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Commodity Costs and Returns Estimation Handbook

A Report of the AAEA Task Force on Commodity Costs and Returns

July 20, 1998

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CHAPTER 2

CONCEPTUAL ISSUES IN COST AND RETURN ESTIMATES

Cost and return (CAR) estimates are developed and used for a variety of purposes. In general, the objective is to accumulate or to develop information about costs and returns that can be used in making or analyzing decisions. Such decisions are made by individuals at the firm level or by society through their representatives. The appropriate procedures for calculating these estimates, the sources of data, and the format in which the estimates are presented depend upon both the question being addressed and the intended audience. This chapter discusses the major conceptual issues that influence the components of CARs, methods of calculation, and types of data used. The recommendations of the Task Force are shown in ***bold italics***.

DEFINITION OF AN ENTERPRISE

Commodity CARs in agriculture are commonly summarized by production enterprise. A **production enterprise**, referred to as an **enterprise** in this report, is any coherent portion of the general input-output structure of the farm business that can be separated and analyzed as a distinct entity. Such an entity uses inputs and incurs costs while producing products or services. The entity is usually defined based on a unit of measurement such as an input (sorghum production per acre of land or total pork production per sow), an output (a ton of peas or 100 board feet of lumber), or some fixed set of resources (orange production for a grove). The appropriate unit of measurement is often dependent on the use for which the estimates are intended.

A farm or ranch business can be divided into enterprises in several different ways depending on the products produced, the technology used, or the restrictions on the uses of various inputs. A common delineation of enterprises is along commodity lines (for example, the barley enterprise, the dairy enterprise, or the rice enterprise). In many instances such a neat division is not possible or not desirable. For example, there is not a meaningful way to separate barley grain and barley straw enterprises, or milk production and cull dairy cow enterprises. Similarly, given the rotation effects of growing corn and soybeans in sequence, there may be little economic sense in separating these entities even if it were feasible technically to do so. For some analyses, such as comparing labor use or revenue in crops versus livestock, the enterprises may be defined as broadly as crops and livestock. An enterprise can then consist of one of many entities: a single commodity such as apples or lettuce; double crops such as wheat and soybeans in the same year; different production practices for the same commodity such as no-till versus conventional till barley; multiple crops over several years such as corn and soybeans; a livestock feeding operation such as cattle or sheep; an integrated breeding and finishing operation such as farrow-to-finish swine; a production activity such as slaughter hogs with manure by-products; an add-on activity such as grazing of wheat pasture; a crop with a nurse crop enterprise such as alfalfa hay and oats; or a sideline activity such as custom harvesting. A given farm or ranch may well be divided into enterprises differently for different purposes of analysis. These examples show good cause for allowing for considerable flexibility in defining enterprises.

Chapter 2. Conceptual Issues in Cost and Return Estimates

The Task Force recommends that presentations of CAR estimates clearly indicate the unit of measurement and that they define the set of products, by-products, and/or the services generated by the enterprise.

TYPES OF CAR ESTIMATES AND THEIR USES

Cost and return estimates may be reported at many different levels of aggregation. While more specific definitions are presented in the next section, at the most basic level, a cost is simply the value of resources consumed, frequently given by the price of an input (such as the price of nitrogen fertilizer per ton), whereas a return is the value received (frequently in cash) for an economic good (such as the price of a ton of hay). Costs can be aggregated in many different ways. Examples of different cost aggregations include the cost of all fertilizer used in growing 800 boxes of bell peppers, the cash costs of producing a hundred weight of milk, the costs of rented land to the whole farming operation, the total costs of producing all the corn in Iowa, or the costs of labor in U.S. agriculture. Similarly, returns can be aggregated in different ways. One of the most common ways to aggregate CAR is by enterprise, but estimates can just as easily be made for aggregations other than enterprises. For example, aggregate U.S. net farm income is an estimate of the CAR to all U.S. agriculture during a given period.

Cost and return estimates can also be reported for different periods or points in time. Most commonly, CAR estimates are reported for the previous or the next production period. Estimates for a previous period are called **historical estimates** because they are based on actual costs and returns that were incurred over the period, while estimates for future periods are called **projections** because they are based on forecasted magnitudes. Record summaries prepared by accounting firms and management services are an example of historical estimates. The CAR summaries prepared the Economic Research Service (ERS) are another example of historical estimates. Projections are regularly made at the individual commodity and whole-farm levels (for production and financial planning) and at the sector level (projected farm income).

The diversity of information required for agricultural decision making has spawned the development of a variety of CAR estimation procedures and formats for presentation of results. Arguably, no particular CAR estimate is suitable for all purposes at all times.

The Task Force recommends distinguishing between historical and projected CAR estimates. The Task Force further recommends differentiating estimates prepared for a single farm enterprise from those summarized for a composite of farms.

Definitions of Specific Types of Estimates

Concise definitions of the different types of estimates are shown immediately below. More detailed background to the definitions is given in the subsection that follows.

Historical CAR estimates for production enterprises are a summary of enterprise CARs for some historical period such as the past calendar year, crop year, or production cycle.

Projected CAR estimates for production enterprises are forecasts of enterprise CARs for some future period such as the coming calendar year or crop year and are based on information available at a certain point in time.

An **individual farm** is either a specific farm currently or previously in operation or a representative farm that has a set of resources, production practices, objectives, and enterprises similar to some class of actual farms. An example of a representative farm would be a 350-acre small grain, hay, and dairy farm in Cache County, Utah, patterned after farms in the county.

A **composite of farms** is a simple or weighted average of enterprise CARs for some period for some group of individual or representative farms. An example would be the production costs for all current wheat farms in Kansas.

Background to Definitions

A **historical CAR estimate for an individual farm** is based on the CAR recorded and allocated to the several enterprises on the farm for a previous time period. This type of estimate could be calculated and used by farm operators to make quantitative evaluations of past performance of a specific enterprise in relation to other enterprises on the farm, with projections, or in comparison with other standards. Just as an income statement or balance sheet provides a source of information for whole-farm management, marketing, or financing decisions, the historical CAR estimate for an enterprise allows the producer to evaluate past management decisions involving that particular enterprise. A combination of enterprise CAR estimates can be used to evaluate the relative performance of various enterprises as part of the total operation. Historical CAR estimates for individual farms are often used by policy analysts to evaluate commodity programs, by lenders as guides to help them make decisions regarding loans to producers, and by extension specialists in providing guidance and counseling on specific production problems.

A **historical CAR estimate for a composite of farms** is a simple or weighted average of enterprise CARs for some historical period. A combination of production practices, sizes of operations, land tenure relationships, crop varieties, or livestock breeds may be represented in a single summary of CARs. For heterogeneous enterprises, the relative weights that are applied to aggregate the parts into a summary affect the outcome. The most common aggregation method is to use population weights that are proportional to acreage, sales, or production. A common but less satisfactory alternative is to use equal weights. Composite CAR estimates are prepared by the United States Department of Agriculture (USDA) to represent the entire United States, the major production regions, and selected states. Data from university or private farm record systems are often summarized in a composite format at the state level and for different groups of farms within a state. Common uses of composite historical CAR estimates are evaluation of the effects of government programs, analysis of changes in technology or investment on net returns, and comparison of interregional differences in agricultural production.

A **projected CAR estimate for an individual farm** is a forecast of CARs for a specific size, location, and system of production. In many instances the forecast of components of CARs is based on an evaluation of the farmer's expectations relative to other general information. Projected CAR estimates are used by producers to determine financial requirements, plan for profit-increasing production adjustments,

make marketing decisions, and resolve numerous other business management problems. Projected CAR estimates may also be made for representative farms. Such estimates can be used to evaluate alternative production practices and management systems for educational purposes or to provide a starting point for individual producers. The estimates are often used by researchers in evaluating new technologies, the feasibility of new products, or the off-site (environmental) effects of alternative cropping and livestock systems. Projected composite estimates may be useful for projecting regional comparative advantage or evaluating the potential effects of a particular government policy on a group of farms.

Projected CAR estimates are sometimes developed for **composite farms**. These estimates represent an average or weighted average of the CARs a set of farms is expected to experience during some future time period. Projected farm income is an example of this type of estimate.

SCOPE OF CAR ESTIMATES

It is important to prepare both historical and projected enterprise CAR estimates with a clearly defined beginning and ending point in order to make meaningful comparisons across farms, regions, and countries.

> ***The Task Force recommends estimating CAR for the production period when it does not exceed 12 months in length. For enterprises with overlapping production periods (such as breeding livestock) or production periods longer than 12 months, the Task Force still recommends using a 12-month period. In situations when a longer period might be warranted for some purposes (cow-calf operations, sugarbeets, or tree crops), the Task Force recommends that such estimates also be reported on an annualized basis for comparison with other enterprises.***

If other periods are used, as may sometimes be appropriate for a given type of analysis, clear specification of beginning and ending points is important. The production period covered begins with the first resource use (and associated costs incurred) by the enterprise, such as first tillage operation, first purchased input, or preparation of facilities. The period ends at the time of physical transfer of the saleable product(s) from the enterprise and includes all costs required to produce the saleable product(s). Marketing then begins when production ends. In many instances there may not be a clear delineation between the CARs associated with production and those associated with marketing. Certain commodities require some processing to produce saleable commodities (e.g., cotton ginning, or cleaning and grading of fruits and vegetables); with other commodities, part of the production process constitutes considerable value added (e.g., field boxing of lettuce).

> ***The Task Force recommends that although CAR estimates for periods longer than one year may sometimes be appropriate (e.g., cow-calf operations), or a clear distinction between production and marketing activities cannot be made, any deviation from the beginning and ending points recommended above should be clearly noted on the statement of CARs.***

Once the production period is defined, a specific point (or points) must be chosen at which to value all CARs. Historical CAR estimates, particularly those generated from accounting systems, typically record the nominal dollars of receipts and expenses when they occur. A similar approach is often used for projected CAR estimates. With inflation, the entries for several different points in time are expressed in dollars that have different purchasing power. Expressing all CARs at one point in time corrects for this problem, making comparisons across enterprises more accurate.

The Task Force recommends that projected CAR estimates establish the end of the production period as the reference point in time at which to value all CARs, and that historical estimates also use this end of period conversion when possible.

DEFINING FACTORS OF PRODUCTION AND PRODUCTS

Economic theory and accounting principles provide the foundation upon which CAR estimates are developed. For economic analysis, the definition of cost is broader than for financial accounting. An "economic cost" is the compensation received by the owners of capital and the units of factors of production, which ensures that the inputs continue to be supplied. The amount of this payment is usually determined by market forces. In some situations markets may not be functioning or no formal market may exist. In these cases, the amount of payment to the factor of production must be determined by other methods. In practice, the measurement of CAR (particularly historical estimates) requires using accounting information because farmers maintain their information in that way. In accounting, CAR are derived using principles that guide the construction of basic financial statements such as the statement of cash flows, the balance sheet, or income statement. In accounting, the concept of actual historical cost is central, but it ignores several important components of economic costs. These items are costs associated with the use of financial (including equity) capital, long-lived factors such as equipment and buildings owned and used by the business, and the contribution of unpaid time and effort provided by the farm operator and family members. Estimates of such implicit costs must be obtained using the economic concept of "opportunity costs"[1].

Clear definitions and distinctions of the important concepts associated with the measurement of economic CARs as opposed to accounting costs will be helpful in preparing and using CAR estimates. The first set of concepts is related to the physical production process.

A **production system or method** is a description of the set of outputs that can be produced by a given set of factors of production or inputs using a given production process.

A **factor of production (input)** is a good or service that is employed in the production process.

A **product** is a good or service that is the output of a particular production process.

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[1]"Opportunity cost" is defined and discussed further in the next section under "Valuing Factors for which there is no Market Transaction."

Chapter 2. Conceptual Issues in Cost and Return Estimates

Economists typically view the production system as a set of outputs and the associated inputs that are capable of producing them, and often assume a continuous production process where alternative combinations of inputs can be used to produce a given level of output. In preparing costs of production estimates, the analyst must specify the production system and the specific input levels used to produce the desired level of output. In other words, the analyst must choose one point in the producible output set on which to base CARs. The typical economic assumption is that the producer will minimize the cost of a given level of output by judicious choice of inputs and technology. For historical estimates, the levels used by the analyst are the actual levels used, whether they represent optimized choices or not. For projected or synthetic estimates, the most common assumption is to choose either a "best management" level of inputs or some "representative" level of inputs. The important point is that for the purposes of CAR estimation the input-output point on the production surface is fixed at either a historical or an "optimal" level, and CARs are estimated as if the technology is of the fixed coefficient "Leontief" type at this point. Estimates based on alternative input-output points can also be constructed for comparison.

Factors of production may be categorized in many ways. A common delineation is between labor and materials, where all inputs other than labor are considered materials. Materials can also be classified in different ways. One common distinction is between primary factors (natural resources such as land and extractables such as oil), which are considered to be nonreproducible, and capital, which is defined as being produced from other factors (labor, primary factors, and other capital). In this sense all produced factors are called capital. A more modern classification differentiates inputs based on stock and flow concepts. This more modern approach defines capital as a stock that yields a stream of services (utility) in the current and future periods. These services have value either as inputs into a production process, for direct consumption, or for sale in the market. The services flowing from a stock of capital are considered distinct from the capital itself. In contrast to capital, factors whose services are exhausted in one period and have no value other than in being used up are called **expendables**.

In the more modern approach, capital refers to stock resources that provide service flows over more than one time period. A number of resources fit this classification: land, equipment, buildings, and machinery are clearly considered capital goods according to this definition. In a more general sense, education and experience—as they enhance the productive capacity of workers—are considered human capital. In a free society, however, ownership of human capital is restricted to the person in whom it is embodied. At a societal level, stocks of knowledge and information are also capital. Some of these stocks can be owned whereas others are in the form of public goods. Legal rights such as the right to remove water from a stream are also a form of capital. Inventories can be considered capital to the extent that they may not be depleted in a single time period.

The production potential of capital can be modified in many ways. These modifications take the form of changes in the service capacity or potential future productivity. Service capacity can be reduced in a variety of ways. This reduction in service potential can be wear and tear associated with the passage of time or use. For example, the roof of a barn deteriorates due to exposure to the elements and the valves on an engine wear out with use. Reduction in service potential might also be due to depletion in the case of natural resources or inventories, obsolescence in the case of knowledge, or deterioration in skills in the case of human capital.

Service capacity can be enhanced by additional investment in the capital asset. Examples include overhauling an engine, reroofing a barn, replacing several sections of a concrete ditch, or terracing an erodible hillside. The service potential of a given stock of human capital can be enhanced by additional education, training, and investment in health; or, it can be reduced by poor coordination and supervision, or extended exposure to damaging environmental factors such as noise, pesticides, and intense heat or cold. Expenditures to enhance the stock of human capital and its service flow can be thought of as analogous to enhancing the service flow of other capital. The increased service capacity is usually embodied in the labor, and thus cannot be owned by anyone else. Owners of capital also take actions that are intended neither to reduce nor to enhance service capacity, but simply to promote optimal productive use. Such actions are usually called **maintenance** or **upkeep**. Examples are lubricating bearings, rotating tires, or mending a fence. Of course these actions do have an impact on long-run service potential and so they must be considered along with wear and tear and service operations in evaluating the productive capacity of a capital asset.

Factors of production are then categorized as being either labor, capital (including land and human capital), or expendables. Since human capital is embodied in the worker, factors are often categorized as being either capital assets or expendables. Capital is useful only to the extent that it provides services. And the services of capital are expendable in the sense that once a given service such as 10 hours of tractor time is used, those specific hours are exhausted. This report makes the following distinctions between factors of production.

> **Expendable factors of production** are raw materials, or produced factors that are completely used up or consumed during a single production period. Common examples of these factors that lose their identity with a single use are seed, fuel, lubrication, some pesticides and fertilizer, feed, and feeder animals.
>
> **Capital** is a stock that is not used up during a single production period, provides services over time, and retains a unique identity. Examples include machinery, buildings, equipment, land, breeding livestock, stocks of natural resources, production rights, and human capital.
>
> **Capital services** are the flow of productive services that can be obtained from a given capital stock during a production period. These services arise from a specific item of capital rather than from a production process. It is usually possible to separate the right to use services from ownership of the capital good. For example, one may hire the services of a potato harvester to dig potatoes, a laborer (with embodied human capital) to provide milking services for a given period, or land to grow crops.

A number of examples will illustrate the argument. Land is considered a capital asset, but the right to use the land for a specific period is an expendable service flow. A laborer and the embodied human capital is considered capital, but the service available from that laborer is considered an expendable capital service. Similarly, a professional such as an accountant, veterinarian, or lawyer is a capital good in the sense that he or she provides services over time, but these services are usually hired on a fee per unit of time or project basis. Shares in an irrigation company are considered capital but the acre feet available for use in a given season is an expendable input. There is often a certain arbitrariness in defining an input as expendable versus a capital service. For example, gravel excavated from an on-farm pit could be considered

either as the capital service of the stock of gravel or as an output, because it requires a production process (excavation and hauling) to obtain the service. In general, only factors that arise directly from a capital stock should be considered capital services, but some looseness of definition is inevitable.

Some inputs that last more than one period lose their unique identity upon use. Examples include paint applied to machinery and buildings, repair parts, hay fed to dairy cattle, subsoiling, spraying of ditches, application of lime, and fertilizers with no appreciable carry-over. Such inputs are usually not treated as capital but as expendable inputs used to maintain the productive capacity of other inputs. The costs of such inputs are usually allocated (prorated) across the periods they provide service. Inputs such as terraces and tile drains may be handled either as separate capital items, because they are quite unique, or as part of the land base when rented in conjunction with the land. Some factors of production produce more than one kind of service. For example, a fire extinguisher loaded and readily available provides fire protection services. The extinguisher provides these services over time and does not lose its identity in the process (thus fitting the capital category), but when used to put out a fire can be used only once. For this service, the extinguisher may be considered an expendable. The classification factors of production that produce more than one kind of service are arbitrary, but they are commonly considered capital assets because they show up on the balance sheet and provide service flows for more than one period.

Differences in classification of factors are important for valuing their contribution to production. Only the actual value contributed to the production process of a specific output during a given period is considered as a relevant cost for a factor. For capital factors that are employed for several periods, one must make an assumption about the contribution that the factor contributes in each period. For a granary, this may be cubic feet of storage space of uniform quality per period. If the quality of this space is fairly uniform over time and can be maintained in this quality with known annual maintenance, then the cost of granary space per unit of grain stored can be computed as a constant. A tractor may have a known purchased price and salvage value, constant fuel and lubrication costs based on hours of use, and increasing repair costs, also based on hours of use. If the quality of an hour of tractor time (with appropriate repairs and maintenance) does not change over the life of the tractor and the tractor is used the same number of hours per year over its life, then the analyst can compute an annual annuity representing the annual cost of the tractor that can be broken down easily on a per hour of service basis. If the production of a ton of sweet corn using a specific production system requires 3.5 hours of tractor time, then the tractor cost per ton can be computed easily using this constant cost per hour of service.

If the productivity of a capital input depends on its age and level of use, then more complicated procedures are needed. For example, consider a capital asset such as an apple orchard. The orchard will have several years of preproductive costs with no output, including a large expenditure in the establishment year. Once production begins, it will typically rise, reach a plateau, and then fall. The cost per bushel for the apples for each year will vary depending on the number of years the orchard is in production, the yields per year, and the operating costs. In this case it is not reasonable to compute a constant capital cost per bushel as with the granary or possibly the tractor, because the productivity of the orchard varies over time. Instead, it makes sense to develop a unit cost of capital that varies with time. Cost of production studies typically assume constant productivity across time for most inputs including machinery, equipment, and buildings. The justification for constant productivity of machinery is that appropriate and increasing repair expenditures can compensate for decreased service capacity. This assumption is probably reasonable in most situations but should be evaluated on a case-by-case basis. The assumption of constant productivity is much

less reasonable in the case of breeding livestock, most perennial crops, some types of wells, and some land or range resources. This report will generally compute capital costs for machinery, equipment, and buildings assuming constant productivity over time. Discussions of appropriate ways to handle variable productivity are contained in Appendix 6A and in Chapter 10 on multiyear enterprises.

VALUING FACTORS OF PRODUCTION

The economist's classical theory of the firm distinguishes between owners of resources and the operator of the firm. The firm is viewed as purchasing expendable inputs such as seed, fuel, and feed, and capital good services such as hours of labor and human capital, machinery and equipment, or the services of land, buildings, and other structures, in exchange for fixed payments. When these inputs can be used over several production periods, the owner of the firm pays a fixed fee for use in a given period. Thus, the actual costs of inputs can be determined by market prices and quantities or expenditures, if the market is assumed to value correctly the contribution of any good or service to the welfare of economic agents. For example, the cost of seed depends on the price per pound and the number of pounds used, the cost of land per acre is the rental rate, the cost of machinery per hour is the custom rate, and the cost of human capital is the wage rate times hours worked or compensation including benefits. In this framework, all factors of production except the operator of the firm are compensated in full for their contribution.

> ***The Task Force recommends that when there are active markets for a given factor of production and there are no constraints on factor use, the preferred value to use for all CAR estimation is the current market price (or compensation) of that specific factor.***

Although the valuation of homogeneous factors traded in active markets is straightforward based on this recommendation, numerous complications arise in practice when factors are not homogeneous and/or not purchased in a competitive market. The remainder of this section will consider general valuation principles for factors of production. After discussing time preferences, interest, and inflation in the following section, a more complete analysis of some of the more complicated issues will be presented.

Valuing Factors that Differ in One or More Attributes

The economic law of one price applies to goods and services that are exactly the same in all relevant dimensions. Some of the most common dimensions are quality, time, and space.

Clearly, costs and revenues must be adjusted to account for quality differentials such as discounts for damaged produce. A discussion of some of these issues, particularly with respect to products, is contained in Chapter 3: Revenues and Government Programs Participation. Issues related to time are discussed in the next major subsection.

With respect to differences in location, it is important to include as a cost of producing and marketing the product, the cost of getting the product to the market from which the product price is obtained. Conversely, the price can be adjusted to compensate for this expense. Otherwise, the net returns to the firm will be overstated. Spatial equilibrium implies that price differences across location of commodities that are otherwise identical should be equal to transportation costs.

The Task Force recommends that all CAR estimates should reflect goods and services that are identical, or that are cost-adjusted (revenue-adjusted) for any differences in location, quality, or time of delivery.

Valuing Expendable Factors that are Purchased

A purchased expendable factor is bought and used during the current production period and so its cost is obtained by multiplying the quantity used by the market-determined purchase price. If there are volume or other discounts or additional payments such as fringe benefits for workers, these should be considered in computing the cost. Adjustments for time, quality, and location should also be made in keeping with the idea of pricing all inputs and outputs at a uniform quality level for a given price, at the same time, and at the same place. More specific discussion on expendable inputs is contained in Chapter 4.

Valuing Capital Services that are Purchased

The market price for capital services is the appropriate charge for CAR estimation if the owner of the capital is distinct from the operator of the firm and the capital services are obtained in a market transaction. All of the adjustments for time, quality, and space, as in the case of expendable factors, apply here as well.

Valuing Factors for which there is no Market Transaction

When the operator of the firm is also the producer of an expendable input used in the production of another output or the owner of the capital used to provide a service, there is no market transaction to reflect the cost of using these factor services, and an implicit cost and revenue must be computed because no market transaction takes place. This situation requires use of the concept of opportunity cost.

> The **opportunity cost** of any good or service is its value in its next best alternative use. For example, the opportunity cost of the service of an input used in the production of any particular commodity is the maximum amount that the input would produce of any other commodity. Opportunity costs are usually measured in monetary terms so that the opportunity cost of any good or service is the maximum amount the good or service could receive elsewhere for use as a production input or for final consumption.

When a market transaction is not available to value a given expendable factor or capital service, methods that will approximate the opportunity cost of the service are used. These methods are not as reliable as direct market valuation; therefore, as long as well established (or regular) markets exist for the given services and the amount of service that is used can be determined, the best estimate of the cost for the services of an operator-owned factor in preparing CAR estimates is the market price of that factor service. But when markets are nonexistent or very "thin," the other methods of estimating costs associated with the ownership and use of an asset must be employed to approximate the market solutions. These methods usually take the form of using market prices for similar expendables or determining implicit rental rates for capital services.

Valuing Produced Expendables

Produced expendables utilized on the farm should be valued at the cost of purchasing the factor from off-farm as the cost of the factor to a utilizing enterprise because this reflects the opportunity cost of the factor to the utilizing enterprise. As an example, consider a farmer who raises feeder pigs for use in a finishing operation. The appropriate cost for these feeder pigs to the finishing operation is the cost of purchasing the pigs off the farm. An alternative for the factor cost is the price the farmer could obtain for the feeder pigs if they were sold in the local market. Alternatively, consider a dairy farmer who produces more corn silage than needed for his dairy herd and who sells the excess to a neighbor who picks it up on the farm. The price the neighbor pays for the silage is an estimate of the value of the corn silage to the dairy enterprise.

Valuing the Capital Services of Owned Capital

Capital services provided by the owner of the operation of a given enterprise should be valued at the cost of obtaining these services from an alternative source in an arms's length market transaction. For example, in situations where there are active cash rental markets for land, these rental rates provide a good estimate of the cost of land services. In situations where cash rental arrangements are not common, share rental rates can sometimes be used to approximate the actual factor cost. In some states there are active markets in machinery rental that can be used to approximate factor cost of machines, although in much of the country such markets are very small and specialized. In many areas, a number of capital services are offered on a custom basis. These custom rates provide an estimate of the cost of the capital service. There are few situations where an active market in general purpose buildings exists. In the case of labor, there may be active markets for unskilled workers allowing use of commonly reported wage rates; however, the market for skilled managers may be much smaller, requiring the use of opportunity cost calculations.

> ***The Task Force recommends that market-determined costs of inputs should be used when they are available and that other methods should attempt to reflect what the market solution would be if it existed. In general, the cost of purchasing inputs from off the farm as opposed to their on-farm production cost should be used in pricing these inputs to other on-farm activities. Similarly, custom rates for machinery should be used when markets for these items are well established and custom operations can be performed in a timely manner.***

These other market-based methods should reflect the CARs associated with the long-term ownership of assets and the market-determined equilibrium cost of obtaining the factor services of those assets.

Accounting for Transactions Costs

In markets with no transactions costs, the purchase and sale price of a given good or service will be the same. Most markets, even those that operate efficiently, will have some transactions costs associated with minimal transportation, brokerage and handling fees, short-term storage, insurance premiums, checkoff assessments, shrinkage, or other loss. A common example is the difference between the buy and sell price at a grain elevator. When transactions costs are not zero, the purchase price of a factor will exceed the sale price by the transactions costs. The correct value to use in assessing the return to the selling enterprise,

assuming outside sale, is the sales price net of any transactions costs assumed by the seller. Alternatively, the actual selling price can be used and the transactions costs included in the cost of production. The cost of a factor purchased from outside the firm is the purchase price plus any additional transactions costs assumed by the buyer. If there are unavoidable costs associated with getting a product to market, they should be included as a cost of production. If the product is used internally, these costs should not be included, however. Similarly, if there are costs associated with purchasing a product externally, they should be included when the product is purchased externally but ignored if obtained internal to the firm. The price used for internal transactions should be conceptually the same for both purchase and sale because the factor (product) is at the same time and place at the point of internal sale. The difficulty is that market prices are often for the good or service at a slightly different time or place, and perhaps in a different form. Simply using the market price may implicitly attribute a higher return to one of the enterprises because the actual costs of getting the product to or from the market may not be the same and may not be explicitly counted in the costs of either enterprise.

To make the issue of transactions costs clear, consider an example where the market price of a feeder steer at the local auction market is $250. Assume the cost of transporting the steer from the farm to or from the market is $15 so that the implicit price at the farm is $235. All other costs of production for the feeder steer are $200 so that the net profit to the feeder steer enterprise is $35. The auction charges a fee of $5 which is paid by the buyer of the feeder steer. If all feeder steers produced on the farm are sold at this market then gross revenue to the feeder steer enterprise is $250 and the net price is $235. If the costs of transportation are not explicitly included in the estimate then the net price should be used as the sale price per head for the feeder steer enterprise. Suppose the slaughter steer enterprise on the same farm purchases the feeder steers. The purchase cost of the feeder steers produced on the same farm is $235 per head, assuming no transportation costs. If the slaughter steer enterprise purchases some or all feeder steers at the local market (assuming no closer available source), then the total cost of the purchased feeders steers is $270 (250+15+5). Assume that the revenue minus all other costs for the slaughter steer enterprise is $350. Then the net revenue for the slaughter steer enterprise for the purchased feeder steer is $80, and the net revenue for the slaughter steer enterprise on feeders transferred from the feeder steer enterprise is $115 ($350-235) per head. Using the site-specific net price of $235, the feeder steer enterprise has returns of $35 and the slaughter steer enterprise has returns of $115. The site-specific price is the opportunity cost of the feeder steer produced on the same farm and it is the recommended method of valuing those steers. An alternative method of valuing the feeder steers produced on the farm is to use the market price of $250 as both the selling and buying price. This method may be used when the transportation and auction charges are not well documented, making calculation of the site-specific price somewhat arbitrary. Although the site-specific method has some theoretical backing, assuming well-functioning markets, there is arbitrariness in any such allocation.

The Task Force recommends that, when transactions costs are small, for simplicity, the local market price be used to value the factor (product) and transactions costs not be charged to either enterprise. When transactions costs are large, it is more important that the allocation rule chosen not distort relative factor returns. In such cases, the allocation rule used should be made explicit and the sensitivity of the results to the allocation rule discussed.

A more detailed discussion of allocation rules for handing transfer pricing is contained in Chapter 4: Purchased and Farm-Raised Expendable Inputs. Before considering valuation of these various types of factors in more detail, some discussion of adjustments to both expendable and capital costs to account for time differences is needed.

TIME PREFERENCES, INTEREST, AND INFLATION

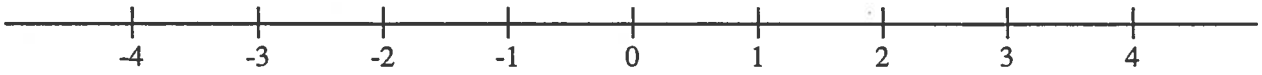
Most agricultural production occurs with a time lag so that costs are often incurred months or even years before the end product is completed and sold. Some factors of production (a tractor, for instance) are used to produce many sets of output over many different production periods. In order to make sense of CARs that occur at different points in time and combine them effectively to make optimal decisions, a clear understanding of issues related to time preferences and interest rates is important. Dealing with this time lag is one of the thorny issues in CAR estimation.

Individuals have preferences over the timing of CARs. Economic theory usually assumes that an individual has a positive rate of time preference, meaning that one dollar today is preferred over one dollar one year from now. This is usually attributed to impatience or quasi concavity of the utility function. Exceptions to this positive rate can occur easily if relative income and wealth levels differ across time periods, if financial markets are not complete, or if there are significant costs for carrying goods between periods. The rate of time preference for an individual commodity is the implicit relative price that would induce an individual to consume or hold equal amounts in adjacent periods and is implied by the shape of indifference curves. When applied to an individual commodity, the rate of time preference is called the **own rate of interest**; when applied to a numeraire commodity such as money, it is called the **discount rate** or the **rate of interest**. Just as the interaction of individual preferences for commodities and the production technology determine the relative prices of goods, the interaction of individuals' time preference, commodity preference, and the technology determine a market rate of discount or interest rate. There are clearly different discount rates for time periods of different lengths. These rates reflect the market's evaluation of the relative worth of the same income flows (or money) occurring in different time periods.

An individual's rate of time preference is determined independently of the market rate of interest, but is a factor in determining the market rate. In an economic equilibrium, where individuals can trade freely on commodity and financial markets, they will make production and consumption decisions such that (at the margin) their individual rate of discount between income in different periods is equal to the market rate of interest. The Fisher separation theorem (Copeland and Weston: 11-12) implies that production decisions can be made independently of consumption decisions when markets are complete. This theorem further implies that individuals will make production decisions based on this market rate of interest, and partially justifies the common practice of using the market rate of interest (discount) for evaluating the relative contributions of returns and costs to an individual's welfare at different periods in time. When markets are not complete or fully functioning, a rate of discount other than the market rate may be applicable. This may be particularly important when estimating costs for individual firms when full access to financial markets may not be available or the risk characteristics of the firm make published discount rates inappropriate.

Discounting CAR Flows

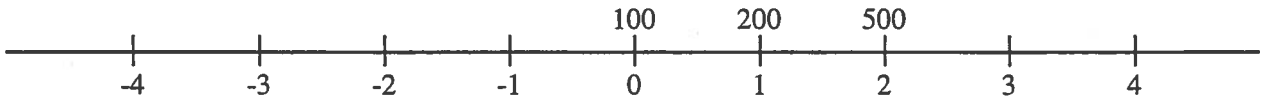
The practice of adjusting all CAR streams to a common point in time to account for time preferences is usually called discounting or present value analysis. The idea is that with properly functioning markets, funds received in one period can be invested at the market discount rate and earn that rate of return over the period. Thus one dollar received today is worth more than one received tomorrow because it can be invested at this usually positive market rate. In CAR estimation, it is important to reflect the value of all CARs at a common point in time so that the values are strictly comparable. If the desire is to reflect all future monetary flows on an equivalent current period basis, **present value formulas** are used. When income streams are adjusted to a future point in time, the practice is sometimes called **compounding** or **future value analysis** to contrast it with **discounting** income flows back to the current period. This report will use the terms **present value analysis** and **discounting** to reflect any adjustments of income streams to account for time preference, whether these adjustments are forward or backward from the base period. The literature on capital budgeting and financial decision making provides a useful reference for this discussion (Copeland and Weston; Lee; Levy and Sarnat). In order to make the analysis clear, consider a number line taking values from $-\infty$ to ∞ as below. Time 0 is considered to be the present time, time 1 is one period in the future, -2 is two periods in the past, and so forth.



Of course, the line can be renumbered so that any point on it is time 0. Consider now an income (cost) stream that begins at the present time 0 (or the beginning of the first period) and ends at time n. The value of this stream at time 0 is given by

$$V_0 = \sum_{t=0}^{n} \frac{R_t}{(1+i)^t} \qquad \textbf{(2.1)}$$

where V_0 is the present value of the payment stream (of income or costs) on the right-hand side of the equal sign. The notation R_t represents the net return or cost at the end of period t, where t denotes the time period 0, 1, 2, 3, ..., n. The discount rate, which is constant over time, is given by i. If the initial period of the income stream is considered to be the base, as in this example, the discounted value is called the present value of the future income stream. For example, consider an income stream with values (100, 200, 500) at the points 0, 1, and 2. This is represented on the number line by placing the returns above the line as follows.



The present value at point 0 of the above stream is given by

$$V_0 = 100 + \frac{200}{1+i} + \frac{500}{(1+i)^2}.$$

If the interest rate is 5%, this will give

$$V_0 = 100 + \frac{200}{1.05} + \frac{500}{(1.05)^2}$$
$$= 743.991.$$

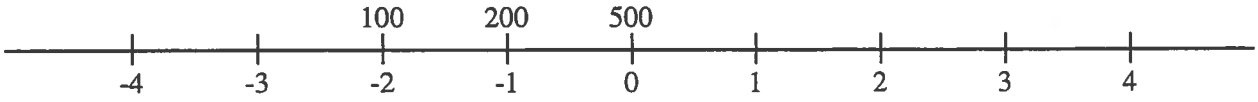
In many instances it is useful to adjust CARs to points in time other than the present. This can be accomplished using the above formula and allowing the index t to take on both positive and negative values in relation to the point of time considered to be the present or the base (0) for the analysis. For example, consider adjusting all income flows to the end of the last period (the n^{th} period) as is done in future value analysis. The value at the end of the n^{th} period (V_0) of a CAR stream occurring over the n periods is given by

$$V_0 = \sum_{t=-n}^{0} \frac{R_t}{(1+i)^t} \qquad (2.2)$$

where V_0 represents the value of the payment stream at the end of the period (time 0). If one prefers to use positive values for the index t and treat the n^{th} period as the base, as in standard future value calculations, the above formula would read

$$V_n = \sum_{t=0}^{n} \frac{R_t}{(1+i)^{t-n}} = \sum_{t=0}^{n} R_t (1+i)^{n-t}. \qquad (2.3)$$

The value on the right-hand side of 2.2 and 2.3 remains the same, but is represented in a slightly different way. For example, suppose the above stream of returns is to be evaluated at the end of the second period (at point 2 on the original line). The line can be renumbered, making the end of the second period (point 2 on the original line) point 0, as shown below



where 100 occurs at -2 and 500 occurs at 0. The present value at point 0 is

$$V_0 = \frac{100}{(1+i)^{-2}} + \frac{200}{(1+i)^{-1}} + 500$$
$$= 100(1+i)^2 + 200(1+i) + 500 .$$

Sometimes it is useful to value an income stream at a point in the middle of the time horizon. For example, one might choose the end of the current year as the point to value CARs for a cow-calf operation even though returns occur next year. In this case, rather than continually modifying the formulas and notation, it may be simpler always to consider the point in time to which the streams are adjusted to be zero in the sense that the discount factor for the period has an exponent of zero and number all periods from that point so that future periods have a positive index (and positive exponent on the discount factor) and prior periods have a negative index. In this case the formula to discount the return streams to the k^{th} period is given by

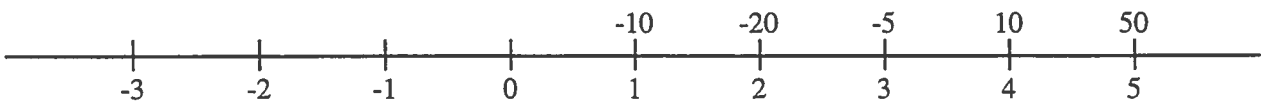
$$V_k = \sum_{t=j}^{n} \frac{R_t}{(1+i)^{t-k}} = \sum_{t=j}^{n} R_t\,(1+i)^{k-t} \qquad \textbf{(2.4)}$$

where j is the first period considered and n is the last. When k is greater than t, flows are adjusted forward to period k; when k is less than t, flows are adjusted back to k; and when k is equal to t, the flow is not adjusted. Consider the value at the end of the first period (or time 1 on the number line) for the above payment stream. The formula will give

$$V_1 = \sum_{t=0}^{2} \frac{R_t}{(1+i)^{t-1}}$$
$$= \frac{100}{(1+i)^{(0-1)}} + \frac{200}{(1+i)^{(1-1)}} + \frac{500}{(1+i)^{(2-1)}}$$
$$= 100(1+i) + 200 + \frac{500}{(1+i)}$$

where V_1 represents the value at the end of the first period.

To clarify the discussion, consider a stream of CAR flows occurring at the end of each period. Let the flow at the end of period 1 be -10 with a further return at the end of period 2 of -20. Let the returns at the end of periods 3 through 5 be -5, 10, and 50. The number line is as follows:



Assume a discount rate of 10%. The adjusted (discounted) values of each flow and the total for the entire stream at the end of each period are given in Table 2.1 below. The columns give the cash flow adjusted to the end of the period in the column title. For example, consider the first line of the table which reflects cash flow of -10 at the end of the first period. This cash flow has value -10 at the time 1, but declines in value (grows in absolute value) to -11 (11) by the end of period 2. The value at the beginning of period 1 (end of period 0) is -9.091. The adjusted value of this flow at the end of the fifth period is -14.641. Similarly, the value at the end of period 0 of the 50 dollar return occurring is 31.046 and the value of the 50 dollar return at the end of the fifth period valued at the end of the fifth period is 50. The Total row at the bottom of the table gives the total of the cash flows for all periods adjusted to the end of the period in the column title. Thus, for example, the total value of all five cash flows at time 0 is $8.499, while at the end of the first period (time 1) it is 9.35 and at the end of the fifth period it is 13.689. The diagonal elements of the table are the same as the actual cash flows, because the diagonal represents adjustments to that period as the base. Furthermore, the amounts in the Total line can be adjusted to any other period using similar procedures. For example, the value of the entire stream at the end of the fourth period ($12.445) is properly discounted to the end of the first period using the relation $V_0 = 12.445/(1.1)^3 = 9.35$.

TABLE 2.1 Discounted Values of a Cost and Return Stream

Value at Point in Time		0	1	2	3	4	5
Period	Cash Flow at End of Period						
1	-10.000	-9.091	-10.000	-11.000	-12.100	-13.310	-14.641
2	-20.000	-16.529	-18.182	-20.000	-22.000	-24.200	-26.620
3	-5.000	-3.757	-4.132	-4.545	-5.000	-5.500	-6.050
4	10.000	6.830	7.513	8.264	9.091	10.000	11.000
5	50.000	31.046	34.151	37.566	41.322	45.455	50.000
Total		$8.499	$9.350	$10.285	$11.313	$12.445	$13.689

The point is that all CAR streams can be adjusted to reflect the same point in time using an appropriate discount rate. These adjusted CARs can then be summed to compute net income, return on investment, and other financial measures.

Measuring Growth Rates of Economic Variables and Compounding of Interest

When analyzing economic variables that are growing over time, an important issue is how to measure the rate of growth. Growth rates are usually expressed as a percentage rate over some time period. For example, if average corn yields in a county were 100 in 1980 and 110 in 1990, the growth over the ten-year

period is 10% ({110-100}/100). The annual rate of growth is not 1%, however, because if yields were 100 in 1980 and 101 in 1981, a 1% growth rate would imply yields of 102.01 [(101)(1.01)] in 1982, 103.03 in 1983 and 110.46 in 1990. This is of course due to the compounding of the growth over time. The annual rate of growth that is consistent with a 10% rate of growth over the ten-year period is .9576% because $(100)(1.009576)^{10} = 110$. Thus when computing growth rates of any type, a period for compounding the rate must be considered and be made explicit in the analysis. For example, one can talk of a quarterly rate of growth that is consistent with a given annual rate, an annual rate that is consistent with a biennial rate, etc. For example, a 1% rate of quarterly growth is equivalent to a 4.06% $[\{(1.01)^4-1\}\{100\}]$ rate of annual growth or a 4% rate of annual growth is equivalent to a .9853% $[\{(1.04)^{.25}-1\}\{100\}]$ quarterly growth rate.

Whereas many economic variables have a natural defining time period, such as yields for an annual crop, for others the appropriate period is not always obvious or even the same for different types of questions. For example, it is not clear whether an annual rate of productivity growth is appropriate for broiler or almond production. In analyzing the growth of farm income, monthly, quarterly and annual rates all make sense for different types of questions. When considering financial variables where interest (and discount) rates are often applied, it is crucial to decide the appropriate period for compounding and correctly convert subperiod rates to annual rates and vice versa. This is especially important when some variables may earn interest under different compounding rules such as daily versus monthly versus annual compounding in the case of production loans.

Real and Nominal Magnitudes

The value of a commodity can be expressed in terms of other goods or in terms of prices (dollars). When commodities are measured in terms of other commodities or in terms of their purchasing power, the stated value is in real terms since it reflects the "real" purchasing power of the commodities. When the value is stated in terms of current prices, the value is in nominal terms. For a single commodity, real values can be expressed in terms of bilateral exchange ratios or in terms of a numeraire commodity. The most common numeraire is the price of money in some base period. For example, the relative price of corn and soybeans can be stated as 2 bushels of corn for 1 bushel of beans or, alternatively, that corn sells for $2.50 and soybeans sell for $5.00. For aggregate output, real magnitudes are expressed in terms of some base period price level. Thus for example, we talk about real Gross Domestic Product (GDP) as being current output at base period prices. So, nominal magnitudes reflect values in current period prices and real values reflect values in base period prices. The change in the overall price level between any period and the base is called the **general rate of inflation**. When the overall price level does not change between periods, real and nominal values will be the same. Just as with other prices, real interest rates are specified in terms of some base period and nominal interest rates are stated in terms of the current period. In an economy with constant prices (no inflation), the market-determined rate of interest is both a real and a nominal rate. When there is inflation, the real and nominal rates of interest differ because the higher price level in later periods reduces the future value of other goods in relation to the numeraire good (money). When interest rates are specified in terms of the current monetary unit, the nominal interest rate on a loan is more than the real rate (when inflation is positive) because the real cost of a loan is less than the nominal cost.

Real and nominal rates of interest are related by the Fisher equation. If π is the inflation rate between two periods, r is the real interest rate and i is the nominal interest rate, then the following identity (Copeland and Weston: 65; Fisher; Patinkin) holds

$$(1 + \pi)(1 + r) = (1 + i). \tag{2.5}$$

Notice that (1+i) ≠ (1+π+r) because inflation and the real interest rate interact over the time period. Specifically, the interest rate applies to the inflating dollars, not just the beginning of period dollars. When r and π are small, (1+π+r) is approximately equal to (1+i). The Fisher relation can be rewritten to solve for the nominal interest rate, i, as a function of the real rate, r, and the inflation rate, π, as

$$i = r + \pi + \pi r \tag{2.6}$$

or for the real rate as a function of i and π as

$$r = \frac{(i - \pi)}{(1+\pi)} = \frac{(1 + i)}{(1+\pi)} - 1 \tag{2.7}$$

or for the inflation rate as a function of i and r as

$$\pi = \frac{(1 + i)}{(1 + r)} - 1 = \frac{i - r}{(1 + r)} \tag{2.8}$$

where all rates are stated for the same time period and there is no compounding of interest within the stated periods[2]. For example, with an annual inflation rate of 5% and a nominal interest rate of 8%, the implied annual real interest rate is (1.08)/(1.05) -1 = .0286. Similarly, with an inflation rate of 5% and a real rate of 3% the implied nominal annual interest rate is 8.15 %.

The nominal rate of interest is appropriate for use in comparing nominal magnitudes, but the real rate is correct for use in comparing real magnitudes. The nominal rate is, of course, made up of the real rate and an inflation adjustment. Adjustments to cash flows for time preference thus have a component related to the real interest rate or the real cost of holding money and a component related to changes in prices due to inflation. The combined effects of the inflation component and the real interest component can be calculated using the nominal interest rate. It is appropriate to use the nominal interest rate to discount nominal CARs within a given production year as long as all the analysis proceeds on a nominal basis. These adjustments can be arbitrarily divided into real interest and inflation components.

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[2]If interest is continuously compounded, then the Fisher relation is given by $i = e^{\pi t}e^{rt} - 1$ where t is the number of periods of compounding and π and r refer to the inflation and interest rates per period. Thus if annual inflation is 5% per year and the interest rate is 3% per year, the implied annual nominal rate is 8.33%, which is higher than the rate of 8.15% computed using annual compounding.

The Task Force suggests that, when CAR estimates are computed on an annual basis in nominal magnitudes, the nominal interest rate be used to adjust all within-year magnitudes to a common point in time. As mentioned earlier, the Task Force recommends that this point in time be the end of the production period or the end of the year, whichever is sooner. The Task Force further recommends that the estimates explicitly state this nominal rate, and the items and length of time to which it applies.

Once production period values are adjusted to a common point in time using a nominal discount rate, they can be decomposed into real and inflation components or converted to real terms for comparisons among periods, for long run analyses, or for capital budgeting, etc. If the end of the production period is used as the base period for prices, then the end of year prices/costs or returns are both a nominal magnitude and a real magnitude in these year-end prices.

Whether economic analysis should be performed on a nominal or a real basis is an often debated issue. As long as the analysis is performed in a careful and accurate manner, it is immaterial which approach is used as far as the end result is concerned. There are often reasons for performing it in one way or another, usually to be comparable with other estimates. The issues relate to ease of computation, interpretation, and comparison. It is sometimes easier to interpret real magnitudes because inflation distortions are eliminated, but more commonly it is easier to interpret nominal magnitudes because that is the way most values are reported. For example, in considering net farm income per farmer in 1920 to evaluate the welfare of today's farmers, it is probably better to consider this in real terms so that what the income will buy is the same. But if one is interested in obtaining a production loan, the nominal projected value of this year's income is the easiest value to use. In addition, some issues such as taxes and subsidy payments are related explicitly to nominal magnitudes. If returns are changing over time due to inflation, then performing analysis with nominal return values and nominal interest rates will give the same present value as using real returns and real interest rates. This becomes clear if one rewrites equation 2.1 assuming that all magnitudes are real. The discounted value of a real return stream at time 0 is

$$V_0^r = \sum_{t=0}^{n} \frac{R_t^r}{(1+r)^t} \qquad \textbf{(2.9)}$$

where R^r_t is the real return at time t, V_0^r is the real present value of the value stream using a real discount rate, and r is the real interest rate. If the inflation rate is given by π, then the nominal return at time t, assuming that the base period is period 0, is given by $R_t = R^r_t\,(1+\pi)^t$. For example, if the real return in the first period is \$300 and the inflation rate is 4% then the nominal return for the period is \$312. If the real return in the second period is again \$300 and inflation is unchanged, then the nominal return relative to the base period is $(300)(1.04)^2 = \$324.48$. Alternatively, a nominal return of \$324.48 in the second period is equivalent to a real return of \$300 because $\$324.48/(1.04)^2 = 300$. Now consider a nominal return stream obtained using the above relations and then discounted by a nominal interest rate. This gives

$$
\begin{aligned}
V_0 &= \sum_{t=0}^{n} \frac{R_t}{(1+i)^t} \\
&= \sum_{t=0}^{n} \frac{R_t^r (1+\pi)^t}{(1+r)^t \ (1+\pi)^t} \\
&= \sum_{t=0}^{n} \frac{R_t^r}{(1+r)^t} \\
&= V_0^r
\end{aligned}
\qquad \textbf{(2.10)}
$$

which is the same as real present value in equation 2.9. The value stream in real terms, V_0^r, and the value of the stream in nominal terms, V_0, are the same because we are considering point 0 to be the base for the computation of real values. **Thus real and nominal discounted values will be the same if the base period for the real values is the period to which the flows are discounted.** If a given investment is subject to different rates of inflation than the general rate, then the above analysis must be modified so that the real rates of return to this asset reflect its returns relative to other assets in the economy. Cost and return estimates often assume that the goods under question are subject to the same rates of inflation as other goods in the economy and so these problems are not a real issue. Given the long run trend toward declining relative prices in agriculture, this common assumption should probably be reconsidered. An alternative, as suggested later in this report, is to conduct all analysis outside the current period in real terms.

Implicit and Explicit Interest Charges and Time Adjustments for Within-Period CARs

Implicit and Explicit Discounting of CAR Flows

The market rate of interest is important not only for adjusting CARs received in different periods, but also for computing the explicit and implicit interest charges accumulated on financial capital used to carry out the firm's operations. Most farming enterprises apply inputs during a time period and receive revenues at the end of the period. Such CARs must be accumulated to a common point in time to make them comparable for decision making. As stated earlier, ***the Task Force recommends that projected CARs establish the end of the production period as the reference point in time.*** This means that all expenditures and revenues should be accumulated to the end of the production period using time adjustment calculations. If all costs were incurred at the beginning of the year and all revenues received at the end, this would entail multiplying all costs by (1+i) where i is the nominal market rate of interest. Because revenues are assumed to occur at year end, they would not be adjusted. Because costs and revenues do not conveniently occur at the beginning and the end of the period, some adjustments for timing and compounding must be made.

There is a different market rate of time discount between time periods of different lengths. For example, there are one-month rates, one-year rates and five-year rates of discount. The rate most commonly quoted is the annual rate, and that is the rate assumed unless otherwise stated. Rates for longer periods are related to the rates for shorter periods, but the relationship is not additive as was discussed in the section on growth rates of economic variables. Interest can be calculated over periods different than the one to which

the rate applies using the simple rate or using compounding. Compounding is theoretically correct in almost all situations and so is the suggested procedure. The correct interest charge with compounding in effect is given by the following general formula

$$ic = R(1+i)^k - R \tag{2.11}$$

where ic is the interest charge, R is the amount of a cash flow at the beginning of the first period, i is the constant interest or discount rate for a single period, and k is the number of periods. For example, the interest charge on \$500 for six months with a 1% monthly rate, compounded monthly, is given by $\{500(1.01)^6 - 500\}$ = \$30.76. If there were no compounding the charge would be $\{500(1.06) - 500\}$ = \$30.00. A compounded one-month rate compatible with a given annual rate is not that annual rate divided by 12, but is given by the formula

$$\begin{aligned} (1 + i_m)^{12} &= (1+i) \\ \Rightarrow i_m &= (1+i)^{\frac{1}{12}} - 1 \end{aligned} \tag{2.12}$$

where i is the annual rate and i_m is the monthly rate compatible with the given annual rate i. In a similar fashion, the annual rate consistent with a given monthly rate can be computed using the formula

$$i = (1+i_m)^{12} - 1\,. \tag{2.13}$$

Similar formulas hold for other compounding periods.

Some examples may help clarify the above formulas. If the one-month rate is 1%, then the equivalent annual rate assuming compounding is $(1.01)^{12} - 1 = 12.6825\%$ and not the simple annual rate of 12%. The monthly rate equivalent to an annual rate of 12% is $(1.12)^{1/12} - 1 = 1.009488 - 1 = .009488 = .9488\%$. The annual interest on a one-year loan of \$500 with a monthly interest rate of 1% is $500(1.01)^{12} - 500 = 563.41 - 500 = \$63.41$. Alternatively, the annual interest on a \$500 loan with 12% annual interest and no compounding (or .9488% monthly interest with compounding) is \$60.00.

Now consider the case of a cash expenditure (loan) that is made with some months remaining in the year where compounding is assumed to take place monthly and the monthly rate is known. The interest that will be due on the loan at the end of the year is given by the formula

$$ic = R(1+i_m)^n - R \tag{2.14}$$

where n is the number of months remaining in the year (the number of months the loan is outstanding). For example, consider a loan of \$500 held for six months from the beginning of the year. The interest charge assuming a 1% monthly rate of interest is $500(1.01)^6 - 500 = \$30.76$. If this amount plus the original amount

of \$500 were held an additional six months until the end of the year, the total interest would be $530.76(1.01)^6 - 500 = 563.41 - 500 = \$63.41$, which is the same interest that would accrue if the loan were held for one year instead of six months.

> ***The Task Force recommends that, when a monthly interest rate is given, interest be computed using the following formula:***
>
> $$ic = R(1+i_m)^n - R$$
>
> ***where ic is the interest charge, R is cash flow at the end of a given month i_m is the monthly interest rate, and n is the number of months interest on the cash flow will be charged.***

Similarly, the appropriate **discount factor is $(1+i_m)^n$** where n is the number of months to the end of the year.

Often the rate used in preparing CAR estimates is the annual rate of interest. In this case an equivalent monthly rate must be determined in order to adjust expenditures and revenues to a common point in time. As stated above, the appropriate formula is given by equation 2.12 and the relevant interest charge on a loan (or opportunity cost on a cash expenditure) made during the year is computed by substituting equation 2.12 into equation 2.14 as follows:

$$\begin{aligned} ic &= R(1+(1+i)^{\frac{1}{12}}-1)^n - R \\ &= R(1+i)^{\frac{n}{12}} - R \end{aligned} \qquad \textbf{(2.15)}$$

where i is the annual interest rate and n is the number of months the cash flow is being adjusted. The interest on \$500 with an annual rate of 12% when held for six months is given by $500(1.12)^{6/12} - 500 = \$29.15$, which is lower than the interest charge of \$30.76 that would result if the monthly rate were 1% because the implied monthly rate with an annual rate of 12% is .9489%.

> ***The Task Force recommends that, when the annual interest rate is specified, the equivalent monthly rate be computed using the formula $i_m = (1+i)^{1/12} - 1$ and that interest charges be computed using this rate as the monthly rate or that the direct formula $ic = R(1+i)^{n/12} - R$ be used. Similarly, the appropriate discount factor is $(1+i)^{n/12}$ where n is the number of months to the end of the year.***

A Comparison of the Recommended Method of Discounting with Two Alternative Methods

When loan lengths or discount intervals are less than a full period, two other practices have been commonly used. The first is to compute interest (or the time value adjustment) based on the per period rate and the applicable proportion of the period and not include compounding for subperiods. This means that if the interest rate is stated as an annual level, the rate for different subperiods will be the proportion of the

year over which the cash flow is discounted, multiplied by the annual rate. Specifically, for a loan held for n months the approximate interest charge is

$$ic \approx R(1 + (\frac{n}{12})(i)) - R \qquad \textbf{(2.16)}$$

where n is the number of months loan is outstanding. So if the loan amount is $500 with an annual rate of 12% and the loan is made for six months, the interest charge would be given by ic = 500(1 + .06) - 500 = $30.00. This method gives a higher interest charge than the correct method (which implied interest charges of $29.15) even without compounding because the implied subperiod interest rate is higher than the correct rate.

The second method that is occasionally used is to compute a proportional monthly rate and then use monthly compounding. Specifically, for a loan held for n months the approximate interest charge is

$$ic \approx R(1 + (\frac{1}{12})(i))^n - R \qquad \textbf{(2.17)}$$

where n is the number of months the loan is outstanding. So if the loan amount is $500 with an annual rate of 12% and the loan is made for six months, the interest charge would be $500(1.01)^6 - 500 = $30.76. This method gives a much higher interest charge than the correct method (which yielded an interest charge of $29.15) because the implied subperiod interest rate is higher than the correct rate and is compounded. This second method is seldom used and is not recommended.

In order to clarify issues regarding discounting, consider an example: a farmer produces cotton and wants to compute the costs of fertilizer, seed, and insecticides. Production begins in February and ends the first of December. The expense items, time of use, and actual costs are given in the first three columns of Table 2.2. The total cost of the items is $101.73. The interest on each is computed using the formula ic = $R(1+i)^{(n/12)} - R$. For example, the interest cost for the cotton seed is given by $17.28\ (1.1)^{8/12} - 17.28 = \$1.13$ and the interest cost of the last insecticide treatment is $20(1.1)^{3/12} - 20 = \$0.482$. The total of these interest charges is $5.09. Total costs are then given by the sum of actual and interest costs for a total of $106.823.

TABLE 2.2 Suggested Method of Computing Within-Year Interest Charges

Enterprise termination date is 1 Dec.
Implied monthly nominal rate of interest is applied to actual expense with compounding
Annual nominal interest rate is 0.10 = 10%

Implied monthly nominal interest rate is $\left[(1+0.10)^{\frac{1}{12}} - 1\right] = 0.007974 = 0.7947\%$

Interest charge = (Actual cost)$(1+i)^{n/12}$ - (Actual cost)

Item	Time of Use	Actual Cost	Months of Use	Nominal Interest Charge
Fertilizer	1-Feb	$24.45	10	$2.021
Cotton Seed	1- Apr	$17.28	8	$1.134
Insecticide	1-Jul	$20.000	5	$0.810
Insecticide	1-Aug	$20.000	4	$0.646
Insecticide	1-Sep	$20.000	3	$0.482
Total		$101.73		$5.093
Total Actual Cost + Interest		$106.823		

In alternative method 1, presented in Table 2.3, a proportional nominal interest rate representing the number of months the loan is out is used, assuming no compounding during the year. The interest on each expense item is computed using the formula ic = [R(1+(n/12)(i)) - R] where n/12 is the proportion of the year for which the interest is calculated. For example, the interest on the cotton seed is given by [17.28 (1+ (8/12)(0.1)) -17.28] = $1.152 and the interest on the fertilizer is given by [24.45(1 + (10/12)(0.1)) - 24.45] = $2.038. The total interest charge is given by $5.19, which is larger than before because a proportional rate implies a higher interest charge for subperiods. The total costs of $106.92 are also higher by this increased interest charge.

TABLE 2.3 Alternative Method 1 for Computing Within-Year Interest Charges

Enterprise termination date is 1 Dec.
Proportional monthly nominal rate of interest is applied to actual expense with no compounding
Annual nominal interest rate is 0.10 = 10%

Implied monthly nominal interest rate is $\left[(1+(\frac{1}{12})(0.10))-1\right] = 0.008333 = 0.8333\%$

Interest charge = $\left[(\text{Actual cost})\,[1+(\frac{n}{12})(i)]\right]$ - Actual cost

Item	Time of Use	Actual Cost	Months of Use	Nominal Interest Charge
Fertilizer	1-Feb	$24.45	10	$2.038
Cotton Seed	1- Apr	$17.28	8	$1.152
Insecticide	1-Jul	$20.000	5	$0.833
Insecticide	1-Aug	$20.000	4	$0.667
Insecticide	1-Sep	$20.000	3	$0.500
Total		$101.73		$5.19
Total Actual Cost + Interest		$106.92		

In Table 2.4, alternative method 2 uses a proportional nominal monthly interest rate along with compounding during the year. In this method a proportional monthly rate is calculated and then used as in the base case. The interest on each expense item is computed using the formula $ic = R(1+(1/12)(i))^n - R$ where n is the number of months that interest accrues. For example, the interest on the cotton seed is given by $17.28\ (1+\ (1/12)(0.1))^8 - 17.28 = \$1.186$ and the interest on the fertilizer is given by $24.45(1 + (1/12)(0.1))^{10} - 24.45 = \$2.116$. The total interest charge is given by $5.328 which is much larger than before because a proportional monthly rate implies a higher interest charge than the equivalent compound rate. The total costs of $107.058 are also higher.

Although the use of proportions is a common practice and easily implemented using hand calculations, the more correct formulas are just as easy to implement using computers and thus are preferred. **If a different procedure than that recommended by the Task Force is used, it should be made explicit in the presentation of results and the magnitude of any approximation errors should be discussed.**

TABLE 2.4 Alternative Method 2 for Computing Within-Year Interest Charges

Enterprise termination date is 1 Dec.
Proportional monthly nominal rate of interest is applied to actual expense with compounding
Annual nominal interest rate is 0.10 = 10%

Implied monthly nominal interest rate is $\left[(1+(\frac{1}{12})(0.10))-1\right] = 0.008333 = 0.8333\%$

Interest charge = $\left[(\text{Actual cost})\,[1+(\frac{n}{12})(i)]^n\right]$ - Actual cost

Item	Time of Use	Actual Cost	Months of Use	Nominal Interest Charge
Fertilizer	1-Feb	$24.45	10	$2.116
Cotton Seed	1- Apr	$17.28	8	$1.186
Insecticide	1-Jul	$20.000	5	$0.847
Insecticide	1-Aug	$20.000	4	$0.675
Insecticide	1-Sep	$20.000	3	$0.504
Total		$101.73		$5.328
Total Actual Cost + Interest		$107.058		

If the recommended method of calculating interest charges is used, then the implicit time value of money adjustments reflects the economic cost of financing the operation if all money is borrowed at the market rate of interest, and it is assumed that any revenues received at any time before the end of the period are invested at this same market rate of interest until the end of the period. Thus it may be useful to think of this time value adjustment as an implicit interest charge. In the real world, however, the producer may borrow only part of the money, the rate at which borrowing occurs may be different than the market rate of interest, and the rate at which revenues can be invested could be different than the rate at which funds are borrowed. A reasonable approach is to adjust all input costs (self- and externally financed) and any revenues to the end of the period using the time value formulas discussed with a specific interest rate. Explicit interest charges can be included for those items where interest is paid and implicit interest charges included as a time adjustment for unpaid (self-financed) interest. This unpaid interest would then not be considered in cash flow analyses. Alternatively, end-of-period prices could be used for all items that are not financed so that the implicit interest charge is contained in the price. This may be particularly useful in the case of owned equipment, buildings, or land, if an implicit cost of ownership is to be included. **Although there is some argument for using different rates of interest for the externally and internally financed items, a common practice is to use a weighted average rate for projected budgets. In any case, the assumptions used should be made explicit.**

The Task Force recommends that CAR estimates specify explicitly what rate of interest was used and to which items it was applied over what time period, so that estimates can easily be recomputed using alternative interest rate assumptions.

In preparing historical CAR estimates, there is less clear direction on appropriate procedures to account for explicit and implicit interest charges. One alternative is to use the actual interest paid to reflect the cost of borrowed funds and use the suggested adjustment procedures incorporating market interest rates to account for other implicit interest charges. A more theoretically pleasing alternative is to apply the same procedures to historical and projected budgets and treat actual financing as separate from estimation of CARs. This alternative, however, is open to criticism in that it ignores the actual situation and may be difficult to explain to farmers or policy makers.

The Task Force recommends that historical budgets explicitly state how all interest charges and time adjustments are applied so that alternative assumptions can be implemented easily.

Separating Within-Period Inflation and Real Interest Costs [3]

In periods of high inflation, such as the late 1970s and early 1980s in the United States and the 1990s in Eastern Europe and the former Soviet Union, it is useful to be able to separate out from nominal interest the costs that are due to inflation and those that are due to real interest. Costs associated with inflation are often compensated for by rising product prices in periods of high general inflation whereas real interest costs receive no such compensating adjustment. During periods of low and stable inflation such issues are of lesser concern. In order to adjust expenditures and revenues within a period and compute implicit interest charges it is necessary to adopt conventions for compounding and separating out the effects due to inflation and those due to implicit real interest. This is done correctly using the time adjustment techniques already discussed. Although such analysis is straightforward, building on previous discussion, the computations can become tedious; therefore, the exact procedures are discussed in Appendix 2A. The bottom line is that any nominal interest charge can be (somewhat arbitrarily) divided into real interest and inflation components.

Implicit and Explicit Interest Charges and Time Adjustments for Between-Period CARs

The costs of all inputs and the prices of all outputs in CAR estimation should be adjusted to the same point in time. Previous sections discussed how to adjust CAR flows within a given period. In the section entitled Real and Nominal Magnitudes, the **Task Force recommended that the nominal interest rate be used to adjust all within-year magnitudes to a common point in time and that point in time generally be the end of the production period or the end of the year, whichever is sooner.** This section discusses the adjustment of cash flows between periods. As discussed in connection with equation 2.10, real and nominal discounted values will be the same if the base period for the real values is the period to which all flows are discounted.

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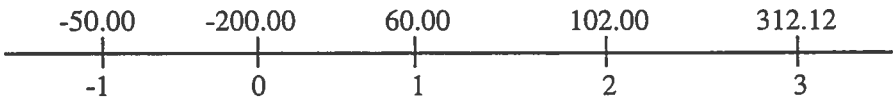
[3]This section may be skipped if the reader is not interested in separating inflation and real interest costs.

To ensure that real and nominal values are equivalent at the base time point, the Task Force recommends that the base point in time for the computation of all real values be the end of the current production period or the end of the current year, whichever is chosen as the base time point for CAR estimation.

Nominal CAR flows for periods other than the current one should be adjusted to the end of the current period using the appropriate interest rate. Real CAR flows for periods other than the current one should also be adjusted to the end of the current period using the appropriate interest rate.

The use of this procedure guarantees that all CAR flows are valued in the same terms at the same point in time. Consider the following simple example of five cash flows expressed as nominal values on a time line. The first period begins at zero and ends at one.



If the nominal interest rate is constant at 7.1% over this period, then the value of these cash flows at the end of period one using equation 2.4 is as follows:

$$V_1 = \sum_{t=-1}^{3} \frac{R_t}{(1.071)^{t-1}}$$

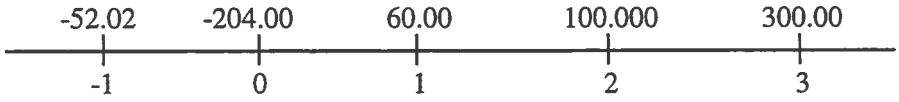
$$= \frac{-50}{(1.071)^{(-1-1)}} + \frac{-200}{(1.071)^{(0-1)}} + \frac{60}{(1.071)^{(1-1)}} + \frac{102}{(1.071)^{(2-1)}} + \frac{312.12}{(1.071)^{(3-1)}}$$

$$= (-50)\ (1.071)^2 + (-200)(1.071) + 60 + \frac{102}{(1.071)} + \frac{312.12}{(1.071)^2}$$

$$= -57.352 - 214.2 + 60 + 95.238 + 272.109$$

$$= 155.795\ .$$

If the inflation rate during the entire time period from -1 to 3 was equal to 2%, then we can compute real values for each of these cash flows for any base period. If we assume that the base period is at point 1, then the value of 60 does not change. The value at point 0 of -200 is inflated to be -204 [(-200)(1.02)] in real terms. The value at point 3 of 312.12 is deflated to be 300 [$(312.12)(1.02)^{-2}$] at the base point. The time line in real values is then



If the nominal interest rate is constant at 7.1% with a constant 2% rate of inflation, then the real interest rate is 5% for each period. With a 5% real rate of interest the value of these real cash flows at the end of period one is computed as

$$V_1^r = \sum_{t=-1}^{3} \frac{R_t^r}{(1.05)^{t-1}}$$

$$= \frac{-52.02}{(1.05)^{(-1-1)}} + \frac{-204}{(1.05)^{(0-1)}} + \frac{60}{(1.05)^{(1-1)}} + \frac{100}{(1.05)^{(2-1)}} + \frac{300}{(1.05)^{(3-1)}}$$

$$= (-52.02)\ (1.05)^2 + (-204)(1.05) + 60 + \frac{100}{(1.05)} + \frac{300}{(1.05)^2}$$

$$= -57.352 - 214.2 + 60 + 95.238 + 272.109$$

$$= 155.795.$$

The value at the base point in time is the same as before.

Now consider a situation where the real interest rate is constant at 5% but inflation and nominal interest are as on the following time line, with the same nominal value flows as in the first case.

π		.01	.02	.02	0
i		.0605	.071	.071	.05
Value Flow	-50.00	-200.00	60.00	102.000	312.12
Time Point	-1	0	1	2	3

Thus the cash flow of -50 at point -1 would be adjusted from the point -1 to 0 at a nominal rate of 6.05% and from point 0 to point 1 at a rate of 7.1%. The value of this return stream at the end of period 1 must be computed using the individual nominal rates for each year as follows:

$$V_1 \equiv (-50)\ (1.065)(1.71) + (-200)(1.071) + 60 + \frac{102}{(1.071)} + \frac{312.12}{(1.071)(1.05)}$$

$$= -56.790 - 214.200 + 60 + 95.238 + 277.551$$

$$= 161.799\,.$$

This is of course a different value than computed previously because the interest rates are different. But if we were to convert the nominal values in the example to real values using the latter stream of inflation rates and then discount them using the constant real interest rate of 5%, the value at end of period one would be the same. This can be verified by first computing the real values as follows:

$$V^r(-50) = (-50)(1.01)(1.02) = -51.51$$

$$V^r(-200) = (-200)(1.02) = -204$$

$$V^r(60) = 60$$

$$V^r(102) = \frac{102}{1.02} = 100$$

$$V^r(312.12) = \frac{312.12}{(1)(1.02)} = 306.$$

Discounting these real values with a 5% real rate as before will give

$$V_1 = \sum_{t=-1}^{3} \frac{R_t}{(1.05)^{t-1}}$$

$$= (-51.51)\ (1.05)^2 + (-204)(1.05) + 60 + \frac{100}{(1.05)} + \frac{306}{(1.05)^2}$$

$$= -56.790 - 214.200 + 60 + 95.238 + 277.551$$

$$= 161.799.$$

This gives the same value as the analysis using nominal values and nominal interest rates because both real and nominal value are discounted to the base period for defining real values.

The prediction of period-by-period inflation rates and asset price movements is not an easy task and is probably of second order concern in estimating production costs. Thus, rather than deal with nominal values and potentially different inflation and interest rates for each period of the analysis, it may be simpler to assume that all values outside of the current one are in real terms. This is especially true for projected estimates.

The Task Force recommends for projected estimates that all values outside of the period of analysis (current period in most cases) be denominated in real terms as of the end of the period of analysis. The Task Force also recommends that real interest rates be used for discounting flows between these outside periods. For historical estimates, the Task Force recommends the use of real values and real cash flows whenever feasible and straightforward to compute.

In summary, then, the preferred approach is to use nominal rates within the year and real rates between years.

The Task Force recommends that CARs associated with production processes or assets lasting more than one year be calculated in nominal terms at the end of the production period and that nominal interest rates be used to discount such CARs within the given

year. The Task Force suggests the end of the production period to be the base period for real values. The Task Force also suggests that CARs for years other than the production period (year) be computed on a real basis and that the real interest rate be used for discounting these returns between periods.

Risk Premiums

When different income streams have different risk attributes, they will be evaluated differently by individuals. Similarly, in the market, investments having the same expected income but different risk characteristics will be priced differently. For example, the market price for farmland with an expected cash rental per acre per year of $120 may be different from the market price of a bond which guarantees $120 per year in perpetuity. And market rates of interest for income streams with risk properties similar to those in agriculture may be different from those for sectors of the economy that have different risk properties. Therefore, the general market rate of interest should not be used as the rate for most agricultural applications, but rather, a market rate adjusted for the risk inherent in agriculture should be used. This is not the rate of interest farmers pay for agricultural loans inasmuch as the commercial interest rate for agricultural loans may contain loan management fees and other distortions that reduce its value as a measure of the true discount rate. The preferred approach is to start with a risk-free market interest rate from the general economy and adjust it upwards for risk in agriculture. This topic along with a discussion of how to choose real and nominal interest rates for use in CAR estimation is discussed in the next section.

Choosing Rates of (Opportunity) Interest for CAR Estimation

An important issue in estimating CARs is choosing an appropriate opportunity cost of capital to use for discounting various income and cost flows. When nominal interest rates are high, the choice of a nominal rate and the associated inflation rate can have a significant impact on the magnitude of CARs. The rate that is appropriate for one type of analysis may not be the best rate to use for other purposes. For example, the rate to use in discussing peanut production in rural Georgia may not be appropriate for computing the average returns to Great Plains winter wheat. Furthermore, the rate to use for composite historical budgets may be different from the rate to use for a planning budget for an individual vegetable farmer. The key factor is to select a rate that reflects the actual market evaluation of alternatives to the cost, return, and risk associated with a given expenditure or revenue. There are two basic approaches to determining appropriate interest rates. The first is the so-called **bottom up** method, which starts from a risk-free real rate for the general economy, adds in a factor to account for riskiness of agricultural investments, and then another to account for inflation. Finally, the rate may be adjusted to account for transactions costs associated with investments. The second approach is the **top down** approach, which starts with the nominal interest rate charged on agricultural loans and attempts first to back out charges for transactions costs, and then adjust for inflation and riskiness to compare with non-risky real rates. Although the two approaches should give similar answers for the real and nominal rates to use in CAR estimation, there will be some differences due to a variety of errors in estimation. The biggest problem with using the agricultural loan rate in the top down approach is the difficulty in determining the portion of the rate due to transactions costs.

The Task Force recommends that the bottom up approach of building from a risk-free general rate to a risky nominal rate for agriculture be used whenever feasible. This means starting from a risk-free rate for the general economy, adding a factor to account

for riskiness of agricultural investments, and then another to account for inflation. Specifically, the Task Force recommends using the chained price index for the consumption component of GDP as the inflation factor.

Determining the Risk-free Real Rate of Interest from a General Nominal Rate

The first step in the process is to obtain a risk-free real rate of interest as the basis for the other calculations. We think of a riskless asset as one with a nearly zero probability of default, and that is frequently traded at a negligible transactions cost, e.g., a Treasury bill (T-bill) or note. The risk-free rate of return will usually be different depending on the period of time the investment (asset) is to be held. A plot of the yield on government bonds with differing times to maturity but the same risk, liquidity, and tax considerations is called a **yield curve** (Mishkin, 1995: Chapter 7). The rate of return on a longer-term bond is related to the expected yield on shorter-term bonds because investors who have no inherent preference for one maturity over another will trade in equilibrium such that the expected rate of longer-term bonds will equal the average expected rate on shorter-term bonds that could be held. The yield curve generally slopes upwards because of the price risk associated with holding bonds for longer-term periods as opposed to holding shorter-term bonds and rolling them over. This liquidity premium associated with longer-term bonds premium leads to a higher risk-free rate for most long-term assets. For CAR estimation, if a particular expenditure commits capital for a long time period, the appropriate opportunity cost of that capital may be different than if the capital is only committed for a few months. For example, the rate for three-month T-bills might be used to proxy the riskless interest rate for money invested in producing vegetable crops and feeding enterprises, the rate for either six-month or one-year T-bills could be used for annual crops, and the rate for longer-term government bonds could be used for multiyear investments. It is most common, however, to choose a single rate for CAR estimation.

In the United States, the *ex ante* real short-term interest rate for a riskless asset (expressed in purchasing power over consumer goods) is usually approximated by the average nominal interest on a U.S. T-bill, adjusted by the expected rate of inflation. The *ex post* real interest rate on U.S. T-bills is the average annual nominal interest rate on these T-bills minus the actual rate of inflation. The actual rate of inflation is usually computed from some type of price index. For a one-year T-bill issued in January 1995 and redeemed in January 1996, the annual rate of inflation in 1995 is the appropriate adjustment factor. However, for a one-year T-bill issued in August 1995, the annual rate of inflation over the period August 1995-July 1996 is appropriate. Thus, there is some difficulty in using reported annual inflation rates to adjust annual average T-bill rates. One alternative is to always use the T-bill rate for the first month of the year. The more common method is to use the average annual rate on T-bills and simply use average calendar year inflation and assume the error from using the wrong period for the inflation adjustment is minimal. For projected budgets, the ex ante rate is most appropriate. Given the difficulties in forecasting inflation rates, however, a common practice for determining an expected real rate of interest is to average the ex post real rates for several years and use this as a forecast rather than use the Livingston index (Croushore, Bomberger and Frazer) or an econometrically estimated (Engle, Diba and Oh) expected rate of inflation. For example, in 1996 the average nominal interest rate on U.S. treasury securities at constant one-year maturity was 5.52% and the annual rate of inflation based on the price index for the consumption component of GDP was 2.2%. Using the Fisher equation (2.5) the implied **U.S. real rate of interest (return) on the "riskless" asset** (i.e., a one-year note) can be computed as 3.2485 % {([.0552 - .022]/1.022)(100)} for the year. If the interaction

term in the Fisher equation rate is ignored, the ex post rate of return for the year was 3.32% {(.0552 -.022)(100)}.

Choosing Appropriate Nominal Rates of Interest from which to Construct a Risk-free Real Rate

The most commonly traded risk-free assets are various forms of U.S. government securities such as T-bills, notes, and bonds.

> ***The Task Force recommends that the nominal annual returns on U.S. government securities of various lengths be used as the basis for risk-free real rates and that the risk-free real rate be estimated as*** $r = \frac{(1 + i)}{(1+\pi)} - 1$ ***where i is the nominal rate, r is the real rate, and*** π ***is the rate of inflation computed using the chained price index for the consumption component of GDP.***

The nominal rates of return on U.S. securities of various terms are given in Table 2.5. The change in the chained price index for all of GDP and its personal consumption component, as well as changes in the implicit price deflators for both series, are also reported. The change in the price index is probably the better measure to use for reflecting inflation. Ex post real rates of return on each security using the change in the personal consumption price index to adjust for inflation are also reported. Note that for T-bills the rate quoted is a discount rate that can be converted to a simple interest rate using the formula i = d/(1-d) (d is the discount rate expressed in hundredths) because T-bills are sold at a discount from face value and thus earn more than the discount rate. For example, a discount rate of 5% (.05) is equivalent to a 5.26% (.0526) simple interest rate. Notice that the real rate of return tends to be higher for assets with a longer maturity. For example, the average real return from 1987 through 1996 was 2.25% for three-month T-bills, 2.12% for one-year T-bills, and 4.03% for thirty-year Treasury notes. Estimates for the real rate of return on assets from current income in agriculture (a risky income stream) prepared by the ERS (USDA, Economic Indicators of the Farm Sector: National Financial Summary) are generally higher than the rate on ten-year notes and less than the rate on thirty-year notes. The rate of return in agriculture including capital gains is higher, averaging 4.05% over the period 1964-95. This risky nominal rate of return is also quite variable with a standard deviation above 5.

The data in Table 2.6 illustrate that although the derived real rates have fluctuated from year to year, the average over a period of years is relatively constant for each length of security other than during periods such as the 1970s when government policies resulted in negative real interest rates (Wilcox). If averages such as those in the top section of Table 2.6 are recomputed eliminating all years with negative real interest rates (1971-78), the results seem even more stable. Results obtained by eliminating the years 1971-78 from the multiyear averages are contained in the bottom section of Table 2.6. The years in the first column represent the first year of an average. For example, the fifteen-year average from 1982-1996 is reported in the 1982 row. Comparing the numbers for the averages ending in 1996, they range from 2.553 for the twenty-year average of the six-month rates to 4.7463 for the ten-year average of the thirty-year rates. Although recent work in monetary economics has indicated that the real rate may not be stationary over long periods (Mishkin 1981, 1992; Herndershott and Peek; Rose; Fried and Howitt; Gagnon and Unferth; Patel

and Akella), there is still much debate on the subject. Garcia and Perron indicate that the ex post real rate was essentially random with means and variances that are different for the periods 1961-73, 1973-80, and 1980-86.

Given these studies, the most recent ten-year moving average of real interest rates computed from the comparable nominal rates is a good alternative. Based on the data in Tables 2.5 and 2.6, **a reasonable riskless real rate for U.S. investments in most crop and livestock inputs is in the range of 2.0% to 3.5%**. This riskless rate is also very consistent with long-held beliefs that the real interest rate is between 2% and 4% (Simon). The risk-free rate applicable to investments with long maturities may be higher due to the term premium.

Risk Differentials and Risky Discount Rates

The real (and nominal) interest rates or rates of return on all types of assets for any time period are generally different and not perfectly correlated. Thus, individuals, households, and businesses that are risk averse can reduce their income risk by diversifying their holding of assets. Among risky assets, competitive market forces cause equilibrium-compensating real rate of return differentials to emerge. Assets that have greater risk (e.g., corporate bonds, venture capital, shares of stock, or shares in a mutual fund) than a riskless asset usually have a higher real rate of return than the riskless asset. Among risky assets, competitive forces insure that, on average, the expected (and average actual) rate of return will be higher for more risky assets. Thus the expected return to investing in limited partnership office buildings may be larger than the expected return from investing in a "conservative" mutual fund based on the associated return patterns. But given the ability of diversification to reduce risk, the return premium demanded by an investor to commit funds to a particular asset depends not only on variability of returns associated with that asset, but also on how that asset contributes to the variability of the total investment portfolio.

An individual (producer or outside investor) considering an investment in agriculture must then consider the distribution of returns on the investment and their interaction with the other returns in the individual's total asset portfolio. Returns to many agricultural enterprises and operations are quite variable due to weather, disease, incidence of pests, and market prices. Whether these variable returns increase the risk associated with the investor's total portfolio depends on the composition of assets held. Many agricultural producers are not well diversified outside of agriculture and so bear considerable risk by holding assets and operating a farm business. On the other hand, many outside investors may add little risk to a well-diversified portfolio by adding agricultural investments. In considering the opportunity cost of funds invested in the farm business, one must then consider the type of asset portfolios to which the funds will be added. For a well-diversified portfolio, the premium above the risk-free rate that the investor expects may be low, but for a portfolio comprised primarily of other agricultural assets, the risk premium above the risk-free rate may be substantial.

The Capital Asset Pricing Model (CAPM) and Risk-Adjusted Discount Rates

The capital asset pricing model developed by Sharpe, Lintner, and Mossin is a market-based model that attempts to predict the equilibrium rate of return on an asset based on its contribution to a total market wealth portfolio. The model argues that individual capital assets are priced in equilibrium to reflect the asset's contribution to the risk of a well-diversified portfolio, and that risk premiums are paid only to an

asset's owner for bearing the systematic, or market, risk that is pervasive in the universe of assets. Given its assumptions, the model implies that investors will be able to diversify away all risk of holding a particular asset except the covariance of that asset with the market portfolio. The model then implies that as the covariance between an asset's returns and market returns becomes larger, the asset's price is adjusted to provide higher rates of return. The empirical version of the model implies that the expected rate of return of an individual asset above the risk-free rate of return is a linear function of the excess of the expected rate of return of the "market" portfolio over the same risk-free rate. Let the random rate of return to asset j be given by $\tilde{R}_j$, the random rate of return to the market portfolio by $\tilde{R}_m$, and the risk-free rate of interest by R_f. Let a bar (-) above a rate of return denote the expected excess rate of return for either the asset or the market portfolio so that $\bar{R}_j = E(\tilde{R}_j) - R_f$ and $\bar{R}_m = E(\tilde{R}_m) - R_f$. Then we have as the empirical version of the CAPM $\bar{R}_j = \alpha_j + \beta_j \bar{R}_m$. The constant term ($\alpha_j$) is hypothesized to be zero, and the slope coefficient (beta) is equal to σ_{jm}/σ_m^2 where j indexes the j^{th} asset, m indexes the market portfolio, and σ_{jm} is the covariance between returns to asset j and the market portfolio. Estimates of the model parameters can be obtained using ordinary least squares by appending a serially uncorrelated zero mean normal random disturbance to the right-hand side of the equation. It is also assumed that the contemporaneous correlation across assets is stationary. Estimates of β then provide information of the relative riskiness of alternative assets with higher levels of β implying higher risk. While α_j is hypothesized to be zero, non-zero estimates can be used to compare the expected returns of a particular asset to those of assets with similar values for β. As a general alternative to CAPM, arbitrage pricing theory (APT) developed by Ross argues that the price of an asset depends linearly on k factors rather than the single factor represented by the rate of return on the market portfolio. These factors are common to the returns of all assets under consideration. The CAPM and APT models can be used to determine how an investment in a particular stock, type of real estate, or other asset contributes to the performance of a well-diversified market portfolio. They can also be used to determine if the observed rate of return on a particular investment is similar to returns with the same level of risk.

Empirical Evidence on Riskiness of Asset Returns in Agriculture

Several authors (Barry; Irwin et al.; Bjornson and Innes; Bjornson) have applied CAPM and APT models to agricultural assets. The purpose of these studies was to determine whether or not investments in agriculture can help diversify away risk for holders of the market portfolio, and to compare the agricultural returns to nonagricultural returns with similar riskiness. Both Barry and Irwin et al. find that there is little, if any, risk premium for holding agricultural assets using CAPM and an inflation-adjusted CAPM. They also find that risk-adjusted returns to agricultural assets are slightly higher (the constant term [α_j] is positive in the regression) than expected under CAPM. Irwin et al. also suggest that these returns are sensitive to inflation. Bjornson and Innes attempt to obtain separate effects for landlords and owner-operators using both CAPM and APT. They find similar CAPM results for landlords but different results for owner-operators. They find a positive β for owner-operators but a negative α, implying less than expected risk-adjusted returns for this group. Using a cross section regression, they find that returns to agricultural owner-operators are significantly lower than returns to owners of nonagricultural assets with the same level of systematic risk. They find that returns on farmland ownership are higher than on nonagricultural assets, but only statistically so at the 20% level. Their APT model implies that returns to both landlords and owner-operators are sensitive to some systematic (market) risk. Land, in particular, may be a hedge against unexpected inflation. Again, the returns to land ownership seem to be higher than for similar risk nonagricultural assets, but the returns to owner-operators seem to be lower. The required returns to landholders may be larger than on the

market portfolio due to the illiquid nature of land or to the fact that many land owners may have poorly diversified investments. The lower rate accepted by owner-operators may be due to psychic benefits from farming (Brewster). **Based on these results, it is not clear that the risky rate of return for all of agriculture is any higher than for comparable risk nonagricultural assets (of similar β), and it may be slightly lower.**

Adjusting the Risk-free Real Interest Rate for Use in Agricultural CAR Analysis

Given a well-specified CAPM for returns to agricultural assets in which α is close to zero, an estimate of the excess rate of return to be used for discounting can be obtained by multiplying the market excess rate of return by the estimated β. If α is significantly different from zero and one believes that this is a structural phenomenon common to assets in agriculture, the predicted value from the CAPM model using both α and β coefficients could be used. If α is positive, this implies that investments in agriculture yield a higher rate of risk-adjusted excess return than the market portfolio. This could possibly be due to limited portfolio diversification by individuals typically investing in agriculture. When α is negative, however, this implies that the agricultural producer is accepting a rate of return less than that possible from choosing a well-diversified portfolio of similar β risk.

A simple, but inexact, alternative for CAR estimation may be to consider the real market rate of return as a ballpark estimate of the risk-adjusted rate of return for agriculture with the idea that the well-diversified investor should be able to get at least this rate of return with normal risk. Given the relatively low β's typically estimated for agricultural assets (or higher β's with negative α's) and the opportunity for most agricultural investors to diversify if they so choose, it may be appropriate to view the market rate of excess returns as an upper bound on this risk adjustment. **Therefore, an estimate of the upper bound for the risky real discount rate for agricultural cash flows (or "assets") can be obtained by simply adding estimates of the market excess rate of return to the chosen real risk-free interest rate.**

A crude estimate of the risk premium for a specific type of agricultural production where person receiving the returns is not diversified outside this investment could be calculated using a series of annual returns on assets used for the particular type of production and region of interest. The series of returns on agricultural assets should be expressed in real terms. A nominal series could be deflated by the change in a price index to obtain a real series. The average difference over a series of production periods between this real return on agricultural assets and the real return on government securities is a crude estimate of the risk premium. This relatively simple approach could be used to estimate the risk premium for various sizes and types of agricultural production when the required data are available. But because investors in agricultural production have access to the general capital markets, the excess returns associated with these markets are probably more relevant.

Estimates of the Market Excess Rate of Return

In their paper investigating returns in agriculture, Bjornson and Innes estimate a mean excess return for their constructed "market portfolio" over the years 1963-84 of 3.2%. Fama and French, in a paper on common risk factors in returns to stocks and bonds, find an average excess monthly return of 0.43% for their market portfolio over the period 1963-1990. This is equivalent to a 5.28% [1.0043^{12} -1] annual excess rate of return. They report an excess return for AAA-rated corporate bonds of 0.06% per month with an excess return on BAA-rated bonds of 0.14% per month. The annual equivalents to these monthly rates are 0.70%

and 1.69%. Carhart, in a paper on persistence in mutual fund performance, uses a value-weighted stock index prepared by the Center for Research in Security Prices (CRSP) minus the one-month T-bill return as the market excess rate of return. This averages 0.44% over the period July 1963-December 1993. This translates into a 5.41% [1.0044^{12} -1] annual rate. Data on historical returns on stocks, bonds, and bills prepared in 1997 by Ibbotson Associates indicate annual real rates of returns on large company (S&P 500) stocks above the risk-free interest rate of 4.14% for the period 1964-96, 5.88% for the period 1982-96, and 7.41% during the recent high return period of 1987-96. Rates on a set of small company stocks averaged 9.78%, 9.38%, and 4.55% for the same periods. **Based on these studies and others, a reasonable estimate of an additive risk adjustment for agricultural investments would be from 3 to 6%.**

Suggested Risky Real Discount Rates for Agriculture

Given a long-term real rate of 2.0 to 3.5%, and an additive risk adjustment in agriculture from 3 to 6%, the long-term risky real rate for investments in agriculture probably ranges from 5.0 to 9%. This is significantly higher than the average rate of return on assets from current income for all of U.S. agriculture of 3.29% as reported in Table 2.5 for the years 1964-95. It is also higher than the rate of return on assets including capital gains which averaged 5.4% over the same period. Thus, the opportunity costs of funds invested in agriculture operations may tend to be higher than their own rate of return if the capital gains do not accrue to the investor.

Operating a farm business is a risky venture. Returns in any one year are highly variable due to weather, biological catastrophe, labor problems, and prices. The probability of economic failure during any time period is larger than zero. For individual farms, especially those that are in "poor" financial condition, this risk may be substantial. Institutions loaning money to agricultural enterprises may demand a premium because of the probability of default, particularly if the lender is not well diversified. Thus the price charged for agricultural loans may be higher than for loans in some other sectors of the economy. But this loan risk premium is not directly relevant for analyzing the opportunity cost of funds invested in agriculture. The opportunity cost for agricultural funds should be based on alternative investments in the rest of the economy. If there is some desire to account for this cost of loanable funds, some type of weighted cost of capital might be used instead of the opportunity cost (Levy and Sarnat 1994: Chapter 17). This approach, however, is not generally recommended for risky investments by practitioners in capital budgeting (Bierman and Smidt: 397).

TABLE 2.5 Nominal and Ex Post Real Interest Rates 1964-1996 (Averages begin in year noted)

Year	3-month T-bill	6-month T-bill	1-year T-bill	1-year Note	3-year Note	5-year Note	10-year Note	30-year Note	Δ GDP Price‡ Index	Δ GDP Deflator	Δ PCE† Price‡ Index	Δ PCE Deflator	3-month T-bill (Real)§	6-month T-bill (Real)	1-year T-bill (Real)	1-year Note (Real)	3-year Note (Real)	5-year Note (Real)	10-year Note (Real)	30-year Note (Real)	ROR¶ Assets U.S. Ag.
1964	3.56	3.69	3.75	3.85	4.03	4.07	4.19		1.5	1.5	1.4	1.4	2.030	2.258	2.318	2.416	2.594	2.633	2.751		2.52
1965	3.95	4.05	4.06	4.15	4.22	4.25	4.28		1.9	2	1.6	1.6	2.012	2.411	2.421	2.510	2.579	2.608	2.608		3.54
1966	4.88	5.08	5.07	5.2	5.23	5.11	4.93		2.8	2.8	2.6	2.6	2.023	2.417	2.407	2.534	2.563	2.446	2.446		3.82
1967	4.32	4.63	4.7	4.88	5.03	5.1	5.07		3.2	3.2	2.7	2.7	1.085	1.879	1.947	2.123	2.269	2.337	2.337		2.9
1968	5.34	5.47	5.46	5.69	5.68	5.7	5.64		4.4	4.4	4	4	0.900	1.413	1.404	1.625	1.615	1.635	1.635		2.65
1969	6.68	6.85	6.79	7.12	7.02	6.93	6.67		4.7	4.7	4.1	4.1	1.891	2.642	2.584	2.901	2.805	2.719	2.719		3.21
1970	6.43	6.53	6.49	6.9	7.29	7.38	7.35		5.3	5.3	4.7	4.7	1.073	1.748	1.710	2.101	2.474	2.560	2.560		3.09
1971	4.35	4.51	4.67	4.89	5.66	5.99	6.16		5.2	5.2	4.5	4.5	-0.808	0.010	0.163	0.373	1.110	1.426	1.426		3.15
1972	4.07	4.47	4.76	4.95	5.72	5.98	6.21		4.2	4.2	3.5	3.5	-0.125	0.937	1.217	1.401	2.145	2.396	2.396		4.33
1973	7.04	7.18	7.02	7.32	6.96	6.87	6.85		5.6	5.6	5.4	5.4	1.364	1.689	1.537	1.822	1.480	1.395	1.395		7.82
1974	7.89	7.93	7.72	8.2	7.84	7.82	7.56		8.9	9	10.1	10.1	-0.927	-1.971	-2.162	-1.726	-2.053	-2.071	-2.307		4.68
1975	5.84	6.12	6.3	6.78	7.5	7.78	7.99		9.4	9.4	8.1	8.1	-3.254	-1.832	-1.665	-1.221	-0.555	-0.296	-0.102		3.73
1976	4.99	5.27	5.52	5.88	6.77	7.18	7.61		5.8	5.8	5.7	5.7	-0.766	-0.407	-0.170	0.170	1.012	1.400	1.400		2.2
1977	5.27	5.52	5.7	6.08	6.68	6.99	7.42	7.75	6.5	6.5	6.6	6.6	-1.155	-1.013	-0.844	-0.488	0.075	0.366	0.769	1.079	1.88
1978	7.22	7.58	7.74	8.34	8.29	8.32	8.41	8.49	7.3	7.3	7.3	7.3	-0.075	0.261	0.410	0.969	0.923	0.951	1.034	1.109	2.46
1979	10.05	10.02	9.73	10.7	9.7	9.51	9.43	9.28	8.5	8.5	9	9	1.429	0.933	0.670	1.514	0.642	0.468	0.394	0.257	2.64
1980	11.51	11.37	10.9	12	11.5	11.5	11.43	11.27	9.3	9.2	10.9	10.9	2.022	0.427	-0.045	0.992	0.550	0.496	0.478	0.334	1.28
1981	14.03	13.78	13.2	14.8	14.5	14.3	13.92	13.45	9.4	9.4	8.9	8.9	4.232	4.478	3.912	5.418	5.106	4.913	4.610	4.178	2.36
1982	10.69	11.08	11.1	12.3	12.9	13	13.01	12.76	6.3	6.3	5.8	5.8	4.130	4.994	4.981	6.115	6.739	6.815	6.815	6.578	2.29
1983	8.63	8.75	8.8	9.58	10.5	10.8	11.1	11.18	4.3	4.3	4.5	4.6	4.151	4.067	4.115	4.861	5.694	6.019	6.316	6.392	1.41
1984	9.35	9.77	9.94	10.9	11.9	12.3	12.46	12.41	3.8	3.8	3.8	3.8	5.347	5.751	5.915	6.850	7.823	8.150	8.343	8.295	3.34
1985	7.47	7.64	7.81	8.42	9.64	10.1	10.62	10.79	3.4	3.4	3.7	3.7	3.936	3.799	3.963	4.552	5.728	6.191	6.673	6.837	3.81
1986	5.98	6.03	6.07	6.45	7.06	7.3	7.67	7.78	2.6	2.6	2.8	2.8	3.294	3.142	3.181	3.551	4.144	4.377	4.737	4.844	3.34
1987	5.82	6.05	6.33	6.77	7.68	7.94	8.39	8.59	3.1	3.1	3.8	3.8	2.638	2.168	2.437	2.861	3.738	3.988	4.422	4.615	4.33
1988	6.69	6.92	7.13	7.65	8.26	8.48	8.85	8.96	3.7	3.7	4.2	4.1	2.883	2.610	2.812	3.311	3.896	4.107	4.463	4.568	4.02
1989	8.12	8.04	7.92	8.53	8.55	8.5	8.49	8.45	4.2	4.2	4.9	4.9	3.762	2.993	2.879	3.460	3.480	3.432	3.422	3.384	4.64
1990	7.51	7.47	7.35	7.89	8.26	8.37	8.55	8.61	4.4	4.3	5.1	5.1	2.979	2.255	2.141	2.655	3.007	3.111	3.283	3.340	4.29
1991	5.42	5.49	5.52	5.86	6.82	7.37	7.86	8.14	3.9	4	4.2	4.2	1.463	1.238	1.267	1.593	2.514	3.042	3.512	3.781	3.07
1992	3.45	3.57	3.71	3.89	5.3	6.19	7.01	7.67	2.8	2.8	3.3	3.3	0.632	0.261	0.397	0.571	1.936	2.798	3.591	4.230	4.12
1993	3.02	3.14	3.29	3.43	4.44	5.14	5.87	6.59	2.6	2.6	2.6	2.6	0.409	0.526	0.673	0.809	1.793	2.476	3.187	3.889	3.05
1994	4.29	4.66	5.02	5.32	6.27	6.69	7.09	7.37	2.3	2.3	2.4	2.4	1.945	2.207	2.559	2.852	3.779	4.189	4.580	4.854	3.69
1995	5.51	5.59	5.6	5.94	6.25	6.38	6.57	6.88	2.5	2.5	2.4	2.4	2.937	3.115	3.125	3.457	3.760	3.887	4.072	4.375	1.73
1996	5.02	5.09	5.22	5.52	5.99	6.18	6.44	6.71	2.1	2	2.2	2.1	2.860	3.029	2.955	3.350	3.043	3.058	3.385	3.251	
High	14.03	13.78	13.2	14.8	14.5	14.3	13.92	13.45	9.4	9.4	10.9	10.9	5.347	5.751	5.915	6.850	7.823	8.150	8.343	7.820	7.820
Low	3.02	3.14	3.29	3.43	4.03	4.07	4.19	6.59	1.5	1.5	1.4	1.4	-3.254	-1.971	-2.162	-1.726	-2.053	-2.071	-2.307	1.280	1.280
Ave	6.497	6.647	6.67	7.15	7.56	7.74	7.912	9.157	4.724	4.72	4.75	4.75	1.707	1.831	1.855	2.312	2.679	2.849	3.011	3.293	3.293
STD#	2.474	2.415	2.27	2.61	2.46	2.41	2.37	2.036	2.295	2.29	2.4	2.4	1.856	1.784	1.770	1.908	2.033	2.072	2.153	1.213	1.213

Source: Federal Reserve Bulletin (Board of Governors), Survey of Current Business (Department of Commerce), Economic Report of the President (U.S. Government Printing Office), various issues, unpublished data available from the Federal Reserve Board and the Department of Commerce.

† Change in Personal Consumption Component of GDP, ‡ Change in chained price index for series, § Real rates computed using nominal rates and the change in the chained price index for the consumption component of GDP ¶ Real rate of return on current assets in agriculture as computed by ERS # Standard Deviation.

TABLE 2.6 Multiple Year Averages of Ex Post Real Interest Rates 1964-1996 for Years Starting in 1964-1987

Starting Year	3-month T-bill 10 yr	3-month T-bill 15 yr	3-month T-bill 20 yr	3-month T-bill 25 yr	6-month T-bill 10 yr	6-month T-bill 20 yr	1-year Note 10 yr	1-year Note 15 yr	1-year Note 20 yr	1-year Note 25 yr	3-year Note 10 yr	3-year Note 20 yr	5-year Note 10 yr	5-year Note 15 yr	5-year Note 20 yr	10-year Note 10 yr	10-year Note 15 yr	10-year Note 20 yr	30-year Note 10 yr	30-year Note 15 yr
1964	1.1445	0.3513	1.0616	1.5733	1.7405	1.3671	1.980	1.167	1.820	2.3014	2.1634	1.9883	2.2154	1.5003	2.0607	2.2273	1.5378	2.084		
1965	0.8488	0.3112	1.2275	1.6426	1.3175	1.5418	1.566	1.107	2.042	2.3432	1.6987	2.2498	1.745	1.3559	2.3366	1.7214	1.3807	2.3636		
1966	0.3223	0.3119	1.3237	1.6813	0.8932	1.6112	1.193	1.006	2.144	2.349	1.3854	2.4073	1.4546	1.2151	2.5157	1.4504	1.2387	2.5668		
1967	0.0434	0.4591	1.3873	1.6588	0.6108	1.6474	0.956	1.198	2.195	2.3113	1.2302	2.4863	1.35	1.3795	2.6123	1.3458	1.3829	2.6814		
1968	-0.181	0.6621	1.4649	1.6407	0.3216	1.6618	0.695	1.464	2.232	2.2493	1.0109	2.5597	1.1529	1.678	2.6948	1.189	1.6814	2.7856		
1969	-0.278	0.8788	1.5641	1.6211	0.2064	1.7217	0.630	1.680	2.316	2.2166	0.9416	2.6738	1.0845	1.9703	2.8185	1.129	1.9935	2.927		
1970	-0.324	1.1092	1.6576	1.6232	0.0355	1.7393	0.491	1.943	2.344	2.2146	0.7253	2.7075	0.8594	2.3325	2.8542	0.8966	2.3685	2.9622		
1971	-0.23	1.3001	1.7529	1.6978	-0.097	1.7646	0.380	2.106	2.372	2.2689	0.533	2.7342	0.653	2.5745	2.8817	0.6884	2.6427	2.9984		
1972	0.2745	1.5736	1.8664	1.8445	0.3502	1.826	0.885	2.318	2.433	2.3879	0.9325	2.8044	1.0017	2.7713	2.9626	1.0068	2.8635	3.1027		
1973	0.7	1.7578	1.9043		0.756	1.7923	1.356	2.416	2.391		1.3919	2.7939	1.4436	2.8775	2.9826	1.4487	2.9985	3.1625		
1974	0.9787	1.8591	1.8566		0.9938	1.7341	1.660	2.515	2.340		1.8133	2.8096	1.906	3.0583	3.0367	1.9408	3.2031	3.2521		
1975	1.6062	2.1717	2.0002		1.766	1.943	2.518	2.861	2.569		2.8008	3.1012	2.9281	3.4252	3.3497	3.0058	3.585	3.5964		
1976	2.3252	2.5872	2.3098		2.3291	2.1904	3.095	3.119	2.803		3.4292	3.3169	3.5768	3.6523	3.5588	3.6833	3.8106	3.8051		
1977	2.7312	2.7358	2.491		2.684	2.3622	3.433	3.214	2.962		3.7423	3.4185	3.8746	3.7618	3.6417	4.017	3.9514	3.9044	3.9903	3.9728
1978	3.1105	2.855			3.0021		3.768	3.284			4.1086		4.2368	3.9239		4.3823	4.1396		4.3439	4.1829
1979	3.4063	2.8872			3.237		4.002	3.274			4.406		4.5525	4.0256		4.7251	4.2831		4.6898	4.3682
1980	3.6396	2.9217			3.443		4.197	3.363			4.6897		4.8489	4.2737		5.0278	4.5621		5.0026	4.6746
1981	3.7353	2.9826			3.6258		4.363	3.527			4.9354		5.1104	4.4997		5.3083	4.8018		5.3032	4.9441
1982	3.4584	2.8912			3.3019		3.980	3.389			4.6763		4.9234	4.3761		5.1986	4.7201		5.2635	4.8823
1983	3.1087				2.8286		3.426				4.196		4.5217			4.8763			5.0287	
1984	2.7344				2.4745		3.021				3.8059		4.1673			4.5634			4.7783	
1985	2.3943				2.12		2.621				3.4016		3.7712			4.1871			4.4342	
1986	2.2943				2.0516		2.512				3.2047		3.5408			3.927			4.188	
1987	2.2509				2.0404		2.491				3.0946		3.4089			3.7918			4.0287	
Ave	1.6706	1.7161	1.7049	1.6648	1.7514	1.7788	2.301	2.366	2.354	2.2936	2.6799	2.718	2.847	2.8764	2.879	2.9891	3.0076	3.0137	4.641	4.5041
High	3.7353	2.9826	2.491	1.8445	3.6258	2.3622	4.363	3.527	2.962	2.3879	4.9354	3.4185	5.1104	4.4997	3.6417	5.3083	4.8018	3.9044	5.3032	4.9441
Low	-0.324	0.3112	1.0616	1.5733	-0.097	1.3671	0.380	1.006	1.820	2.2146	0.533	1.9883	0.653	1.2151	2.0607	0.6884	1.2387	2.084	3.9903	3.9728

TABLE 2.6 (continued)

---------- Data truncated to eliminate the years 1971-1978 ----------

Starting Year	3-month T-bill 10 yr	3-month T-bill 15 yr	3-month T-bill 20 yr	6-month T-bill 10 yr	6-month T-bill 20 yr	1-year Note 10 yr	1-year Note 15 yr	1-year Note 20 yr	3-year Note 10 yr	3-year Note 20 yr	5-year Note 10 yr	5-year Note 15 yr	5-year Note 20 yr	10-year Note 10 yr	10-year Note 15 yr	10-year Note 20 yr	30-year Note 10 yr	30-year Note 15 yr
1964	1.8697	2.6371	2.6641	2.0607	2.6813	2.4134	3.3375	3.1971	2.3196	3.4980	2.2814	3.6244	3.6024	2.2538	3.6948	3.7262		
1965	2.0798	2.6776	2.5942	2.3343	2.5814	2.7833	3.3671	3.1049	2.7342	3.4651	2.6996	3.7148	3.6106	2.6601	3.8062	3.7682		
1966	2.2937	2.7357	2.5141	2.4999	2.4872	3.0184	3.4205	3.0198	3.0457	3.4258	3.0407	3.8147	3.6040	3.0309	3.9298	3.7971		
1967	2.6261	2.8517	2.5102	2.8333	2.4767	3.4500	3.4823	3.0357	3.5716	3.4866	3.6111	3.8804	3.6911	3.6205	3.9948	3.9038		
1968	2.9112	2.9779	2.6027	3.0253	2.5385	3.6929	3.5178	3.1024	3.9176	3.5612	3.9965	3.9321	3.7686	4.0542	4.0579	3.9906		
1969	3.1506	3.0154	2.7007	3.1982	2.6193	3.8854	3.5156	3.1886	4.1704	3.6325	4.2707	4.0259	3.8398	4.3644	4.1831	4.0781		
1970	3.2253	2.9315		3.1508		3.8814	3.3603		4.2637		4.3977	4.0312		4.5348	4.2413			
1979	3.4063	2.8872		3.2370		4.0024	3.2742		4.4060		4.5525	4.0256		4.7251	4.2831		4.6898	4.3682
1980	3.6396	2.9217		3.4430		4.1971	3.3633		4.6897		4.8489	4.2737		5.0278	4.5621		5.0026	4.6746
1981	3.7353	2.9826		3.6258		4.3634	3.5277		4.9354		5.1104	4.4997		5.3083	4.8018		5.3032	4.9441
1982	3.4584	2.8912		3.3019		3.9809	3.3898		4.6763		4.9234	4.3761		5.1986	4.7201		5.2635	4.8823
1983	3.1087			2.8286		3.4265			4.1960		4.5217			4.8763			5.0287	
1984	2.7344			2.4745		3.0212			3.8059		4.1673			4.5634			4.7783	
1985	2.3943			2.1200		2.6214			3.4016		3.7712			4.1871			4.4342	
1986	2.2943			2.0516		2.5120			3.2047		3.5408			3.9270			4.1880	
1987	2.2509			2.0404		2.4919			3.0946		3.4089			3.7918			4.0287	
Ave	2.8237	2.8645	2.5771	2.7641	**2.5530**	3.3588	3.4142	3.0920	3.7771	3.4873	3.9464	4.0181	3.6554	4.1328	4.2068	3.8372	**4.7463**	4.7173
High	3.7353	3.0154	2.6641	3.6258	2.6813	4.3634	3.5277	3.1971	4.9354	3.5612	5.1104	4.4997	3.7686	5.3083	4.8018	3.9906	5.3032	4.9441
Low	1.8697	2.6371	2.5102	2.0404	2.4767	2.4134	3.2742	3.0198	2.3196	3.4258	2.2814	3.6244	3.6024	2.2538	3.6948	3.7262	4.0287	4.3682

Sources: Federal Reserve Bulletin, Survey of Current Business, Economic Report of the President, various issues, unpublished data available from the Federal Reserve Board and the Department of Commerce.

† Change in personal consumption component of GDP, ‡ Change in chained price index for series.

The literature on capital budgeting under uncertainty (Bogue and Roll; Fama; Constantinides; Copeland and Weston: Chapter 12; Robison and Barry (1996): Chapter 23; Lee: Chapter 10) argues that risk-free interest rates used for discounting cash flows should be adjusted to account for the riskiness of the various flows, or that the flows should be adjusted to a certainty equivalent basis. There are a number of theoretical problems in doing this for long time horizons (Fama), but general practice in portfolio management and capital budgeting has been to use a constant risk-adjusted discount rate as estimated using an asset pricing model, and proceed as if this were the relevant and correct rate for each item. Although not specifically endorsing this approach, this Task Force feels this is a reasonable alternative for applied work.

Adjusting the Risky Real Discount Rate to Account for Inflation

The risky real discount rate can be adjusted upwards for inflation using the chained price index for the consumption component of the GDP and the Fisher equation. For example, if the real rate is 2.0% and the risk adjustment is 3.0% with 4% inflation, the implied risky nominal rate is

$$i = (.020 + .030) + .04 + (.05)(.04) = .092 = 9.2\%.$$

More precise adjustments, allowing for risk to affect the nominal rate directly, can also be considered if inflation is sufficiently high. Although the Task Force does not recommend specific real and nominal rates of return, it does recommend appropriate procedures.

The Task Force recommends:

(1) Adjusting the nominal rate of return for a class of government securities by the chained price index for the consumption component of GDP to obtain a risk-free real rate of discount for a class of agricultural assets with like maturity. This adjustment should use the Fisher equation (2.5).

(2) Adjusting the estimated risk-free real rate to account for risk in agriculture by either:

> ***(a) Using an asset pricing model to relate the excess rate of return on agricultural assets to the market excess rate of return, or***
>
> ***(b) Adding the market excess rate of return to the estimated risk-free real rate of return.***

(3) Adjusting the estimated risky real rate to account for inflation using the chained price index for the consumption component of GDP. This adjustment should use the Fisher equation (2.5).

VALUING THE SERVICES OF OWNED CAPITAL

Introduction and Example

The most controversial and complex cost calculations are those associated with the service flows from capital assets owned by the producer. As discussed in the section entitled Valuing Factors for which there is no Market Transaction, ***the Task Force recommends that market-determined costs of inputs should be used when they are available***. A market-based definition for the costs of capital services is as follows:

> The **cost (or revenue to owner) of capital services** for a given period is the market price the owner of the capital resource is able to obtain for these services. This is the cost that should be included in CAR estimates.

If there is a market transaction for the capital service, the associated price should be used to compute the service flow cost. When the operator of the firm owns the capital good and a market price cannot be obtained to value the service flow, it can be proxied by the returns that should accrue to that asset in economic equilibrium. This is done by assuming that the capital service will be offered for no less than the full costs of providing that service in an arm's-length market transaction. This can be done using data on similar market transactions (market prices for similar products or services, custom rates, etc.) or through determining the costs of providing the service. The discussion on determining these costs will build on simple examples and elementary concepts. The simplest ownership situation to consider is when the owner of the asset purchases it for use at the beginning of the period, obtains services from the asset which may reduce its service capacity, performs some maintenance and/or service enhancement during the period, incurs some other ownership costs, and then sells the asset at the end of the period. The asset may or may not have the same value at the end of the period as at the beginning depending on prices and use. Maintenance is usually considered an expendable cost that is necessary to maintain the basic service potential of an asset and extract its services; service enhancement costs are those associated with actions that significantly change the service potential. Lubrication is an example of maintenance and remodeling a packing shed would be considered service enhancement. We might say then that the costs of providing the services of the capital asset are as follows:

Capital service cost (CSC) = Opportunity cost of holding the asset
+ service capacity reduction cost
+ change in the price of the capital asset's service capacity
+ service enhancement cost
+ maintenance cost
+ other time costs.

More careful discussion of the definition and the various concepts contained in it will be given after discussing an intuitive first example. If the owner buys the asset and then sells it at the end of the period, all costs are directly observable. The asset has a known fixed service life at the beginning of the period and this

can be valued using the beginning-of-period market prices. The costs associated with maintenance and service enhancement are observed. Given the use, maintenance, and service enhancement that take place, the asset will have a different service life at the end of the period. This new service life also has a market value at end-of-period prices. If the end of the period is the period for which all costs are computed, then beginning and within period expenses can be inflated to the end of the period using the nominal rate of interest. A specific example will be used to illustrate this and other cases.

Suppose there is a tractor with 1,500 hours of useful life at the beginning of the year. The rental price of an hour of tractor time at the beginning of the period is $20. The tractor has beginning-of-period market value of $30,000. Assume that during the year the owner has maintenance costs of $200 for lubrication and minor repairs. At the end of the year the tractor has a useful life of 1,250 hours, either because it was used for 250 hours, or time and use together reduced its useful life by 250 hours. The price of an hour of tractor time at the end of the period is $21. Thus the market value at the end of the period is $26,250. Also assume that the owner performs service enhancement (new hydraulics) at the end of the year that increases the useful life to 1,300 hours. This service enhancement costs $1,050 in end-of-year dollars. With this service enhancement the tractor is now worth $27,300 [(1,300)(21)] at year's end. Assume the real interest rate is 4% and the rate of inflation is 5%. These two rates imply an implicit annual nominal interest rate of 9.2% {(.04+.05+(.04)(.05))(100)} using the Fisher equation. The implied nominal rate for a month is .7361% and the implied real rate for a month is .3274%. The data for this tractor are given in Table 2.7.

TABLE 2.7 Cost Data on Purchase and Sale of Tractor

Real interest rate	.04=4%
Inflation rate	.05=5%
Implied nominal interest rate	.092=9.2%

	Quantity (hours)	Price ($)	Total ($)
Beginning of period useful life	1,500	20	30,000
Midperiod maintenance			200
End-of-period service enhancement			1,050
End-of-period useful life before enhancement	1,250	21	26,250
End-of-period useful life after enhancement	1,300	21	27,300

The cost for the year is found by computing all explicit and implicit costs, and then adjusting them to an end-of-period value. There are a number of ways to do this calculation, each of which gives the same results but slightly different insights. Consider first simply adjusting all values to the end of the period and then comparing costs with revenues. The first cost is the purchase for $30,000. Adjusted to year end by the nominal interest rate, this gives a cost of $32,760. Assume that the maintenance all takes place at midyear (six months) for ease of computation. Also assume that the $200 is a nominal value as of the middle of the year. Then the adjusted maintenance cost is given by multiplying the actual maintenance cost by $(1+i)^{.5}$ which gives $(200)(1.092)^{.5} = \$208.99$. Prior to any service enhancement the tractor has a year-end value of $26,250. The total cost of using the tractor can be obtained by adding the adjusted purchase cost and the

adjusted maintenance cost and then subtracting the sale price of the tractor. Table 2.8 gives the data in tabular form.

TABLE 2.8 Cost of Using a Tractor Assuming Purchase and Sale Ignoring Service Enhancement (in $)

Item	Actual Cost/Return	End-of-Period Cost/Return
Purchase	30,000.00	32,760.00
Maintenance	200.00	208.99
Sale	-26,250.00	-26,250.00
Total cost in end-of-period $		6,718.99

An alternative approach is to include the cost of service enhancement, but also increase the projected sale price of the tractor to reflect this increased value. The service enhancement takes place at the end of the year and so need not be adjusted in value for time. At the end of the year, after service enhancement, the tractor has a useful life of 1,300 hours, which has a value of $27,300 [(1,300)(21)]. In tabular form this approach gives the data in Table 2.9.

TABLE 2.9 Cost of Using a Tractor Assuming Purchase and Sale Incorporating Service Enhancement (in $)

Item	Actual Cost/Return	End-of-Period Cost/Return
Purchase	30,000.00	32,760.00
Maintenance	200.00	208.99
Service Enhancement	1,050.00	1,050.00
Sale	-27,300.00	-27,300.00
Total cost in end-of-period $		6,718.99

The total cost of using the tractor can be obtained by adding the adjusted purchase cost, the adjusted maintenance cost, and the service enhancement cost, and then subtracting the sale price of the tractor. The total cost of $6,718.99 is the same as before. If maintenance is considered an expendable cost item, the cost of ownership and use is just $6,510.

The above example illustrates how to compute the costs of purchasing an asset, holding it for one period, and then liquidating it. In most situations an asset owner will not buy and sell an asset each period and so an alternative approach is needed. The suggested approach is based on the idea that the costs obtained should be the same as if the asset owner bought and sold the asset each period assuming efficient markets and no transactions costs. It is possible to divide these costs as follows: components associated with the

opportunity cost of holding financial wealth in the tractor, the real interest and inflation components of that cost, the costs associated with the tractor losing service capacity over the period, and the costs (revenues) associated with changes in the value of service capacity of the tractor due to price changes. Although such division is not necessary if the tractor is purchased and sold, it is essential in imputing costs if the tractor is held for several periods by the owner. Costs incurred for expendable items during the year have a direct component and an opportunity cost component for the funds tied up in the purchase. Costs for expendables at the end of the year have only a direct component because there is no explicit or implicit interest charge. Capital items will have only an opportunity cost because they are still available at the end of the year (though perhaps with a different service potential).

Estimating the Costs of Capital Services

The basic equation for estimating capital service costs is the standard present value recursion

$$V_1 = (1 + i)V_0 - R_1 \qquad \textbf{(2.18)}$$

where V_1 is the nominal value of the asset at the end of the first period, V_0 is the nominal value at the end of the 0th period, and R_1 is a net cash flow occurring at the end of period 1. If the value of the asset in the two periods is known, then an implicit value for R_1 can be obtained by rearranging equation 2.18 as follows:

$$R_1 = iV_0 + (V_0 - V_1)\,. \qquad \textbf{(2.19)}$$

The change in the value of an asset (V_0 - V_1), plus the opportunity cost of holding the asset (iV_0), is sometimes called **ownership cost**. Thus equation 2.19 implies that net cash flows are equal to ownership cost.

The **change in the value of an asset over a period (V_0 - V_1) is called economic depreciation**. For the general time period t, economic depreciation is given by (V_{t-1} - V_t). For an asset that is declining in value this will be a positive number and reflect a cost to the owner. Economic depreciation, which reflects changes in the market value of an asset between periods, is different from financial depreciation as computed for income tax purposes. Financial depreciation associated with buildings and equipment is the only type of depreciation that can be deducted for tax purposes. A landowner would consider changes in the productive capacity of land due to use in an economic analysis, but should not consider these in forming an income statement for tax purposes. An individual worker may consider a decline in her human capital as a hazard of holding a particular job, but her employer cannot usually deduct such an implicit cost for tax purposes.

The beginning value of the capital asset multiplied by the opportunity interest rate (iV_0) is called the **opportunity cost** of holding the asset and reflects compensation to the owner of the asset for the funds tied up in the asset over the period. Thus equation 2.19 implies that ownership costs are equal to opportunity cost plus economic depreciation.

Consider the example tractor where maintenance costs are treated as an expendable accounted for elsewhere rather than as a capital expense. The initial value is $30,000 and the final value before enhancement is $26,250. The implicit cost of holding the asset is then R_1 = (0.092)(30,000) + (30,000 - 26,250) = (1.092)(30,000) - 26,250 = $6,510. Alternatively, if the service enhanced value is used, the service enhancement cost of 1,050 is added to the implicit cost of the enhanced asset. The implicit cost of the service enhanced asset is (1.092)(30,000) - (27,300) = $5,460. The total cost is 5,460 + 1,050 = $6,510 as before.

Some of the costs associated with holding a capital asset occur simply because the capital is owned, some occur depending on its use, and some depend on the changes in the market price of a particular service capacity. Costs that occur simply **because the asset is held over a period are referred to as time costs.** The opportunity costs associated with the financial resources tied up in the capital asset are one form of time costs. The owner of the asset incurs an opportunity cost equal to the rate of return that the capital asset could earn if it were liquidated in the market and the funds reinvested. **Other time costs** include those costs associated with property taxes, general overhead, licenses, and insurance.

Measuring the Opportunity Costs of Capital

The opportunity cost for owned capital may be calculated by multiplying the beginning period value of the asset by the nominal next best rate of return. This next best rate of return is often proxied by the nominal interest rate so we obtain (iV_0) as in equation 2.19. This opportunity cost can also be obtained in a two-step procedure that measures the inflation component and then adds a measure of the real interest component. Alternatively, the opportunity cost can be calculated by measuring the real interest rate component and then adding an inflation component. The total opportunity cost will be the same in either case, but the division between components will differ depending on which adjustment was made first. The first method inflates the asset's beginning value to the end of the period using the inflation rate and then subtracts the beginning value to get the inflation component or equivalently multiplies the beginning-of-period value by the inflation rate. The inflation-adjusted end-of-period value is then multiplied by the real rate of interest implied by the next best investment opportunity to get the real interest component. The second approach multiplies the beginning-of-period value by the real interest rate then subtracts the beginning-of-period value to get a measure of the interest rate component. The real interest rate adjusted value is then multiplied by the inflation rate to get the inflation component. Both approaches assume that any capital gains implied by the nominal interest rate are accounted for in computing the asset's end-of-period market value. The first method is illustrated in equation 2.20a where the inflation adjustment is made first and it is assumed that the nominal interest rate is the next best available rate,

$$
\begin{aligned}
\textit{Opportunity cost} &= V_0\, i \\
&= [V_0\,(1+\pi)]\, r + [V_0(1 + \pi) - V_0] \\
&= V_0 r + V_0 \pi r + V_0 + V_0 \pi - V_0 \qquad \textbf{(2.20a)} \\
&= V_0 [r + \pi + \pi r] \\
&= V_0\, i\,.
\end{aligned}
$$

The second method is illustrated in equation 2.20b where the real interest rate adjustment is made first.

$$\begin{aligned} \textit{Opportunity cost} &= V_0\, i \\ &= [V_0(1 + r) - V_0] + [V_0(1+r)]\,\pi \\ &= V_0 r + [V_0(1+r)\,\pi] \\ &= V_0[r+\pi+r\pi] \\ &= V_0\, i\,. \end{aligned} \qquad \textbf{(2.20b)}$$

Now consider computing the opportunity cost for the example tractor already discussed as presented in Table 2.10. The opportunity cost of the initial investment of $30,000 can be computed in one of two ways. The first is to multiply the initial investment amount by the nominal rate of interest. This gives an end-of-period opportunity cost of holding the tractor of $2,760. This amount represents the real interest cost of holding the asset plus inflationary increase in V_0 over the period. This can also be obtained by inflating the value of the tractor to the end of the period using the inflation rate and then applying the real interest rate to this amount. Specifically, the $30,000 is inflated to an end-of-period value of $31,500. Thus the inflation cost of holding the tractor is $1,500. The end-of-period value is then multiplied by the implied real rate of return of 4% to obtain the real interest cost of $1,260. The total cost is $2,760, as before.

TABLE 2.10 Cost of Using a Tractor with Division of Components and No Service Enhancement (in $)

	Actual Cost/Ret.	Direct Period Cost	Opportunity Cost	Inflation Component	Interest Component	Direct + Opportunity Cost
Purchase	30,000.00	0.00	2,760.00	1,500.00	1,260.00	2,760.00
Maintenance	200.00	200.00	8.99	4.94	4.05	208.99
Serv. Decline	5,000.00	5,000.00	0.00	0.00	0.00	5,000.00
Price Change	-1,250.00	-1,250.00	0.00	0.00	0.00	-1,250.00
Total		3,950.00	2,768.99	1,504.94	1,264.05	6,718.99

It is of course arbitrary whether the real interest or inflation adjustment is made first. A different order will lead to a slightly different division among the components. For example, if the real interest adjustment were made first, the real interest component would be (1.04)30,000 - 30,000 = $1,200 and the inflation component would be (1.05)(31,200) - (31,200) = $1,560. The total is $2,760, as in the previous case.

Measuring Economic Depreciation

Economic depreciation is the change in the present value of an asset as time passes ($V_0 - V_1$). It is often useful to divide economic depreciation into costs that occur because of a **reduction in service potential** and those that occur due to **changes in market prices**. Costs that occur because the asset loses some of its service capacity during the period are called **service reduction costs**.

> The **service reduction costs** of holding a capital asset are the decline in the service capacity of the asset due to use and/or time. These costs are computed assuming constant real prices for the asset, and are given by multiplying the beginning-of-period market price for a unit of service by the amount that service potential (hours, years, quality-adjusted acres, etc.) declines during the period. Such service reduction can occur because of use or time, and may not be simply the number of hours the machine was used during the period. The amount of service capacity reduction that occurs in a given time period can be modified by use and/or care and maintenance.
>
> > **Service reduction due to use** is a decline in the service capacity of a capital asset due to operating, as opposed to not operating. These implicit costs occur because the use of the factor alters its future service potential. These costs are the real decline in service capacity and are not related to market prices. For example, using a tractor for more hours (or more intensive hours) during a period may reduce its expected useful life and its market value.
> >
> > **Service reduction due to time** is a decline in the original service capacity of a capital asset that occurs only as a result of the passage of time. Service reduction costs associated with time include only those that occur independent of market prices. For example, weather may reduce the life of a barn due to wear. Capital assets may also lose value over time due to obsolescence. A laborer's skills may no longer be adequate to perform previously performed tasks due to changes in technology (for example, the advent of computers).

The division of economic depreciation into service reduction and changes in market prices is seen most easily in the case where service potential is measured in a single dimension such as hours of remaining service. Let the market price of a unit of this asset service at the beginning of the period be given by p_b, the beginning service potential by q_b, and the ending service potential by q_e. The value of the amount of service reduction that occurs during the period is then computed as follows:

$$\begin{aligned} \textit{Service reduction cost} &= (p_b)\ (\textit{amount of service reduction}) \\ &= (p_b)\ (q_b - q_e)\,. \end{aligned} \tag{2.21}$$

Consider the service reduction costs for the example tractor. These costs are given by multiplying the decline in use potential (250 hours) by the price per hour of use ($20) for a cost of $5,000 as shown in Table 2.10. The service reduction costs for the full decline in service potential of 250 hours are all charged in this case, and the costs associated with enhancing the service capacity back to 1,300 hours are not included.

Capital goods can change in value independent of service potential due to changes in the market price of the asset's services. The opportunity cost computed for a capital good should reflect the market value of a specific service capacity. The market value of a capital good at the end of a given period should reflect both the service reduction and service enhancement that occurred during the period, along with any changes in the market value. This leads to a definition of the price change costs of a capital asset.

> The **price change costs** of a capital good include costs associated with changes in the market value of a good with a fixed service flow during a single production period that occur because of general inflation or deflation, or changes in market conditions related to that specific capital item. However, there may be other market forces that must be accounted for separately. For example, the discovery that in ten years a road will be built on a particular farm changes the market value of the farm even though the services extracted in the current period have not changed. Or, there may be a change in the price of the product produced by a capital asset that changes the asset's value. These capital gains or losses are usually accounted for separately from the other costs of holding capital.

Price change costs for an asset are computed using the service potential at the end of the period and the change in price over the period. Specifically, if q_e is the service potential at the end of the period, and beginning and ending prices are given by p_b and p_e, respectively, the cost associated with a change in price is

$$Price\ change\ cost = q_e (p_b - p_e) . \tag{2.22}$$

With rising prices, the price change cost will be negative. The total cost due to service reduction and price changes is given by the beginning-of-period value minus the ending value, or

$$\begin{aligned} Service\ reduction\ cost + price\ change\ cost &= economic\ depreciation\ (ED) \\ &= beginning\ value - ending\ value \\ &= V_0 - V_1 \\ &= p_b q_b - p_e q_e \end{aligned} \tag{2.23}$$

where q_b is the beginning service potential. This can be clearly decomposed into the two components in equations 2.21 and 2.22 by subtracting and adding $p_b q_e$ from equation 2.23 as follows:

$$
\begin{aligned}
\textit{Service reduction cost} + \textit{price change cost} &= \textit{economic depreciation } (ED) \\
&= p_b q_b - p_e q_e \\
&= p_b q_b - p_b q_e + q_e p_b - q_e p_e \\
&= p_b (q_b - q_e) + q_e (p_b - p_e) .
\end{aligned}
\tag{2.24}
$$

Aggregate or representative farm CAR projections usually assume that market prices of capital assets increase or decrease only by the general rate of inflation. Given declining real prices of agricultural goods and increased productive potential of new technologies, this practice may only be reasonable for short-run analysis covering three to five years.

Consider now the price adjustments for the example. The general inflation will cause some increase in the tractor's value. This will help offset the other costs of the tractor. The return from inflating prices can be computed by multiplying the end-of-period useful life of the tractor (1,250 hours) by the change in price (-1) for a return of $1,250 or a cost of -$1,250. The costs associated with the change in the value of the tractor over the period, $3,750 (30,000 -26,250), are thus clearly given by the decline in service capacity ($5,000) plus the change in value due to the price increase (-$1,250), for a total of $3,750.

Measuring Service Enhancement Costs

The next category of costs is that associated with enhancing the productive capacity of an asset. The **service enhancement** costs of holding a capital good are the **direct costs of increasing the service capacity of the asset**. These are the costs of expendables and other capital services that are used to alter the productive capacity of the asset. Because these costs allow for the provision of services for more than the current time period, they are normally treated as an investment in a capital asset and not as a period expense when the asset is not sold at the end of the period but is held for future use. The most common way to do this is to consider them as an adjustment to the service capacity of the asset to which they are applied, and then use this adjusted service capacity as the basis for all future cost calculations for that asset. Alternatively, the service reduction cost can be reduced if the service enhancement cost is charged in the current period and the enhanced service capacity is used to compute that decline in cost and also the change in market value, if any.

Consider computing service enhancement costs for the example tractor. One way to handle this computation is to use the calculations as in Table 2.10 but increase the value of the tractor when performing the analysis for future periods. Thus, rather than using the ending period value of $26,250, a higher value reflecting the enhanced service capacity could be used. For the example, this higher market value is $27,300 [(1,300)(21)]. An alternative is to reduce the service reduction costs to the amount necessary to cover the net decline (after enhancement) in value, and then include the service enhancement costs in the calculation. This is done in Table 2.11.

TABLE 2.11 Cost of Using a Tractor with Division of Components and with Service Enhancement Costs Included (in $)

	Actual Cost/Return	Direct Period Cost	Opportunity Cost	Inflation Component	Interest Component	Direct + Opportunity Cost
Purchase	30,000.00	0.00	2,760.00	1,500.00	1,260.00	2,760.00
Maintenance	200.00	200.00	8.99	4.94	4.05	208.99
Serv. Decline	4,000.00	4,000.00	0.00	0.00	0.00	4,000.00
Serv. Enhanc.	1,050.00	1,050.00	0.00	0.00	0.00	1,050.00
Price Change	-1,300.00	-1,300.00	0.00	0.00	0.00	-1,300.00
Total		3,950.00	2,768.99	1,504.94	1,264.05	6,718.99

Service decline costs are given by net decline in use hours (now only 200 hours) multiplied by the beginning-of-period price ($20). Thus the total service decline costs are $4,000. Service enhancement costs are now included and the adjustment for price changes is based on the enhanced capacity of 1,300 hours. Specifically, the price change effect is based on the change in price of $1 (21-20) multiplied by the enhanced service capacity of 1,300 hours. The result is the same total cost of $6,718.99, as before.

Maintenance Costs

The **maintenance costs** of holding a capital asset are the expenses required to maintain the service potential of the asset at a reasonable level and to extract services for a single time period. Activities associated with these costs are not usually viewed as enhancing the service capacity of the capital asset in any significant way when determining its end-of-period value. For example, expenses such as fuel, oil, and other lubricants are usually considered operating costs associated with the use of machinery and are treated as expendable inputs. Fence repair on land might be considered a maintenance cost of holding land, and mandatory pesticide education classes might be considered a maintenance cost for a farm employee. These costs usually are charged to the user of the capital service rather than the owner, although the distribution can differ by rental arrangement and custom.

Consider maintenance costs for the example case. The direct cost of this expendable is $200. Because this cost occurs at midyear, it implies an opportunity cost equal to the amount ($200) multiplied by $(1+i)^{.5}$ minus the original amount ($200) for an inflation plus real interest cost of $8.99 $[(200)(1.092)^{.5} - 200]$. This can also be obtained by adjusting the value to the end of the year using the inflation rate and then applying the implied real rate of interest to the inflation-adjusted amount. The inflation component is then $4.939 $[(200)(1.05)^{.5} - 200]$ and the real interest component is $4.058 $[(204.939)(1.04)^{.5} - 204.939]$. The sum of these two is $8.99, after rounding.

As pointed out previously, activities that restore a capital asset's lost service capacity should not be considered an expense in the current period because the lost capacity is often charged against the asset as a service reduction cost. Such activities should be treated as service enhancement costs, which can then be

treated as part of the potential service flow of the capital good. Care must be given to the estimation of service reduction, service enhancement, and maintenance costs as they affect the service potential in an interdependent manner. For example, if an engine loses 10% of its potential capacity during the period with regular maintenance, the 10% reduction in potential and the maintenance cost should be charged to the current period. If at the end of the period the owner makes a repair to restore 5% of the lost capacity, this should not be considered a cost in the current period unless an adjustment is made to the cost charged for reduced capacity. The most common procedure is to charge the full 10% service reduction cost and treat the 5% enhancement as an investment rather than a cost.

Combining the Costs of Capital Services

During a given production period, the owner of a resource incurs all the costs just outlined. Included are those costs associated with holding the asset over the period (including opportunity interest and other time costs), service reduction due to use and time, service enhancement, maintenance, and changes in price. A definition for the costs of owning and using a capital asset can be given as follows:

$$\begin{aligned} \textit{Capital service cost } (CSC) = {} & \textit{Opportunity cost} \\ & + \textit{ service capacity reduction cost} \\ & + \textit{ change in price of the capital asset's service capacity} \\ & + \textit{ service enhancement cost} \\ & + \textit{ maintenance cost} \\ & + \textit{other time costs} \\ & = iV_0 + (V_0 - V_1) + C_1 \\ & = iV_0 + ED + C_1 \end{aligned} \qquad (2.25)$$

where economic depreciation (ED) is defined as service reduction plus price changes and is given by (V_0 - V_1), and C_1 represents maintenance, service enhancement, and other time costs adjusted to the end of period 1. Service enhancement costs are in parentheses to remind the reader that these costs are usually handled in conjunction with service reduction costs or the price change adjustments. The costs of using the example tractor can then be divided into the opportunity cost of invested capital at the original capacity (opportunity cost), the decline in useful value at the beginning-of-period prices (service reduction cost), the decline or increase in market price due to inflation, the costs of maintenance adjusted to the end of the period, service enhancement, and other time costs. The costs can also be written as the sum of direct costs \$3,950 (200+4,000+1,050-1,300) and opportunity costs \$2,768.99 (2,760+8.99). This gives a total cost of \$6,718.99.

Based on these CSCs, the capital good is then offered for use during a production period. A market-based definition for the costs of capital services specifies that the **cost of capital factor services for a given period is the market price the owner of the resource is able to obtain for these services**. In simplistic terms this is just the rental rate the owner is able to obtain for the use of the asset for a given time period.

This is the cost that should be included in CAR estimates. When the firm operator owns a capital good and a market price is not available to value the service flow, the value can be proxied by the returns that should accrue to that asset in economic equilibrium. This is done by assuming that the capital service will be offered on the market for no less than the full costs of providing the service. Thus capital ownership and use cost can be used to proxy capital service cost. Preparers of CAR estimates often disregard maintenance costs in computing capital service costs because maintenance costs are usually included as an expendable item paid for by the user of the capital rather than the owner. This common practice may be suspect if repair and maintenance costs vary significantly over the life of the asset so that older assets have higher costs. It is also common to regard other time costs such as property taxes as an expendable if they are similar from year to year and can be accounted for as a general overhead expense that may or may not be allocated to a specific enterprise or use. Further, it is usually assumed that any service enhancement is treated as a separate investment. Thus, the most common approximation to use for capital services is

$$\begin{aligned} \textit{Capital service cost}\,(CSC) \approx\ & \textit{Opportunity cost} \\ & + \textit{service reduction cost} \\ & + \textit{change in price of the capital asset's service capacity}. \end{aligned} \tag{2.26}$$

Chapter 5 of this report contains more detail on computing maintenance costs, and Chapter 6 discusses other time costs plus those costs explicitly included in equation 2.26. For the example tractor, equation 2.26 gives a capital service cost of \$6,510 (2,760+5,000-1,250). This total is less than the previous calculations by the cost of maintenance.

Sometimes it is useful to combine the opportunity cost and changes in price into a measure that gives the real cost of holding the asset accounting for price changes. This might be called **net opportunity cost**. In this case the formula is modified to read

$$\textit{Capital service cost}\ (CSC) \approx \textit{Net opportunity cost} + \textit{service reduction cost}. \tag{2.27}$$

In our tractor example, the net opportunity cost would be 2,760-1,250 = \$1,510. Adding the service reduction costs of \$5,000 gives the total cost of \$6,510.

An alternative approach is to combine the terms concerning changes in value and add them to the opportunity costs as

$$\begin{aligned} \textit{Capital service cost}\ (CSC) &\approx \textit{Opportunity cost} + \textit{service reduction cost} + \textit{change in price} \\ &= \textit{Opportunity cost} + \textit{Economic depreciation}\ (ED) \\ &= iV_0 + (V_0 - V_1) \end{aligned} \tag{2.28}$$

which is the basic present value recursion used in equation 2.19. Substituting R_1 for capital service cost, this can be written as

$$R_1 = (1 + i)V_0 - V_1.$$

Using Annuities to Value Owned Capital

Although the above procedures are appropriate for estimating the current costs of using specific capital assets with known beginning and ending values, it is often useful to estimate a representative cost of using more generic capital over several time periods. This is particularly true for assets with a fixed life that lose value due to both use and time. The most common examples are machinery and equipment. Because of the decline in value of these assets due to use or time, the opportunity costs associated with ownership will tend to decline. The rising value of a given quantity of remaining usage due to inflation will, however, tend to compensate for this fact. Thus it is sometimes useful to use as the cost of the capital asset, not its current cost as computed above but rather, an annuity payment that has the same present value. The cost will then be the same for all years of the asset's life and there is no arbitrariness in picking a given year to assess costs. This can be either a real annuity that has constant real but changing nominal value or a nominal annuity that is constant in nominal dollars. This annuity is often referred to as the capital service cost (CSC) of the asset because it represents the annual cost of obtaining the asset's services. The discussion that follows assumes that maintenance and other time costs are excluded from computation of the annuity and are accounted for elsewhere. A discussion of the more general case is contained in Appendix 2C and in Burt (1992).

The formula for a nominal annuity, a^{nom}, that has the same discounted value as the actual costs of an asset over an n period horizon is derived in Appendix 2B (2B.10). This assumes that the asset is purchased at a cost of V_0 at the beginning of year 1 and is sold with value V_n at the end of year n. The resulting annuity (a^{nom}) is given by

$$a^{nom} = CSC = \frac{\left(V_0 - \frac{V_n}{(1+i)^n} \right)}{\left(\frac{1 - \frac{1}{(1+i)^n}}{i} \right)}. \qquad (2.29)$$

The numerator in equation 2.29 is just the present value at the beginning of the first period of the stream of payments associated with holding the asset for n years. As an example, consider the tractor discussed previously, assuming 1,500 hours of useful life to this firm and a five-year time horizon. Assume that after five years the tractor is sold having a useful life of 250 hours. The useful life of the tractor when it is sold or traded is often called the **salvage life** of the tractor. The value of this salvage life is called **salvage value.** Using straight line physical depreciation over the five years gives annual depreciation of 250 hours. Alternative assumptions concerning depreciation are discussed in Chapter 6. Table 2.12 shows the initial investment, the service reduction costs, and market price change costs for each year over the five-year period assuming an inflation rate of 5% and a real interest rate of 4%. The annual capital cost is computed from equation 2.26 and is equal to ownership cost plus service reduction cost plus the change in the price.

The first year cost is \$6,510, as before. In the second year the cost is 26,250(0.092)+250(21) + (−1.05)(1,000) = \$6,615. The reduction in service hours during this year is 250, and the beginning-of-year price of service is \$21.00 per hour. At the end of the second year the tractor has 1,000 remaining service hours and the price of an hour of tractor time increases from \$21.00 to \$22.05 dollars per hour. The price increase thus helps reduce costs. The capital cost in the fifth year is \$6,891.92. The value of these costs discounted to the end of the first period using the nominal interest rate of 9.2% is \$28,272.278 and to the beginning of the first period is \$25,890.3645. A nominal annuity paid at the end of each period beginning with the first that has the same value as this stream of \$25,890.3645 is \$6,690.7945. Thus a constant nominal payment of \$6,690.7945 at the end of periods 1 through 5 has the same present value as the actual cost stream. This amount can be determined without computing the costs for each year by using equation 2.29. In this case V_0 is \$30,000. The salvage life of the asset is 250 hours. To obtain the salvage value, this quantity is multiplied by the price (adjusted for inflation) for the fifth period or $V_n = (250)(20)(1.05)^5 = (250)(25.53) = \$6,381.407$, which is the same as the ending value for the fifth year in Table 2.12. Substituting these values into equation 2.29 we have

$$a = \frac{\left(30000 - \frac{(20)(1.05)^5(250)}{(1.092)^5}\right)}{\left(\frac{1 - \frac{1}{(1.092)^5}}{.092}\right)}$$

$$= \frac{\left(30000 - \frac{6381.4078}{1.55279}\right)}{\left(\frac{1 - \frac{1}{(1.092)^5}}{.092}\right)} = \frac{(30000 - 4109.6355)}{\left(\frac{1 - \frac{1}{(1.092)^5}}{.092}\right)} \qquad \textbf{(2.30)}$$

$$= \frac{25890.3644}{3.86955} = 6690.7945\,.$$

This constant nominal amount accounts for the cost of using the asset over the five-year time horizon. This is not the actual cost for any one period, but is a constant amount (an annuity) with the same present value as the stream of actual costs. This annuity can also be obtained using the standard annuity functions available on business calculators or in spreadsheet programs (such as PMT in EXCEL). In using such canned procedures, $\left(V_0 - \frac{V_n}{(1+i)^n}\right)$, which equals 25,890.3644 in this problem, should be used as the present value of the annuity with the assumption that the payment is made at the end of the period.

An alternative to computing this constant nominal cost is to compute the real annuity that has the same value as a noninflationary return stream and then inflate the value of this annuity each year in the cost

estimation. Thus, rather than using the nominal interest rate in equation 2.29, the real rate is used and the salvage value is expressed in constant end-of-year dollars. Because there is no inflation, V_n is computed assuming that prices are the same as at the beginning of the first period. This gives

$$a^r = \frac{\left(V_0 - \frac{V_n^r}{(1+r)^n}\right)}{\left(\frac{1 - \frac{1}{(1+r)^n}}{r}\right)}. \qquad (2.31)$$

It is important to note that V_n^r in equation 2.31 is computed assuming no inflation but V_n in equations 2.29 and 2.30 assumes a constant inflation rate over the entire time horizon. For the example, the real annuity is given by

$$a^r = \frac{\left(30000 - \frac{(20)*(250)}{(1.04)^5}\right)}{\left(\frac{1 - \frac{1}{(1.04)^5}}{.04}\right)} = 5815.6778. \qquad (2.32)$$

This is the real amount paid at the end of each period that has the same present value as the nominal stream in equation 2.30. Because inflation is 5% in the example, the actual amount to be charged in each period is given by the stream $a_j^r = a(1+\pi)^j$ or \$6,106.46, \$6,411.784, \$6,732.374, \$7,068.992, and \$7,722.4424. Rather than assuming a constant nominal amount in all years of \$6,690.79, this approach allows a real amount that grows with the rate of inflation. Thus for the first year the cost is \$6,106.46 rather than \$6,690.794. Note that the present value of this increasing stream is the same as the value of the constant stream of \$6,690.794. The first year cost of this increasing stream is also the cost that would be obtained if one were to consider inflation to occur during the first year and no inflation to occur thereafter. The annuity equivalent in this case is given by

$$a^m = \frac{\left((1+\pi)\,V_0 - \frac{V_n^m}{(1+r)^n}\right)}{\left(\frac{1 - \frac{1}{(1+r)^n}}{r}\right)} \qquad (2.33)$$

where a_m denotes a mixed nominal and real annuity and V_n^m is the salvage value assuming that inflation occurs only during the first year. This annuity has the same present value as a return stream with no inflation after the first year discounted to the present. This is the same as the real annuity given in equation 2.31 multiplied by $(1+\pi)$.

Consider now the example tractor where inflation is assumed to occur only for one year. The annuity is given by

$$a^m = \frac{\left((1.05)(30000) - \frac{(20)(1.05)(250)}{(1.04)^5}\right)}{\left(\frac{1 - \frac{1}{(1.04)^5}}{.04}\right)} = 6106.46 \qquad (2.34)$$

which is the same as the real annuity, $5,815.677, multiplied by $(1+\pi)$. The present value of this stream where the discount rate for the first year is i and for subsequent years is r is the same as for the previous two cases.

There is thus a choice when using an annuity to reflect the costs of a multiyear asset in cases where some magnitudes are in nominal terms. The nominal approach uses equation 2.29 and finds the constant nominal annuity that is equivalent to the nominal return stream where it is assumed that inflation continues at the current rate over the life of the asset. If this approach is used, all other costs and returns for future periods must also be in nominal terms. The adjusted real approach, which allows for inflation in the current period only, uses equation 2.31 to obtain a real annuity that is adjusted for inflation in the current period or uses the mixed annuity equation (2.33) to obtain an answer directly. The easiest solution is to compute a real annuity using 2.31 assuming that V_0 and V_n are both in beginning-of-period dollars, and then multiply this annuity by the assumed inflation rate. In the first case, a constant nominal amount will be used in all subsequent periods but in the latter case the amount will rise with the rate of inflation. Neither annuity is an exact cost for a given period but has the same present value as the exact stream.

The preceding discussion assumed the only costs associated with holding the asset over the five-year time horizon were the initial purchase costs plus the opportunity interest on the money tied up in this asset minus the present value of the income from salvaging the asset at the end of the time period. Thus the present value of the cost/income stream was simply $V_0 - \frac{V_n}{(1+i)^n}$. If other costs such as maintenance, service enhancement, or other time costs are part of the cost profile, then a year-by-year tabulation of the present value of costs/returns as demonstrated in Table 2.12 should be undertaken. Appendix 2C contains a more complete discussion.

VALUING THE CONTRIBUTION OF OPERATOR LABOR

All factors of production except the operator of the firm can be accounted for using the above concepts. Compensation for the operator of the firm is based on opportunity cost of off-farm work, or the return available in the next best alternative use of his time and effort. For example, the operator of a farm has an implicit cost of his farm hours that is the opportunity costs associated with the nonfarm use of these hours. The opportunity cost for the operator of a farm firm who also has the skills and experience equivalent to a factory worker is the going wage for manufacturing workers in the area. Ways of estimating the costs of the owner-operator's time are discussed in more detail in Chapter 8 on labor costs.

COLLECTING, CREATING, AND USING PRICE SERIES

Most historical data is collected in nominal terms for a specific month and year. When an historical estimate is created for a given year, this reported nominal data for that year is appropriate for developing a nominal CAR estimate. For projected estimates a monthly nominal value for the previous year might be used as a base projection that then be adjusted ahead by the annual rate of inflation. Another alternative is to collect nominal data for several past years, convert these to real terms as of month on interest in the base year, average them and then adjust them for inflation in the base year. Another option is to use an econometric forecasting model that accounts for seasonality and monthly inflation rates. Another method is to obtain dealer estimates for the month of use as compared to the time the data is collected prior to the preparation of the estimate. A common situation is one where there is a single nominal estimate for the previous year or the current year. A nominal estimate for a previous year may be updated using the inflation rate. Often the price reported or to be used for a given year is a nominal value for the entire year computed by averaging daily or monthly prices with equal weights as compared to a nominal value in the month of a given expenditure. Given this single observation and a rate of inflation, one may want to estimate monthly prices for the year that rise at the rate of inflation. What is wanted then is a real (and also nominal given the base period convention) price at the end of the year that when converted to monthly nominal prices has a simple average equal to the reported nominal average. Let $\bar{p}^{\,n}$ be the average nominal price for the year, $p_j^{\,n}$ the nominal price in the j^{th} month and π_m the monthly rate of inflation computed from equation 2.12 where π. replaces i. We can then find the real (nominal) price at the end of the year (p^r) as follows

$$
\begin{aligned}
\bar{p}^{\,n} &= \frac{\sum_{j=1}^{12} p_j^{\,n}}{12} \\
p_j^{\,n} &= p^{\,r}(1+\pi_m)^{j-12} \\
\Rightarrow \bar{p}^{\,n} &= \frac{\sum_{j=1}^{12} p^{\,r}(1+\pi_m)^{j-12}}{12} \\
&= \frac{p^{\,r}\sum_{j=1}^{12}(1+\pi_m)^{j-12}}{12} \\
\Rightarrow p^{\,r} &= \frac{(12)(\bar{p}^{\,n})}{\sum_{j=1}^{12}(1+\pi_m)^{j-12}} \\
&= \frac{(12)(\bar{p}^{\,n})}{(1+\pi_m)\sum_{j=1}^{12}(1+\pi_m)^{(j-12-1)}} \\
&= \frac{(12)(\bar{p}^{\,n})}{(1+\pi_m)\,US_0(\pi_m,12)} .
\end{aligned}
\tag{2.35}
$$

where the last equality comes from equation 2B.7 in Appendix 2B where π replaces i in the summation. Writing the expression this way allows the use of canned annuity procedures for computing p^r. The nominal price for each month is then computed as

$$
p_j^{\,n} = p^{\,r}(1+\pi_m)^{j-12} \tag{2.36}
$$

where $p_{12}^{n} = p_r$.

PROFITS AND RESIDUAL RETURNS

The difference between the farm's revenue and costs leads to the concept of profit or residual returns.

Profits (residual returns) to the firm (or enterprise) are the revenues from production minus all the market-determined costs of factors and the opportunity cost of the operator's time and any other unaccounted for resources. With equilibrium in competitive markets, costs of production should, on average, just equal returns. Thus, this residual return or profit has an expected value of zero. Deviations from zero are due to randomness such as

unusual geoclimatic conditions, market imperfections, errors in measurement, inclusion or exclusion of government program payments, or risk-averse behavior by some individuals or to the simple fact that the firm is not in an equilibrium situation. For these reasons, the profit of any one farm or even the average of all farms is probably not equal to zero in a given production period or year.

If the operator of a specific firm consistently obtains positive profits in a competitive environment, the opportunity cost of resources such as land or this person's unpaid labor are not being valued highly enough. For example, if this person has unusual allocative skills in farming and farming alone, the opportunity cost measure that is based on off-farm earning potential will understate the individual's true contribution to the profits of the firm. Even in situations where abnormally high profits are maintained by artificial means (government subsidy, tariffs, or quotas), these returns are normally bid into the costs of factors so that excess profits will be eliminated.

Residual returns to a given factor of production are the revenues from production minus the opportunity cost of the operator's time and the market-determined costs of all but that factor of production. With all other factors accounted for, any residual returns are said to accrue to this factor.

If the market-based costs of more than one factor are not accounted for then residual returns to the unvalued factors are exaggerated. As an example, analysts sometimes speak of a return to labor and management, or a return to operator-owned resources. Allocating this residual among the unvalued resources requires information concerning the marginal contributions of these resources to production. A difficulty with the residual method of imputing value is that all of the elements that cause economic profit to deviate from its long-run equilibrium get included in this unallocated residual. For example, if the farmer had exceptionally low barley yields this year due to drought and the resource being priced was operator labor, these low yields would all be attributed to operator labor, giving it a low value. In the same sense, if the unvalued resource were land, the land would have a low value. Year-to-year variations thus make imputations rather arbitrary and of limited usefulness. In addition, individual producers or groups of producers are rarely, if ever, in a long-run equilibrium, so that in any given situation residual returns measure more than a long-run return to management or entrepreneurial skill even if all other inputs are correctly measured and included.

The Task Force recommends that factors of production be valued based on market transactions and that the residual, if any, simply be denoted residual returns to unvalued resources.

OTHER CAR CONCEPTS

Accountants often use the concept of cash versus noncash costs in preparing cash flow statements. Cash costs also are often used in capital budgeting. Such a distinction is important for planning borrowing needs and the timing of operator withdrawals for own consumption, but is not a key factor in estimating CARs.

> **Cash costs** are costs that require a cash payment at the time the transaction occurs or during a specified reporting period such as a week or month. **Noncash** costs are those in which the timing of the physical use of resources and the cash payments differs.

Most costs associated with the acquisition of expendable inputs are cash costs. Some counter examples are feed produced during the current period that is fed to livestock and landlord-paid costs of inputs in a sharecropping arrangement. Depreciation costs associated with operator-owned capital goods such as equipment are always considered noncash costs, as are opportunity costs associated with holding capital goods.

It is also important to distinguish between economic cost concepts and finance terminology. Economic costs represent the valuation of all resources consumed during the course of a production period, regardless of ownership. Whether an individual production input is owned, financed, or leased is immaterial to the estimation of CARs, though it is very important to the management of an individual operation. From a resource perspective, the costs of one hour of service for a tractor that is owned and a tractor that is financed are identical because the values of the economic contributions of each to the production process are similar. The cash costs of each tractor do vary with asset ownership, however, because of the difference between interest and lease payments. The cash flow statements of two farmers who have debt levels of 0% and 50%, respectively, may differ because of financial payments, even though the two farmers may be using identical inputs and production practices in their farming operation (and therefore, have the same economic costs of production).

> ***The Task Force recommends that all costs and revenues associated with a given enterprise be adjusted (discounted) to the same point in time for the purposes of CAR estimation. The Task Force recommends that this point in time be the end of the production period. Because this approach applies implicit interest to all costs and returns, any costs associated with financing a given enterprise should not be included in the estimates.***

Another common distinction is between fixed and variable costs. This distinction depends on the range of choices considered available to the firm in the currently defined decision period. Currently available choices are inputs whose level of use and thus cost is not already determined. For example, once a feeder lamb reaches 100 pounds, the farmer cannot decide to change the amount and cost of the oats consumed. A specific time period is often associated with the decision problem so that what is fixed and variable changes depending on the time period considered.

> **Fixed costs** are those costs that the firm is committed to pay to factors of production regardless of the firm's action in the currently defined decision period. They are costs that are not affected by the current set of decisions. If some choices are fixed for a given decision problem, then costs associated with them are also fixed.
>
> **Variable costs** are those costs that are affected by the firm's actions in the currently defined decision period. Variable costs occur because of the decision to purchase additional factors or factor services for use in production.

The time period under consideration clearly affects the delineation of fixed and variable factors and associated costs. For example, if a tractor is leased (with no possibility of re-leasing) on an annual basis, the cost of the lease is fixed when deciding whether to produce cotton or tomatoes, but the per acre charge for custom harvesting is variable when deciding whether to harvest a damaged crop. Once the owner of a resource decides to assume ownership for another period, the ownership costs, service reduction costs due to time, and potential price gains are fixed. If the owner considers selling the services of a capital good along with using the services internally then the portion of the fixed charges to allocate to internal operations is variable depending on use. As irreversible decisions on input use are made, costs that were previously variable become fixed. In this vein, the costs of all expendable inputs are variable until they are contracted for use. For operator-owned capital goods, the costs are fixed once the operator decides to maintain the asset for another period.

The fact that operators of farms often own some of the resources used in production has led many analysts to classify the associated service flows as being fixed in the sense that the owner of the resource (and in this case the operator of the firm) incurs the ownership costs regardless of the amount of product produced. These analysts have then called these ownership costs "fixed costs" because they are associated with the "fixed" factors. This has caused great confusion as to the meaning of fixed and variable costs, the costs of ownership, and the costs of use. The difficulty found in labeling costs as fixed or variable has led some researchers to use the categories "ownership" and "operating costs." However, because most farm and ranch operators combine ownership with use, the categorization and measurement of CARs in these categories is less clear.

Furthermore, each firm or composite of firms operates with a different mix of owned and purchased inputs and different combinations of fixed and variable factors of production. Problems with the categorizations of fixed and variable costs are further compounded by the fact that accounting measures typically include all variable costs, some fixed costs, and direct use costs (but not returns) if the operator is an owner of factors. Accounting measures rarely (except in the case of depreciation) include the imputed CARs of operator-owned resources. Accountants, in particular, prefer the distinction between cash and noncash costs. As a result, the terminology commonly used tends to be confusing.

> ***The Task Force therefore recommends that costs should be categorized only as to whether they are associated with expendable factors or the services of capital assets. The division of costs into categories such as fixed and variable should generally be avoided in preparing CAR estimates. For the purpose of preparing CAR estimates for specific enterprises, the Task Force recommends that all the costs of all expendables be allocated to the generic group OPERATING COSTS and that all other costs be allocated to the group ALLOCATED OVERHEAD.***

TABLE 2.12 Annuities and Multiperiod Costs

Annual real interest rate	4%	Salvage life	250 hrs.
Annual inflation rate	5%	Life in years	5
Annual nominal interest rate	9.2%	Annual depreciation	250 hrs.
Original life of asset	1,500 hrs.	Initial price	$20 per hour

Year	Item	Beg. Price	Beg. Quantity	Beg. Value	Opp. Cost	Inflation	Interest	End Price	End Quantity	End Value	Cost
1	Investment	20	1,500	30,000	2,760	1,500	1,260	21	1,250	26,250	2,760
	Service reduction										5,000
	Price change										-1250
	Total cost										6,510
	Constant nominal annuity										6,690.795
	Infla. adj. real annuity										6,106.462
2	Investment	21	1,250	26,250	2,415	1,312.5	1,102.5	22.05	1,000	22,050	2,415
	Service reduction										5,250
	Price change										-1,050
	Total cost										6,615
	Constant nominal annuity										6,690.795
	Infla. adj. real annuity										6,411.784
3	Investment	22.05	1,000	22,050	2,028.6	1,102.5	926.1	23.15	750	17,364.3	2,028.6
	Service reduction										5,512.5
	Price change										-826.875
	Total cost										6,714.225
	Constant nominal annuity										6,690.795
	Infla. adj. real annuity										6,732.374
4	Investment	23.1525	750	17,364.3	1,597.5	868.219	729.304	24.31	500	12,155.0	1,597.523
	Service reduction										5,788.125
	Price change										-578.813
	Total cost										6,806.835
	Constant nominal annuity										6,690.795
	Infla. adj. real annuity										7,068.993

TABLE 2.12 (continued)

5	Investment	24.3101	500	12,155.0	1,118.2	607.753	510.513	25.53	250	6,381.40	1,118.266
	Service reduction										6,077.531
	Price change										-303.877
	Total cost										6,891.920
	Constant nominal annuity										6,690.795
	Infla. adj. real annuity										7,422.442
	$US_0(.04, 5)$	4.45182233									
	$US_0(.092, 5)$	3.86955005									
	P.V. of annual costs at end of 1st period										28,272.278
	P.V. of annual costs at beginning of 1st period										25,890.365
	P.V. of nominal annuity at end of 1st period										28,272.278
	P.V. of inflated real annuity at end of 1st period										28,272.278
	Annual nominal annuity with present value beginning of 1st period of 25,890.86										6,690.795
	Annual real annuity with present value at beginning of 1st period of 25,890.86										5,815.68

APPENDIX 2A

Separating Real Interest Charges and Inflation from Nominal Interest Charges

The appropriate way to adjust any cost or expenditure (R) occurring n months from the end of the period to the end of the period (year or last year in the case of multiyear periods) year is to use the formula given in equation 2.2,

$$V_0 = \sum_{t=-n}^{0} \frac{R_t}{(1+i)^t}. \qquad (2.2)$$

If there is only one payment and it occurs j-months from the end of the year, then the value of this payment at the end of the year is given by

$$V_0 = \frac{R_t}{(1+i_m)^{-j}}$$

where i_m is the monthly interest rate and j denotes the number of months that the expenditure occurs from the beginning of the year. We can write this in several alternative ways as follows:

$$V_0 = \frac{R}{(1+i_m)^{-n}} = R(1+i_m)^n = R(1+i)^{n/12}$$

where V_0 is the value of the expenditure at the end of the period, and i_m is the monthly interest rate, and n now denotes the number of months the expenditure occurs from the end of the year. The interest cost for this adjustment is given by either equation 2.14 or 2.15,

$$ic = R(1+i_m)^n - R \qquad (2.14)$$

$$ic = R(1+i)^{\frac{n}{12}} - R. \qquad (2.15)$$

The results of using this procedure for the cotton cost example were contained in Table 2.2 of Chapter 2. In order to divide the nominal interest cost into inflation and real interest rate components it is necessary to compute an inflation rate compatible with the given real and nominal interest rates. This can be done on both an annual and a monthly basis. The first step is to find the annual inflation rate using the Fisher formula $\pi = (i - r)/(1 + r)$. Consider the example in Table 2A.1 (Inflation and Real Interest Division). Here an annual nominal interest rate of 10% and an annual real interest rate of 3% are assumed. The implied annual inflation

rate is then (.1 - .03)/(1.03) = .067961. Once the annual inflation rate is known, implied monthly rates for nominal and real interest and inflation can be obtained using the relations

$$\begin{aligned} i_m &= (1+i)^{\frac{1}{12}} - 1 \\ r_m &= (1+r)^{\frac{1}{12}} - 1 \\ \pi_m &= (1+\pi)^{\frac{1}{12}} - 1 \end{aligned} \qquad \textbf{(2A.1)}$$

where no subscript implies an annual rate and subscript m denotes a monthly rate. For the example case this gives an implied monthly real rate of $r_m = (1.03)^{1/12} - 1 = .002466$ and an implied monthly inflation rate of $\pi_m = (1.0679)^{1/12} - 1 = .005494$. The Fisher formula implies that $(1 + r)(1 + \pi) = 1 + i$. Using the above identities, it also implies that $(1+r_m)(1+\pi_m) = 1 + i_m$ because $(1+r)^{1/12}(1+\pi)^{1/12} = (1+i)^{1/12}$. These relations are used in allocating the nominal interest charges to inflation and real interest.

The Fisher relation specifies that the product of $(1+r)$ and $(1+\pi)$ equals $1+i$. The relationship is thus multiplicative and not additive and so any division between inflation and real interest is somewhat subjective for any discrete time period. Specifically, part of the adjustment of a cost or return variable is due to real interest (r), part is due to inflation (π), and part is due to the cross product term (πr). Any additive division of this cross product term is arbitrary. Rather than arbitrarily allocate this factor, the common practice is to explicitly attribute it to either the real interest or inflation component by sequentially making the adjustments. An example helps make this clear. Consider an expense of $500 occurring six months before the end of the year with a nominal interest rate of 8% and a real rate of 3%. Using equation 2.15 and a nominal interest rate of 8% gives a nominal interest cost of $(500)(1.08)^{½} - (500) = \$19.615$. The annual inflation rate compatible with an 8% nominal rate and a 3% real rate is given by (.08-.03)/(1.03) = .04854 = 4.854%. Consider making the inflation adjustment first. The inflation adjusted value of $500 for six months is given by $(500)(1.04854)^{½} = \$511.992$. This gives an inflation cost of $(500)(1.04854)^{½} - 500 = \$11.992$. This inflation-adjusted amount is then adjusted using the real interest rate. This will give an inflation- and real interest-adjusted amount of $511.992(1.03)^{-.5} = \$519.615$, which is exactly the same as obtained using the nominal rate. The real interest component is then computed as this **inflation- and real interest-adjusted** amount minus the **inflation-adjusted** amount. For the example this gives 519.615 - 511.992 = $7.623. The total of the inflation costs (11.992) and the real interest costs (7.623) equals the total nominal interest cost of $19.615. What is arbitrary is performing the inflation adjustment first because this implies that the real interest is assessed on a larger value than the original unadjusted amount. An alternative is to make the real interest adjustment first. This gives a real interest-adjusted amount of $500(1.03)^{-.5} - 500 = \$507.445$ or a real interest cost of $7.445, which is less than before. This real interest-adjusted amount is then adjusted using the inflation rate and yields a total adjusted value of $507.445(1.04854)^{.5} = \$519.615$ or a total nominal interest cost of $19.615. The inflation adjustment is given by subtracting the real interest-adjusted value from the total or 519.615 - 507.445 = $12.17, which is larger than before because the inflation adjustment is applied to the larger real interest-adjusted amount. The total of the inflation ($12.17) and real interest ($7.445) cost is equal to the total nominal cost ($19.615).

Now consider the example in the first part of Table 2A.1 where the inflation adjustment is made first. In the first step the actual charge is adjusted to the end of the year using the implied monthly inflation rate.

The adjustment factor is $(1 + \pi)^{n/12}$. For example, the inflation-adjusted cost of fertilizer is $(24.45)(1.0679)^{10/12} = \$25.827$. This could also be computed using the implied monthly rate and the formula $(1 + \pi_m)^n$, which gives $(24.45)(1.005494)^{10} = \$25.827$. The inflation cost is then found by subtracting the initial unadjusted cost or

$$\pi c = \textit{actual cost}\ (1 + \pi)^{n/12} - \textit{actual cost} \qquad \textbf{(2A.2)}$$

where πc is the cost associated with inflation. For the example this gives \$1.3771. Once all costs are adjusted to the end of the year using the implied inflation rate, the real interest cost can be obtained using the formula

$$ric = \textit{inflation-adjusted cost}\ (1 + r)^{n/12} - \textit{inflation-adjusted cost} \qquad \textbf{(2A.3)}$$

where ric is the real interest cost and r is the real annual interest rate. For example, the real interest on the fertilizer expense is given by $(25.827)(1.03)^{10/12} - 25.827 = .644$. The total of the real interest costs and inflation costs is $.644 + 1.3771 = 2.021$, which is the same as that computed using the direct nominal rate. Thus the nominal interest can be divided into real interest and inflation components using the suggested procedure.

Now consider making the real interest adjustment first in the second portion of Table 2A.1. In the first step the actual charge is adjusted to the end of the year using the implied monthly real interest rate. The adjustment factor is $(1 + r)^{n/12}$. For example, the real interest-adjusted cost of fertilizer is $(24.45)(1.03)^{10/12} = \$25.06$. The real interest cost is then found by subtracting the initial unadjusted cost or

$$ric = \textit{actual cost}\ (1 + r)^{n/12} - \textit{actual cost} \qquad \textbf{(2A.4)}$$

where ric is the cost associated with real interest. For the example this gives \$0.6097. Once all costs are adjusted to the end of the year using the real interest rate, the inflation cost can be obtained using the formula

$$\pi c = \textit{real interest-adjusted cost}\ (1 + \pi)^{n/12} - \textit{real interest-adjusted cost} \qquad \textbf{(2A.5)}$$

where πc is the inflation cost and π is the annual inflation rate. For example, the inflation cost on the fertilizer expense is given by $(25.06)(1.0679)^{10/12} - 25.06 = \$1.411$. The total of the real interest costs and inflation costs is $.6097 + 1.411 = \$2.021$, which is the same as that computed using the direct nominal rate and the inflation first assumption. Clearly, the two assumptions lead to slightly different allocations of real interest and inflation. The more common approach is to make the real interest rate adjustment first because with low inflation, real interest is the more important issue; however, there is no compelling argument for doing so.

The Task Force recommends that when decomposing nominal interest magnitudes into real interest and inflation components, one of the above procedures which compound interest during the year and take explicit account of the interactions of interest rates and

inflation be used. Other procedures using proportional interest or ignoring the interaction effects should be viewed as approximations only.[4]

[4]A difficulty with using the proportional methods of computing interest is that the decomposition is inconsistent. This leads to problems because the Fisher formula $(1+r)(1+\pi) = 1+i$ cannot hold at both an annual and subperiod level if the rule for determining monthly rates is $r_m = (n/12)(r)$ rather than $r_m = (1 + r)^{1/12} - 1$. To see this, multiply out the implied monthly Fisher relation. The only way to obtain consistency is to allow the implied inflation rate to differ for each subperiod and not be computed using $\pi_m = (n/12)(\pi)$ or $\pi_m = (1 + \pi)^{1/12} - 1$.

TABLE 2A.1 Inflation and Real Interest Division

Enterprise termination date is 1 Dec.
Annual nominal and real interest rates are used to impute an annual inflation rate using the Fisher equation
Actual costs are adjusted to the end of the period using implied monthly inflation rates
Implied monthly real rates are applied to the inflation-adjusted costs
Annual nominal interest rate is 0.1 = 10%
Implied monthly rates are computed using the formula $i_m = (i + 1)^{n/12} - 1$
Implied monthly nominal rate is 0.007974 = .7494%
Annual real interest rate is 0.030000 = 3%
Implied monthly real interest rate is 0.002466 = .2466%
Implied annual inflation rate is 0.067961 = 6.796%
Implied monthly inflation rate is 0.005494 = .5494%
Inflation-adjusted cost = (Actual cost) $(1+\pi)^{n/12}$
Real interest charge = (Adjusted cost)$(1+r)^{n/12}$ - (Adjusted cost)

A. Inflation and Real Interest Division with Inflation Adjustment First

Item	Time of Use	Actual Cost	Months Used	Real Interest-Adjusted Cost	Inflation Cost	Inflation on Adjusted Cost	Nominal Interest
Fertilizer	1 Feb	24.45	10	25.8271	1.3771	0.644	2.021
Cotton Seed	1 Apr	17.28	8	18.0543	0.7743	0.359	1.134
Insecticide	1 Jul	20.00	5	20.5555	0.5555	0.255	.810
Insecticide	1 Aug	20.00	4	20.4432	0.4432	0.202	.646
Insecticide	1 Sep	20.00	3	20.3315	0.3315	0.151	.482
	Total	101.73		105.212	3.4815	1.6113	5.093
	Inflation Adjustment	3.482					
	Real Interest	1.611					
Total Cost		106.823					

Table 2A.1 (continued)
B. Inflation and Real Interest Division with Real Interest Adjustment First

Item	Time of Use	Actual Cost	Months Used	Real Interest-Adjusted Cost	Real Interest Cost	Inflation on Adjusted Cost	Nominal Interest
Fertilizer	1 Feb	24.45	10	25.060	.6097	1.411	2.021
Cotton Seed	1 Apr	17.28	8	17.624	.3439	0.790	1.134
Insecticide	1 Jul	20.00	5	20.248	.2478	0.562	.810
Insecticide	1 Aug	20.00	4	20.198	.1980	0.448	.646
Insecticide	1 Sep	20.00	3	20.148	.1483	0.334	.482
	Total	101.73		103.278	1.5479	3.5450	5.093
	Inflation Adjustment	3.545					
	Real Interest	1.547					
Total		106.823					

APPENDIX 2B

Derivation of Annuity Formulas

Preparers of CAR estimates may prefer to represent the capital service cost of capital assets with an annuity payment rather than the period-by-period costs for ownership, service reduction, and change in price. This can be either a real annuity that has a constant real but changing nominal value or a nominal annuity that is constant in nominal dollars. The annuity formulas are derived here. Numerical examples are given in Chapter 2.

Present Value of a Return Stream

One can compute the present value of an infinite stream of payments using the present value recursion given in Chapter 2, equation 2.18 where V_n is a value at the end of the n^{th} period, V_0 is a value at the beginning of the first period, and R_n is a payment at the end of the n^{th} period. Beginning with n =1 and continuing to substitute for V_n we obtain

$$\begin{aligned} V_1 &= (1+i)V_0 - R_1 \\ \Rightarrow V_0 &= \frac{R_1}{1+i} + \frac{V_1}{1+i} \\ &= \frac{R_1}{1+i} + \frac{R_2}{(1+i)^2} + \frac{V_2}{(1+i)^2} \\ &= \frac{R_1}{1+i} + \frac{R_2}{(1+i)^2} + \cdots + \frac{R_n}{(1+i)^n} + \frac{V_n}{(1+i)^n} \\ &= \sum_{t=1}^{\infty} \frac{R_t}{(1+i)^t}. \end{aligned} \qquad \textbf{(2B.1)}$$

In a similar way one can compute the value at the end of period n of a stream of payments beginning at the end of period n+1 as

$$\begin{aligned} V_n &= \frac{R_{n+1}}{1+i} + \frac{V_{n+1}}{1+i} \\ &= \frac{R_{n+1}}{1+i} + \frac{R_{n+2}}{(1+i)^2} + \cdots + \\ &= \sum_{t=1}^{\infty} \frac{R_{t+n}}{(1+i)^t} \\ &= \sum_{t=n+1}^{\infty} \frac{R_t}{(1+i)^{t-n}} \\ &= (1+i)^n \sum_{t=n+1}^{\infty} \frac{R_t}{(1+i)^t}. \end{aligned} \qquad \textbf{(2B.2)}$$

The formula for V_0 can then be written as the sum of R_t over n time periods plus the residual value for V_n as follows:

$$V_0 = \sum_{t=1}^{n} \frac{R_t}{(1+i)^t} + \frac{V_n}{(1+i)^n}. \qquad \textbf{(2B.3)}$$

This can be rearranged to express the present value of the returns at the end of each of n time periods as a function of V_0 and V_n as

$$\sum_{t=1}^{n} \frac{R_t}{(1+i)^t} = V_0 - \frac{V_n}{(1+i)^n}. \qquad \textbf{(2B.4)}$$

Notice that the left-hand side of 2B.4 is the present value of the payment stream discounted to the beginning of the first period (end of period 0). Multiplying equation 2B.4 by (1+i) gives the value of the payment stream at the end of period 1 as

$$\begin{aligned} (1+i)\left[\sum_{t=1}^{n} \frac{R_t}{(1+i)^t}\right] &= (1+i)\left[V_0 - \frac{V_n}{(1+i)^n}\right] \\ \Rightarrow \sum_{t=1}^{n} \frac{R_t}{(1+i)^{t-1}} &= (1+i)\left[V_0 - \frac{V_n}{(1+i)^n}\right]. \end{aligned} \qquad \textbf{(2B.5)}$$

Calculation of an Annuity Payment Representing a Present Value

We can calculate an annuity (a) with n equal payments at the end of each period having the same value as the left-hand side of equation 2B.4. Specifically, we find a uniform payment (a) to be received (or dispersed) at the end of each period that has the same present value at time zero as the sum of the R_t each discounted to time zero. The annuity (a) is implicitly defined by writing out this identity for $\sum_{t=1}^{n} \frac{R_t}{(1+i)^t}$,

$$\frac{a}{1+i} + \frac{a}{(1+i)^2} + \cdots + \frac{a}{(1+i)^n} = \frac{R_1}{1+i} + \frac{R_2}{(1+i)^2} + \cdots + \frac{R_n}{(1+i)^n}$$

$$\Rightarrow a \sum_{t=1}^{n} \frac{1}{(1+i)^t} = \sum_{t=1}^{n} \frac{R_t}{(1+i)^t}$$

(2B.6)

$$\Rightarrow a = \frac{\sum_{t=1}^{n} \frac{R_t}{(1+i)^t}}{\sum_{t=1}^{n} \frac{1}{(1+i)^t}}.$$

The expression in the denominator of 2B.6 is a geometric series that can be simplified. Let this denominator be denoted by $US_0(i,n)$ meaning a uniform series having interest rate i and n periods.
Specifically, let $US_0(i,n)$ be defined as

$$US_0(i,n) = \left(\frac{1}{1+i} + \frac{1}{(1+i)^2} + \cdots + \frac{1}{(1+i)^n} \right)$$

$$= \sum_{t=1}^{n} \frac{1}{(1+i)^t}.$$

(2B.7)

Now multiply $US_0(i,n)$ by $1/(1+i)$ and then subtract from $US_0(i,n)$ as follows:

$$US_0(i,n) - \left[\frac{1}{1+i}\right] US_0(i,n) = \left[\left(\frac{1}{1+i} + \frac{1}{(1+i)^2} + \cdots + \frac{1}{(1+i)^n}\right) - \left(\frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \cdots + \frac{1}{(1+i)^{n+1}}\right)\right]$$

$$= \frac{1}{1+i} - \frac{1}{(1+i)^{n+1}}$$

$$\Rightarrow (1+i)US_0(i,n) - US_0(i,n) = 1 - \frac{1}{(1+i)^n}$$

$$\Rightarrow i\ US_0(i,n) = 1 - \frac{1}{(1+i)^n} \qquad \textbf{(2B.8)}$$

$$\Rightarrow US_0(i,n) = \frac{\left(1 - \frac{1}{(1+i)^n}\right)}{i}.$$

Thus by sequentially substituting 2B.8 into 2B.7 and then into 2B.6 we obtain

$$a = \frac{\sum_{t=1}^{n} \frac{R_t}{(1+i)^t}}{\sum_{t=1}^{n} \frac{1}{(1+i)^t}} \qquad \textbf{(2B.9)}$$

$$= \frac{\left(\sum_{t=1}^{n} \frac{R_t}{(1+i)^t}\right)}{\left[\frac{1 - \frac{1}{(1+i)^n}}{i}\right]}.$$

Equation 2B.9 gives an annuity payable at the end of each period that has the same discounted value at the beginning of the time frame as the actual payments over the n period time horizon. If we then substitute 2B.4 for the numerator in 2B.9 we obtain

$$a^{nom} = \frac{\left(V_0 - \frac{V_n}{(1+i)^n}\right)}{\left(\frac{1 - \frac{1}{(1+i)^n}}{i}\right)}. \qquad \textbf{(2B.10)}$$

If all values are expressed in real terms, then a real annuity with equivalent present value to 2B.10 is given by

$$a^{r} = \frac{\left(V_0 - \frac{V_n}{(1+r)^n}\right)}{\left(\frac{1 - \frac{1}{(1+r)^n}}{r}\right)} \qquad \textbf{(2B.11)}$$

where V_0 and V_n are expressed in real dollars. This is the annuity payment in real terms in the base period. To find the nominal payment that is to be made in other periods, this amount is adjusted by the inflation rate. Specifically, the nominal payment in the j^{th} period from the base is $a_j = a^r(1 + \pi)^j$ where π is the constant rate of inflation per period.

The nominal first year payment of this increasing stream is also the payment that would be obtained if one were to assume that inflation occurs only during the first year and no inflation occurs thereafter. To see this, recompute the present value recursion using a nominal interest rate for the first year and a real interest rate for subsequent years.

$$
\begin{aligned}
V_1 &= (1+i)V_0 - R_1 \\
\Rightarrow V_0 &= \frac{R_1}{1+i} + \frac{V_1}{1+i} \\
V_2 &= (1+r)V_1 - R_2 \\
\Rightarrow V_1 &= \frac{R_2}{1+r} + \frac{V_2}{1+r} \\
\Rightarrow V_0 &= \frac{R_1}{1+i} + \frac{R_2}{(1+i)(1+r)} + \frac{V_2}{(1+i)(1+r)} \\
&= \frac{R_1}{1+i} + \frac{R_2}{(1+i)(1+r)} + \cdots + \frac{R_n}{(1+i)(1+r)^{n-1}} + \frac{V_n}{(1+i)(1+r)^{n-1}} \\
&= \frac{1}{1+i} \sum_{t=1}^{n} \frac{R_t}{(1+r)^{t-1}} + \frac{V_n}{(1+i)(1+r)^{n-1}} \\
&= \frac{1}{1+i} \sum_{t=1}^{\infty} \frac{R_t}{(1+r)^{t-1}}.
\end{aligned}
\qquad \textbf{(2B.12)}
$$

We can rearrange the next to last expression in 2B.12 to give the present value of the payment stream at the beginning of the time horizon assuming inflation in only the first period:

$$
\begin{aligned}
\frac{1}{1+i} \sum_{t=1}^{n} \frac{R_t}{(1+r)^{t-1}} &= V_0 - \frac{V_n}{(1+i)(1+r)^{n-1}} \\
\Rightarrow \sum_{t=1}^{n} \frac{R_t}{(1+r)^{t-1}} &= (1+i)V_0 - \frac{V_n}{(1+r)^{n-1}}.
\end{aligned}
\qquad \textbf{(2B.13)}
$$

An annuity with n equal payments at the end of each period that has the same present value as the left-hand side of 2B.13 is computed from

$$\frac{R_1}{1+i} + \frac{R_2}{(1+i)(1+r)} + \cdots + \frac{R_n}{(1+i)(1+r)^{n-1}} = \frac{a^m}{1+i} + \frac{a^m}{(1+i)(1+r)} + \cdots + \frac{a^m}{(1+i)(1+r)^{n-1}}$$

$$\Rightarrow a^m \left(\frac{1}{1+i}\right) \sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}} = \left(\frac{1}{1+i}\right) \sum_{t=1}^{n} \frac{R_t}{(1+r)^{t-1}}$$

$$\Rightarrow a^m = \frac{\sum_{t=1}^{n} \frac{R_t}{(1+r)^{t-1}}}{\sum_{t=1}^{n} \frac{1}{(1+r)^{t-1}}}$$

$$= \frac{\sum_{t=1}^{n} \frac{R_t}{(1+r)^{t-1}}}{(1+r) \sum_{t=1}^{n} \frac{1}{(1+r)^{t}}}$$

$$= \frac{\sum_{t=1}^{n} \frac{R_t}{(1+r)^{t-1}}}{(1+r)\ US_0(r,n)} \qquad \textbf{(2B.14)}$$

where the superscript m on "a" denotes a mixed annuity that will be the same for all periods assuming inflation in the first year and none thereafter. If we substitute the expression for $US_0(r,n)$ into 2B.14 and then substitute 2B.13 for the numerator in 2B.14 we obtain

$$
\begin{aligned}
a^m &= \frac{\sum_{t=1}^{n} \frac{R_t}{(1+r)^{t-1}}}{(1+r)\left(\frac{1-\frac{1}{(1+r)^n}}{r}\right)} \\
&= \frac{(1+i)V_0 - \left(\frac{V_n}{(1+r)^{n-1}}\right)}{(1+r)\left(\frac{1-\frac{1}{(1+r)^n}}{r}\right)} \\
&= \frac{\frac{(1+i)}{(1+r)}V_0 - \left(\frac{V_n}{(1+r)^n}\right)}{\left(\frac{1-\frac{1}{(1+r)^n}}{r}\right)} \\
&= \frac{(1+\pi)V_0 - \frac{V_n}{(1+r)^n}}{\left(\frac{1-\frac{1}{(1+r)^n}}{r}\right)} \\
&= \frac{(1+\pi)\left[V_0 - \frac{V_n}{(1+\pi)(1+r)^n}\right]}{\left(\frac{1-\frac{1}{(1+r)^n}}{r}\right)}.
\end{aligned}
\qquad \textbf{(2B.15)}
$$

This then is a constant annuity payable at the end of each period that has the same present value as a return stream having inflation in the first year and no inflation thereafter discounted to the present. It is easy to see that this is the same as the real annuity given in equation 2B.11 multiplied by $(1+\pi)$ because V_n in equation 2B.15 (where there is one year of inflation) will be the same as V_n in equation 2B.11 multiplied by $(1+\pi)$.

General Annuities

Annuities can also be developed for subperiods of time and for alternative compounding scenarios. For example, we might choose to create an annuity making payments every six months to represent the present value of an income stream with payments at the end of each year. Alternatively, we may want to create a fractional annuity that makes one payment a year for 5 years and makes a final payment at 5 ½ years. This type of annuity may be useful for income or cost streams associated with assets that are sold or traded at noninteger time intervals.

The easiest way to compute such annuities is to use the fractional period interest formulas given in equations 2.12 and 2.13. We can always find a fractional (including improper fractions) interest rate such that the following generalization of 2.12 is appropriate:

$$\begin{aligned} (1 + i_p)^p &= (1+i)^q \\ \Rightarrow 1 + i_p &= (1+i)^{\frac{q}{p}} \\ \Rightarrow i_p &= (1+i)^{\frac{q}{p}} - 1 \end{aligned} \qquad \textbf{(2B.16)}$$

where p is the number of times that i_p is compounded in q years and it is assumed that i is the annual nominal interest rate. For example, if p = 12 and q = 1 we get equation 2.12.

Now consider an annuity that is paid at the end of each period with a final payment at some fraction of a period. Let n be a noninteger with int(n) representing the integer part of n and frac(n) the fractional part where n = int(n) + frac(n). Now assume a payment stream with present value at time zero of V_0. The annuity is defined implicitly by

$$\frac{a}{(1+i)^1} + \frac{a}{(1+i)^2} + \cdots + \frac{a}{(1+i)^{int(n)}} + \frac{as}{(1+i)^n} = V_0 \qquad \textbf{(2B.17)}$$

where "as" is a payment made at the termination point. The general formula for an annuity in equation 2B.9 implies that

$$a = \frac{V_0}{\sum_{t=1}^{n} \frac{1}{(1+i)^t}} = \frac{V_0}{\left[\frac{1 - \frac{1}{(1+i)^n}}{i}\right]} = V_0 \left[\frac{i}{1 - (1+i)^{-n}}\right] = V_0 \frac{i(1+i)^n}{\left[(1+i)^n - 1\right]}. \qquad \textbf{(2B.18)}$$

Now write out equation 2B.17 substituting for the summation from 2B.9 as follows:

$$\begin{aligned} a \sum_{t=1}^{int(n)} \frac{1}{(1+i)^t} + \frac{as}{(1+i)^n} &= V_0 \\ \Rightarrow a \left[\frac{1 - (1+i)^{-int(n)}}{i}\right] + \frac{as}{(1+i)^n} &= V_0 \\ \Rightarrow a \left[\frac{(1+i)^{int(n)} - 1}{i(1+i)^{int(n)}}\right] + \frac{as}{(1+i)^n} &= V_0. \end{aligned} \qquad \textbf{(2B.19)}$$

We can then solve 2B.19 for the fractional payment (as) as follows:

$$
\begin{aligned}
a\left[\frac{1-(1+i)^{-int(n)}}{i}\right] + \frac{as}{(1+i)^n} &= V_0 \\
\Rightarrow as &= (1+i)^n V_0 - (1+i)^n\, a\left[\frac{1-(1+i)^{-int(n)}}{i}\right] \\
&= (1+i)^n V_0 - a\left[\frac{(1+i)^n - (1+i)^{frac(n)}}{i}\right] \\
&= (1+i)^n V_0 - V_0\left[\frac{i}{1-(1+i)^{-n}}\right]\left[\frac{(1+i)^n - (1+i)^{frac(n)}}{i}\right] \\
&= V_0\left[\frac{(1-(1+i)^{-n})(1+i)^n - (1+i)^n + (1+i)^{frac(n)}}{1-(1+i)^{-n}}\right] \\
&= V_0\left[\frac{(1+i)^n - 1 - (1+i)^n + (1+i)^{frac(n)}}{1-(1+i)^{-n}}\right] \\
&= V_0\left[\frac{(1+i)^{frac(n)} - 1}{1-(1+i)^{-n}}\right] \\
&= \frac{1}{i} V_0\left[\frac{i}{1-(1+i)^{-n}}\right]\left[(1+i)^{frac(n)} - 1\right] \\
\Rightarrow as &= a\,\frac{[(1+i)^{frac(n)} - 1]}{i}.
\end{aligned}
\qquad \textbf{(2B.20)}
$$

We can verify that this definition of the partial payment (as) is correct by substituting from 2B.20 into 2B.19 as follows:

$$
\begin{aligned}
a\sum_{t=1}^{int(n)} \frac{1}{(1+i)^t} + \frac{a\left(\dfrac{(1+i)^{frac(n)} - 1}{i}\right)}{(1+i)^n} &= V_0 \qquad \textbf{(2B.21)} \\
\Rightarrow a\left[\frac{1-(1+i)^{-int(n)}}{i}\right] + \frac{a\left((1+i)^{frac(n)} - 1\right)}{i(1+i)^n} &= V_0.
\end{aligned}
$$

Now use 2B.18 to define V_0 as follows:

$$a = V_0 \frac{i}{\left[1 - (1 + i)^{-n}\right]}$$
$$\Rightarrow V_0 = a\left(\frac{1 - (1 + i)^{-n}}{i}\right). \qquad \textbf{(2B.22)}$$

Then set 2B.21 and 2B.22 equal to each other and show that the left-hand side equals the right-hand side:

$$a\left[\frac{1 - (1+i)^{-int(n)}}{i}\right] + \frac{a\left((1+i)^{frac(n)} - 1\right)}{i(1+i)^n} = a\left(\frac{1 - (1+i)^{-n}}{i}\right)$$
$$\Rightarrow \left[\frac{1 - (1+i)^{-int(n)}}{i}\right] + \frac{\left((1+i)^{frac(n)} - 1\right)}{i(1+i)^n} = \left(\frac{1 - (1+i)^{-n}}{i}\right)$$
$$\Rightarrow \left(\frac{(1+i)^n - (1+i)^{fracn} + (1+i)^{frac(n)} - 1}{i\,(1+i)^n}\right) = \left(\frac{1 - (1+i)^{-n}}{i}\right) \qquad \textbf{(2B.23)}$$
$$\Rightarrow \left(\frac{(1+i)^n - 1}{i\,(1+i)^n}\right) = \left(\frac{1 - (1+i)^{-n}}{i}\right) = \left(\frac{(1+i)^n - 1}{i\,(1+i)^n}\right).$$

We can also find the fractional payment (as) using the equation 2B.16 and the definition of an annuity. First rewrite 2B.16 with $p = \frac{1}{\text{frac(n)}}$ and q = 1 as follows:

$$(1 + i_f)^{\frac{1}{frac(n)}} = (1+i)$$
$$\Rightarrow 1 + i_f = (1+i)^{frac(n)} \qquad \textbf{(2B.24)}$$
$$\Rightarrow i_f = (1+i)^{frac(n)} - 1$$

where i_f is the interest rate that when compounded $\frac{1}{\text{frac(n)}}$ times per period is equivalent to the interest rate i compounded once per period. Now write the equation for V_0 (equation 2B.22) for two different annuities covering one period.

$$V_0 = a\left(\frac{1 - (1 + i)^{-1}}{i}\right) = as\left(\frac{1 - (1 + i_f)^{\frac{-1}{frac(n)}}}{i_f}\right). \qquad \textbf{(2B.25)}$$

If we substitute the expressions in 2B.21 into 2B.22 we obtain

$$\begin{aligned} a\left(\frac{1 - (1 + i)^{-1}}{i}\right) &= as\left(\frac{1 - (1 + i_f)^{\frac{-1}{frac(n)}}}{i_f}\right) \\ &= as\left(\frac{1 - (1 + i)^{-1}}{(1+i)^{frac(n)} - 1}\right) \\ \Rightarrow \frac{a}{i} &= \frac{as}{(1+i)^{frac(n)} - 1} \\ \Rightarrow as &= a\left[\frac{(1+i)^{frac(n)} - 1}{i}\right] \end{aligned} \qquad \textbf{(2B.26)}$$

which is the same as 2B.20.

APPENDIX 2C

Using Annuities to Represent the Costs of a Capital Asset: Example

As discussed in the body of Chapter 2, it is often useful to represent the variable cost stream associated with a capital asset over its lifetime using a constant annuity payment. The idea is to compute the net present value of all CARs associated with ownership and use of the asset and then construct an annual end-of-period annuity having the same present value. The example in the chapter considered costs associated with economic depreciation (service reduction and price changes) and opportunity interest cost but ignored maintenance, service enhancement, and other time costs. This is a common practice because the present value of economic depreciation and opportunity interest can be computed using the formula $\left(V_0 - \frac{V_n}{(1+i)^n} \right)$ where V_0 is initial value and V_n is the value at the end of n periods without having to consider the year-by-year cost/return flows. Economic depreciation (ED) in year t is given by the sum of service reduction and price change costs and is equal to $V_{t-1} - V_t$. Opportunity interest cost (OC) in year t is given by $(i)(V_{t-1})$. If we discount these terms back to the beginning of period 1 and then sum them for the n years we obtain

$$
\begin{aligned}
NPC(ED+OC)_0 &= \frac{V_0 - V_1 + iV_0}{(1+i)} + \frac{V_1 - V_2 + iV_1}{(1+i)^2} + \cdots + \frac{V_{n-1} - V_n + iV_{n-1}}{(1+i)^n} \\
&= \frac{V_0(1+i)}{(1+i)} - \frac{V_1}{(1+i)} + \frac{V_1(1+i)}{(1+i)^2} - \frac{V_2}{(1+i)^2} + \frac{V_2(1+i)}{(1+i)^3} - \frac{V_3}{(1+i)^3} + \cdots \\
&= V_0 - \frac{V_{n-1}}{(1+i)^{n-1}} + \frac{V_{n-1}(1+i)}{(1+i)^n} - \frac{V_n}{(1+i)^n} \\
&= \left(V_0 - \frac{V_n}{(1+i)^n} \right)
\end{aligned}
\qquad \textbf{(2C.1)}
$$

where $NPC(ED+OC)_0$ is the net present value of ED and opportunity cost at the end of period zero.

When other costs such as maintenance or service enhancement are considered, a year-by-year accounting is required to find the net present value. For an asset that is held n years, the net present value at the end of period 0 for the cost stream is given by

$$
NPC_0 = \left(V_0 - \frac{V_n}{(1+i)^n} \right) + \sum_{t=0}^{n} \frac{C_t}{(1+i)^t} \qquad \textbf{(2C.2)}
$$

where NPC_0 is the present value of costs at the beginning of period 1, V_0 is the initial purchase cost, V_n is the salvage value at the end of the n^{th} year, C_t is expenses such as maintenance and taxes associated with the asset in period t, and i is the nominal interest rate. It is assumed that all costs in period t occur at the end of

the period. These costs can be converted to an annual nominal annuity with n payments, one at the end of each year, by dividing equation 2C.2 by $US_0(i,n)$ as follows:

$$a^{nom} = \frac{\left[\left(V_0 - \frac{V_n}{(1+i)^n}\right) + \sum_{t=0}^{n} \frac{C_t}{(1+i)^t}\right]}{US_0(i,\ n)}$$

$$= \frac{\left[\left(V_0 - \frac{V_n}{(1+i)^n}\right) + \sum_{t=0}^{n} \frac{C_t}{(1+i)^t}\right]}{\left(\frac{1 - \frac{1}{(1+i)^n}}{i}\right)}. \qquad \text{(2C.3)}$$

A real annuity could be constructed in a similar manner.

Consider now an example similar to the one in the body of Chapter 2 where a tractor with 1,500 hours of useful life is purchased at the beginning of the first period for $30,000 or $20.00 per hour of potential service. The tractor is assumed to be used for 250 hours each year. Based on this purchase price and 250 annual hours of use, the real value of maintenance at the end of each year is assumed to follow the pattern in Table 2C.1. This maintenance cost will be larger with inflation. This pattern assumes that a tractor with fewer remaining hours of service will have higher maintenance costs. For purposes of this example, assume that the maintenance will take place at midyear rather than at the end of the year so that interest will accrue during the year at a rate of $(1+i)^{.5}$. The property tax rate is assumed to be 1% of market value at the beginning of the period, but paid at the end of the period. The producer is planning on some major service enhancement at the end of the third period to restore 250 hours worth of service potential. Maintenance will change after the service enhancement because the tractor will now have a longer service life. Specifically, the real value of maintenance in period 4 will be the same as in period 3.

TABLE 2C.1 Data on Purchase, Use, and Sale of Tractor

Item	
Initial service capacity (hours)	1,500.00
Real price of 1 hour of service potential in period 1	$20.00
Initial purchase price (V_0)	$30,000.00
Use per year (hours)	250.00
Service enhancement at end of period 3 (hours)	250.00
Property tax rate	.01
Real interest rate	.04
Inflation rate	.05
Implied nominal interest rate	.092

Real value of maintenance performed at end of period t based on cumulative hours of use and list price in current dollars

Year	Maintenance Cost	Cumulative Use	Inflated List Price
1	$75.00	250	31,500
2	$225.00	500	33,075
3	$375.00	750	34,728.75
4	$525.00	1,000	36,465.108
5	$675.00	1,250	38,288.447

Table 2C.2 is similar to Table 2.12 in Chapter 2 and documents the costs for this tractor for each of the five years, the present value of these costs, and the equivalent annual nominal and real annuities. Consider the first year. Based on a purchase price of $30,000, the opportunity cost is given by ($30,000)(.092) = $2,760. The decline in service capacity of 250 hours valued at beginning-of-year prices of 20 dollars per hour gives a cost of (250)($20) = $5,000. The end of the year service capacity is 1,250 hours. With 5% inflation, the price of a unit of service at the end of the period is 21 dollars. The price change cost is then [($20-$21)(1,250)] = -$1,250. The sum of service reduction and price change costs is equal to economic depreciation and given by ($5,000 - $1,250) = $3,750. The value of the tractor at the end of the year is ($21)(1,250) = $26,250. Economic depreciation can also be computed as $V_0 - V_1$, which gives ($30,000 - $26,250) = $3,750 as before. With 5% inflation the real end-of-period value of the maintenance must be adjusted upwards to $78.75 [($75)(1.05)]. Because the producer is incurring this expense at midyear rather than at the end, we must account for the earlier commitment of funds. The end-of-period cost is then $(\$78.75)(1.092)^{-.5} = \$82.29$. With property taxes of $300, total costs for the period are $6,892.29 ($2,760+5,000-$1,250+$82.29+$300).

Computations for the second year are similar to the first. For example, service reduction is given by ($21)(250) = $5,250. Given 5% inflation, maintenance costs paid at the end of period 2 would have value $(\$225)(1.05)^2 = \$248.06$. Given that they must be paid at midyear, the cost is $(\$248.06)(1.092)^{-.5} = \$259.22$. Opportunity costs of $2,415 can be divided into inflation costs of $1,312.50 [($26,250)(.05)] and real interest costs of $1,102.5 [(1.05)($26,250)(.04)]. The property tax is computed as 1% of $26,250 or $262.5.

The third year is somewhat different because service enhancement takes place. The price of an hour of service at the end of the third year is \$23.1525 [(\$20)(1.05)^3]. The cost of restoring 250 hours of service at the end of the year is assumed to be (250)(\$23.1525) = \$5,788.125. This enhancement will restore the tractors service life to 1,000 hours. Maintenance paid at year's end would be (\$375)(1.05)^3 = \$434.11 and with interest for one-half year is (\$434.11)(1.092)$^{.5}$ = \$453.64. There is no charge for service reduction because the beginning and end-of-year service capacities are the same after the service enhancement. Specifically, the tractor has 1,000 hours of service at the beginning and end of the year which at beginning and ending prices of \$22.05 and \$23.1525 give values of \$22,050 and \$23,152.5. Notice that the value of the tractor at the end of the year is just the value at the beginning adjusted for inflation [\$23,152.5 = (1.05)(\$22,050)]. Property taxes are (\$22,050)(.01) = \$220.50. Total costs for the year are \$7,388.36.

Computations for the fourth and fifth years are similar. Maintenance in the fourth year is just the maintenance value for the third year (\$434.109) adjusted for an inflation rate of 5% because the tractor has the same remaining service life for both years. Specifically, \$453.6391 = (\$434.109)(1.05). The present value of all the annual costs at the beginning of period 1 (end of period 0) is given by discounting each to the beginning of period 1 using the nominal interest rate of 9.2%. This discounted sum is \$28,597.151. This can be converted to a nominal annuity by dividing it by $US_0(9.2, 5)$ or to a real annuity using $US_0(4.0, 5)$. The nominal annuity is \$7,390.304 and the real annuity is \$6,423.6955. This real annuity would be multiplied by the $(1+\pi)$ to adjust for inflation in the first period. The inflation-adjusted annuity is then (6,423.69)(1.05) = \$6,744.8803.

TABLE 2C.2 Annuities and Multiperiod Costs Including Service Enhancement, Maintenance, and Taxes

Annual real interest	0.04	Year	Inflation Rate	Nominal i	Service Enhancement (hrs)	Nominal Annuity including Maintenance	7,390.304
Original life of asset	1,500	1	0.05	0.092	0	Real Annuity incl Maintenance	6,423.695
Salvage life (assuming no enhancement)	250	2	0.05	0.092	0	Nominal Annuity for Maintenance	$368.78
Life in years	5	3	0.05	0.092	250	Real Annuity for Maintenance	$320.55
Annual depreciation	250	4	0.05	0.092	0		
Initial price	20	5	0.05	0.092	0		
Property tax rate	0.01						

Year	Item	Beg. Price	Beg. Quantity	Actual Value	Direct Cost	Opp. Cost	Inflation	Interest	Total Use Before Enhanced	Total Use After Enhanced	End Price	End Quantity	End Value	Cost
1	Investment	20	1,500	30,000	0	2,760	1,500	1,260	250	250	21	1,250	26,250	2,760
	Service enhancement													0
	Service reduction													5,000
	Maintenance			78.75	78.75	3.542808	1.94474	1.5981						82.29281
	Price change													-1,250
	Property taxes													300
	Total													6,892.293
	Constant nominal annuity													7,390.304
	Infla. adj. real annuity													6,744.88
2	Investment	21	1,250	26,250	0	2,415	1,312.5	1,102.5	500	500	22.05	1,000	22,050	2,415
	Service enhancement													0
	Service reduction													5,250
	Maintenance			248.0625	248.06	11.15985	6.12592	5.0339						259.2223
	Price change													-1,050
	Property taxes													262.5
	Total													7,136.722
	Constant nominal annuity													7,390.304
	Infla. adj. real annuity													7,082.124

TABLE 2C.2 (continued)

	Item	Beg. Price	Beg. Quantity	Actual Value	Direct Cost	Opp. Cost	Inflation	Interest	Total Use Before Enhanced	Total Use After Enhanced	End Price	End Quantity	End Value	Cost
3	Investment	22.05	1,000	22,050	0	2,028.6	1,102.5	926.1	750	500	23.15	1,000	23,152.5	2,028.6
	Service enhancement													5,788.125
	Service reduction													0
	Maintenance			434.10938	434.11	19.52973	10.7204	8.8094						453.6391
	Price change													-1,102.5
	Property taxes													220.5
	Total													7,388.364
	Constant nominal annuity													7,390.304
	Infla. adj. real annuity													7,436.231
4	Investment	23.1525	1,000	23,152.5	0	2,130.03	1,157.63	972.41	750	750	24.31	750	18,232.59	2,130.03
	Service enhancement													0
	Service reduction													5,788.125
	Maintenance			455.81484	455.81	20.50622	11.2564	9.2498						476.3211
	Price change													-868.219
	Property taxes													231.525
	Total													7,757.782
	Constant nominal annuity													7,390.304
	Infla. adj. real annuity													7,808.042
5	Investment	24.3101	750	18,232.594	0	16,77.399	911.63	765.77	1,000	1,000	25.53	500	12,762.82	1,677.399
	Service enhancement													0
	Service reduction													6,077.531
	Maintenance			670.04782	670.05	30.14414	16.5469	13.597						700.192
	Price change													-607.753
	Property taxes													182.3259
	Total													8,029.695
	Constant nominal annuity													7,390.304
	Infla. adj. real annuity													8,198.444

TABLE 2C.2 (continued)

	Value	Cost
US_0 Real	4.4518223	
US_0 Nominal	3.8695501	
P.V. of total annual costs (end of period 1)		31,228.09
P.V. of total annual costs (beg. of per. 1)		28,597.15
P.V. of nominal annuity (beg. of 1)		31,228.09
P.V. of inflation adjusted-real annuity (beg. of 1)		31,228.09
P.V. of maintenance (end of period 1)		1,558.297
P.V. of maintenance (beg. of period 1)		1,427.012
P.V. of nominal annuity less mnt. (beg. of 1)		29,801.08
P.V. of inflation adjusted-real annuity less mnt. (beg. of 1)		29,801.08