On the Coexistence of Spot and Contract Markets: A Delivery Requirement Explanation

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ABSTRACT

A model is presented in which spot and contract market exchange co-exist. A contract consists of a delivery requirement between an upstream and a downstream party. Contract formation determines to a certain extent the probability distribution of the spot market price. This contract formation externality entails the removal of high reservation price buyers and various sellers from the spot market. The first effect decreases the expected spot market price when the number of contracts is small, whereas the decrease in the number of sellers and additional residual contract demand increase the expected spot market price beyond a certain number of contracts. It implies an endogenous upper bound on the number of contracts. Contract prices are positively related to the number of contracts. Finally, additional contracts reduce the variance of the spot market price when the number of contracts is sufficiently large.

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1 Introduction

Many industries consist of enterprises using different forms of exchange. First, many markets are characterized by the co-existence of spot and contract markets. Menard and Klein (2004, p752) report that ‘In France, over 80% of the growers in the poultry industry operated under contracts in 1994. In the U.S. pork industry, about 72% of total hogs were sold through marketing contracts in 2001.’ This phenomenon is not limited to agricultural markets. Electricity markets are an example (Newbery, 1998). Second, partial vertical integration regarding internal shipments to manufacturing establishments is a widespread phenomenon. McDonald (1985) reports that it varies from 10% in industries like tabacco, furniture and leather to more than 50% in the transportation equipment and petroleum refining industries. Finally, the coexistence of investor owned firms and producer cooperatives can be found in agricultural markets throughout the world (Hendrikse, 1998 and Sexton and Lavoie, 2001).

The empirical significance of the co-existence of exchange forms is not reflected in the theoretical literature. One of the main problems of the models is the stark predictions they generate. Either all exchange via contracts in the industry or no contracting at all is privately optimal. There are a few exceptions. Perry (1978) obtains co-existence in equilibrium by having a dominant manufacturer adopting a policy of price discrimination. This firm contracts with the stages having more elastic derived demands in order to be able to charge the remaining stages higher prices. Carlton (1979a) models the assurance of supply argument. Contracts arise from a desire to avoid input rationing and it transfers risk from one sector of the economy to another. Firms contract partially in equilibrium in order to assure a sufficient probability of sale. Hendrikse and Peters (1989) establish co-existence in a world characterized by rationing, differences in the reservation prices of buyers, and differences in the risk attitudes of buyers and seller. Carlton (1979b) assumes that the cost of transacting in the spot market are higher than those internally because there is a cost due to the variability of spot market exchange besides the fixed and variable production cost. Contract exchange doesn't incur these variability cost. Uncertainty and transaction cost create incentives for firms to use both markets for the sale of their output. The short-run equilibrium is attained when both the spot and contract market clear. These markets equilibrate when firms are indifferent at the margin between supplying on the spot and contract market. Products exchanged via contracts sell at a lower price than those exchanged via the spot market because the marginal cost of satisfying contract demand is lower than the marginal cost of satisfying spot demand. The comparative statics show that if the spot market variability increases, then the expected spot market price increases and the contract price decreases. Bolton and Whinston (1993) consider the choice of governance structure in a setting with one seller and two buyers; and uncertainty about the ability of the seller to deliver. The main focus is on supply assurance concerns when several downstream firms are competing for inputs in limited supply. Our model has many sellers and buyers, focusing on the relationship between the spot and contract market. Finally, Xia and Sexton (2004) look at the (anti) competitive implications of contract pricing clauses in a duopsony model. This article addresses the impact of a contractual delivery requirement on the coexistence of spot and contract markets.

Perry (1989) distinguishes three broad determinants of contracting: technological economies, transactional economies and market imperfections. Market imperfections entail both imperfect competition and imperfect or asymmetric information. Our model treats transactional economies and market imperfections, with an emphasis on the latter in the form of imperfect competition. More specifically, the interdependence between spot and contract market exchange will be examined with a model in which the market is characterized by asymmetric buyers, stochastic supply and costs associated with spot market trade. Each buyer is characterized by a different reservation price and always wants to buy one unit. Sellers are assumed to be identical and have a unit for sale only with a certain probability. Buyers and sellers have to decide whether to exchange products via contracts or in the spot market. This analysis of endogenous contract formation and endogenous uncertainty establishes results about the size of the spot market and which firms are in the spot market.

A contract consists of a delivery requirement and a contract benefit parameter. Both features are exogenous in our model in order to focus on the impact of (real-world) contracts on market structure. First, delivery requirements are a common feature of contracts between buyers and sellers. Nilsson (1998, p42) writes regarding agricultural cooperatives ‘The delivery obligation for members is the dominating practice everywhere; in some countries it is even an obligation bylaw.’ Cook and Tong (1997) identify as one of the main organizational characteristics of New Generation Cooperatives that that each member has members have the right, but also the obligation, to deliver a specific quantity of the commodity each season. If the quantity delivered is lower than initially agreed in the delivery right, the cooperative has the right to buy the commodity on the producer’s behalf and charge them for the difference in price. Cook and Iliopoulos (1999, p526) cite a cooperative expert saying ‘Farmers are required to deliver according to plan regardless of the open market.’

Second, exchange via a contract faces different cost than exchange in the spot market. A contract is an institutional arrangement by which some of the spot market exchange costs can be reduced, but other costs emerge. Contract exchange requires resources in order to establish and police incentive systems for good performance, to coordinate, and to plan. However, buying and selling in external markets requires also resources. For example, Carlton (1979b) argues that some benefits are not captured when exchange is done in the spot market (‘real costs are associated with operating in a variable market’) because search costs are involved in establishing a match between a buyer and a seller.

The focus of transaction cost economics is on the different costs associated with different governance structures (Williamson, 1986). Each mode of exchange has different coordination and incentive problems. This article will not specify a new model with respect to these differences, but take them for granted and summarize them by a parameter. It reflects the net benefit associated with contract exchange compared to spot market exchange. This reduced form specification is chosen, like in Riordan and Williamson (1985), in order to focus on the market effects of contracts. Empirical research has to determine the value of this parameter based on a model capturing more fundamental, structural technological, coordination and incentive features, like in Crocker and Reynolds (1993).

Contract formation determines to a certain extent the probability distribution of the spot market price in our model. This contract formation externality entails the removal of high reservation price buyers and various sellers from the spot market. The first effect decreases the expected spot market price when the number of contracts is small, whereas the decrease in the number of sellers and additional residual contract demand increase the expected spot market price beyond a certain number of contracts. Contract prices are positively related to the number of contracts. Finally, additional contracts reduce the variance of the spot market price when the number of contracts is sufficiently large.

The model entails an endogenous upper bound on the number of contracts. The extent of contracting is disciplined by the opportunities provided by the spot market to each buyer and seller. It determines which buyers and sellers will be in contracts as well as the distribution of benefits. The buyers with the high reservation prices will be in contracts because they are able to compensate the seller for not being in the spot market. The spot market provides these buyers with an extra opportunity to satisfy their unfulfilled demand. Not all firms can afford a contract. Buyers with a low reservation price are not able to compensate sellers for not being in the spot market when spot market demand is high. The role of these buyers is to support the existence of the spot market. These buyers are able to survive because it is sufficient for them to get at least once in a while the product. Sellers in the spot market earn the same profits as their counterparts in the contracting mode of exchange because they have the opportunity of making an exchange with a high reservation price buyer, once in a while.

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Sellers in the spot market earn the same profits as their counterparts in the vertical integration mode of exchange because they have the opportunity of making an exchange with a high reservation price buyer, once in a while.

The comparative statics analysis with respect to the effect of a change in the benefit of contracting on the expected spot market price depends on three economic forces: the change in the spot market supply, the change in the composition of buyers not vertically integrated and the change in residual contract demand in the spot market. The endogeneity of the probability distribution of the spot market price is responsible for these forces. A change in the extent of contracting will alter the composition of buyers and sellers in the spot market and therefore the distribution of the spot market price. It is shown that a switch from no contracting to some contracting will decrease the expected spot market price. The reduction in spot market demand due to contracting (second effect) dominates the reduction in supply effect (first effect), because the high reservation price buyers are less often active in the spot market. An increase in the contract benefit at larger values of the contract benefit parameter will increase the expected spot market price. Albeit high reservation price buyers will switch to the contracting mode of exchange, they will still sometimes be in the spot market when the contract partner is not able to deliver a unit of the product (third effect). This demand effect dominates the reduction in spot market supply effect beyond a certain level of the contract benefit parameter. This result establishes that there is an endogenous upperbound on the number of contracts that can be formed. The decision to contract changes the composition of agents in the spot market and therefore the probability distribution of the spot market price. This external effect limits the use of vertical integration as the primary mode of exchange.

The variance of the spot market price exhibits a similar pattern, except for high values of the contract benefit parameter. An increase in the costs of spot market exchange takes a high reservation price buyer out of the spot market, given that the costs of spot market exchange are small. The remaining buyers in the spot market are more similar, which reduces the variance of the spot market price. Additional contracting reduces the differences between the buyers in the spot market even further, but it increases the variety in the residual contract demand. This second effect dominates for intermediate values of the contract benefit parameter and therefore increases the variance of the spot market price. Finally, a further increase in the contract benefit parameter will reduce the variance when almost all exchange goes via contracts. Spot market supply is at such a low level that only the high reservation price buyers are able to get a unit (at a high price) in the spot market when their contract partner is not delivering.

The article is organized as follows. Section 2 presents the model. Section three states and proves the results. The comparative statics results are formulated in section four. Section five formulates conclusions and directions for further research.

2 MODEL

The coexistence of spot and contract markets is analysed with a model consisting of σ sellers and β buyers. Denote the set of sellers by $S = \{1,...\sigma\}$ and the set of buyers by $B = \{1,...,\beta\}$. All sellers are assumed to be identical. Two possible states are distinguished for each seller i, $i \in S$. Seller i has either one unit for sale or nothing at all. The probability that seller i does not have a unit for sale is μ. Define a σ-dimensional vector e such that $e_i = 0$ when seller i does not have a unit for sale and $e_i = 1$ otherwise. Define R_j as the reservation price of buyer j for the product, and assume that $R_1 > R_2 > ... > R_\beta$. Only one state is considered for each buyer j, i.e. buyer j always wants to buy one unit. (Notice that with this specification no assumption is made with respect to the slope of the demand function.) Figure 1 presents these features of the market.

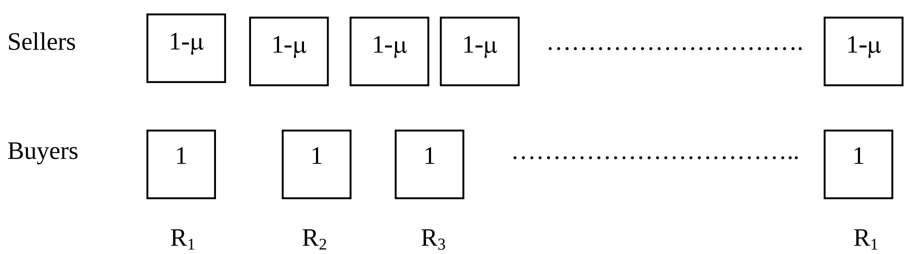


Figure 1: Sellers and buyers

A market structure M is a partition of the set of buyers and sellers. The set of buyers and sellers in contracts is defined to be C_M. The alternative to a contract is to be in the spot market. The spot market price p(M,e) is defined to clear the market, i.e. the spot market price allocates the available goods in the spot market to the buyers with the highest reservation price. The level of the market clearing price may be anywhere between the reservation price of the marginal buyer and the highest reservation price of the buyers who don't get a unit. Price formation in markets is the result of a bargaining process, where the price rule p(M,e) summarizes the distribution of bargaining power in the spot market. Vincent (1992) illustrates with a number of simple many-person bargaining games that many subtleties are involved in the modelling of competitive forces in a strategic setting. The equilibrium price turns out to be quite sensitive with respect to the model being adopted and the equilibrium concept that is used. However, the most demanding equilibrium requirements point towards the buyers having all the bargaining power in our model. We assume therefore that the spot market price p(M,e) is equal to the reservation price of the buyer with the highest reservation price among the buyers who don't get a unit of the product. If supply is equal to, or larger than, demand, then the spot market price is equal to zero.

A contract between a buyer and a seller is a binding agreement, i.e. each party abides the rules of the contract. Contracts are only allowed between one buyer and one seller. This captures the belief that there are limits to the extent of contracting. The assumption of efficient rationing (Tirole, 1988) is used to break ties in situations where the demand for contracts exceeds supply. The contract has to address all possible states of the economic environment. We adopt the following rules for a contract between seller i and buyer j:

- if seller i has a unit for sale, then this unit has to be delivered to buyer j;
- buyer j receives a contract benefit α when seller i delivers the unit
- if seller i does not have a unit for sale, then buyer j will try to buy a unit from another seller not in a contract
- seller i receives a contract price c_i, regardless of the ex-post realization of e.[1]

The vector of contract prices is defined to be c.

The contract benefit is defined to be α. The exogenous parameter α summarizes the difference between a contract and the spot market mode of exchange to deal with coordination and incentive problems. Higher values of α imply that contract exchange becomes more attractive than spot market exchange. Denote the expected payoff of seller i in market structure M as:

$$s_i(M) = \begin{cases} c_i & ,i \in C_M \\ (1-\mu)E\{p(M,e) \mid e_i = 1\} & ,i \notin C_M \end{cases}.$$

Define the one-to-one correspondence $\varphi_M : C_M \cap S \to C_M \cap B,$ i.e. each seller in a contract is assigned a particular buyer. Denote the expected payoff of buyer j in market structure M as:

$$b_j(M) = \begin{cases} (1-\mu)(R_j+\alpha)-c_i+\mu E\{\max\{0, R_j - p(M,e)\} \mid e_i = 0\}, & j = \varphi_M(i),\ j \in C_M \\ E\{\max\{0, R_j - p(M,e)\}\} & ,j \notin C_M \end{cases}.$$

[1] This contract rule implies that payments are not contingent on the state. This seems to be a natural way of modelling vertical integration when we think of it as the buyer making a bid for the seller. However, contract rules could be easily changed into state contingent payments. For example, the payment c_i could be made contingent on the seller having a unit for sale simply by multiplying by $1/(1-\mu)$. Although this scheme is harder to accept than the interpretation used here, it does not change the qualitative nature of our results. Notice that the specification is not at all close to a complete contingent contract. Bajari and Tadelis (2001) provide evidence and arguments why these contracts are not observed in many sectors.

The sequence of decisions is as follows. All sellers are assumed to choose simultaneously a contract price in a non-cooperative way. Subsequently, buyers decide simultaneously which contract to accept (or to be in the spot market), given the vector of contract prices c. Third, nature chooses for each seller independently the value of e_i. Finally, trade takes place and the spot market price is determined to clear the market. An outcome (c,M) is defined to be an equilibrium when it is a Nash equilibrium of the above game. We refer to M of this equilibrium as an equilibrium market structure.

3 EQUILIBRIUM

The equilibrium features of the model will be derived in this section. Theorem 1 establishes that the buyers with the high reservation prices are in contracts.

Theorem 1: Suppose $R_j > R_{j+1}$. If buyer j+1 is in a contract in equilibrium, then buyer j is as well.

Proof: Suppose contrary to the proposition that there exists an equilibrium market structure M in which buyer j+1 is in a contract and buyer j is not. Denote by M′ and alternative market structure that differs from M only by the fact that buyer j+1 is not in a contract. By assumption of a Nash equilibrium $M \geq_{j+1} M'$, i.e. market structure M is weakly preferred to market structure M′ by buyer j+1. Also define market structure M" as the market structure that differs from M only by the fact that buyer j is in a contract and buyer j+1 is not. By assumption of a Nash equilibrium in the acceptance game $M \geq_j M''$. The efficient rationing assumption entails that buyer j can always decide to replace buyer j+1. This will be done because buyer j strictly prefers M" above M, when buyer j+1 weakly prefers M above M′. This is a contradiction and implies that M can not be an equilibrium. The conclusion is therefore that a contract is attractive for buyer j when it is attractive for buyer j+1.

Theorem 2 establishes that the equilibrium contract prices are identical and equal to the equilibrium payoff of a seller in the spot market.

Theorem 2: If (c,M) is an equilibrium, then $c_i = s_l(M)$, where $i, l \in S$, $i \in C_M$ and $l \notin C_M$.

Proof: Assume that the set C_M of equilibrium outcome (c,M) consists of k buyers and k sellers. Theorem 1 has shown that the buyers with $R_1,...,R_k$ will be in C_M. Define the ranking of contract prices of the sellers such that $c(1) \leq c(2) \leq ... \leq c(\sigma)$. Observe that $c_i < s_l(M)$ can not be part of an equilibrium outcome, because a contract price $s_l(M)$ is strictly preferred by seller i. A contract price $c_i > s_l(M)$ can also not be part of an equilibrium outcome, because an offer $s_l = s_l(M) + (c_i - s_l(M))/2$ strictly improves the expected payoff of seller j and upsets (c,M) as an equilibrium outcome. The proof is completed by observing that no player can strictly improve his expected payoff by changing his contract price or mode of exchange when $c_i = s_l(M)$.

Define $\alpha(k)$ as the minimum value α for which adopting the k-th contract is advantageous for buyer R_k, given that the k-1 buyers with the highest reservation prices are in contracts.

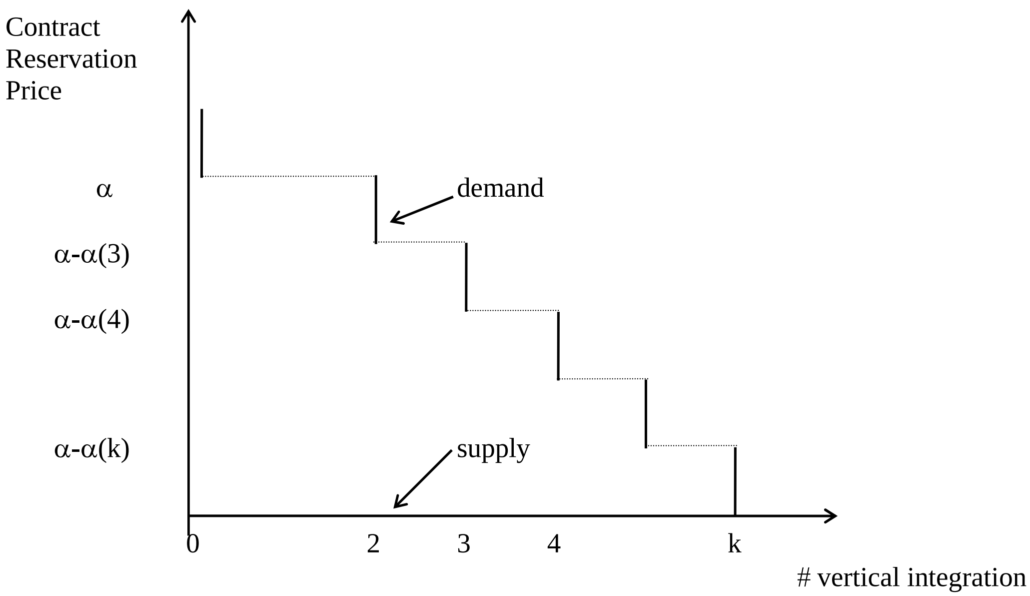
Lemma: $0 = \alpha(1) = \alpha(2) < \alpha(3) < ... < \alpha(N)$.

The intuition is provided here, while the formal proof of this lemma is in the appendix. If there are no contracts, then the move to the contract mode of exchange by buyer R_1 doesn't change the distribution of prices which have to be paid by this buyer. Buyer R_1 acquires the product at the prevailing price in the market, or has to compensate its contract partner for not being in the spot market. The expected cost of buyer R_1 of acquiring a unit is the same for both modes of exchange when there is only one contract. However, the contract generates an additional benefit α for buyer R_1 when the contract partner delivers. The value of α at which vertical integration is at least as attractive as spot market exchange is therefore zero. A similar argument applies to $\alpha(2)$. If there are two or more units produced by the sellers, then the distribution of prices is unaffected by the formation of the second contract. The only possibility for the value of $\alpha(2)$ to be different from $\alpha(1)$ is therefore the situation in which only one unit is available in the market. Buyer R_2 will not obtain this unit when it is either produced by the contract partner of buyer R_1 or by one of the sellers in the spot market. If the contract partner of R_2 is the only one producing, then it will be delivered in this contract. However, the contract partner has to be compensated for not being in the spot market, i.e. R_2 has to be paid. The surplus associated with this possibility for R_2 is therefore zero. The contract benefit α has to be larger than zero in order to have a third contract. The reason is that buyer R_3 may receive a unit when either buyer R_1 or R_2 would have received it at spot market price R_2 and R_3 respectively, without this third contract. Prices R_2 and R_3 will occur less frequently in the spot market, because the contract partner of buyer R_3 has to be compensated for these opportunities. It does not matter for the expected payoff of buyer R_3 when spot market price R_3 would have occurred, but it does for buyer R_2. This results in a positive value $\alpha(3)$. The other inequalities in the lemma are explained in the same way.

Theorem 3 establishes the unique equilibrium market structure for every value of α.

Theorem 3: If $\alpha(k) \leq \alpha < \alpha(k+1)$, then the unique equilibrium market structure consists of k contracts, involving the k buyers with the highest reservation prices.

This result follows immediately from theorem 1, 2 and the definition of $\alpha(k)$. It shows that the coexistence of the spot and contract market is driven by some kind of market failure (, i.e. α) in our model. There are no contracts when $\alpha < 0$. The difference $\alpha - \alpha(j)$ reflects the reservation price of buyer j for having a contract, i.e. $\alpha - \alpha(j)$ is the demand curve for contracts. The supply curve of contracts is perfectly elastic in our set-up because there are no costs involved in starting or carrying out contract exchange. Figure 2 shows the demand and supply curve for contracts.



Figuur 2: The contract market

Notice that the above result is independent of the reservation prices of the buyers. It is robust with respect to any demand schedule. The spot market enables the high reservation price buyers with an additional opportunity to satisfy their unfulfilled demand. The role of the low reservation price buyers is to support the existence of the spot market. These buyers are not able to compensate the remaining sellers in the spot market for giving up the option of supplying high reservation value buyers in states of low overall supply. However, they survive because it is sufficient for them to get the product at least once in a while. Sellers in the spot market earn the same profits as their counterparts in contracts because they have the opportunity of making an exchange with a high reservation price buyer, once in a while. Notice that our model has uncertainty on the supply side and differences between buyers. This is qualitatively equivalent to differences between sellers and uncertainty on the demand side.

We have taken the number of sellers to be equal to the number of buyers. This is a short run situation. Suppose that the number of buyers is also fixed in the long run, but that the number of sellers is determined by market conditions. There are three kinds of sellers in the long run. First, sellers in contracts having positive average output. Second, sellers without contract having positive average output. Finally, sellers having zero average output and in the spot market (potential entrants). Buyers can contract with any seller who has not already done so. Suppose that sellers have to pay a fee each period in order to participate in this market. The expected spot market price will be equal to this fee in the long-run equilibrium, because the zero profit condition of entrants determines the expected spot market price. Entry has a negative effect on the expected payoff of a seller. This will limit the extent of entry. A decrease in the expected payoff of sellers will increase the number of contracts because more buyers are now able to afford a contract. The formation of additional contracts increases the expected payoff of a seller and will (partially) offset the decrease in the expected payoff due to entry. This will limit entry less.

4 COMPARATIVE STATICS

This section will show the relationship between the number of contracts, the expected payoff of a seller in the spot market, the expected spot market price and the variance of the spot market price. We also show numerical comparative statics results with respect to the probability of having a unit for sale.

Theorem 4: The equilibrium expected payoff of a seller in the spot market as a function of the extent of contracting k is constant when $k \in \{0,1,2\}$ and increases otherwise.

A matrix S(k) is constructed in the appendix. It characterizes the probability distribution of the spot market price as a function of the extent of contracting k from the viewpoint of a seller in the spot market, given that this seller is producing and the k buyers with the highest reservation prices have contracts. It is shown that S(0) = S(1). The only way in which S(0) and S(1) could possibly differ is that the probability of spot market price R_2 is different. R_2 only emerges when all suppliers are not producing, except for the supplier being considered. This probability is μ^{N-1} when there are 0 or 1 contracts.

The only spot market prices which are candidates for a change in probability weight are R_2 and R_3 when the second contract will be established. The probability of R_2 continues to be μ^{N-1} because it only emerges when both contracts don't produce and only one unit is available in the spot market. R_3's probability weight also doesn't change because it doesn't matter from a combinatorial point of view whether the contract partner of the buyer with R_1, or the buyer with R_2, or both don't produce.

If the number of contracts becomes larger than two, then the expected payoff of a seller in the spot market will increase. The reason is that buyers in contracts with relatively low reservation prices may receive a unit when a high reservation price buyer doesn't because his contract partner doesn't produce and the available units in the spot market are at such a low level that they are obtained by buyers with even higher reservation prices. More probability weight is therefore shifted to higher spot market prices when the number of contracts increases beyond two. This will drive up the spot market price and, therefore, the expected payoff of a seller in the spot market. A positive relationship emerges between the expected payoff of a seller in the spot market and the number of contracts when the number of contracts is larger than two. Figure three illustrates theorem 4.

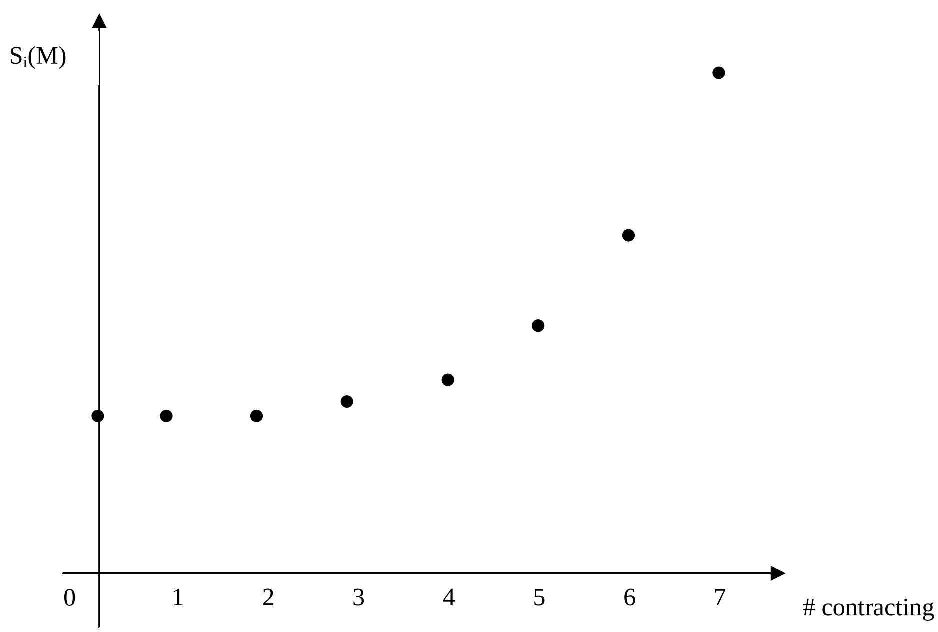


Figure 3: The expected payoff of a seller in the spotmarket

Theorem 5: The expected spot market price decreases as a function of α when α is small and positive and increases for sufficiently large values of α, given that the market is large enough.

The intuition for the first part is that the formation of the first contract implies that the buyer with the reservation price R_1 is not in the spot market when his contract partner is producing. The probability that the spot market price is R_2 is therefore reduced. All other spot market prices continue to have the same probability weight. The appendix has formalizes this by defining a matrix M(k) in the proof of the lemma which characterizes the probability distribution of the spot market price as a function of the number of contracts. It is subsequently straightforward to show that

$$E\{p \mid k=0\} - E\{p \mid k=1\} = (M_{12}(0)-M_{12}(1))\mu^{N-1} (1-\mu)R_2 = \mu^{N-1}(1-\mu) R_2 > 0.$$

The formation of the second contract reinforces this effect. The probability of spot market price R_2 is further reduced and also the probability of R_3 is decreased. All other spot market prices continue to have the same probability weight. Formally,

$$E\{p \mid k=1\} - E\{p \mid k=2\}$$

$$= (M_{12}(1)-M_{12}(2))\, \mu^{N-1}(1-\mu)R_2 + (M_{13}(1)-M_{13}(2))\, \mu^{N-2}(1-\mu)^2R_3$$

$$= \mu^{N-1}(1-\mu)R_2 + \mu^{N-2}(1-\mu)^2R_3 > 0.$$

The second part of the theorem requires that k and N are sufficiently large. This result can be made intuitive by considering the probability that the spot market price is R_2. This requires that the contract partners of the buyers with R_1 and R_2 are not delivering. The probability of this event is μ^2. R_2 emerges when exactly one seller is producing in the spot market. The probability of this event depends on the size of the spot market. It increases when the spot market becomes smaller. A similar argument holds for the probability weight of the other spot market prices.[2] Matrix M(k) has to be used to calculate the exact turning point, which depends on the reservation prices $R_2,...,R_N$ and μ. Figure four provides a graphical illustration of theorem 5.

[2] Table 1 provides an example in which the market is too small for the emergence of the second part of theorem 5, whereas table 2 shows a numerical example where the market is large enough.

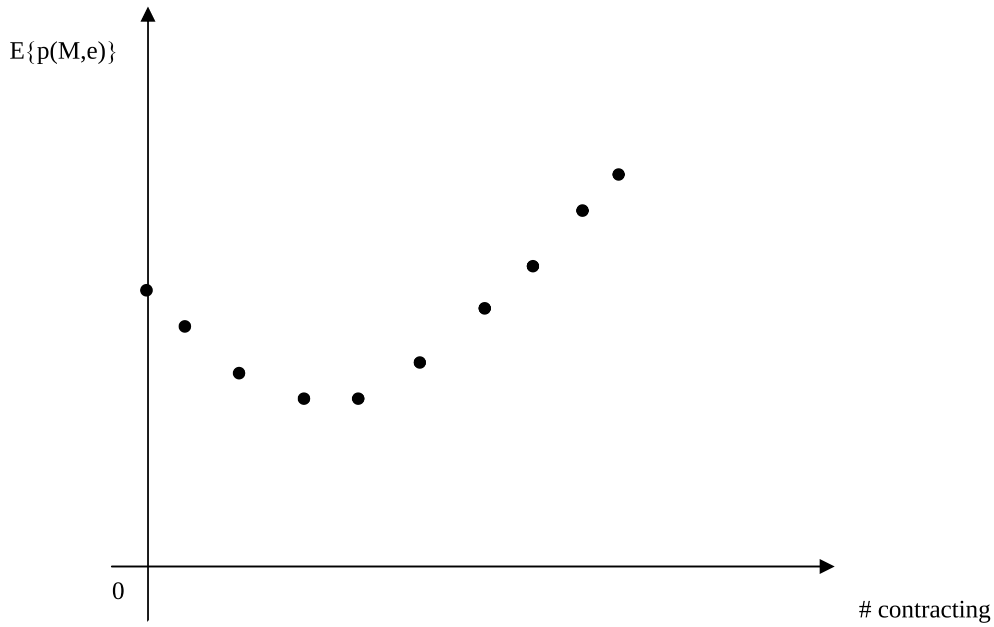


Figure 4: The expected spot market price

Three forces drive the result of theorem 5. First, if the extent of contracting increases, then the supply of the product on the spot market will be lower on average. This has an increasing effect on the expected spot market price. Second, contracting removes the buyers partially out of the spot market. Buyers in contracts are only in the spot market when the upstream contract partner does not deliver. This has a decreasing effect on the expected spot market price. This second effect dominates when α is positive and close to zero (, i.e. the extent of contracting is small), because the high reservation price buyers are less often in the market. Theorem 5 has provided the formal argument for why this is strictly so when a market with zero contracts is compared with one contract and one contract is compared with two contracts. Third, buyers in contracts will be in the spot market when the upstream contract partner does not deliver. Additional contracting entails that residual contract demand gains importance in the spot market. The increased use of the contracting mode of exchange consists of the buyers with the intermediate reservation prices. This implies that the probability of intermediate spot market prices decreases, i.e. the relative probability weight of high spot market prices increases. Additional residual contract demand increases therefore the expected spot market price. This third effect reverses the negative relationship between the spot market price and α, when α is above a certain level. However, the value of the parameters β, σ and μ may be such that the market is too small for the first and third effect to dominate the second. A numerical example with $\beta = \sigma = 2$ and $\mu = 0.5$ illustrates this market size feature in table 1, where Var{p(M,e)} is the variance of the spot market price p(M,e).

j	R_j	k	E{p(M,e)}	Var{p(M,e)}
1	2	0	0.6667	0.2222
2	1	1	0.5000	0.2500

Table 1: The impact of contract formation when the market consists of two buyers and two sellers and $\mu = 0.5$

Our probability distribution argument provides an endogenous bound on contracting in an industry. Additional contracting increases the expected spot market price and therefore reinforces the upperbound. The expected spot market price increases as a function of the extent of contracting because the reduction in supply dominates the reduction in spot market demand. However, an increase in α when α is either small or large decreases the expected spot market price. The demand effect dominates the supply effect when α is small, whereas the probability that at least one unit is available in the spotmarket is responsible for a declining expected spot market price when α is large.

Notice the difference between theorem 4 (figure three) and 5 (figure four). The focus of theorem 4 is on a seller, whereas theorem 5 represents the market point of view. The probability distribution of the spot market price associated with the first perspective is not the same as the probability distribution of the spot market price seen from a market perspective. These probability distributions are only identical when the equilibrium consists of N-1 contracts.

The expressions of the expectation and the variance turn out to be analytically to cumbersome to derive additional comparative statics results. The next four theorems present therefore the results from numerical analyses[3].

Theorem 6: The variance of the spot market price declines, subsequently increases and finally decreases as a function of α, given that the market is large enough.

Three forces determine the comparative statics results of an increasing contract benefit parameter on the variance: reduction of high reservation price spot market demand, size of the spot

[3] The appendix specifies matrices S(k) and M(k) regarding the equilibrium contract prices as a function of the extent of contracting and the expected spot market price. They are programmed, where the numerical values of μ, $R_1,...,R_\beta$ and σ are input parameters. Table 1 serves as a first check of the numerical analysis.

market and reduction of spot market supply. An increase in α from 0 to a positive number induces contracting by the high reservation price buyers. They will compete less often in the spot market against each other and therefore high spot market prices will occur less frequent; the buyers in the spot market become more similar on average. This implies a lower variance of the spot market price. An additional increase in the contract benefit parameter will also drive the buyers with intermediate reservation prices to the contracting mode of exchange. This additional reduction of the spot market supply restores the relative probability weight on high spot market prices; residual contract demand gains importance compared to the spot market demand from the non-integrated buyers. Finally, the reduction in the spot market supply determines the downward pattern in the variance when the level of the contract benefit induces a large extent of contracting. The spot market supply is at such a low level that only the high reservation price residual contract demand can be satisfied; buyers actually getting the product are more similar on average. Spot market prices will almost always be high, which implies a low variance.

Figure five illustrates theorems 5 and 6. Suppose that we have a linear reservation price schedule. The curve D_k represents the expected spot market demand, when there are k contracts. If $p(M,e) > R_{k+1}$, then D_k is a fraction $1-\mu$ of D_0. If $p(M,e) < R_{k+1}$, then D_{k+1} is to the left of D_k because the expected spot market demand of the firm with reservation price R_{k+1} is $1-\mu$, whereas it is one when there are only k contracts. The expected spot market demand decreases when there are more contracts. This has a decreasing effect on the expected spot market price. However, the expected spot market supply also decreases. This is represented by the vertical lines S_k and S_{k+1}. (The expected spot market supply is $(1-\mu)(N-k)$ when there are k contracts). This has an increasing effect on the expected spot market price. If the number of contracts is small, then the first effect dominates. Otherwise the second effect will dominate. In terms of figure four, the expected demand to the left of point a is not affected by the formation of more than k contracts, whereas the expected supply curve is. If the number of contracts is large, then the variance of the spot market price will increase with further contract formation. This is caused by a decrease in the expected spot market supply, whereas demand remains almost the same.

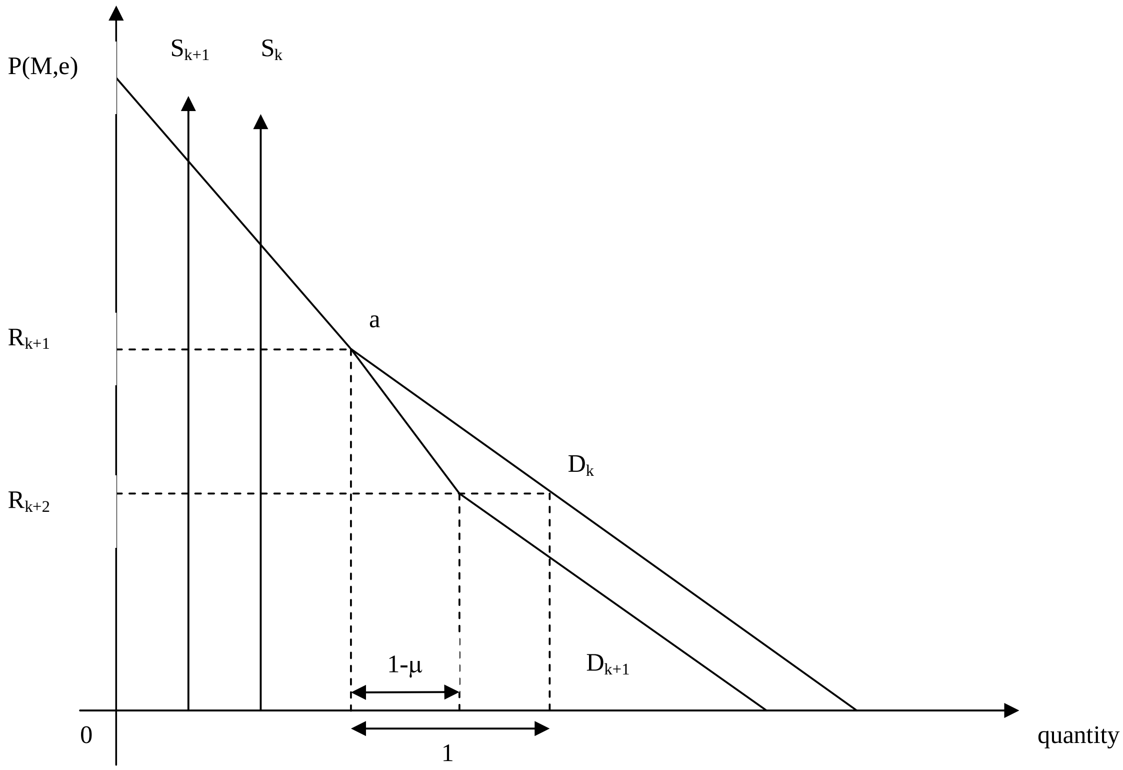


Figure 5: The spot market

Contracts are often formed in order to reduce the variability of the spot market price. Carlton (1979) wrote: "I'm arguing that real costs are associated with operating in a variable market. For example, one reason for contracts is that transaction costs of finding buyers on the spot market are eliminated by a long-term contract. This transaction cost of finding buyers is likely to depend on the variability of the spot price. (More variability implies more likely initial dispersion of prices which in turn implies more search)". Although contracts are not formed in our model to decrease price variability, it should be noted that theorem 6 would reinforce the coexistence result. The incorporation of the variance of the spot market price in the contracting decision implies that α depends on Var{p(M,e)}. The observation by Carlton implies that α is positively related to Var{p(M,e)}. The values of $\alpha(k)$, k=0,...,N-1 will not change when considerations regarding Var{p(M,e)} play a role in the contracting decision. It reinforces the result of theorem 5 that there is an endogenous upperbound on the extent of contracting, because the value of α will decrease due to contracting when many market participants have already integrated.

A numerical example with $\mu = 0.5$ is summarized in table two. The value of the exogenous parameter α does not show up in this table, because there is a one to one correspondence between α and the equilibrium number of contracts k, i.e. there are k contracts when $\alpha(k) \leq \alpha < \alpha(k+1)$. The column with the value of k could therefore be replaced by a column specifying intervals for α, where the endpoints are $\alpha(k)$ and $\alpha(k+1)$.

j	R_j	k	E{p(M,e)}	Var{p(M,e)}	$s_i(j;M)$
1	13	0	6.4992	3.2452	3.0000
2	12	1	6.4985	3.2419	3.0000
3	11	2	6.4973	3.2366	3.0000
4	10	3	6.4963	3.2402	3.0001
5	9	4	6.5022	3.3094	3.0017
6	8	5	6.5377	3.5819	3.0114
7	7	6	6.6507	4.2246	3.0479
8	6	7	6.9034	5.2356	3.1448
9	5	8	7.3373	6.2593	3.3384
10	4	9	7.9387	6.7292	3.6448
11	3	10	8.6390	6.3253	4.0479
12	2	11	9.3506	5.2348	4.5114
13	1	12	10.0034	3.9267	5.0017

Table 2: A numerical example

Some comparative statics results regarding μ are straightforward and not explicitly stated in the form of theorems. First, the extent of contracting at which the variance of the spot market price attains its maximum declines when μ is increased. Notice that it is not claimed that the maximum is reached at $\mu = .5$. This might seem strange because the (stochastic) event that a seller has a unit for sale has a variance of $\mu(1-\mu)$. However, the variance of the spot market price consists of two parts: an endogenous and an exogenous component. The exogenous uncertainty is captured by the probability that a seller has a unit for sale. The variance of this stochastic variable is $\mu(1-\mu)$ for each seller. The effect of the endogenous component is described in theorem 6. A larger value of μ implies that the buyer in a contract will be more often in the spot market and will outbid the permanent spot market buyers. This reduces the variance of the spot market price. A smaller extent of contracting will increase the variability of the spot market price. Second, if μ increases, then E{p(M,e)} increases. The buyers with the highest reservation prices are the only ones to get a unit on the spot market, given that the spot market supply is low. If the probability of not having a unit for sale increases for every seller, then there is on average less for sale on the spot market. This tilts the distribution of buyer valuations in the spot market towards higher values in equilibrium and therefore a higher expected spot market price. Third, an increase in μ reduces the extent of contracting at which the minimum of $E\{p(M,e)\}$ is attained. The initial decrease in the expected spot market price when α is small and increased is due to the buyers with high reservation prices being removed partially from the spot market. An increase in μ has a stronger reverse effect on the expected spot market price because spot market supply will be lower and residual contract demand will be higher on average. Table 2 shows that the lowest expected spot market price for $\mu = .5$ emerges (in figure 4) when the number of contracts is three. This minimum is reached at $k = 2$ when $\mu = .75$ and at $k = 4$ when $\mu = .1$.

5 CONCLUSIONS AND FURTHER RESEARCH

Contracting is explained by differences in the reservation prices of buyers, production uncertainty regarding sellers and a difference in cost between contract exchange and spot market exchange. Firms choose the contracting mode of exchange because it is able to capture certain benefits that can't be absorbed when exchange is done in the spot market. The formation and extent of contracts and the distribution of the contract benefit between the buyer and the seller is disciplined by the opportunities facing each contract participant in the spot market. The buyers with the relatively high reservation prices will have contracts because they are able to compensate the seller for not being in the spot market. The role of the spot market is to provide buyers in contracts with an additional opportunity to satisfy their unfulfilled contract demand. The role of the buyers with the low reservation price is to support the existence of the spot market. These buyers are able to survive because it is sufficient for them to get the product only once in a while. Sellers in the spot market earn the same profits as their counterparts in contracts because they have the opportunity of making an exchange with a high reservation price buyer, once in a while. The comparative statics reinforced the result that there is an upper bound on the number of contracts that can be formed. The expected payoff of a seller does not decrease with contracting (theorem 4), but this is not true for the expected spot market price (theorem 5) since contracting changes which trades occur and changes the probability with which the spot market opens.

There are several extensions of the above analysis possible. All sellers are taken to be identical. Our qualitative results are not influenced by relaxing this assumption. However, the combinatorial difficulties associated with the calculation of the expected spot market price and the expected payoff of the market participants increase considerably. A similar remark holds for a relaxing the assumption that sellers in contracts are not allowed to trade on the spot market. Qualitative results are robust with respect to this specification, because the contract benefit essentially would prevent ex post that buyers with higher valuation obtain the product. A more fundamental extension would be an investigation into whether contracting is an optimal mode of exchange. There is for example no distinction between vertical integration or exchange via contracts (Aghion and Bolton, 1987). The specification of a detailed information structure or the bounded rationality of the players becomes important in order to analyse the rationale for different types of contracts. Vertical integration may change the incentive structure of the integrating parties in a different way than contracts do. The parameter α will be derived endogenously in such a framework. This is important but goes beyond the scope of this paper. However, we expect that our qualitative results regarding market structure continue to hold within more complex economic environments.

Another avenue for research concerns empirical analyses. The introduction section has shown that coexistence is a widespread phenomenon. An attractive feature of our model is that it requires only (the estimation of) a limited number of parameters (β, σ, μ and α) and is robust with respect to the reservation prices of buyers and sellers. Case studies may also shed some light on the relevance of the approach taken in this article. An example is the oil tanker market. The case studies by Porter (1980) and Frankel et al. (1985) seem to support certain aspects of the above model. Porter observed: "The main participants in the oil tanker industry were the oil companies, the independent ship owners, shipping brokers, and the shipbuilding industry. These players were remarkably different in their approaches to doing business." Oil companies can be thought of as the high reservation price buyers in our model. They use continuous process operations, which are difficult as well as extremely costly to shut down. However, they don't rely completely on their own fleet ("To supplement their internal tanker fleets, oil companies utilized both chartered and spot vessels"). The description of the independent ship owners is also not at odds with our model. Porter writes: "Ship owners had a number of options in running their fleets, ranging from operating all ships under long-term charter agreements to running an entire fleet on the spot market. At any given time approximately sixty percent of the independently owned fleet was on term charter." and "The Greeks generally followed a policy of seeking time charters on their newer, larger vessels. They placed only their older, usually smaller vessels on the spot market." Despite these observations, more empirical research is needed to determine whether coexistence in these markets exhibit the features highlighted in the above model.

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Appendix

This appendix provides the proofs of the lemma and theorem 4. We will use $\beta = \sigma = N$ in order to simplify the formulation of the proofs.

Proof of the lemma

Buyer R_k faces a different probability distribution of the spot market price in the situation with a contract partner not delivering than in the situation without a contract. Define g_k as the expected payoff of buyer R_k when his contract partner is not delivering and h_k as the expected payoff of buyer R_k without a contract, given that buyers $R_1,...,R_{k-1}$ have contracts. The reservation contract price $c_k(k-1)$ of buyer R_k when the buyers $R_1,...,R_{k-1}$ have contracts is determined by

$$(1-\mu)(R_k+\alpha) + \mu g_k - c_k(k-1) = h_k$$

$$\Leftrightarrow R_k + \alpha - c_k(k-1)/(1-\mu) = (h_k-\mu g_k)/(1-\mu).$$

Expressions g_k and h_k are determined by the probability distribution of the spot market price. Define a (N+1,N+1)-matrix M(k) such that the probability distribution of the spot market price when buyers j with $R_j \geq R_k$ are the only buyers in contracts is

$$\Pr_k \begin{pmatrix} - \\ R_2 \\ R_3 \\ . \\ . \\ . \\ R_N \\ 0 \end{pmatrix} = M(k) \begin{pmatrix} \mu^N \\ \mu^{N-1}(1-\mu) \\ \\ . \\ . \\ . \\ \mu(1-\mu)^{N-1} \\ (1-\mu)^N \end{pmatrix},$$

where '-' denotes that there are zero units available in the spot market. The sum of the elements of a particular column of M(k) is again independent of k.

If k=0, then $M_{jj}(0) = \binom{N}{j-1}$, $j = 1(1)N+1$. All other elements of M(0) are zero. The formation of the first contract increases the probability that the spot market doesn't exist, i.e. no units are produced by suppliers without contracts. Fewer supplier in the spot market is responsible for this result. Some of the probability weight is shifted from R_2 to the event that the spot market doesn't exist. All other elements of M(1) are the same as in M(0). The formation of the k-th contract involves $M_{jj}(k) - M_{jj}(k-1) < 0$ and $M_{ij}(k) - M_{ij}(k-1) > 0$ for j = 2(1)K+1. The combinatorial difference $M_{jj}(k-1) - M_{jj}(k)$ shifts completely to the event that the spot market doesn't exist when $M_{jj}(k-1) - M_{jj}(k) = M_{1j}(k) - M_{1j}(k-1)$. Otherwise, the difference $M_{jj}(k-1) - M_{jj}(k) - (M_{1j}(k) - M_{1j}(k-1)$ is distributed over $M_{ij}(k)$, where $2 \le i \le j-1$. The probability that the spot market doesn't exist is μ^{N-k} when there are k contracts, which is equal to $\mu^{N-k} \sum_{\ell=0}^{k} \binom{k}{\ell} \mu^{k-\ell}(1-\mu)^{\ell}$. The remaining part of the construction is identical to the one of S(k). This procedure is formally captured by

$$M_{1j}(k) = \begin{cases} \binom{k}{j-1}, & 1 \le j \le k+1 \\ 0, & \text{otherwise} \end{cases}$$

$$M_{ij}(k) = \begin{cases} \left(\binom{N}{i-1} - \sum_{\ell=1}^{i-1} M_{\ell i}(k)\right)\binom{k-i}{j-i}, & i \ge 2, i \le j \le k \\ \binom{N}{i-1}, & i = j \ge k+1 \\ 0, & i \ge 2, \quad \text{otherwise.} \end{cases}$$

This completes the description of M(k).

The size (N+1,N+1) of matrix M(k) has thus far been suppressed in order to simplify the notation. It turns out that this size is important in determining $(h_k-\mu g_k)/(1-\mu)$ and will therefore be made explicit by using the notation $M^{N+1}(k)$ and $Pr_k^{N+1}(.)$ in the remaining part of this proof. The combinatorial features of h_k are completely described by $M^{N+1}(k-1)$. The definition of g_k implies that spot market price 0 will not occur due to the non-delivery of the contract partner of buyer k. Only N-1 sellers determine the probability weight of the spot market price in the expression of g_k, where the combinatorial aspects of the probability weight are described by $M^N(k-1)$.

The combinatorial aspects of the probabilities of the spot market prices in $(h_k-\mu g_k)/(1-\mu)$ are summarized in a (N,N) - matrix $D^N(k)$. Observe that the weight attached to the event that no units are supplied in the spot market in the expression of h_k and μg_k is the same. The difference $h_k - \mu g_k$ has to be divided by $(1-\mu)$, which leaves

$$\Pr d_k \begin{pmatrix} R_2 \\ R_3 \\ . \\ . \\ . \\ R_N \\ 0 \end{pmatrix} = D^N(k) \begin{pmatrix} \mu^{N-1} \\ \mu^{N-2}(1-\mu) \\ . \\ . \\ . \\ \mu(1-\mu)^{N-2} \\ (1-\mu)^{N-1} \end{pmatrix}.$$

The elements of $D^N(k)$ are formally captured by

$k = 1,2 \quad D^N(k) = M^N(0)$

$$k \geq 3 \qquad D^N_{ij}(k) = \begin{cases} M^{N+1}_{i+1j+1}(k-1) - M^N_{I+1j+1}(k-1) & ,1 \leq i, j \leq N-1 \\ M^{N+1}_{i+1j+1}(k-1) & ,i = N \text{ or } j = N. \end{cases}$$

We have therefore that

$$R_k + \alpha - c_k(k-1)/(1-\mu) = \sum_{1=k+1}^{N+1} \Pr d_k(R_l)(R_k - R_l)$$

$$\Leftrightarrow c_k(k-1)/(1-\mu) = \alpha + \sum_{l=2}^{k} \Pr d_k(R_l)R_k + \sum_{1=k+1}^{N+1} \Pr d_k + (R_l)R_l,$$

where $R_{N+1} \equiv 0$. Notice that $c_k(k\text{-}1)$ is negatively related to the value of k.

Define c(k) as the expected payoff of a seller having a unit available in the spot market when there are k contracts. The definition of α(k) entails that buyer R_k can just afford a contract, i.e. the reservation contract price $c_k(k\text{-}1)$ is equal to c(k). Using the expression of $c_k(k\text{-}1)$ results in

$$\alpha(k) = c(k)/(1-\mu) - \sum_{l=2}^{k} \Pr d_k(R_l)R_k - \sum_{1=k+1}^{N+1} \Pr d_k(R_l)R_l.$$

Theorem 2 states that c(1) = c(2) and that c(k) is an increasing function in k for k ≥ 3. Similarly, it has been shown in this proof that $(h_1\text{-}\mu g_1)/(1\text{-}\mu) = (h_2\text{-}\mu g_2)/(1\text{-}\mu)$ and that $(h_k\text{-}\mu g_k)/(1\text{-}\mu)$ is a decreasing function in k for k ≥ 3. It follows immediately from these observations that and α(k) is an increasing function in k. Finally, it is straightforward to calculate that α(1) = α(2) = 0 by using the above expression of α(k). This completes the proof.

Proof of theorem 4

Define a (N,N)-matrix S(k) such that the probability distribution of the spot market price faced from the viewpoint of a seller in the spot market when producing and the buyers $R_1,...,R_k$ having contracts is

$$\Pr s_k \begin{pmatrix} R_2 \\ R_3 \\ . \\ . \\ . \\ R_N \\ 0 \end{pmatrix} = S(k) \begin{pmatrix} \mu^{N-1} \\ \mu^{N-2}(1-\mu) \\ . \\ . \\ . \\ \mu(1-\mu)^{N-2} \\ (1-\mu)^{N-1} \end{pmatrix}.$$

The number on the i-th row and j-th column of S(k) is defined to be $S_{ij}(k)$. It is equal to the number of possibilities in which j-1 units can be produced by the other N-1 suppliers and generates a spot market price R_{i+1} when the buyers j with $R_j \geq R_k$ use the contracting mode of exchange. Notice that the sum of the elements of a particular column j of S(k) reflects the number of ways in which j-1 units can be produced by N-1 suppliers. It does not depend on k. It will be shown how this number is distributed over $R_2,...,R_j$ as a function of the number of contracts. This depends on k because the identities of the firms in contracts having to buy in the spot market matter for the level of the spot market price.

The determination of S(k) will be done first for the case k=0. If there are no contracts, then the spot market price is completely determined by the number of units which are produced. It does not matter which sellers are producing. The probability that the spot market price is R_i, i=2(1)N is equal to $\binom{N-1}{i-2} \mu^{N-1-(i-2)}(1-\mu)^{i-2}$. All off-diagonal elements are therefore zero, whereas the diagonal elements are $S_{ii}(0) = \binom{N-1}{i-1}$, $i = 1(1)N$.

S(k) is computed from the viewpoint of a seller in the spot market having a unit available. This perspective is responsible for the claim that S(2) = S(1) = S(0). The buyer with R_1 is in the first contract. S(1) is always identical to S(0), because the buyer with R_1 will always get a unit when there is one available in either the spot market or the contract. The number of combinations at which a particular spot market price emerges doesn't change. If there are two contracts, then the only way that R_2 emerges as spot market price is that everybody else is broken down. R_3 clears the market when one other unit is available. The number of combinations at which this is realized does not depend on whether this one unit is produced in the spot market or in a contract.

S(k) changes with the formation of the third contract. The spot market price is not only determined by the number of units which are produced, but also by which sellers in contracts are not delivering. Suppose that only the supplier of the third contract is producing and nobody else, except for the particular supplier in the spot market we are considering. The market clearing price will be R_2. The spot market price would be R_3 when there are less than three contracts, because there are two units available in the spot market. So, some of the probability weight of R_3 is shifted to R_2. All other elements of S(3) are the same as in S(2).

The probability weight of R_2 continues to increase when the number of contracts is further expanded. It is equal to the probability that the first two contracts don't produce and only the seller we are considering in the spot market is producing. If there are k contracts, then this probability is equal to μ^2 times μ^{N-k+1}. It can be written as $\mu^{N-k+1} = \mu^{N-k+1}\sum_{\ell=0}^{k-2}\binom{k-2}{\ell}\mu^{k-2-\ell}(1-\mu)^{\ell}$. This determines the first k-1 elements of the first row of S(k). All other elements of the first row are zero. A similar combinatorial procedure is used for the determination of all other rows of S(k). The only difference is that the (combinatorial) numbers of a particular row have to be multiplied by the diagonal numbers of this row. This accounts for the number of combinations which result in this market clearing price. This number is already obtained in the calculation of the previous rows because the numbers of a particular column add up to a number which is independent of the number of contracts.

This procedure is formally captured by

$$\text{k=0,1} \qquad S_{ij}(k) = \begin{cases} \binom{N-1}{i-1} & , i = j \\ 0 & , \text{otherwise} \end{cases}$$

$$\text{k 2} \qquad S_{1j}(k) = \begin{cases} \binom{k-2}{j-1} & , 1 \le j \le k-1 \\ 0 & , \text{otherwise} \end{cases}$$

$$S_{ij}(k) = \begin{cases} \left(\binom{N-1}{i-1} - \sum_{\ell=1}^{i-1} S_{\ell i}(k)\right)\binom{k-1-i}{j-1} & , i \ge 2, \quad i \le j \le k-1 \\ \binom{N-1}{i-1} & , i = j \ge k \\ 0 & , i \ge 2, \quad \text{otherwise.} \end{cases}$$

This completes the description of S(k). The equilibrium expected payoff of a seller in the spot market when there are k firms in contracts is equal to

$$\sum_{i=2}^{N} \Pr s_k(R_i) R_i .$$

The proof is completed by observing that $Prs_{k+1}(R_i)$ - $Prs_k(R_i) = 0$ for every k=1,2 and $i \in \{2,...,N\}$ and $Prs_{k+1}(R_i)$ - $Prs_k(R_i) \geq 0$ for every $k \geq 3$ and $i \in \{2,...,N\}$.