Models

The probability density function for a generalized negative binomial distribution can be expressed as

(9) $$f(y) = \frac{\Gamma(y + \mu^{\theta}\alpha^{-1})}{\Gamma(y+1)\Gamma(\mu^{\theta}\alpha^{-1})} \frac{(\alpha\mu)^{y}\mu^{\theta\mu^{\theta}\alpha^{-1}}}{(\mu^{\theta} + \alpha\mu)^{y+\mu^{\theta}\alpha^{-1}}},$$

for $\alpha > 0$, where $\Gamma(\cdot)$ is the gamma function (Sarker and Surry). The mean is μ and the variance is $\mu(1 + \alpha\mu^{1-\theta})$. Restrictions on θ or α give other negative binomial models.

When $\theta = 0$, the distribution becomes a negative binomial type II (negbin II) distribution with probability density function that can be expressed as

(10) $$f(y) = \frac{\Gamma(y + \alpha^{-1})}{\Gamma(y+1)\Gamma(\alpha^{-1})} \frac{(\alpha\mu)^{y}}{(1 + \alpha\mu)^{y+\alpha^{-1}}},$$

which has mean μ and variance $\mu(1 + \alpha\mu)$. When $\theta = 1$, the distribution becomes a negative binomial type I (negbin I) distribution with probability density function that can be expressed as

(11) $$f(y) = \frac{\Gamma(y + \mu/\alpha)}{\Gamma(y+1)\Gamma(\mu/\alpha)} \frac{\alpha^{y}}{(1 + \alpha)^{y+\mu/\alpha}},$$

which has mean μ and variance $\mu(1 + \alpha)$. Finally, note that $\alpha = 0$ gives the Poisson density, indicating that the negative binomial model nests the Poisson model.

Because the sum of independent negative binomials is also a negative binomial (Evans, Hastings, and Peacock), these negative binomial distribution can be adjusted to account for different sample sizes in the same manner as for Poisson distributions. For the generalized negative binomial distribution, it can be shown that in terms of equation (9), the parameter μ is replaced by $n\mu$, the parameter α by $\alpha/n^{1-\theta}$, and θ does not change, so that the probability density function with different sample sizes can be expressed as

(12) $$f(y)=\frac{\Gamma(y+n\mu^{\theta}\alpha^{-1})}{\Gamma(y+1)\Gamma(n\mu^{\theta}\alpha^{-1})}\frac{(\alpha\mu)^{y}\,\mu^{n\theta\mu^{\theta}a^{-1}}}{(\mu^{\theta}+\alpha\mu)^{y+n\mu^{\theta}a^{-1}}},$$

which has mean $n\mu$ and variance $n\mu(1 + \alpha\mu^{1-\theta})$. From this density, the negbin I and negbin II can be obtained by setting θ equal to 1 and 0 respectively.

For the negbin I distribution, in terms of equation (11), the parameter μ is replaced by $n\mu$, so that the negbin I probability density function with different sample sizes can be expressed as

(13) $$f(y)=\frac{\Gamma(y+n\mu/\alpha)}{\Gamma(y+1)\Gamma(n\mu/\alpha)}\frac{\alpha^{y}}{(1+\alpha)^{y+n\mu/\alpha}},$$

which has mean $n\mu$ and variance $n\mu(1 + \alpha)$. For the negbin II distribution, in terms of equation (10), the parameter μ is replaced by $n\mu$ and the parameter α by α/n, so that the negbin II probability density function with different sample sizes can be expressed as

(14) $$f(y)=\frac{\Gamma(y+n/\alpha)}{\Gamma(y+1)\Gamma(n/\alpha)}\frac{(\alpha\mu)^{y}}{(1+\alpha\mu)^{y+n/\alpha}},$$

which has mean $n\mu$ and variance $n\mu(1 + \alpha\mu)$.

Alternative Single-Parameter Count Data Models

Other single parameter distributions exist as alternatives to the Poisson count data model. Following Sarker and Surry, we present the geometric, Borel, and Yule distributions. Since the Borel and Yule distributions do not include zero among the range of values for y, the distributions are modified by shifting them to the left to include zero in their ranges.

We express the probability density function for the geometric distribution as

(15) $$f(y) = \mu^{y-1}(1 + \mu)^{-y},$$

which has mean μ and variance $\mu(1 + \mu)$. The probability density function for the modified Borel distribution can be expressed as

(16) $$f(y) = (y+1)^{y-1}\mu^{y}(1+\mu)^{-y}\exp(-\mu(y+1)/(\mu+1))/y!,$$

which has mean μ and variance $\mu(1 + \mu)^2$. The probability density function for the modified Yule distribution can be expressed as

(17) $$f(y) = \frac{\mu+1}{\mu}\frac{\Gamma(y+1)\Gamma((2\mu+1)/\mu)}{\Gamma(y+(3\mu+1)/\mu)},$$

which has mean μ and variance $\mu(1 + \mu)^2/(1 - \mu)$.

Hurdle Models

The development of alternatives to the Poisson model has largely been motivated by the desire to develop more flexible models that allow over dispersion and under dispersion. Besides alternative distributions as presented above, hurdle models are possible alternatives that allow over dispersion and under dispersion (Greene pp. 943-946). Two types of hurdle models have been used for analyzing count data. For the first type, a binary outcome determines whether the outcome is zero or positive, and then for positive outcomes, a count data model (properly rescaled) determines the non-zero outcome. Proper rescaling of the count data model requires using count data distribution without zero outcomes, which often in practice is a truncated-at-zero count data distribution. For the second type of hurdle model, a binary outcome determines which of two regimes holds. If the first regime holds, then a zero outcome occurs. If the second regime holds, then a count data model determines whether a zero or positive outcome occurs. The essential difference between these two types of hurdle models is whether the second part of the process allows only positive outcomes, or zero and positive outcomes. Models of the first type are usually called hurdle models, while models of the second type are called zero-inflated or zero-altered models (Greene, pp. 943-945; Cameron and Trevedi, pp. 125-127). In both types of models, independence is assumed between the hurdle distribution and the distribution of positive outcomes.

We present a general version of both types of hurdle models, so that practitioners can combine any density function for the hurdle with any count data model. For notation, $g(z_i)$ is the probability that y_i is zero, where z_i is a vector of regressors that may or may not be equivalent to the regressors x_i used for the count data model. For a hurdle model, the probability of zero and positive outcomes are

$$\text{(18)} \qquad \begin{aligned} &\Pr[y_i = 0] = g(z_i) \\ &\Pr[y_i > k] = (1 - g(z_i)) f(y)/(1 - f(0)) \qquad j > 0 \end{aligned}.$$

Here, the ratio $f(y)/(1 - f(0))$ is the count data distribution $f(y)$ rescaled for truncation at zero. Instead of a truncated-at-zero count data model, distributions such as the unmodified Borel or Yule distributions could be used, since their range is $y_i \in \{1, 2, 3, \ldots\}$ (Sarker and Surry). For a zero-altered hurdle model, the probability of zero and positive outcomes are

$$\text{(19)} \qquad \begin{aligned} &\Pr[y_i = 0] = g(z_i) + (1 - g(z_i)) f(0) \\ &\Pr[y_i > k] = (1 - g(z_i)) f(y_i) \qquad\qquad j > 0 \end{aligned}.$$

The probability density functions reported by equations (18) and (19) allow derivation of the log-likelihood function as explained in the Estimation section.

The hurdle probability $g(z_i)$ is quite flexible. A constant probability can be estimated or standard probit or logit models. In addition, Mullahy describes Poisson and geometric hurdles, while Cameron and Trivedi (p. 124-125) specify a negative binomial hurdle. The count data distribution is any valid count data distribution with $y_i > 0$, including truncated-at-zero distributions $f(y)/(1 - f(0))$, or distributions such as the unmodified Borel or Yule distributions (Sarker and Surry).

Data

A mail survey of prescribed burners was conducted following Dillman's method. The initial mailing list consisted of individuals and private contractors conducting fire suppression and prescribed burns for landowners. A mail survey of 460 potential prescribed burners in eight states (FL, IA, MN, MO, OK, OR, TX, WI) was conducted. The survey process involved sending pre-survey letters, an initial survey mailing, a follow-up postcard and attempted telephone call, and then a second survey mailing. Of the 460 on the initial list, 231 surveys were returned (50%). Of the non-respondents, 57 were contacted by telephone and stated that they did not conduct prescribed burns (only fire suppression) and 43 had incorrect or out-of-date addresses and/or telephone numbers. No contact was made with the remaining 129 non-respondents. Of the 231 returned surveys, 223 were usable, with a total of 109 respondents reporting that they conducted prescribed burns.

The survey asked a variety questions concerning the characteristics of their business (e.g., clients, activities, revenue), their prescribed burning practices (e.g., location, fuel types, training), and the number of prescribed burns, acreage burned, and number of escaped prescribed fires during each of five years (1999-2003). Not all respondents conducted prescribed burns in each year. Table 1 summarizes the collected data and the responses. A copy of the survey instrument is available from the authors.

It is important to note that an escaped fire does not necessarily cause damage. Prescribed burners are supposed to have a burn plan that designates a boundary for the prescribed fire and any burning that occurs outside of this boundary is technically an escape, though it may only burn a small area and cause no property damage or injury. Respondents were asked to report all escapes, since potentially any escape can cause damage or injury. Respondents reporting escapes were contacted in a follow-up telephone survey to collect data on the damage occuring from escapes. However, that study is not the focus of the research reported here.

The data are technically a panel, but we treat the data as a cross section for this analysis, since we are not interested in year effects for insurance purposes. As such, we stack the observations, turning each burner-year combination into a single observation, which makes 441 observations from the 109 respondents reporting prescribed burns. Furthermore, not all available variables are included in the final analysis, since insurance companies would only want to use information from clients if the information significantly affected the likelihood of escapes.

Estimation

Maximum likelihood estimation is for these count data models. For the models specified by equations (1)-(17), the general form of the log-likelihood function is

(20) $$\ln L(\cdot) = \sum_{i=1}^{N} \ln f(y_i),$$

where N is the total number of prescribed burners. To ensure a positive mean, estimation assumes $\mu = \exp(x_i'\beta)$. The log-likelihood function is maximized with respect to the parameter vector β and any variance parameters (i.e., α and θ in the models above). For the two types of hurdle models, the general form of the log-likelihood is slightly different. From equation (18), the general form of the log-likelihood function for a hurdle model is

(21) $$\ln L(\cdot) = \sum_{i=1}^{N} \left[(1-D_i)\ln g(z_i) + D_i \ln[1-g(z_i)] + D_i \left(\ln f(y_i) - \ln[1-f(0)] \right) \right],$$

where D_i is an indicator variable that equals 1 if $y_i > 0$ and zero otherwise. The log-likelihood function is maximized with respect to the parameters of the function $g(z_i)$ and $f(y_i)$. Note that the last term in the summation could be $D_i(\ln h(y_i))$, where $h(y_i)$ is a count data distribution that is not a truncated-at-zero distribution, such as the unmodified Borel or Yule distributions. From equation (19), the general form of the log-likelihood function for a zero-altered hurdle model is

$$\ln L(\cdot)=\sum_{i=1}^{N}\left[(1-D_i)\ln[g(z_i)+(1-g(z_i))f(0)]+D_i\ln[(1-g(z_i))f(y_i)]\right]. \tag{22}$$

Results

We first discuss the model refinement process used to identify significant variable from the survey to be used for predicting the expected number of escapes for a prescribed burner. We then discuss the econometric results and their implications for the various models all using these same variables. We discuss the policy implications of our analysis for the final section.

The estimation process proceeded by first estimating the Poisson model as reported in equation (4) with a mean $\mu_i = \exp(x_i'\beta)$, using as many potential regressors from the survey data used as seemed possible. Our goal was to identify statically significant variables using standard t-tests, since we were looking for variables an insurance company could use to establish a premium for a potential client (i.e., a company would not want to collect extraneous information that had no bearing on risk). Variables were chosen that in consultation with subject matter experts we thought would have significant effects on the likelihood of prescribed burn escapes. Several models were estimated until we developed a refined model that identified the significant variables that had empirically sensible effects on the implied likelihood of escapes. Table 2 described these variables and the abbreviations use for them in the remaining tables.

Table 3 reports the estimation results for the standard Poisson model both with and without the correction for difference in sample size (here the number of burns per year). Tables 4, 5, and 6 do the same for the restricted generalized Poisson model, the generalized Poisson model, and the hybrid generalized Poisson model, respectively. Tables 7, 8, and 9 report comparable results for the negative binomial type II model, the negative binomial type I model, and the generalized negative binomial model, respectively. Finally, table 10 reports results for the geometric, Borel, and Yule models respectively, but only for estimation without any correction for differences in sample sizes. In some cases, we could not get convergence during estimation when using all the variables, but only with a smaller subset. These estimation results are still reported in the tables.

The results in Tables 4-9 support the use of the Poisson model over the other more flexible Poisson and negative binomial models (technically fail to reject the simple Poisson model relative to these other models). For these more flexible models, if the parameter α = 0, the density functions simplify to become the standard Poison density as reported in equation (4). In Table 4-9, the results for the standard models indicate that the parameter α is not significantly different from zero. In addition, the results in Table 10 indicate that none of these alternative single parameter distributions outperform the standard Poisson. The maximized value of the log-likelihood is substantially greater for the Poisson model than for any of the models in Table 10, while the Schwarz-Bayes information criterion is substantially lower, which support the Poison model. Hence, we conclude that these survey data support the use of the standard Poisson model over any of the estimated alternatives.

The results in Tables 3-9 also support the use of the sample size correction. In all cases for which the standard and sample size corrected models converged with the same number of variables, the sample size correction model had a greater maximized value of the log-likelihood and a lesser Schwarz-Bayes information criterion. However, in three cases, we could not achieve convergence during estimation when using all the variables, while this only happened once and less severely for the models without the correction. Hence, we conclude that the sample size correction models are improvements over models without correction, but that convergence can become more difficult to achieve.

Just as for the standard models, among the sample size corrected models, the parameter α is still statistically insignificant, supporting the use of the simple Poisson model over the more flexible Poisson and negative binomial models. The only exception is for restricted generalized Poisson (Table 4). For this model, the maximized value of the log-likelihood function is slightly greater (50.1396 versus 50.1206), while the Schwarz-Bayes information criterion is noticeably lower for the Poisson model (19.9035 versus 22.9289). Given these mixed results and the preponderance of the other information, we still conclude that these survey data support the use of the standard Poisson model over any of the estimated alternatives.

For the hurdle models, we focused on using the Poisson and the sample size corrected Poisson for the count data portion, given the results of the analysis reported in Tables 3-10. The truncated-at-zero Poisson density derived from $f(y)/(1 - f(0))$ is $\mu^y \exp(-\mu)/(y!(1 - \exp(-\mu))$. For the hurdle portion, we used a probit model: $g(z_i) = \Phi(z_i\gamma)$, where γ is a vector of parameters to estimate and $\Phi(\cdot)$ is the standard normal cumulative distribution function. At this time we were unable to get convergence of any such hurdle models using the same variables for both the hurdle and count data distribution (i.e., $z_i = x_i$). We hope to gain convergence by using a more parsimonious model for both z_i and x_i, since conceptually, the determinants of both processes likely needs re-evaluation relative to the original refined model using the variables reported in Table 2. In addition, we intend to try using different models for the hurdle, including a logit model, as well as geometric, Poisson, and negative binomial hurdles (Mullahy, Cameron and Trivedi). In addition, we may examine using corrections for different sample sizes. We leave this work for later.

Interpretation and Policy Implications

The expected number of escapes for prescribed burner i is $\mu_i = n_i \exp(x_i'\beta)$ for the sample size corrected model, where n_i is the number of burns conducted by burner i during a year. Simple calculus indicates that $\frac{\partial \mu_i}{\partial x_{ij}} = \mu_i \beta_j$, where x_{ij} is the j^{th} element of the vector x_i and β_j is its estimated coefficient. Hence, the effect of increasing each variable (or switching from a value of 0 to 1 for an indicator variable) on the expected number of escapes has the same sign as the estimated coefficient for each variable. As a result, the effect of each variable on the expected number of escapes can be directly interpreted from the table of estimated coefficients and the greater the magnitude of the coefficient, the greater the variable's effect. We use this result to interpret our findings and infer policy implications from our analysis, using the coefficients for the sample size corrected Poisson model in Table 3 unless stated otherwise.

The large negative coefficients for Midwest and West imply that the expected number of escapes is lower in these regions relative to the South (the excluded category). Landowners in the South have traditionally used prescribed burning to manage timber stands and residents of the region generally have greater tolerance for smoke and escapes, so that prescribed burners have less incentive to be as careful as in other regions.

The large negative coefficient for Burns ≥ 25% implies that firms specializing or relying substantially on prescribed burns for revenue have a substantially lower expected number of escapes. Reputation effects likely create incentives for such firms to take actions to reduce the likelihood of escapes, while greater familiarity with prescribed burning also likely has an effect.

The relatively large and positive coefficient for Suppression ≥ 10 yrs implies that prescribed burners with substantial experience with fire suppression can be expected to have more escapes. Perhaps substantial experience with fire suppression makes prescribed burners less fearful of escapes and more willing to risk their occurrence, since they believe they can control such escapes and still prevent property damage or injury.

The relatively large negative coefficient on Consulting ≥ 5% indicates that prescribed burners who also are professional consultants on prescribed burning (teaching or helping others do burn plans or fire/smoke modeling) have fewer expected escapes. Such individuals know the prescribed fire "dos and don'ts" and why such rules are important, and so are more likely to follow the recommended safety procedures. In addition, reputation effects may have a role.

The moderately large and positive coefficients for Revenue < $250,000 and Revenue > $1,000,000 indicate that firms in these revenue categories have a greater expected number of escapes relative to the more moderated size firms (revenue ranging $250,000 to $1,000,000: the excluded group). Perhaps smaller firms are more inexperienced or have less or lower quality equipment and so have a greater number of expected escapes and larger firms, since they have more experience and more and better equipment and the financial capacity, can conduct riskier burns and so have a greater number of expected escapes.

To control for this latter effect (certain types of firms conducting riskier burns), we used the six revenue and fuel type interaction terms at the bottom of tables. As a result, the complete effect of firm size as measure by revenue is not captured solely by the Revenue < $250,000 and Revenue > $1,000,000 coefficients. Hence, a larger firm has potentially a greater number of expected escapes, but the expected escapes decreases the more the large firm conducts burns consisting primarily of brush or slash fuels and increases the more the firm conducts burns of primarily grass fuels. In fact, for the estimated coefficients, a larger firm conducting more than

50% of its prescribed burn in brush has the same contribution to the expected number of escapes as a firm with moderate revenue burning only in timber.

The coefficients for the percentage of burns by the different fuel types indicate that, relative to timber (the excluded fuel type), firms conducting more burns in brush have a lower expected number of escapes, while those doing so in grass and slash have a greater number of escapes. These results fit the expectations of subject matter experts with whom we discussed our results. Slash is particularly prone to escapes, since piles of logging residues often continue to smolder for days and can flare up latter causing an escape. Note that as discussed in the previous paragraph, the complete effect of fuel type is determined in interaction with a firm's revenue category in order to control for larger firms potentially conducting riskier burns (e.g., slash) and being less risky than smaller firms conducting such burns.

If the person in charge of the prescribed burn (called the "burn boss") has training equal to or exceeding the National Wildfire Coordinating Group's Burn Boss II designation, the expected number of escapes decreases. This decrease is likely due to similar factors as discussed for the effect of Consulting $\geq$ 5%. It would seem that more trained individuals know and are more likely to follow the recommended safety procedures.

The relatively large negative coefficient for Wildland/Urban > 50% indicates that firms conducting prescribed fires primary in areas where wild lands and urban residents meet have fewer expected escapes. Subject matter experts found this result consistent with their expectations. Because the potential for catastrophic damage is much higher in these situations, the prescribed burns are extra careful, and so have fewer escapes. We tried several interaction terms to capture this extra care effect, but none in the survey were statistically significant.

The large and negative coefficient for Never/Rarely after Sunset implies that firms never or rarely conducting prescribed burns that are ignited after sunset have fewer expected escapes. Again, this result was consisting with the expectations of subject matter experts. Fires ignited after dark are generally riskier and so more prone to escapes.

The coefficient for Burns and Acres imply that the more burns a firm conducts, the greater the expected number of escapes, but the more acres a firm burns, the fewer the expected number of escapes. The positive effect of the number of burns is not surprising. We interpret the negative effect of acres as evidence that firms conducting larger burns (and so accumulating more total acres burned) have fewer expected escapes, because such large burns are conducted in sparsely populated areas and so less risky.

Finally, the slightly positive effect of Prescribed Fire Liability indicates that firms with insurance providing coverage for property damage and personal injury have more expected escapes than affirm without such insurance coverage. This result would seem to be evidence for a moral hazard effect of insurance coverage (insured firms conducting riskier activities). However, not only is the coefficient very small, but it is also statistically insignificant, and so should be given little credence. Interestingly, if the Poisson model without the correction for difference in sample sizes is used, the coefficient is statistically significant. This switch in significance between the standard and corrected models occurs for the other more flexible Poisson models and for the negative binomial models in which convergence was achieved with all variables. Thus, in this case, it seems that not correcting for the difference in sample size leads to an erroneous conclusion concerning the effect of insurance coverage on the behavior of prescribed burners.

Conclusion

In this paper we specify several count data models, generally following Sarker and Surry, but re-parameterizing all the probability densities in terms of their means for easier comparison. In addition, following Cameron and Trivedi, we specified two types of hurdle models. Finally, we derived a correction of the probability densities for differences in sample sizes, which is a contribution to the count data literature as far as we are aware.

We then estimated several of these models and tried to estimate several others, but did not achieve convergence. Despite estimating all these models, we found that the simplest model (the single parameter Poisson) statically performed the others. Of course this is not a general conclusion, but merely an empirical reality for our data. We found that the sample size correction made convergence more difficult to achieve for the more flexible Poisson and negative binomial models with our data. We believe this difficulty occurred because the likelihood function contains a term $\ln(\alpha)$ and has α as a denominator is other terms and the implied value of a for the uncorrected models and simpler models was $\alpha = 0$, indicating that the Poisson model was the preferred model. Hence the difficulty we found for achieving convergence may not be a general result, but an empirical reality for our data. Finally, our analysis provided evidence that not correcting for the difference in sample size can lead to erroneous conclusion concerning the statistical significance of variables used in estimation.

References

Cameron, A. C., and P. K. Trivedi. *Regression Analysis of Count Data*. Cambridge: Cambridge University Press, 1998.

Consul, P. C. *Generalized Poisson Distributions*. New York: Marcel Dekker, Inc., 1989.

Dillman, D. A. *Mail and Internet Surveys: The Tailored Design Method*, 2nd ed. New York: John Wiley Company, 2000.

Evans, M., N. Hastings, and B. Peacock. *Statistical Distributions*, 2nd ed. New York: John Wiley, 1993.

Greene, W. H. *Econometric Analysis*, 3rd ed. Upper Saddle River, NJ: Prentice-Hall, Inc. 1997.

Maddala, G. S. *Limited Dependent and Qualitative Variables in Econometrics*. Cambridge: Cambridge University Press, 1983.

Mullahy, J. "Specification and Testing of Some Modified Count Data Models." *Journal of Econometrics* 33(1986):341-365.

Sarker, R., and Y. Surry. "The Fast Decay Process in Outdoor Recreational Activities and the Use of Alternative Count Data Models." *American Journal of Agricultural Economics* 86(August 2004): 701-715.

Table 1. Summary of survey questions and responses.

Question description	Response Options	Average or Mode (Standard Deviation, % Respondents, or Range)
Use written burn plan	Five choices: never to always	Always (54%)
Predict smoke behavior		Always (77%)
Wear personal protection equipment		Always (54%)
Number of burns conducted by year	Fill in blank for 1999 to 2003	30.6 (1 to 814)
Total acres burned by year		2061 (2 to 34,100
Burns in wildland/urban interface	Five choices: 0% to 100%	1-25% (44%)
Burns next to public lands		1-25% (45%)
Burns in sparely populated areas		76-100% (33%)
Primary fuel type (% of burns): Grass	Fill in blank	44% (37%)
Primary fuel type (% of burns): Brush		8% (14%)
Primary fuel type (% of burns): Timber		33% (31%)
Primary fuel type (% of burns): Slash		17% (28%)
Start burn after sunset	Five choices: never to always	Never (53%)
Burn more than 24 hours		Never (43%)
Extinguish after sunset		Sometimes (34%)
Size of burns: low end of range	Fill in blank	32.4 (0 to 1,000)
Size of burns: high end of range		708.9 (2 to 30,000)
Number of escapes by year		0.58 (0 to 20)
Number of smoke claims with escapes		0.0023 (0 to 1)
Number of smoke claims without escapes		0.14 (0 to 2)
Business has general liability policy	Yes, no, don't know	Yes (84%)
Current general liability premium	Fill in blank	$21,650 ($0 to $1,800,000)
Policy specifically covers prescribed fire	Five choices	Yes, no claims filed (48%)
Years of prescribed fire experience	Fill in blank	18.3 (9.3)
Years of fire suppression experience		10.5 (11.7)
Training of burn boss	Five choices	Don't know (47%)
Business' average gross income		< $100,000 (35%)
Gross revenue from prescribed fire	Fill in blank (%)	12.5% (20.3%)
Gross revenue from mechanical clearing		8.9% (20.0%)
Gross revenue from chemical treatment		5.3% (9.5%)
Gross revenue from fire suppression		6.4% (18.8%)
Gross revenue from consulting (teaching)		2.9% (10.9%)
Gross revenue from other activities		60.6% (36.4%)
Length of burn season (months)	Fill in blank	4.0 (2.9)
Business age (years)		20.2 (15.6)
States where conduct prescribed burns		Florida (29 different states)

Table 2: Abbreviations and descriptions of survey responses used as regressors.

Abbreviation	Description
Midwest	Conduct burns in Midwest
West	Conduct burns in West
Burns ≥25%	At least 25% of revenue from conducting prescribed burns
Suppression ≥ 10 yrs	At least 10 years of fire suppression experience
Consulting > 5%	At least 5% of revenue from consulting on prescribed fire (modeling, teaching, preparing burn plans)
Revenue < $250,000	Annual business revenue < $250,000
Revenue > $1,000,000	Annual business revenue > $1,000,000
% Brush	Percentage of burns with brush (including chaparral and pocosins) as primary fuel type
% Slash	Percentage of burns with slash (logging residues from partial or clear cuts) as primary fuel type
% Grass	Percentage of burns with grass, scatter sagebrush, savannas and open pine with grass understory as primary fuel type
Training ≥ BB II	Training equals or exceeds National Wildfire Coordinating Group Burn Boss II designation
Wildland/Urban > 50%	Conduct more than 50% of burns in wildland/urban interface
Never/Rarely after Sunset	Never or rarely start prescribed burns after sunset
Burns	Total number of burns this year
Acres	Total acres burned this year
Prescribed Fire Liability	Currently have liability policy specifically addressing prescribed fire activities

Table 3. Estimation results for the Poisson model.

	Standard Model			Sample Size Correction		
Parameter	Estimate	Standard Error	P-value	Estimate	Standard Error	P-value
Intercept	-0.982	0.517	0.058	-3.125	0.568	0.000
Midwest	-1.518	0.364	0.000	-1.654	0.364	0.000
West	-2.890	0.806	0.000	-3.116	0.943	0.001
Burns ≥25%	-2.764	0.763	0.000	-2.818	0.746	0.000
Suppression ≥ 10 yrs	1.156	0.265	0.000	1.056	0.300	0.000
Consulting > 5%	-1.313	0.537	0.014	-1.291	0.630	0.040
Revenue < \$250,000	0.696	0.406	0.086	1.124	0.450	0.013
Revenue > \$1,000,000	0.329	0.396	0.406	0.738	0.415	0.076
% Brush	-0.00698	0.024	0.775	-0.00182	0.024	0.940
% Slash	0.023	0.007	0.002	0.0205	0.008	0.011
% Grass	0.0162	0.004	0.000	0.0134	0.005	0.003
Training ≥ BB II	-0.657	0.255	0.010	-0.496	0.257	0.054
Wildland/Urban > 50%	-1.198	0.399	0.003	-1.071	0.413	0.009
Never/Rarely after Sunset	-1.402	0.319	0.000	-1.157	0.320	0.000
Burns	0.795	0.284	0.005	0.485	0.300	0.106
Acres	-0.00311	0.002	0.171	-0.0193	0.004	0.000
Prescribed Fire Liability	0.0000765	0.000	0.000	0.0000236	0.000	0.264
% Slash x Revenue < \$250,000	0.0152	0.025	0.535	0.0126	0.024	0.598
% Slash x Revenue > \$1,000,000	-0.0244	0.027	0.361	-0.0198	0.027	0.457
% Grass x Revenue < \$250,000	-0.0202	0.010	0.051	-0.0180	0.011	0.105
% Grass x Revenue > \$1,000,000	0.00473	0.008	0.563	0.00181	0.009	0.840
% Brush x Revenue < \$250,000	-0.0265	0.008	0.001	-0.0255	0.008	0.002
% Brush x Revenue > \$1,000,000	-0.0119	0.005	0.030	-0.0149	0.006	0.014
Maximized value of log-likelihood function			40.7336			50.1206
Schwarz-Bayes information criterion			29.2904			19.9035

Table 4. Estimation results for the restricted generalized Poisson model.

	Standard Model			Sample Size Correction		
		Standard			Standard	
Parameter	Estimate	Error	P-value	Estimate	Error	P-value
Intercept	-1.001	0.524	0.056	-3.149	0.557	0.000
Midwest	-1.514	0.367	0.000	-1.682	0.356	0.000
West	-2.899	0.816	0.000	-3.051	0.910	0.001
Burns ≥25%	-2.775	0.766	0.000	-2.817	0.742	0.000
Suppression ≥ 10 yrs	1.170	0.271	0.000	1.089	0.292	0.000
Consulting > 5%	-1.296	0.540	0.016	-1.300	0.615	0.034
Revenue < $250,000	0.702	0.411	0.088	1.161	0.440	0.008
Revenue > $1,000,000	0.317	0.404	0.433	0.779	0.394	0.048
% Brush	-0.00699	0.024	0.775	-0.00142	0.024	0.952
% Slash	0.0228	0.008	0.002	0.0212	0.008	0.005
% Grass	0.0163	0.004	0.000	0.0141	0.004	0.001
Training ≥ BB II	-0.659	0.259	0.011	-0.539	0.248	0.030
Wildland/Urban > 50%	-1.199	0.401	0.003	-1.100	0.408	0.007
Never/Rarely after Sunset	-1.405	0.322	0.000	-1.155	0.314	0.000
Burns	0.814	0.288	0.005	0.483	0.293	0.099
Acres	-0.00347	0.003	0.172	-0.0197	0.004	0.000
Prescribed Fire Liability	0.0000779	0.000	0.001	0.0000192	0.000	0.177
% Slash x Revenue < $250,000	0.0150	0.025	0.544	0.0113	0.023	0.628
% Slash x Revenue > $1,000,000	-0.0247	0.027	0.357	-0.0206	0.026	0.428
% Grass x Revenue < $250,000	-0.0199	0.011	0.060	-0.0183	0.011	0.083
% Grass x Revenue > $1,000,000	0.00471	0.008	0.576	0.000863	0.008	0.918
% Brush x Revenue < $250,000	-0.0267	0.008	0.001	-0.0257	0.008	0.002
% Brush x Revenue > $1,000,000	-0.0118	0.006	0.035	-0.0150	0.006	0.009
Alpha	0.0214	0.026	0.419	-0.561	0.152	0.000
Maximized value of log-likelihood function			42.0104			50.1396
Schwarz-Bayes information criterion			31.0581			22.9289

Table 5. Estimation results for the generalized Poisson model.

	Standard Model			Sample Size Correction		
Parameter	Estimate	Standard Error	P-value	Estimate	Standard Error	P-value
Intercept	-0.835	0.552	0.130	-3.097	0.589	0.000
Midwest	-1.520	0.381	0.000	-1.659	0.368	0.000
West	-2.922	0.853	0.001	-3.114	0.949	0.001
Burns ≥25%	-2.734	0.768	0.000	-2.808	0.748	0.000
Suppression ≥ 10 yrs	1.137	0.278	0.000	1.049	0.305	0.001
Consulting > 5%	-1.295	0.554	0.019	-1.293	0.633	0.041
Revenue < \$250,000	0.620	0.423	0.143	1.110	0.459	0.016
Revenue > \$1,000,000	0.204	0.429	0.634	0.723	0.426	0.090
% Brush	-0.00890	0.025	0.725	-0.00200	0.024	0.934
% Slash	0.0205	0.008	0.010	0.0201	0.008	0.016
% Grass	0.0161	0.004	0.000	0.0133	0.005	0.004
Training ≥ BB II	-0.633	0.271	0.019	-0.491	0.261	0.060
Wildland/Urban > 50%	-1.107	0.420	0.008	-1.050	0.429	0.014
Never/Rarely after Sunset	-1.415	0.341	0.000	-1.155	0.324	0.000
Burns	0.709	0.303	0.019	0.470	0.313	0.133
Acres	-0.00331	0.002	0.165	-0.0194	0.004	0.000
Prescribed Fire Liability	0.0000788	0.000	0.000	0.0000239	0.000	0.263
% Slash x Revenue < \$250,000	0.0183	0.025	0.470	0.0130	0.024	0.587
% Slash x Revenue > \$1,000,000	-0.0225	0.028	0.416	-0.0195	0.027	0.466
% Grass x Revenue < \$250,000	-0.0163	0.011	0.143	-0.0172	0.012	0.144
% Grass x Revenue > \$1,000,000	0.00790	0.009	0.383	0.00230	0.009	0.807
% Brush x Revenue < \$250,000	-0.0291	0.009	0.001	-0.0261	0.009	0.004
% Brush x Revenue > \$1,000,000	-0.0127	0.006	0.033	-0.0150	0.006	0.014
Alpha	0.0726	0.055	0.189	0.00889	0.046	0.848
Maximized value of log-likelihood function			41.1942			52.7766
Schwarz-Bayes information criterion			31.8743			20.2919

Table 6. Estimation results for the hybrid generalized Poisson model.

	Standard Model			Sample Size Correction		
Parameter	Estimate	Standard Error	P-value	Estimate	Standard Error	P-value
Intercept	-0.546	0.495	0.270	-2.877	0.271	0.000
Midwest	-1.226	0.351	0.000	-1.680	0.313	0.000
West	-3.288	0.871	0.000	-4.032	0.959	0.000
Burns ≥25%	-2.331	0.781	0.003	-2.849	0.733	0.000
Suppression ≥ 10 yrs	0.976	0.286	0.001	1.029	0.232	0.000
Consulting > 5%	-0.879	0.498	0.078	-0.719	0.476	0.131
Revenue < \$250,000	-0.0181	0.357	0.960	0.267	0.240	0.265
Revenue > \$1,000,000	0.642	0.360	0.074	1.266	0.256	0.000
% Brush	-0.0127	0.008	0.117	-0.0111	0.004	0.013
% Slash	0.0176	0.008	0.023	0.00729	0.002	0.000
% Grass	0.0126	0.004	0.004	0.00423	0.003	0.197
Training ≥ BB II	-0.565	0.284	0.047	0.736	0.245	0.003
Wildland/Urban > 50%	-1.088	0.421	0.010	-1.761	0.383	0.000
Never/Rarely after Sunset	-1.371	0.330	0.000	-1.176	0.271	0.000
Burns	0.464	0.292	0.112	-0.0201	0.003	0.000
Acres	-0.00470	0.003	0.104	0.0000340	0.000	0.007
Prescribed Fire Liability	0.0000841	0.000	0.000	0.288	0.190	0.128
% Slash x Revenue < \$250,000	--	--	--	--	--	--
% Slash x Revenue > \$1,000,000	--	--	--	--	--	--
% Grass x Revenue < \$250,000	-0.00403	0.010	0.685	--	--	--
% Grass x Revenue > \$1,000,000	0.00741	0.009	0.402	--	--	--
% Brush x Revenue < \$250,000	-0.0134	0.006	0.036	--	--	--
% Brush x Revenue > \$1,000,000	-0.0241	0.005	0.000	--	--	--
Alpha	0.101	0.065	0.122	20103	75251	0.789
Theta	-0.762	0.397	0.055	7.817	2.824	0.006
Maximized value of log-likelihood function			36.2384			9.77330
Schwarz-Bayes information criterion			33.7856			48.0726

Table 7. Estimation results for the negative binomial type II model.

	Standard Model			Sample Size Correction		
		Standard			Standard	
Parameter	Estimate	Error	P-value	Estimate	Error	P-value
Intercept	-1.002	0.526	0.057	-2.636	0.393	0.000
Midwest	-1.512	0.369	0.000	-1.253	0.290	0.000
West	-2.903	0.820	0.000	-2.793	0.927	0.003
Burns ≥25%	-2.776	0.767	0.000	-2.871	0.757	0.000
Suppression ≥ 10 yrs	1.171	0.272	0.000	0.974	0.283	0.001
Consulting > 5%	-1.293	0.542	0.017	-0.668	0.557	0.231
Revenue < \$250,000	0.701	0.413	0.090	-0.0222	0.319	0.945
Revenue > \$1,000,000	0.312	0.407	0.443	0.904	0.318	0.004
% Brush	-0.00700	0.024	0.775	--	--	--
% Slash	0.0228	0.008	0.003	--	--	--
% Grass	0.0163	0.004	0.000	--	--	--
Training ≥ BB II	-0.659	0.260	0.011	-0.266	0.302	0.379
Wildland/Urban > 50%	-1.199	0.402	0.003	-1.866	0.422	0.000
Never/Rarely after Sunset	-1.404	0.323	0.000	-0.879	0.295	0.003
Burns	0.817	0.289	0.005	0.540	0.271	0.047
Acres	-0.00350	0.003	0.173	-0.0221	0.004	0.000
Prescribed Fire Liability	0.0000778	0.000	0.001	0.0000741	0.000	0.015
% Slash x Revenue < \$250,000	-0.0198	0.011	0.063	--	--	--
% Slash x Revenue > \$1,000,000	0.00477	0.009	0.575	--	--	--
% Grass x Revenue < \$250,000	-0.0268	0.008	0.001	--	--	--
% Grass x Revenue > \$1,000,000	-0.0117	0.006	0.037	--	--	--
% Brush x Revenue < \$250,000	0.0150	0.025	0.544	--	--	--
% Brush x Revenue > \$1,000,000	-0.0247	0.027	0.356	--	--	--
Alpha	0.0581	0.072	0.416	13.960	3.847	0.000
Maximized value of log-likelihood function			41.2905			-3.92450
Schwarz-Bayes information criterion			31.7780			49.5923

Table 8. Estimation results for the negative binomial type I model.

	Standard Model			Sample Size Correction		
Parameter	Estimate	Standard Error	P-value	Estimate	Standard Error	P-value
Intercept	-0.840	0.551	0.127	-3.099	0.587	0.000
Midwest	-1.517	0.381	0.000	-1.659	0.368	0.000
West	-2.922	0.853	0.001	-3.114	0.949	0.001
Burns ≥25%	-2.734	0.767	0.000	-2.809	0.748	0.000
Suppression ≥ 10 yrs	1.139	0.278	0.000	1.049	0.305	0.001
Consulting > 5%	-1.295	0.553	0.019	-1.293	0.633	0.041
Revenue < $250,000	0.623	0.423	0.141	1.111	0.458	0.015
Revenue > $1,000,000	0.209	0.428	0.625	0.724	0.426	0.089
% Brush	-0.00886	0.025	0.726	-0.00199	0.024	0.934
% Slash	0.0206	0.008	0.010	0.0201	0.008	0.016
% Grass	0.0160	0.004	0.000	0.0133	0.005	0.004
Training ≥ BB II	-0.635	0.270	0.019	-0.491	0.261	0.059
Wildland/Urban > 50%	-1.112	0.420	0.008	-1.052	0.428	0.014
Never/Rarely after Sunset	-1.413	0.340	0.000	-1.155	0.323	0.000
Burns	0.712	0.303	0.019	0.472	0.312	0.131
Acres	-0.00331	0.002	0.165	-0.0194	0.004	0.000
Prescribed Fire Liability	0.0000786	0.000	0.000	0.0000238	0.000	0.263
% Slash x Revenue < $250,000	-0.0165	0.011	0.139	-0.0173	0.012	0.141
% Slash x Revenue > $1,000,000	0.00777	0.009	0.389	0.00226	0.009	0.809
% Grass x Revenue < $250,000	-0.0291	0.009	0.001	-0.0260	0.009	0.004
% Grass x Revenue > $1,000,000	-0.0127	0.006	0.034	-0.0150	0.006	0.014
% Brush x Revenue < $250,000	0.0183	0.025	0.471	0.0130	0.024	0.588
% Brush x Revenue > $1,000,000	-0.0225	0.028	0.416	-0.0195	0.027	0.466
Alpha	0.145	0.115	0.209	0.0165	0.090	0.855
Maximized value of log-likelihood function			41.9593			50.1381
Schwarz-Bayes information criterion			31.1092			22.9304

Table 9. Estimation results for the generalized negative binomial model.

Parameter	Standard Model Estimate	Standard Model Standard Error	Standard Model P-value	Sample Size Correction Estimate	Sample Size Correction Standard Error	Sample Size Correction P-value
Intercept	-0.861	0.560	0.124	-2.147	0.422	0.000
Midwest	-1.520	0.382	0.000	-1.552	0.341	0.000
West	-2.923	0.856	0.001	-5.006	1.066	0.000
Burns ≥25%	-2.756	0.779	0.000	-2.655	0.788	0.001
Suppression ≥ 10 yrs	1.145	0.281	0.000	0.0557	0.349	0.873
Consulting > 5%	-1.297	0.557	0.020	-0.848	0.573	0.139
Revenue < $250,000	0.635	0.427	0.138	-0.767	0.353	0.030
Revenue > $1,000,000	0.220	0.432	0.611	1.813	0.402	0.000
% Brush	-0.00870	0.025	0.732	--	--	--
% Slash	0.0209	0.008	0.010	--	--	--
% Grass	0.0161	0.004	0.000	--	--	--
Training ≥ BB II	-0.641	0.272	0.019	0.222	0.371	0.549
Wildland/Urban > 50%	-1.131	0.426	0.008	-2.110	0.467	0.000
Never/Rarely after Sunset	-1.418	0.340	0.000	-1.743	0.334	0.000
Burns	0.731	0.312	0.019	0.545	0.274	0.047
Acres	-0.00335	0.002	0.169	--	--	--
Prescribed Fire Liability	0.0000790	0.000	0.000	--	--	--
% Slash x Revenue < $250,000	-0.0170	0.011	0.136	--	--	--
% Slash x Revenue > $1,000,000	0.00731	0.009	0.428	--	--	--
% Grass x Revenue < $250,000	-0.0287	0.009	0.001	--	--	--
% Grass x Revenue > $1,000,000	-0.0125	0.006	0.037	--	--	--
% Brush x Revenue < $250,000	0.0179	0.026	0.484	--	--	--
% Brush x Revenue > $1,000,000	-0.0227	0.028	0.414	--	--	--
Alpha	0.165	0.142	0.245	61.125	46.861	0.192
Theta	0.850	0.605	0.160	-0.473	0.360	0.189
Maximized value of log-likelihood function			41.9871			-29.5422
Schwarz-Bayes information criterion			34.1259			72.1656

Table 10. Estimation results for the geometric, Borel, and Yule models.

	------- geometric -------			---------- Borel ----------			---------- Yule ----------		
Parameter	Estimate	Standard Error	P-value	Estimate	Standard Error	P-value	Estimate	Standard Error	P-value
Intercept	-0.928	0.610	0.128	-0.837	0.731	0.252	-0.415	0.867	0.633
Midwest	-1.355	0.419	0.001	-0.950	0.512	0.063	-0.702	0.709	0.322
West	-3.181	1.011	0.002	-11.185	5.339	0.036	-4.648	1.968	0.018
Burns ≥25%	-2.767	0.812	0.001	-2.820	0.861	0.001	-3.358	1.134	0.003
Suppression ≥ 10 yrs	1.148	0.330	0.000	1.043	0.418	0.013	1.385	0.611	0.023
Consulting > 5%	-1.196	0.626	0.056	-1.084	0.774	0.161	-1.435	0.883	0.104
Revenue < $250,000	0.562	0.481	0.243	0.362	0.580	0.533	-0.143	0.742	0.847
Revenue > $1,000,000	0.100	0.525	0.849	0.0612	0.839	0.942	0.588	1.045	0.573
% Brush	-0.00816	0.026	0.753	-0.00713	0.029	0.803	-0.00211	0.015	0.886
% Slash	0.0208	0.010	0.040	0.0164	0.013	0.202	0.00842	0.016	0.608
% Grass	0.0142	0.005	0.005	0.00759	0.006	0.232	0.00621	0.007	0.385
Training ≥ BB II	-0.590	0.321	0.066	-0.403	0.416	0.333	-0.491	0.582	0.399
Wildland/Urban > 50%	-1.177	0.437	0.007	-1.179	0.492	0.016	-1.285	0.576	0.026
Never/Rarely after Sunset	-1.374	0.373	0.000	-1.337	0.463	0.004	-1.549	0.588	0.008
Burns	-0.00390	0.003	0.254	-0.00477	0.005	0.371	-0.0169	0.011	0.109
Acres	0.0000784	0.000	0.055	0.000095	0.000	0.244	0.000409	0.000	0.084
Prescribed Fire Liability	-0.0158	0.013	0.227	-0.0099	0.016	0.535	0.0106	0.021	0.621
% Slash x Rev. < $250,000	0.0098	0.012	0.397	0.0937	0.052	0.071	0.0251	0.022	0.261
% Slash x Rev. > $1,000,000	-0.0268	0.009	0.002	-0.0268	0.010	0.006	-0.0225	0.011	0.032
% Grass x Rev. < $250,000	-0.0102	0.007	0.168	-0.0199	0.014	0.143	-0.0366	0.013	0.006
% Grass x Rev. > $1,000,000	0.0162	0.026	0.537	0.0204	0.029	0.483	--	--	--
% Brush x Rev. < $250,000	-0.0261	0.029	0.373	-0.0902	0.053	0.092	--	--	--
% Brush x Rev. > $1,000,000	0.815	0.330	0.013	0.924	0.397	0.020	0.835	0.483	0.084
Maximized value of log-likelihood function			-258.092			10.6269			-614.345
Schwarz-Bayes information criterion			328.116			59.3971			638.701