On the spatial nature of the groundwater pumping externality

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Abstract

Most existing economic analyses of optimal groundwater management use single-cell aquifer models, which assume that an aquifer responds uniformly and instantly to groundwater pumping. This paper demonstrates how spatially explicit aquifer response equations from the water resources engineering literature may be embedded in a general economic framework. Calibration of our theoretical model to published economic studies of specific aquifers demonstrates that, by averaging basin drawdown across the entire resource, existing studies generally understate the magnitude of the groundwater pumping externality relative to spatially explicit models. For the aquifers studied, the drawdown predicted by single- cell models may be orders of magnitude less than that predicted by a spatially explicit model, even at large distances from a pumping well. Our results suggest that single-cell models may be appropriate for analyses of the welfare effects of groundwater management policies either in small aquifers or in larger aquifers where average well spacings are tens of miles or more. However, in extensive aquifers where well spacings are on the order of a few miles or less, such as many of those of concern to groundwater managers and policy makers, use of single-cell models may result in misleading policy implications due to understatement of the magnitude and spatial nature of the groundwater externality.

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1 Introduction

Groundwater resources are a major source of agricultural, potable, and industrial water throughout the world. In response to ongoing public concern about overextraction and rapid resource depletion, the optimal management of groundwater resources has received much attention from hydrologists, water resources engineers, and economists over the last several decades. Unsurprisingly, disciplinary studies in the engineering and economics literatures have taken quite different approaches, both philosophically and operationally, to the analysis of how groundwater should be allocated across space and time.

Economic analyses of groundwater have focused on the externalities associated with groundwater pumping, and on policies that could increase welfare through addressing these externalities. Most theoretical and empirical economic studies of optimal groundwater management have represented groundwater dynamics using a single-cell aquifer, implying commonality and uniform water levels throughout the resource, in both theoretical and empirical modeling. Early contributions derived optimization rules for the management of groundwater resources [6, 8]. More recent, and influential, studies have sought to quantify the magnitude of potential welfare gains from groundwater management using parameters from real aquifers and comparing myopic, socially optimal, and non-cooperative strategic pumping trajectories [5, 7, 14, 15, 18, 25]. In general, these studies have found very small or negligible gains to optimal groundwater management, implying that from an economic standpoint – and contrary to public opinion – intervention in this particular resource is unwarranted.

Engineering analyses of the optimal management of groundwater start with the continuity equations that characterize groundwater flow, and then generally use finite difference, finite element, or numerical integration methods to allow embedding of the aquifer response equations in an optimization framework [2, 4, 19, 28]. As such studies involve problem-specific initial and boundary conditions and well locations, the recommendations of these studies are also limited to their respective study areas. Additionally, although some of these papers consider the influence of hydrological parameters on groundwater management options [2], they do not analyze economic concepts such as the nature and magnitude of externalities or the welfare impacts of specific policies. A few studies have combined distributed parameter modeling of groundwater with economic analysis using simulation and linearization [22, 23]; however these studies are also calibrated to particular groundwater basins and thus results and policy implications are difficult to apply broadly.

In this paper, we take a different approach by incorporating analytical aquifer response equations directly into an economic optimization framework. We use a relatively simple and well known response equation for confined aquifers, the Theis equation [27]. Although this entails several simplifying assumptions, the first-order behavior embodied in the Theis equation represents realistic groundwater flow much more closely than the single-cell aquifer models currently used in economic analyses. The advantage of using an analytic expression for groundwater flow is that completely general economic optimality conditions can be derived and analyzed. This allows both explicit consideration of the role of hydrologic parameters in the optimal economic management of groundwater and a direct comparison with existing economic models of groundwater extraction that use single-cell models. Using hydrological parameter data for several aquifers that have been studied by economists, we show that in many cases, the optimal pumping behavior predicted by single-cell models and our spatially explicit model differ by a large amount. This implies that care should be exercised when using single-cell models to analyze the economic effects of alternative groundwater management policies.

The paper is organized as follows. Section 2 describes how transient flow equations for a confined aquifer may be used to generate equations of motion for aquifer potentiometric surfaces when there are multiple wells and non-constant pumping.[1] Section 3 derives optimality

[1]The material in this section is familiar to water resources engineers and is included because it is completely unfamiliar to most economists.

conditions for the welfare maximization problem involving multiple groundwater users and spatially explicit groundwater flow. The following section analyzes the steady state externality as a function of hydrological parameters and distance from a pumping well. An extension of the model to allow economic analysis of flow in unconfined aquifers is presented in Section 5. Section 6 compares published estimates of the economic impact of the groundwater externality with estimates calculated for the same aquifers using the spatially explicit flow model developed in this paper, and discusses the policy implications of differences in these estimates. Finally, Section 7 concludes.

2 Transient well response to pumping

Theoretical analyses of groundwater flow in the water resources engineering and hydrology literature are based on the physics of water flow towards a well during pumping, with water flowing from regions of higher potential to those with lower potential.[2] Theis [27] was the first to derive an analytical solution for transient well response to pumping. In a well-known result, he showed that for a well pumping water at a constant rate u from a confined aquifer[3] with storativity S and transmissivity T,[4] the drawdown x, at a distance r from the well, at time t after pumping commences is given by

$$x_t(r) = \frac{u}{4\pi T} \int_{\frac{r^2 S}{4Tt}}^{\infty} \frac{e^{-z}}{z} dz \tag{1}$$

[2]See Domenico [9], Freeze and Cherry [13], or Willis and Yeh [28] for more detailed derivations of the groundwater flow equations.

[3]For analytical tractability, Theis assumed that the aquifer is horizontal, has infinite areal extent, is of constant thickness with impermeable layers above and below, and is homogeneous and isotropic. He also assumed that the pumping well penetrates the entire depth of the aquifer, has an infinitesimal diameter, and that before the start of pumping, hydraulic head is uniform throughout the aquifer.

[4]The storativity of a confined aquifer is the volume of water released from storage per unit of surface area per unit decrease in the hydraulic head. Storativity is dimensionless and may be thought of as the capacitance of the aquifer. The storativities of confined aquifers are generally in the range 0.00005 to 0.005. Aquifer transmissivity is defined as the hydraulic conductivity of the aquifer multiplied by its thickness, where the hydraulic conductivity is a constant of proportionality relating specific discharge from a region to the hydraulic gradient across it. The range of values of observed transmissivities varies enormously depending on formation lithology, sedimentology, and fracturing. In this study we consider transmissivities in the range from 100 ft^2/day to 100,000 ft^2/day, which encompasses values generally found in aquifers used as significant water supplies.

For simplicity of notation, we abbreviate the integral in equation (1) as $w(t, r)$, with storativity and transmissivity taken as constant; the function $w(t, r)$ is often referred to as the well function in the hydrological literature.

The Theis solution assumes a single pumping well and constant pumping rates. However, it can easily be extended to include both pumping rates that vary through time and multiple wells [9]. Because the underlying transient flow equations are linear in pumping rate, arithmetic summation of independent well functions can be used to calculate the drawdown through time at any point in the aquifer with multiple wells whose pumping rates vary. For example, if there are J wells pumping at constant rates $u^1, u^2, \ldots, u^J$ with well j starting to pump at time t_j, then for a point that is at distances $r_1, r_2, \ldots, r_J$ from the pumping wells, drawdown at time $t > \max[t_1, \ldots, t_J]$ is given by

$$x_t(r_1, r_2, \ldots, r_J) = \frac{u^1}{4\pi T} w(t_1, r_1) + \frac{u^2}{4\pi T} w(t_2, r_2) + \cdots + \frac{u^J}{4\pi T} w(t_J, r_J) \tag{2}$$

Superposition may also be used for the case of a single well with pumping rates that change through time. Consider a well j with initial pumping rate u_1^j at time t_1, changing to rates of $u_2^j, u_3^j, \ldots, u_N^j$ at times $t_2, t_3, \ldots, t_N$. Assuming that no pumping occurs before t_1 (so that $u_0^j = 0$), the drawdown at a distance r from the pumping well at time $t > t_N$[5] is given by

$$x_t^j(r) = \frac{u_1^j}{4\pi T} w(t - t_1, r) + \frac{u_2^j - u_1^j}{4\pi T} w(t - t_2, r) + \cdots + \frac{u_N^j - u_{N-1}^j}{4\pi T} w(t - t_N, r) \tag{3}$$

Intuitively, equation (3) is derived from equation (2) by assuming that there are a sequence of wells pumping different amounts, but all located in exactly the same place.[6]

[5] For $t \leq t_N$, equation (3) does not include well functions for which the first argument is zero or negative, as future pumping changes do not affect the current state of the aquifer.

[6] Note that as $r \to 0$, $x_t(r) \to \infty$. In order to calculate the drawdown at a given wellhead from pumping at that well, a value of r equivalent to the effective well radius is used.

Following from equations (1) to (3), the drawdown at any point in an aquifer depends on both the location and sequence of *all* past pumping. Thus, correct specification of the potential surface of the aquifer at any point in time requires that both the location and the entire pumping history of each well be made explicit. Equations (2) and (3) may be combined to give the drawdown at any point in an aquifer resulting from pumping by multiple wells with variable pumping rates through time.

3 Optimal groundwater pumping

Consider an aquifer from which water is to be extracted by J separate users over an N-period time horizon. These users are spatially distributed with known, fixed locations relative to each other and to the resource, and each owns a single well.[7] Each user $j = 1, \ldots, J$ extracts water at a rate u_t^j per time period, and for simplicity we assume that pumping rates are constant during each time period but may change between periods. The pumping lift at well j during period t is given by x_t^j. Note that in general, $x_t^i \neq x_t^j$, as the potential surface of the aquifer may vary across space based on the distribution and pumping rates of wells.

We define the per-period net benefit of each water user by the function $f(u_t^j, x_t^j)$, which captures both the benefits and costs of resource extraction. For simplicity, we assume that each water user is engaged in the same economic activity with the same scale of operation, and thus every user has an identical benefit function (though the realized benefits in any period may vary spatially as u_t^j and x_t^j vary across users). We assume that $f \geq 0$, $f = 0$ when $u = 0$, $\partial f/\partial u > 0$ and $\partial^2 f/\partial u^2 < 0$. Similarly, because the pumping lift at well j, x_t^j, is defined as a positive quantity, $\partial f/\partial x < 0$, as per-period benefits decrease as the pumping lift increases.

[7] We assume that both the number of resource users and their locations are exogenous. Incorporating endogenous well locations is beyond the scope of the current work, but for a genetic algorithm approach to a very simple well location problem, see Hsiao and Chang [16].

Finally, we also assume that $\partial^2 f/\partial x^2 \leq 0$, so that pumping costs increase at least linearly with depth.

The infinite horizon optimization problem that maximizes benefits for the entire aquifer is then given by

$$\max \sum_{t=1}^{\infty} \beta^t \sum_{j=1}^{J} f(u_t^j, x_t^j) \tag{4}$$

where β is the per-period discount factor, with $\beta < 1$.[8] If drawdown across the aquifer follows the Theis equation (1), then an equation of motion describing the aquifer surface at any point in space and time can be constructed from equations (2) and (3). Defining $r(i,j)$ as the distance between any two wells i and j, the potential surface at time $t+1$ at any well j, x_{t+1}^j, is given by

$$x_{t+1}^j = \sum_{n=1}^{t} \sum_{i=1}^{J} \frac{u_n^i - u_{n-1}^i}{4\pi T} w(t-n+1, r(i,j)) \tag{5}$$

Equations (4) and (5) are a constrained optimization problem for which the associated Lagrangian is

$$\begin{aligned} L \;=\;& \sum_{t=1}^{\infty} \beta^t \sum_{j=1}^{J} f(u_t^j, x_t^j) \\ & + \sum_{t=1}^{\infty} \sum_{j=1}^{J} \lambda_t^j \left[\left(\sum_{n=1}^{t} \sum_{i=1}^{J} \frac{u_n^i - u_{n-1}^i}{4\pi T} w(t-n+1, r(i,j)) \right) - x_{t+1}^j \right] \end{aligned} \tag{6}$$

The first order conditions for an interior solution are:

$$\frac{\partial L}{\partial x_s^l} \;=\; \beta^s \frac{\partial f(u_s^l, x_s^l)}{\partial x_s^l} - \lambda_{s-1}^l = 0 \tag{7}$$

[8] We have assumed that no pumping occurs before $t = 1$.

$$\begin{aligned}\frac{\partial L}{\partial u_s^l} &= \beta^s \frac{\partial f(u_s^l, x_s^l)}{\partial u_s^l} + \sum_{j=1}^{J} \frac{\lambda_s^j}{4\pi T} w(1, r(l,j)) \\ &\quad + \sum_{t=s+1}^{\infty} \sum_{j=1}^{J} \frac{\lambda_t^j}{4\pi T} \left[w(t-s+1, r(l,j)) - w(t-s, r(l,j)) \right] = 0 \qquad (8)\end{aligned}$$

By definition, $w(0, r(l,j)) = 0$ for all l and j, so that first order condition (8) may be rewritten in more compact form as

$$\begin{aligned}\frac{\partial L}{\partial u_s^l} &= \beta^s \frac{\partial f(u_s^l, x_s^l)}{\partial u_s^l} \\ &\quad + \sum_{t=s}^{\infty} \sum_{j=1}^{J} \frac{\lambda_t^j}{4\pi T} \left[w(t-s+1, r(l,j)) - w(t-s, r(l,j)) \right] = 0 \qquad (9)\end{aligned}$$

The adjoint variable λ_t^j is the marginal present value shadow price of the state variable at well j at time t. For the optimization problem stated in (4) and (5), λ_t^j gives the change in the present value of total benefits if the pumping lift at well j at time t *increases* by one unit; consequently the shadow price is negative. Equivalently, λ_t^j can be interpreted as the marginal present value of the groundwater pumping externality. Rearranging first order condition (7) yields an expression for λ_t^j. Then, substituting for λ_t^j in (9) and dividing through by β^s gives the following abbreviated optimality condition:

$$\frac{\partial f(u_s^l, x_s^l)}{\partial u_s^l} = -\sum_{t=s}^{\infty} \sum_{j=1}^{J} \frac{\beta^{t-s+1}}{4\pi T} \frac{\partial f(u_{t+1}^j, x_{t+1}^j)}{\partial x_{t+1}^j} \left[w(t-s+1, r(l,j)) - w(t-s, r(l,j)) \right] \quad (10)$$

As shown in equation (1) and discussed above, any change in pumping will have effects that vary across both space and time, and thus both the spatial and temporal variation of drawdown caused by ongoing pumping must be considered in any optimal management scheme. The difference $(1/4\pi T)[w(t+1, r(l,j)) - w(t, r(l,j))]$ captures the incremental drawdown caused at well j by an additional unit of pumping at well l between time periods t and $t+1$. Equation (10) equates the marginal benefit of pumping an additional unit of water in any period to the discounted sum of marginal costs imposed on all wells, in all future periods, as a result of that additional unit of pumping. It is clear from (10) that the choice of optimal pumping trajectories for a group of wells pumping from a common aquifer depends directly on both their spatial distribution and the hydrological properties of the aquifer. The magnitude of possible externalities across space and time is considered in the next section.

4 Analysis of pumping externalities

Given explicit spatial locations for each pumping well, an appropriate benefit function, and hydrological parameters, equation (10) allows solution of the optimal pumping trajectories. However, even without specifying either a benefit function or well locations, the optimal steady state can be used to analyze how groundwater pumping externalities vary across space. Given the model assumptions, every finite combination of constant pumping rates $u^1, u^2, \ldots, u^J$ will lead towards a steady state with associated pumping lifts $x^1(u^1, u^2, \ldots, u^J), x^2(u^1, u^2, \ldots, u^J), \ldots, x^J(u^1, u^2, \ldots, u^J)$. Defining the pumping combinations at the optimal steady state as $\mathbf{u}_* = \left[u_*^1 \; u_*^2 \; \ldots \; u_*^J\right]$, the associated steady state pumping lifts are $x^1(\mathbf{u}_*), x^2(\mathbf{u}_*), \ldots, x^J(\mathbf{u}_*)$. From (10), the steady state optimality condition for each well is then easily obtained:

$$\frac{\partial f(u_*^l, x^l(\mathbf{u}_*))}{\partial u_*^l} = -\sum_{j=1}^{J} \frac{\partial f(u_*^j, x^j(\mathbf{u}_*))}{\partial x^j(\mathbf{u}_*)} \sum_{t=1}^{\infty} \frac{\beta^t}{4\pi T} \left[w(t+1, r(l,j)) - w(t, r(l,j))\right] \tag{11}$$

Note that because the term $\partial f(u_*^j, x^j(\mathbf{u}_*))/\partial x^j(\mathbf{u}_*)$ is time-invariant, it can be passed through one of the summations. Using equation (7), $\partial f(u_*^j, x^j(\mathbf{u}_*))/\partial x^j(\mathbf{u}_*)$ can be interpreted as the current value marginal shadow price of the groundwater externality at well j: it is negative and gives the per-period loss of benefit at well j from increasing the pumping lift by one unit. Thus, equation (11) relates the optimal steady state marginal value of pumping at well l to the discounted marginal cost imposed on *all* groundwater users by that additional unit of pumping. As shown in (10) and (11), a marginal increase in pumping in any time period will have an effect on pumping lifts throughout the aquifer in all future periods. Equation (11) defines the present value marginal cost imposed on user j as a result of a marginal increase in pumping at well l as the product of the shadow price of the pumping lift and the discounted marginal drawdowns imposed in all future time periods.

The summation $\sum_{t=1}^{\infty}(\beta^t/4\pi T)[w(t+1, r(l,j)) - w(t, r(l,j))]$ can be interpreted as a weighting function that determines – in the optimal steady state – the relative importance of the spatial and temporal interaction between users. For each well site l, the weighting function represents the economic importance placed by user l on externalities imposed on all resource users as a result of l's pumping. It is clear from (11) that as the potential influence of one well's pumping on another well's drawdown decreases, so the influence of that hydrological linkage on optimal steady state pumping at each well also decreases.

Analysis of the weighting function gives insight into the role of aquifer hydrological parameters and spatial relationships between users in determining the optimal steady state. Note that a closely related function, $\sum_{t=1}^{\infty}(1/4\pi T)[w(t+1, r(l,j)) - w(t, r(l,j))]$, is the total drawdown caused at well j by a unit of pumping at well l. However, because the optimization problem (4) discounts future benefits, incremental future drawdowns are also discounted in the determination of optimal pumping rates at each well.

Figures 1, 2, and 3 show contour maps of the optimal steady state weighting function at distances of 1.5 feet (taken to be the effective well radius), half a mile, and five miles from a pumping well, respectively. Values of the weighting function were calculated using a discount rate of five percent, time increments of thirty days, and a constant pumping rate in each well equal to one acre foot per year.[9] A comparison of Figures 1 through 3 shows how distance from a pumping well, transmissivity, and storativity jointly determine the economic importance of

[9] Two thousand time periods, equivalent to 164 years of constant pumping, were used in calculating the weighting function. The difference in calculated weighting functions between $N = 1999$ and $N = 2000$ was in the range of 1.4×10^{-11} to 1.4×10^{-8} across the parameter space and distances considered.

the groundwater externality.[10]

Figures 1 through 3 all show a similar basic pattern of changes in the weighting function as storativity and transmissivity change. Given a constant value of transmissivity, as storativity increases, the weighting function decreases at all distances from a pumping well. This is an intuitive result: the figures represent pumping at a constant rate, and as storativity increases, the drawdown caused by this pumping will decrease, so that the weighting function will also decrease. Analytically, it is straightforward to show that the derivative of the weighting function with respect to S is always negative. Conversely, the derivative of the weighting function with respect to T is ambiguous in sign. This is reflected in changes in the weighting function, given a constant storativity, as transmissivity increases. From Figures 1 through 3, at distances of 1.5 feet and half a mile, the weighting function decreases monotonically as transmissivity increases. However, at a distance of five miles, the weighting function decreases as transmissivity increases for lower storativity values and first increases and then decreases for higher storativity values. This result is related to the geometry of the cone of depression. As transmissivity increases, the cone of depression broadens and shallows. For points that are at a relatively large distance from a pumping well, this may translate to an increase in drawdown, and thus an increase in the weighting function.

From equation (1) it is clear that for well sites j and l, $w(t, r(l,l)) > w(t, r(l,j))$ for all t and $j \neq l$, so that any well's own-effects of pumping are always larger than the effects transmitted to neighboring wells. This result follows immediately from the geometry of a cone of depression, which is centered at the well head of the pumping well. Thus, as shown in Figures 1 through 3, for any given combination of storativity and transmissivity, the weighting func-

[10] Note that for the parameter space of storativity and transmissivity shown in Figures 1 through 3, the difference between calculated values of the weighting function $\sum_{t=1}^{\infty}(\beta^t/4\pi T)[w(t+1, r(l,j)) - w(t, r(l,j))]$ and the total drawdown $\sum_{t=1}^{\infty}(1/4\pi T)[w(t+1, r(l,j)) - w(t, r(l,j))]$ varied between 9.4 percent and 15 percent at a distance of 1.5 feet, increasing to between 27 percent and 700 percent at a distance of five miles. The increasing difference with distance is a result of the increasing time lag for significant drawdown to be transmitted as distance increases.

tion decreases with distance away from a pumping well. For the parameter space considered, representing the range of hydrological parameters found in confined aquifers commonly used as sources of water, the own-weighting function is two to five times more than the weighting function at a distance of half a mile from a pumping well, and three to ninety-eight times more than the weighting function at a distance of five miles. The largest relative variation in weighting function with distance – approximately corresponding to the tightest, though not the deepest, cone of depression – occurs when storativity is high and transmissivity is low.

5 Extension to optimal pumping from unconfined aquifers

The analysis presented in the previous sections assumes that the aquifer of interest is confined, allowing the Theis solution (equation (1)) to be incorporated directly into tractable optimality conditions describing the operation of multiple pumping wells across space and time. However, many aquifers of interest to groundwater managers and policy makers are unconfined, with an upper boundary that is a free surface. Because transient flow in an unconfined aquifer involves interaction between flow in the saturated and unsaturated zones and a dynamically moving boundary (the water table), most solutions for transient unconfined flow involve complex numerical methods [28]. In general, such numerical solution concepts are difficult to incorporate in an economic framework that seeks to analyze optimal groundwater extraction at a general level.

Early studies of transient flow in unconfined aquifers (e.g. Boulton [3], Neuman [20]) suggested that early-time behavior in such systems undergoing pumping follows the Theis solution with relevant hydrological parameters T and S (the transmissivity and storativity, respectively), whereas late-time behavior follows the Theis solution with hydrological parameters T and S_y (the specific yield, replacing storativity).[11] At intermediate times, the drawdown behavior in

[11] For an unconfined aquifer, the specific yield is defined as the volume of water drained by desaturation of the aquifer from a column of unit base area. In unconfined aquifers, storativity and specific yield broadly refer to

an unconfined aquifer is between early- and late-time solutions, where the transition between behaviors is defined by several other parameters of the system being studied, such as thickness of the saturated aquifer. With the additional assumption that the depth of water in the aquifer is large compared to observed drawdowns, superposition may be applied as in equation (2) to calculate drawdown from multiple wells [28].

The processes by which water is released from storage in unconfined aquifers are different from those operating in confined aquifers. In a confined aquifer, the amount of water released from storage depends on the compressibility of water and the porous media, and not pore space. Conversely, in an unconfined aquifer, water is released by gravity from saturated pore space in response to water level gradients in the aquifer. As a result, encountered values of specific yield, S_y, are in the range 0.05 to 0.3, which is much larger than the range of storativity values for confined aquifers. As specific yields govern the behavior of unconfined aquifers over longer timescales, we can use them to calculate steady state weighting functions as before by replacing values of storativity with specific yield as appropriate. Note that in doing so, we ignore short-term behavior where water table drawdown follows the Theis solution with parameters S and T. Because storativity values are always much smaller than specific yields, this assumption means that our estimates of the unconfined steady state weighting function may be thought of as lower bounds.

Figures 4, 5, and 6 show contour maps of the optimal steady state weighting function for an unconfined aquifer at distances of 1.5 feet (taken to be the effective well radius), half a mile, and five miles from a pumping well, respectively. As in the previous figures, values of the weighting function were calculated using a discount rate of five percent, time increments of thirty days, and a constant pumping rate in each well equal to one acre foot per year.[12] The

water released from storage by different processes.

[12] As with confined aquifers, two thousand time periods were used in calculating the weighting function. The difference in calculated weighting functions between $N = 1999$ and $N = 2000$ was in the range of 1.5×10^{-12} to 1.4×10^{-8} across the parameter space and distances considered.

calculated weighting functions for unconfined aquifers show a very similar basic pattern to that seen in Figures 1 through 3 for confined aquifers. For any given transmissivity, the steady state weighting function decreases as specific yield increases, and weighting functions decrease with distance from the pumping well. Because specific yields found in unconfined aquifers are much larger in value than storativities found in confined aquifers, at any particular transmissivity and distance from a pumping well, the weighting function in an unconfined aquifer will be less than that in a confined aquifer. The relative difference between aquifer types is largest at the largest distances. Overall, this means that the spatial extent of the groundwater externality is less in unconfined aquifers than it is in confined aquifers.

6 Modeling and policy implications

Most economic studies of optimal groundwater management have relied on single-cell aquifer models (for example, Burness and Brill [7], Feinerman and Knapp [12], and Gisser and Sanchez [15] among many others; see Koundouri [18] for a comprehensive overview). Single-cell aquifers are lumped parameter models in which the state of the groundwater resource is captured by a single parameter, usually either the total volume of water remaining in the aquifer or the pumping lift. Implicit in the single-cell aquifer are two related assumptions about the nature of both the groundwater resource and of pumping externalities. First, because only one parameter describes the resource state, the pumping lift in the aquifer is assumed to be constant at every point in the aquifer. Second, spatial location of wells does not matter, and a unit of water withdrawn from the aquifer will have the same marginal impact at every point in the aquifer – including the well at which that pumping occurs.

Despite modeling groundwater as a common property resource, economic analyses generally suggest that the quantitative difference between myopic and socially optimal groundwater management outcomes is either very small or negligible. Thus, in contrast to the everyday perception of groundwater overextraction and depletion, existing empirical studies imply that there is little *economic* rationale for public intervention in groundwater management. However, the validity of this conclusion depends critically on the extent to which single-cell models accurately reflect the responses of real aquifers – with features such as cones of depression, well interference, and heterogeneous well distributions across space – to pumping. In this paper, we have developed an economic model of groundwater management that explicitly incorporates the spatial nature of the groundwater pumping externality. Does such a model produce implications for groundwater management policy different to those emerging from single-cell models?

A simple way to compare single-cell aquifer models with the spatially explicit model presented in this paper is to consider a steady state weighting function, analagous to that derived in Section 4, for the single-cell aquifer. Recall that the function $\sum_{t=1}^{\infty}(\beta^t/4\pi T)[w(t+1, r(l,j)) - w(t, r(l,j))]$ represents the weight placed on the marginal pumping externality imposed by user l's pumping on user j in the steady state optimality condition (11), and is the sum of discounted incremental drawdowns. In a confined single-cell aquifer with surface area A and storativity S, the potential surface will be *instantaneously* lowered by an amount equal to $1/AS$ when one unit of water is pumped [11]. Thus, all effects from drawdown are transmitted throughout the aquifer in the following period, and only one future period needs to be considered, so that the single-cell optimal steady state weighting function for a confined aquifer is given by β/AS, where β is the per-period discount factor.[13] Similarly, for an unconfined aquifer, the single-cell steady state weighting function is given by β/AS_y.

Table 1 shows storativities or specific yields and calculated values of β/AS or β/AS_y for six aquifers that have been previously analyzed using single-cell models. These range from 3.81×10^{-5} to 9.90×10^{-3} for confined aquifers and from 1.48×10^{-6} to 1.06×10^{-5} for unconfined aquifers. Because a pumping rate of one acre foot per year was used to generate

[13] For example, see Rubio and Casino [25]. Published literature generally uses pumping rates of acre feet per year and time periods of years.

the values in both Table 1 and Figures 1 through 6, the values of weighting functions may be compared directly. It is clear that in general, a spatially explicit groundwater model predicts that at the optimal steady state, groundwater users place a much higher weight on the effects of their pumping on their neighbors. This follows directly from the larger drawdowns modeled across space using equation (1) rather than a single-cell aquifer model. Perhaps surprisingly, however, even at a distance of five miles from a pumping well, the predicted effects of the groundwater pumping externality are generally much larger for the spatially explicit model than for the single-cell model, which assumes uniform drawdown across the entire aquifer. This result is driven by the large length-scale of some of the aquifers modeled as single cells. For example, the unconfined single-cell Roswell Basin is taken to have a surface area of 790,000 acres (1,200 square miles) [14], the Kern County unconfined aquifer to have a surface area of 1,290,000 acres (2,000 square miles) [12], and the Texas High Plains aquifer a surface area of 4,300,000 acres (6,700 square miles) [21]. When aquifers of this size are modeled using the single-cell assumption, the drawdown resulting from each marginal unit of pumping is spread over an extremely large area (as shown in Table 1).

In order to undertake a more precise comparison between single-cell and spatially explicit models, it is necessary to define aquifer transmissivities. Although some studies view single-cell aquifers as having an infinite transmissivity, strictly speaking transmissivity has no physical meaning in a single-cell aquifer. This is because the state of a single-cell aquifer is fully described by a single parameter (volume or depth to water), so that there is no length-scale defined. As a result, most economic studies using single-cell models do not report transmissivity estimates. However, as many of the groundwater basins in Table 1 have been studied extensively by hydrologists as well as economists, it is possible to estimate transmissivity for them [24, 26]. With a relevant range of transmissivity defined, the ratio of spatially explicit to single-cell weighting functions, $\sum_{t=1}^{\infty}(\beta^t/4\pi T)[w(t+1,r(l,j))-w(t,r(l,j))]/(\beta/AS_{(y)})$ can then be calculated and is an estimate of the extent to which a single-cell aquifer model over- or understates the economic importance of the externality due to groundwater pumping.[14] Table 2 shows values for transmissivity and weighting function ratios for four of the aquifers considered in Table 1. For these aquifers, the importance of the externality implied by spatially explicit and single-cell models varies dramatically.

For the confined carbonate-rock aquifer of the Roswell Basin in New Mexico, a single-cell model understates by a large amount the economic impact of the groundwater pumping externality at all relevant distances (Table 2). Once again, the pattern observed as distance from a well increases follows the basic geometry of a cone of depression: the greatest difference between spatially explicit and single-cell models is at the wellhead of a pumping well. For example, the spatially explicit model predicts that the impact on the steady state optimality condition of the groundwater pumping externality is 120 to 824 times more at the wellhead than the single-cell model. At a distance of five miles, the spatially explicit weighting function is still 24 to 218 times larger than the single-cell weighting function. Even at a distance of twenty miles from a pumping well, the economic impact of the drawdown predicted by a spatially explicit model is 11 to 132 times more than that predicted by a single-cell model.

Conversely, for the confined Crow Creek Valley aquifer in Montana, the agreement between spatially explicit and single-cell models is much closer. At the wellhead of a pumping well, the estimated impact of pumping is 1.78 times greater with a spatially explicit model than with a single-cell model. At a distance of half a mile, the impact predicted by a single-cell model is nearly fifty percent larger than that predicted by the spatially explicit model, and as distance increases further, a single-cell model overstates the externality compared to a spatially explicit model even more. What explains the major difference between results for the Crow Creek Valley aquifer and Roswell Basin aquifers, given that their transmissivities are in the same range?

[14]Note that an annualized discount rate of five percent is used in both calculations, but a time period of thirty days was used for the spatially explicit model and a time period of one year was used for the single-cell model, so that the per-period discount factors β in the numerator and denominator of the weighting function ratio are not equal.

With a surface area of 60,000 acres (94 square miles) the Crow Creek Valley aquifer is quite small [29], so that the assumption that effects of pumping anwhere within the aquifer are transmitted equally everywhere in the aquifer is approximately correct. Results from the spatially explicit model suggest that this assumption is not valid for the confined portion of the much larger Roswell Basin aquifer system.

For unconfined aquifers, the difference between estimates of the economic impact of the groundwater externality are even larger at small distances (Table 2). In particular, except at large distances from a pumping well, the impact of the externality predicted using a spatially explicit model may be several orders of magnitude larger than that predicted using a single-cell model. As before, the spatially explicit weighting function decreases with distance. In comparison to the relevant single-cell weighting function, for the two aquifers considered (the Roswell Basin in New Mexico and the Texas High Plains Aquifer) the spatially explicit weighting function is 1577 to 46208 times larger at the wellhead, 391 to 8170 times larger at a distance of half a mile, and 11 to 110 times larger at a distance of ten miles. Finally, at a distance from a pumping well of twenty miles, the single-cell weighting function may be either smaller or larger than the spatially explicit weighting function.

One of the major empirical findings of studies of the economics of groundwater extraction is that the ability of any public intervention – such as pumping taxes, pumping quotas, or basin adjudication – to increase social welfare is very limited [15, 18]. This finding follows directly from the very small estimated impact of the groundwater externality in single-cell aquifer models. This paper demonstrates that when groundwater is modeled as a spatially explicit resource, using equations from the engineering literature that describe the transient response of aquifers to pumping and the resulting gradients in potential, estimated externality impacts may be orders of magnitude higher than those calculated with single-cell models. If this is the case, then the user costs associated with ongoing pumping of groundwater, which are negligible in single-cell models, will also be significant in spatially explicit models. Thus, at least in the case of large aquifers where wells are spaced a few miles apart or less, it is likely that when a spatially explicit model is used for economic analysis, there may be large welfare gains from optimal groundwater management when compared with myopic or non-cooperative strategic outcomes. On the other hand, for small aquifers of limited areal extent, single-cell models may be adequate for rough calculations of the welfare effects of changes in groundwater management policy.

Note that the location at which single-cell models underestimate drawdown most when compared with a spatially explicit model is at the wellhead. In an economic optimization framework that includes non-cooperative strategic behavior by individual pumpers (rather than myopia), simple intuition might suggest that if the externality from one's own pumping is largest at one's own wellhead, then knowledge of this would reduce the overall externality; it is possible that this reduction would be enough once again to close the gap between scenarios with and without socially optimal policies in place. In an aquifer with a very small number of users, this may well be the case, but if there are hundreds or thousands of well users, then even with a spatially explicit model, the contribution of each individual to the externality is small. If *most* users of an aquifer are influencing each other more than a single-cell model would suggest (for example, as shown in Table 2 for the large aquifers, for well spacings of the order of ten miles or less), then it still possible to have large gains from optimal groundwater management. Further investigation of the welfare gains from management in a spatially explicit groundwater model requires the location of well sites in relation to each other in space, and is left to future work.

7 Conclusion

Most existing analyses of optimal groundwater management in the economic literature use single-cell aquifer models, which assume that an aquifer responds uniformly and instantly to groundwater pumping. This paper demonstrates how spatially explicit aquifer response equations from the water resources engineering literature may be embedded in a general economic framework. Using this framework, we develop and describe an analytical expression that is the sum of discounted future economic impacts of the marginal groundwater externality imposed by one groundwater user on all users. Because this analytical expression represents a weighting function in the steady state optimality conditions, it links relevant hydrological parameters and distance from a pumping well to the marginal benefits and costs of groundwater use.

The model presented in this paper may be compared with the results of existing economic studies in specific aquifers. Comparison of the economic impacts of groundwater pumping implied by single-cell and spatially explicit models suggests that for many aquifers, single-cell models understate the groundwater pumping externality relative to a spatially explicit model. In particular, in aquifers that have large surface areas, such as the Roswell Basin in New Mexico or the Texas High Plains Aquifer, estimated externality impacts with a spatially explicit model may be thousands or tens of thousands of times more than those calculated with a single-cell model. Our results suggest that single-cell models may be appropriate for analyses of the welfare effects of groundwater management policies either in small aquifers or in larger aquifers were average well spacings are tens of miles or more. However, in extensive aquifers where well spacings are on the order of a few miles or less, such as many of those of concern to groundwater managers and policy makers, use of single-cell models may result in misleading policy implications due to understatement of the magnitude and spatial nature of the groundwater externality.

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Table 1 Estimated steady state weighting functions based on single-cell aquifer models in existing studies

Study site [Citation(s)]	**Aquifer type**	S **or** S_y	**Weighting function** β/AS **or** β/AS_y [a]
Roswell Basin, NM [14]	Confined	$0.0001 - 0.005$ [b]	3.81×10^{-5}
Crow Creek Valley, MT [29]	Confined	0.0016	9.90×10^{-3}
Roswell Basin, NM [1, 14, 15, 18]	Unconfined	0.15	$7.05 \times 10^{-6} - 7.94 \times 10^{-6}$
High Plains Aquifer, TX [17, 21]	Unconfined	0.15	1.48×10^{-6}
Ogallala Aquifer, NM [5, 7]	Unconfined	0.15	1.06×10^{-5}
Kern County, CA [11, 12]	Unconfined	0.10	7.38×10^{-6}

[a] In calculating the implied weighting function, A was taken as the reported surface area of the aquifer, and a pumping rate of one acre foot per year was used. It was assumed – as is the case in existing all single-cell models – that all drawdown occurred instantaneously, so that only one time period needs to be considered. An annual discount rate of five percent was used.

[b] Gisser and Mercado [14] report that storativity of the confined carbonate aquifer in the Pecos Basin is 'negligible'. The reported range of storativities are for the equivalent San Andres formation in the nearby Upper Rio Hondo Basin, NM [10].

Table 2 Comparison of steady state weighting functions for single-cell and spatially explicit aquifer models

Study site	**Transmissivity** (ft^2/day)	**Distance** $r(l,j)$	$\frac{\sum_{t=1}^{\infty} \frac{\beta^t}{4\pi T}[w(t+1,r(l,j)) - w(t,r(l,j))]}{\beta/AS_{(y)}}$ [a]
Roswell Basin, NM Confined	$8000 - 50000$ [b]	Wellhead	$120 - 824$
		0.5 miles	$47 - 360$
		5 miles	$24 - 218$
		10 miles [c]	$18 - 174$
		20 miles	$11 - 132$
Crow Creek Valley, MT [d] Confined	13000	Wellhead	1.78
		0.5 miles	0.68
		5 miles	0.35
Roswell Basin, NM Unconfined	$5000 - 15000$ [e]	Wellhead	$1577 - 5036$ [f]
		0.5 miles	$391 - 1036$ [f]
		5 miles	$67 - 93$ [f]
		10 miles	$11 - 22$ [f]
		20 miles	$0.27 - 2.34$ [f]
High Plains Aquifer, TX Unconfined	$2500 - 60000$ [g]	Wellhead	$2263 - 46208$
		0.5 miles	$671 - 8170$
		5 miles	$203 - 442$
		10 miles	$25 - 110$
		20 miles	$0.14 - 27$

[a] Calculated using values of storativity for the confined aquifers and specific yield for the unconfined aquifers.

[b] Taken from Robson and Banta [24] as the general range for the confined carbonate-rock aquifer in the Roswell Basin Aquifer System.

[c] Weighting functions for values of $r(l,j)$ of ten and twenty miles are reported, but not graphed, for the Roswell Basin aquifers and Texas High Plains aquifer, which have relevant surface areas of 1,200 square miles and 6,700 square miles, respectively [14, 21].

[d] Transmissivity value reported in Worthington *et al.* [29]. Because the surface area of the Crow Creek Valley aquifer is less than one hundred square miles, values of $r(l,j)$ greater than five miles are not considered.

[e] Taken from Robson and Banta [24] as the general range for the unconfined alluvial aquifer in the Roswell Basin Aquifer System.

[f] Lower bound of range calculated using higher estimate of 7.94×10^{-6} and upper bound of range calculated using lower estimate of 7.05×10^{-6} from Table 1.

[g] Transmissivity range calculated using reported general ranges of hydraulic conductivities (25 to 100 feet per day) and saturated thicknesses (100 to 600 feet) for the High Plains aquifer in Texas and Oklahoma [26].

Figure 1 Contour map of the steady state weighting function at the wellhead, $\sum_{t=1}^{\infty}(\beta^t/4\pi T)[w(t+1,r(l,j))-w(t,r(l,j))]$, for the range of hydrological parameters commonly found in confined aquifers. In this figure the effective well radius, and thus $r(l,j)$, is assumed to be 1.5 feet. The weighting function was calculated using a discount rate of five percent, time increments of thirty days, and a constant pumping rate in each well equal to one acre foot per year.

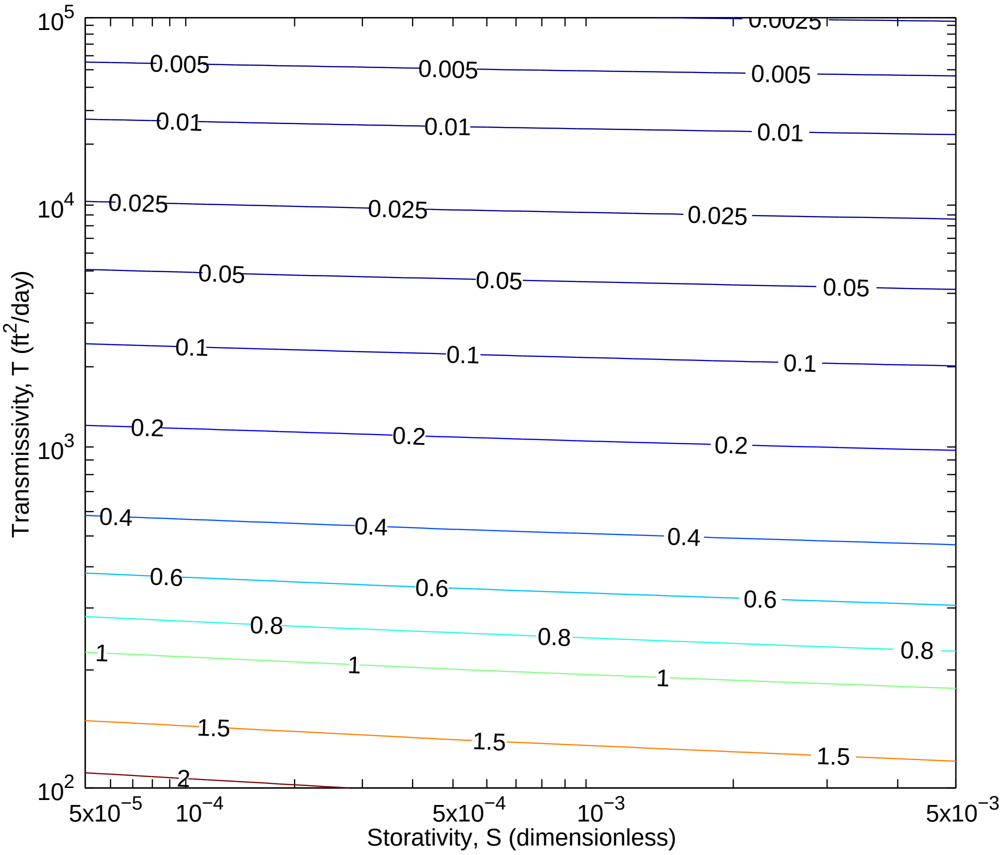

Figure 2 Contour map of the steady state weighting function at a distance of half a mile, $\sum_{t=1}^{\infty}(\beta^t/4\pi T)[w(t+1, r(l,j)) - w(t, r(l,j))]$, for the range of hydrological parameters commonly found in confined aquifers. In this figure $r(l,j)$ is assumed to be half a mile. Parameters used are the same as those in Figure 1.

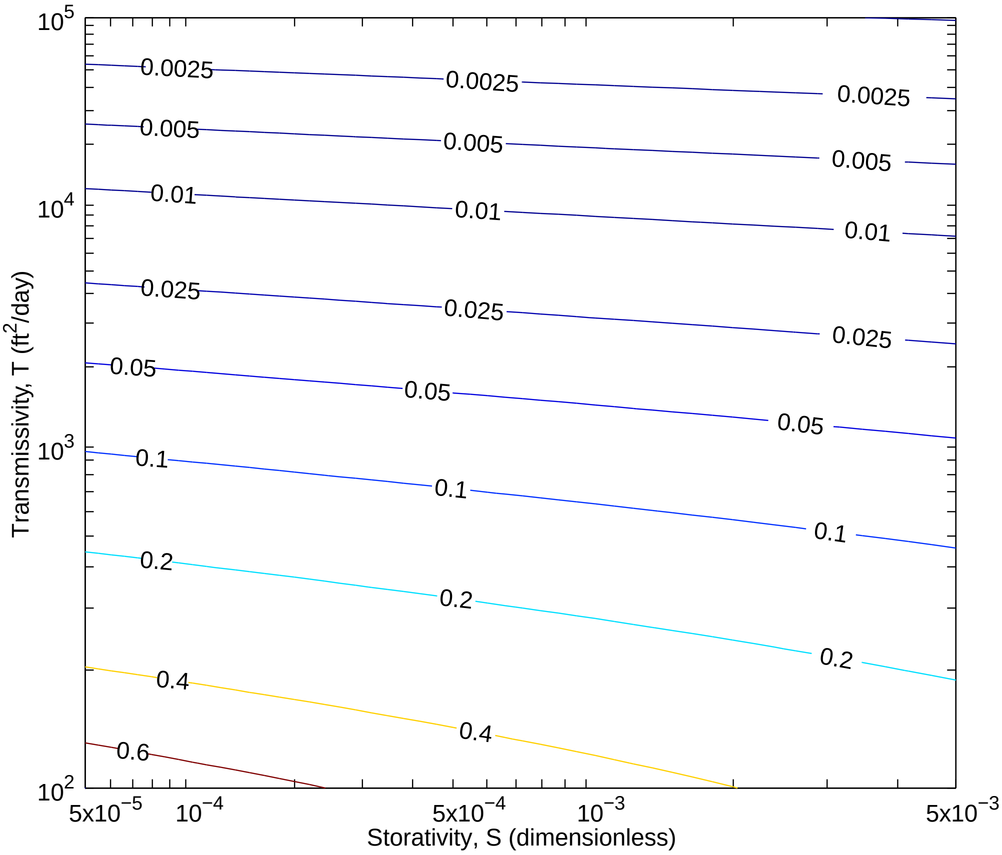

Figure 3 Contour map of the steady state weighting function at a distance of five miles, $\sum_{t=1}^{\infty}(\beta^t/4\pi T)[w(t+1,r(l,j))-w(t,r(l,j))]$, for the range of hydrological parameters commonly found in confined aquifers. In this figure $r(l,j)$ is assumed to be five miles. Parameters used are the same as those in Figure 1.

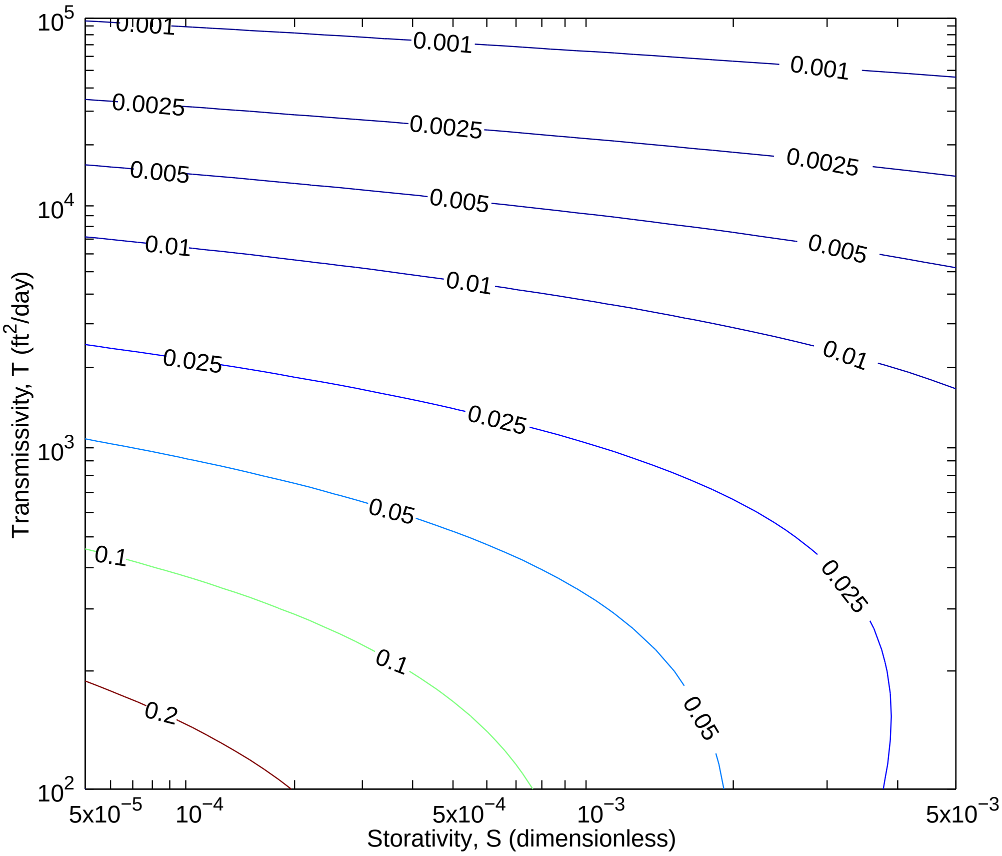

Figure 4 Contour map of the lower bound of the steady state weighting function at the wellhead, $\sum_{t=1}^{\infty}(\beta^t/4\pi T)[w(t+1,r(l,j))-w(t,r(l,j))]$, for the range of hydrological parameters commonly found in unconfined aquifers. In this figure the effective well radius, and thus $r(l,j)$, is assumed to be 1.5 feet. The weighting function was calculated using a discount rate of five percent, time increments of thirty days, and a constant pumping rate in each well equal to one acre foot per year.

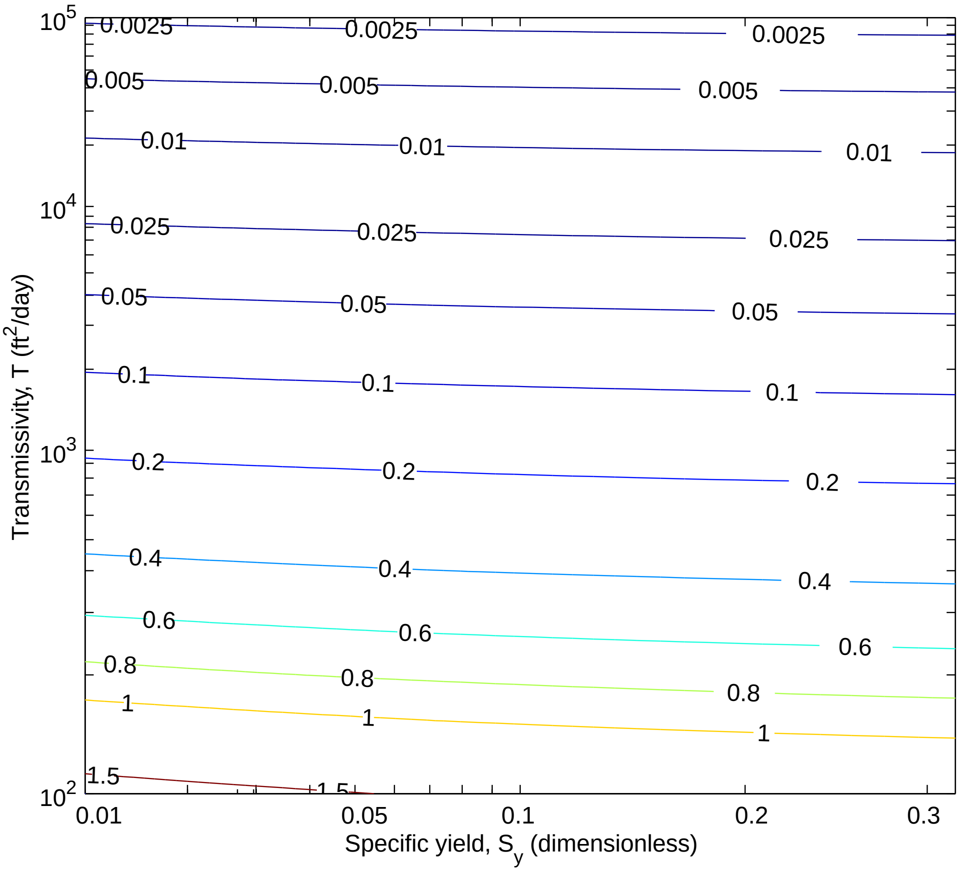

Figure 5 Contour map of the lower bound of the steady state weighting function at a distance of half a mile, $\sum_{t=1}^{\infty}(\beta^t/4\pi T)[w(t+1, r(l,j)) - w(t, r(l,j))]$, for the range of hydrological parameters commonly found in unconfined aquifers. In this figure $r(l,j)$ is assumed to be half a mile. Parameters used are the same as those in Figure 4.

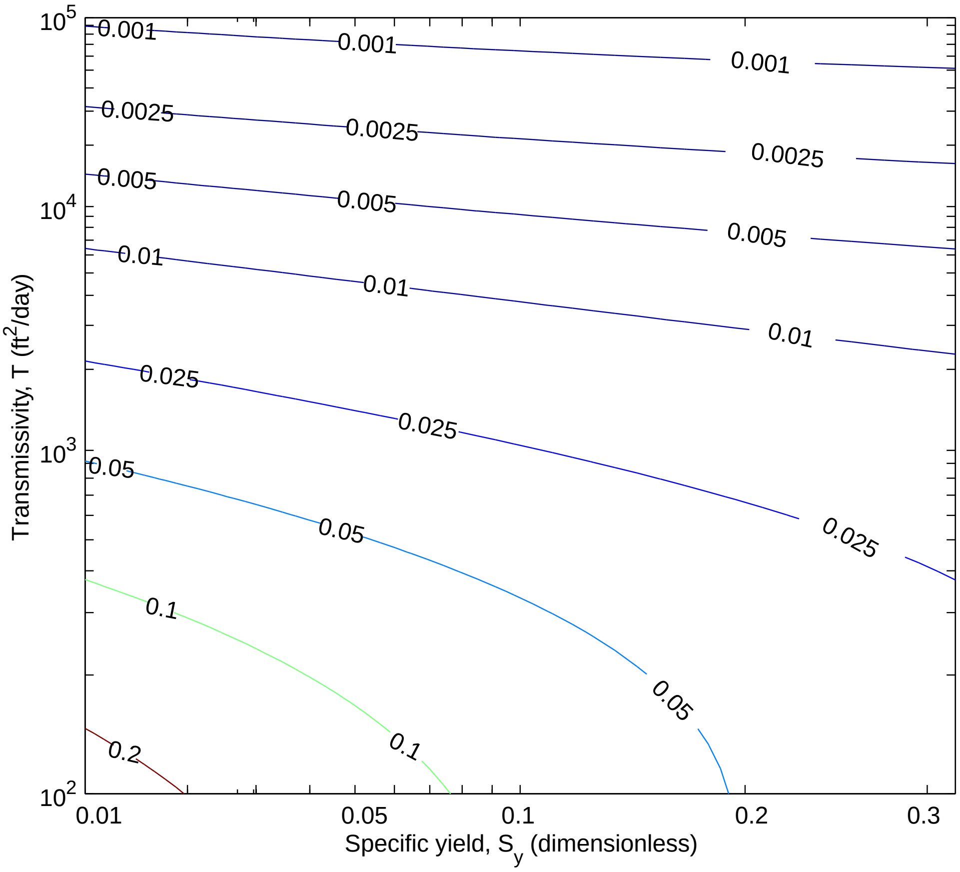

Figure 6 Contour map of the lower bound of the steady state weighting function at a distance of five miles, $\sum_{t=1}^{\infty}(\beta^t/4\pi T)[w(t+1,r(l,j)) - w(t,r(l,j))]$, for the range of hydrological parameters commonly found in unconfined aquifers. In this figure $r(l,j)$ is assumed to be five miles. Parameters used are the same as those in Figure 4.