Precision Nitrogen Fertilization Technology with Micro Grids

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Abstract

Sensor-based precision fertilizer technologies are being developed and researched by production scientists. One such technology uses normalized difference vegetation index (NDVI) reflectance measurements of growing winter wheat plants and a nitrogen fertilizer optimization algorithm (NFOA) to determine nitrogen requirement necessary for plants to reach their yield plateau. A number of precision fertilizer application systems that use this technology are considered in this paper. A linear response stochastic plateau wheat yield function conditional on NDVI reflectance measurements is estimated and used within an expected profit-maximization framework to estimate upper bounds on the returns from the precision nitrogen application systems. The on-the-go precision system that assumes perfect information was approximately $7 per acre more profitable than the convention of applying 80 pounds of nitrogen prior to planting in the fall. The whole-field precision system was break-even with conventional methods.

Key Words: expected profit, NDVI, nitrogen fertilizer, precision farming, site specific, wheat

Introduction

Nitrogen fertilizer is a primary input for winter wheat production, accounting for between 15 and 25 percent of total operating expenses (USDA). A conventional approach to wheat production involves applying nitrogen requirements uniformly to a whole field prior to planting wheat in the fall. Substantial variations in soil nutrient availability both within and across fields and the cost associated with over-application of nitrogen with a whole field management strategy provide justification for using variable rate precision application technologies for wheat production. Since the 1990's, precision application technologies using soil sampling to determine soil nitrogen availability have been proposed. However, adoption of precision soil sampling for nitrogen has been limited (Daberkow and McBride).

Early published studies on the costs and benefits of soil-based precision technologies mostly reported that the benefits from theses technologies were greater than the costs (Lambert and Lowenberg-DeBoer). More recent research argued that technologies and strategies such as combine yield monitors, soil sampling and mapping, and fertilizer applicators equipped with global positioning systems have not been adopted in widespread fashion because significant costs associated with site-specific information management and variable rate application were overlooked (Hurley et al., Swinton and Lowenberg-DeBoer; Bullock and Bullock). Economic theory suggests that if a new technology is unambiguously economical it will be adopted by profit maximizing producers. As a result, alternative techniques for applying fertilizers variably are being explored to solve the problem of over applying fertilizer in some parts of the field, and under applying fertilizer in other parts.

One alternative to soil sampling that has gained substantial interest from the production agriculture community uses reflective sensor measurements of growing wheat plants to determine nitrogen need (Alchanatis et al.; Ehlert et al.; Phillips et al.; Raun et al., 2001; Schachtl et al.). The technology developed by Raun et al., 2001 uses NDVI reflectance measurements of growing wheat plants and a nitrogen fertilizer optimization algorithm (NFOA) to determine plant performance and nitrogen needs on micro grids as small as four square-feet. Two individual systems using this technology have been developed by engineers.

The first system is a precision-based, whole field application system, and the second system is a site-specific variable rate application system that can sample and treat plants on individual four square-foot micro grids instead of the three-acre grids commonly used with soil testing and mapping strategies (Raun et al 1998; Solie et al., 1999). Both systems are commercially available for use in winter wheat production, but adoption has been slow.

Public and private sector investment into this technology, including the two systems described, has been substantial, but an economic analysis of the expected producer benefits from the adoption of these systems is lacking. The objectives of this research are to determine the maximum expected net returns for the whole field system and two special cases of the variable rate system relative to the maximum expected net return from conventional all-before-planting systems. The net value of the plant-based systems above that of the conventional systems would be useful to farm producers in helping them decide whether or not to adopt this technology, and would provide engineers and manufacturers with a target cost to deliver the systems. Data for wheat yield, optical reflectance information, and levels of preplant nitrogen have been collected from on-farm in-season trials over six years and across eight locations in Oklahoma. These data provide the opportunity to develop a yield response to nitrogen function that is conditional on in-season sensor readings taken from growing winter wheat plants in the late winter or early spring.

The NDVI reflectance reading obtained with the optical reflectance sensor is believed to reveal information about plant nutrient availability and hence plant performance.

We first develop a conceptual framework of the producer's optimization problem that describes the interaction between independent variables (such as nitrogen, optical reflectance readings, and stochastic variables) and the dependent variable (wheat yield). Using the panel data set, a wheat yield response to optical reflectance information function and a response function that describes the relationship between optical reflectance information and the level of nitrogen are estimated. Optimal levels of nitrogen for the alternative systems are then derived. Monte Carlo integration is then used to determine whether or not farmers should consider adopting a plant-based precision nitrogen fertilizer application technology. Sensitivity analysis is used to provide insight into how the results change to slight changes in the model's parameters.

Conceptual Framework

Expected profit maximization

The plant-based precision technology requires placing a nitrogen rich strip (NRS) in the field in the fall. The NRS is fertilized with a non-limiting level of nitrogen fertilizer; that is, a level that will ensure that the yield of wheat growing in that strip will reach its plateau level (Frank, Beattie and Embleton; Grimm, Paris, and Williams; Waugh, Cate, and Nelson).

Normalized difference vegetation index (NDVI) sensor measurements of plants growing in the NRS and in nonNRS regions of the field are obtained in late winter (Tucker; Hockheim and Barber; Raun et al., 1999) and used within a nitrogen fertilizer optimization algorithm (NFOA) to compute the optimal level of nitrogen to apply to the growing wheat. A concern regarding the NFOA is that it does not consider the price of nitrogen or the price of wheat. In addition, this particular technology faces a high economic hurdle because it was designed to use urea-ammonium nitrate (UAN), which is historically a more expensive form of nitrogen fertilizer than anhydrous ammonia.

The whole-field precision system uses a portable sensing device that collects NDVI sensor measurements of growing plants that is then entered into the NFOA to obtain the average whole field recommendation of nitrogen fertilizer. Alternatively, the plant-based technology has been incorporated into a site-specific system that has the NFOA stored in a computer on board a self-propelled boom applicator that is equipped with a mix of optical reflectance sensors, computers, and spray nozzles. The applicator assesses plant nitrogen need and applies discrete quantities of liquid nitrogen fertilizer on individual four square-foot grids on the go.

Economic theory suggests that for a precision technology to be adopted into the on-farm production process, the adopters need to be convinced that it is substantially more profitable than the conventional system they are accustomed to using (Lowenberg-DeBoer). Conceptually, the expected farm-level net return associated with the proposed precision technology is the difference between expected crop revenue (expected price times expected yield) and the total cost of nitrogen application (cost of nitrogen plus fixed application costs), or mathematically

$$
\begin{aligned}
&\max_{N^T} \mathrm{E}(\pi_t) = \sum_{i=1}^{n}\sum_{t=1}^{m} \mathrm{E}(p)\mathrm{E}(y_{it}(N_{it}^T)) - r^P \sum_{i=1}^{n}\sum_{t=1}^{m} N_{it}^P - r^R \sum_{i=1}^{n}\sum_{t=1}^{m} N_{it}^R - c^P - c^R,\\
&\text{s.t. } y_{it} = y(N_{it}^T, ORI_{it}, ORI_t^{NRS}, \phi),\\
(1)\qquad &N_{it}^T = N_{it}^P + N_{it}^R,\\
&N_{it}^P, N_{it}^R \geq 0,\\
&c^P, r^P > 0 \text{ if } N_{it}^P > 0,\\
&c^R, r^R > 0 \text{ if } N_{it}^R > 0.
\end{aligned}
$$

where π_t is net return to nitrogen application in field-year t; y_{it} is wheat yield on grid i in field-year t, N_{it}^T is the amount of nitrogen on grid i in field-year t, N_{it}^P is the level of preplant nitrogen on grid *i* in field-year *t*, N_{it}^{R} is the level of topdress nitrogen on grid *i* in year *t*, the symbol ORI_{it} represent optical reflectance readings taken on each grid and field-year, the symbol ORI_{t}^{NRS} represents optical reflectance readings taken off the NRS in year *t*, symbols r^{P} and r^{R} are the price of preplant and topdress nitrogen sources, respectively, symbols c^{P} and c^{R} are fixed costs for preplant application and topdress application, respectively, and ϕ represents a vector of random error terms.

<u>The yield response function</u>

A key element in equation (1) is the yield response to nitrogen function. Because of the data limitations, the yield response function had to be developed and estimated in two parts.[1] The key assumption is that nitrogen is assumed to have the same influence (except for an efficiency adjustment) on ORI and in turn yield whether it is applied preplant or at time of sensing. So, for the first part in developing our yield response function we define wheat yield response to optical reflectance information to be

(2) $$y_{it} = a + bORI_{it} + \theta_{it},$$

where y_{it} is wheat yield in bushels per acre on grid i in field-year t, symbols a and b are the intercept and slope coefficient to be estimated, and the error term θ_{it} is partitioned into an independently and identically distributed random error term θ_{it}^{*} that has mean zero and variance $\sigma_{\theta^*}^{2}$, and year random effect ω_t that has mean zero and variance σ_{ω}^{2}.[2]

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[1] The available data have preplant applications of nitrogen, mid-season readings of ORI, and wheat yield. An ideal experiment would record ORI before applications of varying levels of nitrogen. To-date, such an experiment has not been conducted.

[2] The hypothesis of linear functional form could not be rejected at a 95% level of confidence in favor of an exponential functional form based on the J-test for nonnested models $(H_0 : \alpha = 0, t = 1.33, \Pr > |t| = .7899)$. (Greene, p. 302).

Independence is assumed between the two variance components, and therefore the variance of the overall error term is $\sigma_{\theta}^{2} = \sigma_{\omega}^{2} + \sigma_{\theta^*}^{2}$. The symbol ORI_{it} is defined as the NDVI reflectance reading taken on grid i in field-year t and is adjusted by the number of growing degree days. It is assumed that ORI_{it} is quantifiable information that relates how much nitrogen is available to the plants at the time of sensing, which in turn provides information that is useful in quantifying how much additional nitrogen is needed to reach full yield potential.

The wheat yield response to the optical reflectance information was defined in equation (2). However, the relationship of primary interest for this study is the relationship between wheat yield and the total level of nitrogen, regardless of where it comes from (i.e., residual from previous year, released through soil mineralization, fertilizer application, rain, or lightning). Research suggests that a linear response plateau (LRP) function performs as well, if not better, than polynomial forms (Perrin; Lanzer and Paris), and that the LRP explained crop response to fertilizer at least as well, if not better, than polynomial forms (Grimm, Paris, and Williams; Heady, Pesek, and Brown; Paris; Frank, Beattie, and Embleton; Chambers and Lichtenberg).

A study conducted by Tembo, Brorsen and Epplin used data from a long-term winter wheat experiment (32-years) conducted in Oklahoma to estimate a LRP and a proposed alternative estimated as a linear response stochastic plateau (LRSP), where the plateau is assumed to be a year random variable that is distributed normally. In their paper, they found that the LRSP function improved on the statistical accuracy of the estimates for the optimal level of nitrogen to apply to wheat. Katibie et al. (2003) also utilized both LRP and LRSP functional forms to determine the effect of stocking density on wheat grain yield and average daily gain of steers using seven years of experimental data from a stocking density experiments conducted in

Oklahoma. They used a likelihood ratio test and rejected the conventional LRP in favor of the LRSP function.

Katibie et al. (2005) point out that the primary difference between the LRP and the LRSP forms regards the nonrandom assumption for the plateau. With an LRP the effect is treated as fixed and has often been specified using dummy variables. Tembo, Brorsen, and Epplin argue that when estimating yield response functions using long-term panel data, it is more plausible to assume that the plateau is stochastic due to certain unknown factors over time such as differences in weather patterns, level of rainfall, and mineralization of the organic matter. In addition to the assumption of a stochastic plateau, the Tembo, Brorsen, and Epplin model is a predictive model that allows for identifying unusually low or high yields by estimating random effects for each field-year.

In the case of variable rate nitrogen application, such as the system that Raun et al. (1999) developed, each grid or space in the field is treated as an independent farm with each grid having its own plateau. The plateau on each grid has two random components: a year random effect that is measured with the NRS and an element unique to the grid which is unknown unless measured using the sensors. It is assumed that the plateau in each grid is random due to one or more unknown factors such as weather patterns, rainfall, and/or soil mineralization that all vary across years. Additionally, the plateaus have randomness that results from unknown factors across space, such as uneven rainfall across grids, unequal levels of drainage, poor plant stand, and/or differences in the soil mineralization process that vary across grids within the field (mainly due to different soil types). The nitrogen fertilizer optimization algorithm (NFOA) developed by Raun et al., (2002) implicitly assumes a LRSP function.

The second part, then, uses the approach provided by Tembo, Brorsen, and Epplin to develop and estimate a LRSP function that relates the level of nitrogen to optical reflectance information collected from growing wheat in late winter. This relationship is defined as[3]

$$ORI_{it}^{S} = \begin{cases} \alpha + \beta(N_{it}^{A} + N_{it}^{R}) + u_t + \eta_{it}, \\ \qquad \text{if } \alpha + \beta(N_{it}^{A} + N_{it}^{R}) + u_t \le E\left(ORI_t^{NRS}\right) + v_t + \tau_{it} + u_t, \\ E\left(ORI_t^{NRS}\right) + v_t + \tau_{it} + u_t + \eta_{it}, \\ \qquad \text{if } \alpha + \beta(N_{it}^{A} + N_{it}^{R}) + u_t > E\left(ORI_t^{NRS}\right) + v_t + \tau_{it} + u_t, \end{cases} \tag{3}$$

where ORI_{it}^{S} is optical reflectance information observed in late winter on grid *i* in field-year *t*; α and β are the intercept and slope parameters to be estimated; N_{it}^{A} is the level of nitrogen that is available to the plant at the time of planting (this could be residual N from the previous year, from preplant fertilizer, soil mineralization, or from other possible sources such as rainfall and lightning); N_{it}^{R} is the post-sensing level of nitrogen required in the spring that is necessary for the plants to produce the plateau level of yield; the symbols u_t and η_{it} represent the year random effect and traditional random error component, respectively, and are both assumed to be distributed normal with a mean of zero and variances equal to σ_u^2 and σ_η^2, respectively; and the plateau is defined as $E\left(ORI_t^{NRS}\right) + v_t + \tau_{it}$, which is equal to a constant average of sensor readings taken from the NRS plus a field-year random effect, v_t, and a spatial plateau random effect, τ_{it}, that varies by grid. The plateau random variables are assumed to be independently and identically distributed with means equal to zero and variances equal to, σ_v^2 and σ_τ^2, respectively.

[3] Note that equation (3) can not be estimated in its present state because observations for N_{it}^{R} are not available. That is, the spatial random component τ_{it} can not be estimated because data are not available from experiments in which nitrogen treatments were applied after sensing. Consequently, equation (3) is estimated using observations for preplant nitrogen only for N_{it}^{A}, and assuming that the plateau spatial error component τ_{it} is equal to zero.

As previously mentioned, an important component of this paper is to develop a response equation that sufficiently describes the relationship between wheat yield and the total level of nitrogen that is necessary for the plants to reach their plateau yield. The theoretical derivation of such a relationship can be accomplished in the following steps. The first step is to develop an equation that relates the level of preplant nitrogen to optical reflectance information observed in the late winter. This equation can be expressed as

$$(4) \qquad ORI_{it}^{S} = \begin{cases} \alpha + \beta N_{it}^{P} + u_t + \eta_{it}, \\ \qquad \text{if } \alpha + \beta N_{it}^{P} + u_t \le E\left(ORI_t^{NRS}\right) + v_t + \tau_{it} + u_t, \\ E\left(ORI_t^{NRS}\right) + v_t + \tau_{it} + u_t + \eta_{it}, \\ \qquad \text{if } \alpha + \beta N_{it}^{P} + u_t > E\left(ORI_t^{NRS}\right) + v_t + \tau_{it} + u_t, \end{cases}$$

where N_{it}^{P} is the level of preplant applied nitrogen on grid *i* in year *t*. Equation (4) can be solved for the level of preplant nitrogen, which in this paper is simplified by assuming that the total amount of nitrogen available to the plants at the time of sensing in late winter comes from a preplant source only (i.e., $N_{it}^{A} = N_{it}^{P}$). The solution for this step is written as

$$(5) \qquad N_{it}^{P} = \frac{ORI_{it}^{S} - \alpha - \eta_{it} - u_t}{\beta}.$$

The next step is to derive the relationship between the total level of nitrogen available to the plants and optical reflectance information observed post-topdressing. The challenge here is that that post-topdressing sensor information is never observed with available data. However, it seems reasonable to assume that optical reflectance information taken after topdressing nitrogen in late winter would be the same as the optical reflectance information would be (with an adjustment reflecting an expected gain in nitrogen use efficiency) if the same amount of nitrogen had been applied before planting. With this assumption, the solution obtained in equation (5) can be substituted into equation (4) and simplified. Doing so yields the following

$$(6) \qquad ORI_{it}^{T}\left(N_{it}^{T}\right)=\begin{cases} ORI_{it}^{S}(N_{it}^{P}), \\ \qquad \text{if } ORI_{it}^{S}+\beta N_{it}^{R}-\eta_{it}\le E\left(ORI_{t}^{NRS}\right)+v_{t}+\tau_{it}+u_{t}, \\ E\left(ORI_{t}^{NRS}\right)+v_{t}+\tau_{it}+u_{t}+\eta_{it}, \\ \qquad \text{if } ORI_{it}^{S}+\beta N_{it}^{R}-\eta_{it}> E\left(ORI_{t}^{NRS}\right)+v_{t}+\tau_{it}+u_{t}, \end{cases}$$

where ORI_{it}^{T} represents post-topdressing optical reflectance information on grid *i* in field-year *t* in late winter, which is a function of the optical reflectance information taken prior to topdressing and hence represents the level of nitrogen available to the plants at that time. However, the process is not complete because we are interested in a function that relates total nitrogen level (level of N available plus the level of N required for plants to yield at the plateau) to optical reflectance information. This requires the addition of the variable representing the level of N required back into equation (6). Completion of this step provides a function that relates the total level of nitrogen to optical reflectance information, or more formally

$$(7) \qquad ORI_{it}^{T}\left(N_{it}^{T}\right)=\begin{cases} ORI_{it}^{S}(N_{it}^{P})+\beta N_{it}^{R}, \\ \quad \text{if } ORI_{it}^{S}+\beta N_{it}^{R}-\eta_{it}\le E\left(ORI_{t}^{NRS}\right)+v_{t}+\tau_{it}+u_{t}, \\ E\left(ORI_{t}^{NRS}\right)+v_{t}+\tau_{it}+u_{t}+\eta_{it}, \\ \quad \text{if } ORI_{it}^{S}+\beta N_{it}^{R}-\eta_{it}> E\left(ORI_{t}^{NRS}\right)+v_{t}+\tau_{it}+u_{t}. \end{cases}$$

The final step requires substituting equation (7) into the original yield function described by equation (2) to obtain the desired LRSP function. This LRSP function is expressed as

$$(8) \qquad y_{it}=\begin{cases} a+bORI_{it}^{S}(N_{it}^{P})+b\beta N_{it}^{R}+\theta_{it},\ \text{if} \\ \quad a+bORI_{it}^{S}(N_{it}^{P})+b(\beta N_{it}^{R}-\eta_{it})\le a+bE\left(ORI_{t}^{NRS}\right)+bv_{t}+b\tau_{it}+bu_{t}, \\ a+bE\left(ORI_{t}^{NRS}\right)+bv_{t}+b\tau_{it}+bu_{t}+b\eta_{it}+\theta_{it},\ \text{if} \\ \quad a+bORI_{it}^{S}(N_{it}^{P})+b(\beta N_{it}^{R}-\eta_{it})\le a+bE\left(ORI_{t}^{NRS}\right)+bv_{t}+b\tau_{it}+bu_{t}. \end{cases}$$

Equation (8) represents the production function that will be used to generate yields, levels of nitrogen, and expected profit estimates for the alternative nitrogen fertilizer management systems that are being compared in this study.

Data and Estimation

Parameter estimates for equation (2) and equation (4) (assuming that τ_{it} is equal to zero) are estimated using data gathered from eight on-farm winter wheat experiments conducted at six locations located on or near research stations throughout Oklahoma from 1998-2003. The data set includes observations for wheat yield, optical reflectance information, and level of preplant nitrogen for a total of 624 site years useful for analysis. Locations for each of the experiments included Haskell (Exp. #801), Hennessey (Hennessey AA), Lahoma (Exp. #508), Perkins (Exp. N x P, and Exp. N x S), Stillwater (Exp. #222 and Efaw AA), and Tipton (Exp. N x S). The N rate by spacing (N x S) experiment at Perkins included only nitrogen and was initiated in 1996; however, only data for 1998 was used in this study. The N rate by P rate (N x P) experiment at Perkins included both nitrogen and phosphorus from 1998 to 2003. The Hennessey AA and Efaw AA experiments were designed as anhydrous ammonia fertility experiments. Data were collected at Haskell (Exp. #801) from 1999 to 2002, and at Stillwater (Exp. #222 and Efaw AA) for five years from 1999-2003. At Hennessey data were collected for 2000 and 2002. At Lahoma, data were collected in 1999, 2000, 2002, and 2003, and at Tipton data were only collected in 1998.

Soil types for each locations are: Haskell, Taloka silt loam (fine, mixed, thermic Mollic Albaqualfs); Hennessey, Shellabarger sandy loam(fine-loamy, mixed, thermic Udic Paleustolls); Lahoma, Grant silt loam (fine-silty, mixed, thermic Udic Argiustolls; Perkins, Teller sandy loam (fine-loamy, mixed, thermic Udic Argiustolls); Stillwater, Kirkland silt loam(fine, mixed, thermic Udertic Paleustoll); Stillwater-Efaw, Norge sitl loam (fine-silty, mixed, thermic Udic Paleustoll); and Tipton, Tipton silt loam (fine-loamy, mixed, thermic Pachic Argiustolls).

In each of the experiments, winter wheat was planted at a 70 pounds per acre seeding rate using a 7.5 inch row spacing, excluding the S*N experiment at Perkins where spacing ranged from six inches to ten inches. It was not reported in the paper how spacing affected yields for the Tipton (N x S) and Perkins (N x S) experiments. In addition, the paper did not provide information regarding how phosphorus affected yield for the Perkins (N x P) experiment. All field experiments where sensor and yield data were collected employed randomized complete block designs with 3 to 4 replications (depending on site).

Nitrogen rich strips were placed in each experimental plot prior to planting wheat in late September or early October. All optical reflectance readings were taken during Feekes growth stages 4 (leaf sheaths beginning to lengthen) and 5 (pseudo-stem, formed by sheaths of leaves strongly erect) (Large). Sensor measurements were taken from treatments with varying levels of N nutrition within each replication. NDVI spectral reflectance was measured using a handheld sensor that included two upward and downward directed photodiode sensors that received light through cosine corrected Teflon windows fitted with red (671 ± 6 nm) and near-infrared (NIR) (780 ± 6 nm) interference filters developed by (Stone et al.).

Consistent with different planting times and growing conditions, spectral reflectance readings were from wheat were collected from a 43.03 square-feet (4.0 square-meters) area under natural lighting either in January, February, March, April, or May. Plots were harvested with a self-propelled combine and grain yield was determined from the same 43.03 square-feet area where spectral reflectance data were collected. Additional information regarding the experiments can be found in Mullen et al. (2003).

Parameters in equation (2) are estimated using a linear mixed effects model (PROC MIXED in SAS). The presence of year random effects is tested using a likelihood ratio test.

The LRSP described in equation (3) is estimated using a nonlinear mixed effects model (PROC NLMIXED in SAS). This is required because the randomness associated with year random effects (i.e., v_t in equation 3) enters the response function non-linearly (Tembo, Brorsen, and Epplin). The model illustrated in equation 3 is sufficiently designed to allow for the presence of plot-level plateau spatial randomness, which is denoted by τ_{it} in equation 3. A lack of data prohibits direct estimation of the plot-level plateau randomness; however, an alternative model will be simulated that allows for spatial random effects. In the alternative model, a percentage of the random variation contained in the general error component (η_{it} in equation 3) is subtracted and given to the plateau spatial error component (τ_{it} in equation 3). The two models are compared to determine the effects of spatial variability on profitability.

Levels of Nitrogen

Equation (8) is used to compute the application levels of nitrogen fertilizer for each of several systems, including (System 1) an all-before-planting system based on an economically optimal level of nitrogen computed using the analytical approach provided by Tembo, Brorsen, and Epplin; (System 2) the portable plant-based precision system that gives a uniform, whole field recommendation; (System 3) the on-the-go variable rate precision system; (System 4) the plant-based NFOA system developed by Raun et al. (2004); (System 5) an all-before-planting system that represents the agricultural extension recommendation of 80 pounds per acre preplant system (i.e., two pounds of N per acre based on a 40 bushel per acre yield goal), and (System 6) an all-before-planting system that represents the average of what producers were actually found to be applying in the southern Plains (i.e., 63 pounds per acre) in a survey conducted in 2000 (Hossain et al., 2004). In addition, a check system (System 7) that has no nitrogen applied is included. Optimal application levels of nitrogen for systems 1, 2, and 3 are derived using the response function described by equation (3).

Optimal level of preplant nitrogen

The approach used by Tembo, Brorsen, and Epplin is used to obtain the optimal level of nitrogen to apply in the fall prior to planting wheat, which is the traditional system for applying nitrogen fertilizer in the southern Great Plains region of the United States. This process requires several steps. To account for all nitrogen requirements applied in the fall prior to planting, we need to rewrite equation (3) as

$$(9) \qquad ORI_{it} = \begin{cases} \alpha + \beta N_{it}^{P} + u_t + \eta_{it}, \\ \quad \text{if } N_{it}^{P} \le N_{it}^{NRS} \Rightarrow ORI_t^{NRS} > \alpha + \beta N_{it}^{P}, \\ E\left(ORI_t^{NRS}\right) + v_t + \tau_{it} + u_t + \eta_{it}, \\ \quad \text{if } N_{it}^{P} > N_t^{NRS} \Rightarrow ORI_t^{NRS} \le \alpha + \beta N_{it}^{NRS}, \end{cases}$$

where N_{it}^{P} is the level of nitrogen applied to grid i in field-year t in the fall prior to planting (assumed to be the total level of nitrogen in this case), the symbols α and β represent intercept and slope coefficients to be estimated, N_t^{NRS} is the plateau level of nitrogen. Note, after the sample reflectance readings from the NRS have been taken with the sensors, and an average computed, then N_t^{NRS} will be known.

The next step is to substitute equation (9) into the yield function given by equation (2), which gives the following conditional wheat yield response to nitrogen function

$$(10) \qquad y_{it} = \begin{cases} a + b\alpha + b\beta N_{it}^{P} + bu_t + b\eta_{it} + \theta_{it}, \\ \quad \text{if } N_{it}^{P} \le N_t^{NRS} \Rightarrow \text{(below plateau)} \\ a + bE\left(ORI_t^{NRS}\right) + bv_t + b\tau_{it} + bu_t + b\eta_{it} + \theta_{it}, \\ \quad \text{if } N_{it}^{P} > N_t^{NRS} \Rightarrow \text{(on plateau).} \end{cases}$$

Using the yield function described in equation (10) and following the analytical approach of Tembo, Brorsen, and Epplin, the optimal level of nitrogen to apply as a preplant in the fall $\left(N_{it}^{P*}\right)$ is

(11) $$N_{it}^{P*} = \frac{1}{b\beta}\left[F^{-1}\left(1 - \frac{r}{pb\beta}\right) - a - \alpha b\right],$$

where $F^{-1}(\cdot)$ is the inverse of the normal cumulative distribution function. To complete the computation, the market price for preplant nitrogen (r) and the expected price of wheat (p) are required, and the parameters, $a, b, \alpha,$ and β can be replaced by their statistical estimates. Because N_{it}^{P*} cannot be negative and $b,\ \beta \geq 0,$ equation (11) is valid only if

(12) $$\frac{F^{-1}\left(1 - \frac{r}{pb\beta}\right) - a}{b\beta} \geq \frac{\alpha}{\beta}.$$

An optimal solution can be determined analytically only if a unique inverse exists for the prescribed cumulative distribution function. First, we define

(13) $$y_{it}^{NRS} = a + bORI_t^{NRS},$$

where y_{it}^{NRS} represents the yield that is generated on the NRS, which is expected to be the yield on the plateau. Next, if we assume that $ORI_t^{NRS} \sim N(E(ORI_t^{NRS}), \sigma_v^2 + \sigma_\tau^2)$ then $y_{it}^{NRS} \sim N\left(a + bE(ORI_t^{NRS}), b^2\sigma_v^2 + b^2\sigma_\tau^2\right)$. Furthermore, if we assume the maximum optical reflectance reading is related to the level of nitrogen necessary to achieve the plateau yield

(14) $$ORI_t^{NRS} = \alpha + \beta N_t^{NRS},$$

then $N_t^{NRS} \sim N\left(\frac{E(ORI_t^{NRS}) - \alpha}{\beta}, \frac{\sigma_v^2 + \sigma_\tau^2}{\beta^2}\right)$.

The next step is to obtain an approximate of the inverse in equation (11). However, first convert $E(y_t^{NRS} \mid N_t^{NRS} = N_{it}^{P})$ into a standard normal variant defined as Z_δ, or more formally as

$$Z_\delta = \frac{a + b\alpha + b\beta N_{it}^{P*} - (a + bE(ORI_t^{NRS}))}{b\sigma_v + b\sigma_\tau}, \tag{15}$$

where $\delta = 1 - F(a + b\alpha + b\beta N_{it}^{P*}) = \dfrac{r}{pb\beta}$ is the observed probability in the right-hand tail of the $N(0,1)$ distribution and $F(a + b\alpha + b\beta N_{it}^{P*})$ which is the cdf of y_t^{NRS} evaluated at $a + b\alpha + b\beta N_{it}^{P*}$. The optimal level of preplant nitrogen to apply in the fall prior to planting is obtained by solving (15) for N_{it}^{P*}, which gives

$$N_{it}^{P*} = \frac{Z_\delta\left(\sigma_v + \sigma_\tau\right) - \alpha + E(ORI_t^{NRS})}{\beta}. \tag{16}$$

As an example, assume *r* = \$0.15 and *p* = \$3.00. Further, assume that the slope estimate for *b* in equation 2 is equal to 7.5793 and that the slope estimate for β in equation 3 is equal to 0.031. Using these values we can see that $\delta = \dfrac{.15}{(3.00 \times 7.5793 \times 0.031)} = 0.2128$. Because we are interested in a one-tailed test, we must subtract the $\delta = 0.2128$ from 0.5, which is equal to 0.2872. Unfortunately, the normal distribution function cannot be expressed in an easily invertible form; however, entering the one-tailed version δ into the NORMINV function in Excel provides us with an approximation of the unique inverse desired. After Z_δ is known, solving equation (16) is straightforward. Assuming that $Z_\delta = 0.79$, and that statistical estimates for $\sigma_v = 0.38, \sigma_\tau = 0.20, \alpha = 5.99, \beta = 0.031,$ and $E(ORI_t^{NRS}) = 7.1947$, then the optimal level of preplant nitrogen in equation (16) is equal to 58.52 pounds per acre.

Optimal level of nitrogen for the portable handheld precision system

In this section of the paper we derive a function that describes the uniform level of nitrogen fertilizer that is necessary for plants to produce at the yield plateau. This system makes use of a portable, sensor that obtains average reflectance readings on both the NRS and on individual nonNRS grids throughout the field. After sensing and the optical reflectance information is known, including information from the NRS, the plateau is no longer considered stochastic (assuming as we have that τ_{it} is equal to zero), and therefore optimal levels of nitrogen can be determined using the standard formula for a deterministic plateau. Intuitively, the optimal level of topdress nitrogen required in the late winter is the amount required to achieve the plateau yield.

Under this system, the level of nitrogen required to reach the plateau yield can be thought of as the difference between the level of nitrogen in the NRS and the level of nitrogen applied prior to planting, or

$$N_{it}^{R} = N_{t}^{NRS} - N_{it}^{P}, \tag{17}$$

where the level of nitrogen available in the NRS can be solved using equation (14) and written as

$$N_{t}^{NRS} = \frac{ORI_{t}^{NRS} - \alpha}{\beta}, \tag{18}$$

and the level of preplant nitrogen can be solved using equation (9) and written as

$$N_{it}^{P} = \frac{ORI_{it}^{S} - \alpha}{\beta}. \tag{19}$$

Subtracting equation (19) from equation (18) gives the optimal level of additional nitrogen required in the spring using the portable sensing system, and is written as

$$N_{it}^{R} = \frac{ORI_{t}^{NRS} - ORI_{it}^{S}}{\beta}. \tag{20}$$

Since the optical reflectance information given by the sensor measures the value of the plateau, the plateau is no longer thought of as stochastic and the deterministic solution is appropriate.

Optimal level of nitrogen for variable rate application with perfect information

Determining the optimal level of nitrogen to apply on each grid for each field-year for the variable rate system is an important and challenging task. One of the primary assumptions regarding the on-the-go system is that the cause of any low optical reflectance reading, whether it is from low nitrogen or from another physical factor such as poor soil or a poor stand, can be perfectly identified. This is not achievable in practice at this time, but the NFOA is continually being tweaked based on ongoing research (e.g., Raun et al., 2005).

If all information about plant nitrogen need is known with certainty (i.e., an unachievable, perfect information scenario) then the level of nitrogen required in the spring is thought of as the difference between the plateau yield and the yield at the intercept adjusted by the marginal product of nitrogen. This solution is expressed more formally as

$$N_{it}^{T} = \frac{\text{Plateau Yield - Yield with no N}}{\text{Adjusted Marginal Product of N}}$$

(21)
$$= \frac{a + bORI_{t}^{NRS} + b\tau_{it} - (a + bORI_{it}^{S}) + \eta_{it}}{b\beta}$$

$$= \frac{ORI_{t}^{NRS} - ORI_{it}^{S} + \tau_{it} + \eta_{it}}{\beta}.$$

This result can also be derived directly from the condition outlined in equation (8), and is considered optimal under a situation where perfect information about the random processes is known.

The above result does not assume away uncertainty associated with unfavorable weather that may take place between the time of topdressing and the time of harvesting. However, unknowns associated with soil mineralization, technological problems with the sensors or computers on the system, and other potential problems such as weed and insect problems present at the time of sensing are assumed away. It is unreasonable to assume certainty concerning the random processes, and therefore the results obtained from equation (21) are unachievable in practice. However, the result does place a maximum threshold value on the on-the-go system, barring unusual weather events between topdressing and harvest. Such a value would be useful to producers deciding whether or not to adopt the system.

Optimizing nitrogen using the nitrogen fertilizer optimization algorithm (NFOA)

The nitrogen fertilizer optimization algorithm (NFOA) developed by Raun et al. (2002) is used to determine how much nitrogen is needed in late winter during the topdress application season. Following their work, the optimal level of nitrogen to apply using the plant-based precision technology, N_{it}^{NFOA}, is defined as

(22) $$N_{it}^{NFOA} = \frac{(YPN_{it} - YP0_{it})}{\lambda},$$

where λ is a constant that represents the level of nitrogen use efficiency (NUE) that is expected to be gained from applying only the level of nitrogen that is needed by the plants in the spring with none of it going unused as opposed to applying nitrogen prior to planting in the fall (Raun et al., 2002 used an NUE of 0.70 in the NFOA), $YP0_{it}$ is yield response to optical reflectance information and gives an estimate at the time of sensing for wheat yield potential when no additional nitrogen is added to the plants. Mathematically, $YP0_{it}$ has the following exponential functional form

(23) $$YP0_{it} = c_0 \exp(ORI_{it} c_1),$$

where c_0 and c_1 are the intercept and slope parameters.[4] The symbol ORI_{it} denotes the optical reflectance information taken in the spring on grid *i* in field-year *t*, and the symbol YPN_{it} in equation (22) is defined as the yield potential when additional nitrogen fertilizer is applied in the spring at a level necessary to bring plant growth to the maximum potential. More formally, it is written as

$$(24) \qquad YPN = \begin{cases} \min\left(\left(RI \times YP0\right), YP0\right), & \text{if } \min((RI \times YP0, YP0) < y^{\max}), \\ y^{\max}, & \text{otherwise,} \end{cases}$$

where RI is a response index that is calculated as the ratio of optical sensor readings taken from the NRS to optical senor readings taken from an adjacent nonNRS strip of the field that represents growing wheat when nitrogen is limiting, or defined mathematically as

$$(25) \qquad RI = \frac{ORI \text{ from NRS}}{ORI \text{ from farmer practice}} = \frac{ORI_t^{NRS}}{E(ORI_{it}^{S})} = \frac{ORI_t^{NRS}}{\alpha + u_t};$$

and according to Raun et al. (2002), $y^{\max}$ is the maximum yield that is determined by the farmer, or previously defined as a biological maximum for the specific crop, and grown within a specific region, and under defined management practices (e.g., dryland winter wheat produced in central Oklahoma would be 104 bushels per acre. Substituting equation (25) into equation (24) gives

(26)

$$YPN_{it} = \begin{cases} \min\left\{\left(\left(\frac{ORI_t^{NRS}}{\alpha + u}\right) \times 0.359 \exp(ORI_{it} \times 324.4)\right), 0.359 \exp(ORI_{it} \times 324.4)\right\}, \\ \qquad \text{if } \min((RI \times YP0, YP0) < y^{\max}), \\ y^{\max}, \qquad \text{otherwise.} \end{cases}$$

[4] Note that parameter estimates have been shifted one standard deviation out to the left in an effort by Raun et al. (2004) to describe a yield frontier. Current estimates of c_0 equal to 0.359 and c_1 equal to 324.4 describe the frontier.

The NFOA is very similar to equation (8). The main differences are that *YP0* and *YPN* are based on an exponential function, the plateau level is reduced when YP0 is low, and the value of λ is more than double. In equation (22), λ corresponds to the marginal product of nitrogen $(b\beta \text{ in equation } 8)$ which can be estimated from the data.

Simulation of Expected Net Returns

Equation (8) is simulated in two separate models to determine the expected net return from each of the alternative systems. The first model assumes that no plateau spatial variability exists (i.e., τ_{it} equal to zero), and the second model allows for plateau spatial variability by subtracting variance from the general error component η_{it} and allocating it to the spatial error component τ_{it}. Although this method is crude, it does provide us with an idea of how sensitive net returns are to the presence of spatial variability within the field-year.

Net returns on 250 sample grids within each of 250 sample field-years were simulated using the following steps. First, sample values for the error components in equation (8) are simulated using a random number generator. Errors are assumed normally distributed with mean zero and estimated variances provided from the regression procedures used to estimate equations (2) and (3). Intercepts, slopes, and expected value of optical reflectance information at the plateau are also provided from these regression procedures. In addition to the error components, values of ORI_{it}^{S} and ORI_{t}^{NRS} are simulated for each grid and field-year of the sample. Moreover, application costs, and prices for 82% NH_3 and 28% UAN are included. A zero level of N is assumed when expected net returns from application are negative on average over the entire field.

The process for calculating sample values for the optical reflectance information from the nitrogen rich strip is

(25) $$ORI_t^{NRS} = E(ORI_t^{NRS}) + v_t + u_t,$$

and the process for calculating sample values for the optical reflectance information on an individual grid and field-year is described by equation (3). Note, τ_{it} has not been included in equation (25). Because the NRS is assumed to cover a sufficiently large area of the field, the plateau spatial variability is assumed to average to zero given that a substantial number of readings are taken from it.

After sample values for the errors and the optical reflectance information are simulated for each grid and field-year, then the formulas for the optimal levels of nitrogen (i.e., equations (16), (20), (21), and (22)) for each of the alternative systems can be used to generate samples of optimal nitrogen rates for each grid in each field-year. The yield response function defined in equation (8) is then used to calculate sample values for wheat yield for each system, grid, and field-year in the sample. Net returns are then calculated as the difference between wheat revenue and cost of nitrogen and nitrogen application expenses for each grid in the field-year. The Monte Carlo integration is then completed by averaging net returns across the sample of field-years for each system. The differences in the average profits between the precision systems and the conventional systems provide an estimate for the value of the plant-based precision systems (e.g., the difference between the expected profit from the perfect information system and the expected profit from a uniform application of 80 pounds of nitrogen per acre provides an approximation for how much a winter wheat producer could pay for a variable rate application system).

For each system, a long run average price of $3 per bushel was used for the expected price of wheat grain (USDA), and market prices of $0.15 and $0.25 per pound are used for anhydrous ammonia and 28% UAN, respectively (Oklahoma Department of Agriculture).

Gains in efficiency

It is believed that some gain in efficiency will be obtained when the plant-based sensing technology is used instead of the traditional preplant systems. However, it is not assumed as is by Raun et al. (2002) that a seventy percent gain (i.e., $\lambda = 0.70$) is achievable. For this study, we are assigning a twenty percent gain in efficiency to the marginal product of nitrogen, such that the slope parameter β is multiplied by an efficiency parameter ψ that is set equal to 1.2. Sensitivity analysis on the efficiency parameter and its effect on expected profits are presented later in the paper. The efficiency parameter is assigned to equation (8) as well as for the optimal levels of nitrogen in equations (20), (21), and (22).

Results and Discussion

Regression estimates of equation (2) are presented in Table 1. Rejection of the null hypothesis that no random effects exist were based on the likelihood ratio test. Each of the parameters is significant at the .05 level. Estimates of equation (3) are presented in Table 2. The marginal product of nitrogen $\left(b\beta = (7.5793 \times .031) = .2349\right)$ was smaller than that found by Tembo, Brorsen, and Epplin (0.3075), and is considerably smaller than the 0.7 assumed in the NFOA. This result suggests that approximately 4.3 pounds of nitrogen should be applied to gain an additional bushel of wheat rather than the 3.25 pounds suggested by the Tembo, Brorsen and Epplin model.

Expected yield, optimal levels of nitrogen, and expected profits for each system and without spatial variability are reported in Table 3. As expected, the perfect information variable rate system had the largest expected profit of approximately $114 per acre. Net return to nitrogen application for this system was approximately $3 greater than the average net return for the Tembo, Brosen and Epplin system. A better comparison might be made between the perfect information variable rate system and the state recommendation of applying 80-pounds per acre prior to planting in the fall. The net return to nitrogen and nitrogen application for the state recommendation system was approximately $107 per acre, which is approximately $7 per acre lower than the perfect variable rate system.

The portable system had an average net return of approximately $109 per acre, which was approximately $2 per acre more profitable than the state extension recommendation of applying 80-pounds per acre. The portable system averaged $2 per acre less than that of the Tembo, Brorsen and Epplin system. In this case, the cost saving of the precision technology could not outweigh the gains in additional yield predicted with the Tembo, Brorsen and Epplin system. The portable system used 42% less N on average than the Tembo, Brorsen and Epplin preplant system, but the cost of N for the precision system was only $0.28 less than the cost of N for the Tembo, Brorsen and Epplin preplant system. However, the additional yield obtained with the Tembo, Brorsen and Epplin preplant system relative to the portable system results from using a larger average uniform level of nitrogen (i.e., 57.35 pounds versus 33.3 pounds). Using the average from a set of sensor readings taken from the farmer's field to approximate the uniform level of nitrogen needed to achieve the yield plateau, some areas of the field will still receive less nitrogen than actually needed, keeping some yield in the field from reaching its potential plateau.

Another interesting comparison is the approximate $8 difference in net return between the perfect information variable rate system and the NFOA system. This could be viewed as indication that further improvements could be made to the NFOA. However, it is unlikely that the NFOA could be improved to the point that it performs as well as the perfect information system described in this paper. Note that the marginal product of nitrogen for the NFOA is too high, and adjusting it down to the size of that found using the data, the NFOA outcome would be similar to that given by the profits for the 80 pounds per acre system. Also, note that the production function assumed in the simulation does not exactly match the production function assumed by the NFOA so the NFOA could do relatively better in real-world applications.

Plateau spatial variability is expected to exist within each of the field years resulting from random weather within a field and varying soil type. Table 4 reports average yield, nitrogen, and expected profit for each of the alternative systems assuming that plateau spatial variability is present. In this instance, 50 percent of the variability estimated in the general error component (η_{it} in equation 3) has been subtracted and added to the plateau spatial error component (τ_{it} in equation 3). The presence of plateau spatial variability does not have a large effect on the yields, levels of nitrogen, and expected profits.

Sensitivity values for independent relative changes in the exogenous variables are reported in Table 5. Note that the sensitivity analysis has been conducted on results that were calculated assuming that plateau spatial variability is equal to 25 percent of the total variability estimated for the general error component (η_{it} in equation 3). Sensitivity results indicate that as the marginal product of nitrogen increases (implying that the total level of nitrogen applied decreases) the value of the perfect variable rate system increases relative to the value of the state recommended system that applies 80-pounds of N per acre. In addition, as the marginal product of nitrogen increases, the value of the NFOA system becomes more profitable relative to all other systems. That is, a situation when less nitrogen is needed to obtain an additional unit of yield, the NFOA system becomes the preferred system.

As expected, the value of the perfect variable rate system increases relative to the state recommended system as the price of NH_3 increases relative to the price of UAN. When the price of NH_3 is increased to the point where it is equal to the price of UAN, the value of the variable rate system increased to approximately $11 per acre over that of the state recommended system.

The opposite relationship exists when the price of UAN increases relative to the price of NH_3. If the price of UAN increases to $0.50 per pound, holding the price of NH_3 constant at $0.15 per pound, then the value of the state recommended system is approximately $1 per acre more profitable than the perfect variable rate system. In this situation, a typical producer would not be interested in adopting the plant-based precision system.

As the nitrogen use efficiency adjustment variable is increased, the value of the perfect variable rate system increases over the value of the state recommended system. Note that when the NUE is adjusted upwards from 1.20 to 1.50, the average return of the portable system increases from approximately $106 per acre to $107.50, which is a value larger than the average net return for the state recommended system. Also notice that the estimate for the NUE adjustment factor positively affects the expected profitability of the NFOA system.

As the custom application cost for NH_3 increases relative to the custom application rates for the alternative systems, then the value for the preplant systems is reduced. Similarly, increases cost of custom applying uniform levels of UAN relative to the alternative systems would reduce the value of the portable system relative to the alternative systems. Likewise, if custom variable rate application of UAN increases relative to the custom rates for the alternative systems, then the value of the perfect variable rate system will decline relative to the alternative systems.

Summary and Conclusions

Panel data covering six years and seven locations in Oklahoma were used to estimate wheat yield response to nitrogen conditional on optical reflectance information taken from growing wheat plants in the spring. A linear response stochastic plateau function was assumed to best fit the data. Yield and net return were simulated on a large sample of independent grids and field-years. Under the assumption that the random processes are known perfectly, a maximum, unachievable value for the plant-based precision technology, over and above that of the conventional system, was found to be approximately $7 per acre. The portable precision system was found to be approximately breakeven with the nitrogen application system that represents the 80-pound per-acre state extension recommendation.

Previous economic research has shown that variable rate application technologies that are based on sampling the soil have not been profitable when all economic costs associated with the technology are included in the analysis. A perfect information plant-based precision application technology had a value approximately $7 per acre above that of the conventional preplant system. The implications of this finding would be more promising if the relative prices of nitrogenous fertilizers increase. Currently, the plant-based precision sensing technology is available on a commercial basis, and is being promoted to increase net returns to nitrogen fertilization. However, the findings of this study explain why adoption has been slow. These findings also indicate that the optical sensing technology, including the nitrogen fertilizer optimization algorithm (NFOA), in many cases, does not apply enough nitrogen fertilizer, and therefore could be improved.

In addition to the lack of substantial increases in producer net return, other factors may impede the adoption of this technology such as timing. This specific technology applied all nitrogen in the spring as a topdress. However, during this time adverse weather conditions can limit application to only a few days, and possibly prevent application altogether.

Limitations and Further Research

A limitation to the widespread adoption of this technology in the southern Plains regards the large number of acres that are grazed with stocker cattle in the winter months. In the case of grazing, nitrogen rich strips would have to be fenced off from livestock. Plus, the technology requires a 14 day re-growth period before sensors measurements can be taken. If livestock are not removed at the appropriate time, the window of opportunity for topdressing can narrow or become nonexistent. This too, increases the risk of not being able to apply the necessary level of nitrogen when the plants use it the most efficiently. As a result of these impediments, and the fact that the technology is only marginally profitable may explain the limits of its adoption.

Further research oriented at evaluating the possible economic benefits from using the site-specific system for nitrogen application and application of additional chemicals such as insecticides and herbicides needs to be investigated.

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Table 1. Regression of Wheat Yield Response on Optical Reflectance Information

Statistic	Symbol	Estimates[a]
Intercept	a	-12.25 (3.52)
Optical reflectance	b	7.57 (0.42)
Year random effect	σ^2_{ω}	9.65 (2.58)
Error variance	$\sigma^2_{\vartheta^*}$	103.79 (5.57)

[a] Asymptotic standard errors are in parentheses.
Note, that the parameter estimates for equation (2) were estimated using PROC MIXED in SAS.

Table 1. Stochastic Linear Plateau Model of Optical Reflectance Information as a Function of Nitrogen

Statistic	Symbol	Estimates[a]
Intercept	α	5.99 (.1609)
Level of nitrogen	β	.031 (.3458)
Expected plateau ORI	$E\left(ORI_t^{NRS}\right)$	7.19 (.1958)
Nitrogen at expected plateau	N_t^{NRS} .	58.52 (.1958)
Variance of plateau yield	σ^2_{v}	0.39 (.1378)
Variance of year random effect	σ^2_{u}	0.55 (.1614)
Variance of error term	σ^2_{η}	0.66 (.0385)

[a] Asymptotic standard errors are in parentheses.
Note, the parameter estimates for equation (3) were estimated using NLMIXED procedure in SAS.

Table 2. Average Yield, Nitrogen, and Expected Profits from Alternative Nitrogen Management Systems without Plateau Spatial Variability

	System						
Estimate	0/0[a]	80/0[b]	63/0[c]	0/Port[d]	TBE/0[e]	0/GS[f]	0/NFOA[g]
Average Yield	32.54	41.61	41.23	40.11	41.79	42.11	38.29
Average Nitrogen (lbs)	0.00	80	63	33.38	57.53	31.77	17.73
Average profit ($)	97.63	106.72	108.13	108.88	110.64	113.79	106.28

[a] the check system with no nitrogen added.
[b] the system that represents the state extension recommendation of 80 pounds per acre, or 2 pounds of nitrogen for each bushel of yield goal.
[c] the system that represents the average level of nitrogen applied in the state of Oklahoma that was reported by producers via a survey conducted in 2000.
[d] the system that represents the portable precision system where no nitrogen was applied prior to planting.
[e] the system that represents the analytical approach developed by Tembo, Brorsen, and Epplin to determine the optimal level of nitrogen to apply in the fall prior to planting.
[f] the system that represent the plant-based variable rate precision system that assumes perfect knowledge about the random processes.
[g] the system that represents the NFOA developed by Raun et al. (2004).

Table 4. Average Yield, Nitrogen, and Expected Profits from Alternative Nitrogen Management Systems with Plateau Spatial Variability

	System						
Estimate	0/0[a]	80/0[b]	63/0[c]	0/Port[d]	TBE/0[e]	0/GS[f]	0/NFOA[g]
Average Yield	32.54	41.61	41.23	38.87	41.57	42.10	39.95
Average Nitrogen (lbs)	0.00	80	63	33.27	57.35	32.40	17.65
Average profit ($)	97.63	106.72	108.13	105.01	109.15	113.92	105.25

Note that plateau spatial variability of 50% is assumed to come from the general error component, η_{it}.
[a] the check system with no nitrogen added.
[b] the system that represents the state extension recommendation of 80 pounds per acre, or 2 pounds of nitrogen for each bushel of yield goal.
[c] the system that represents the average level of nitrogen applied in the state of Oklahoma that was reported by producers via a survey conducted in 2004.
[d] the system that represents the portable precision system where no nitrogen was applied prior to planting.
[e] the system that represents the analytical approach developed by Tembo, Brorsen, and Epplin to determine the optimal level of nitrogen to apply in the fall prior to planting.
[f] the system that represent the plant-based variable rate precision system that assumes perfect knowledge about the random processes.
[g] the system that represents the NFOA developed by Raun et al. (2004).

Table 3. Sensitivity Values for Independent Relative Changes in MPN, Prices of Nitrogen, NUE Adjustment, and Custom Application Costs

		System						
Parameter	Coefficient/ Price	0/0[a]	80/0[b]	63/0[c]	0/Port[d]	TBE/0[e]	0/GS[f]	0/NFOA[g]
Marginal Product of Nitrogen	0.114	97.63	100.48	99.68	97.00	101.99	105.43	99.75
	0.189	--------	105.74	106.30	103.81	107.87	111.96	103.15
	0.234*	--------	106.72	108.13	105.80	109.86	113.88	105.97
	0.303	--------	106.99	109.31	107.67	111.86	115.69	110.34
	0.531	--------	107.00	109.55	110.44	115.11	118.35	125.13
	0.682	--------	107.00	109.55	111.26	116.15	119.14	135.01
Price of NH3 ($/lb)	0.15*	97.63	106.72	108.13	105.80	109.86	113.88	105.97
	0.20	--------	102.72	104.98	--------	107.14	--------	--------
	0.25	--------	98.72	101.83	--------	104.66	--------	--------
	0.30	--------	94.72	98.68	--------	102.41	--------	--------
Price of UAN ($/lb)	0.20	--------	--------	--------	107.46	--------	115.49	106.85
	0.25*	97.63	106.72	108.13	105.80	109.86	113.8	105.97
	0.30	--------	--------	--------	104.14	--------	112.28	105.09
	0.40	--------	--------	--------	100.83	--------	109.10	103.38
	0.50	--------	--------	--------	97.55	--------	105.95	101.78
Nitrogen Use Efficiency Adjustment	1.20*	97.63	106.72	108.13	105.80	109.86	113.88	105.97
	1.50	--------	--------	--------	107.46	--------	115.55	109.73
	2.00	--------	--------	--------	109.43	--------	117.16	116.08
Custom, Uniform NH_3 Application Rates	6.11*	97.63	106.72	108.13	105.80	109.86	113.88	105.97
	7.00	--------	105.84	107.25	--------	108.98	--------	--------
	8.00	--------	104.84	106.25	--------	107.98	--------	--------
Custom, Uniform UAN Application Cost ($/acre)	3.74*	97.63	106.72	108.13	105.80	109.86	113.88	105.97
	4.00	--------	--------	--------	105.58	--------	--------	--------
	5.00	--------	--------	--------	104.75	--------	--------	--------
Custom, Variable Rate Application Cost ($/acre)	5.01*	97.63	106.72	108.13	105.80	109.86	113.88	105.97
	6.00	--------	--------	--------	--------	--------	113.02	105.17
	7.00	--------	--------	--------	--------	--------	112.16	104.42
	8.00	--------	--------	--------	--------	--------	111.33	103.74

Note, sensitivity analysis has been conducted on the results that were calculated assuming that plateau spatial variability τ_{it} is equal to 25% of the total variability estimated for the general error component η_{it}.

[a] the check system with no nitrogen added.

[b] the system that represents the state extension recommendation of 80 pounds per acre, or 2 pounds of nitrogen for each bushel of yield goal.

[c] the system that represents the average level of nitrogen applied in the state of Oklahoma that was reported by producers via a survey conducted in 2004.

[d] the system that represents the portable precision system where no nitrogen was applied prior to planting.

[e] the system that represents the analytical approach developed by Tembo, Brorsen, and Epplin to determine the optimal level of nitrogen to apply in the fall prior to planting.

[*] the baseline values for parameters.

f the system that represent the plant-based variable rate precision system that assumes perfect knowledge about the random processes.

g the system that represents the NFOA developed by Raun et al. (2004).