TARGET PRICES, PAYMENT LIMITATIONS AND NON-MARKET CONCERNS IN THE DESIGN OF U.S. AGRICULTURAL POLICY

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ABSTRACT. This paper examines the motivations underlying the government’s choice of particular policy mechanisms for subsidizing agriculture. Optimal policies are analyzed for three alternative government objectives: one where the government wishes to ensure a minimum level of net income for all farmers, a second where the government’s only concern is to transfer income from consumers and taxpayers to the farm sector, and a final “augmented” income-transfer objective. The analysis provides an explanation for observing agricultural policy mechanisms involving overproduction by high-cost producers relative to a free-market equilibrium. Such a distortion might arise from the existence of non-market values for the production of relatively high-cost farmers in the government’s objective.

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1. Introduction

Issues of farm structure, particularly relating to farm size and ownership, have been part of the debate surrounding agricultural policy for more than fifty years, and yet many agricultural economists argue that such issues are only the rhetoric surrounding a more direct policy objective: To transfer income from consumers and taxpayers to the agricultural sector. As one example, Gardner (1987, pg. 347) writes:

In short, the set of farm policies we observe, in the United States and the industrial countries generally, whatever the stated goals may be, appear to be observationally equivalent to policies intended to support the incomes of farmers as an interest group.

This conclusion is partly reached by observing that agricultural policies are generally not tied to specific characteristics of farms. For example, in relation to the often-mentioned goal of preserving the family farm, one might ask the question: If the goal of agricultural policy is to support or promote family farms, why are payments not made contingent on farm ownership structure? One possible answer is that payments tied explicitly to producer characteristics are not politically acceptable. That is, although the government would like to affect industry structure through its policy, it can not do so in a way that explicitly favors one type of producer over another. An excellent example of when this type of constraint might be violated comes from Heady (1983, pg. 33) who suggests a policy that ties payments directly to farm size. Under his proposal, only farms below a certain size limit (measured in acres) would receive payments, and those above the limit would receive nothing. Although growth in farm size is clearly discouraged by such a policy, an obvious obstacle to its implementation is that farmers near the cutoff might complain that the policy is unfair.

However, even ignoring political obstacles of this sort, if existing policy is purely the result of an income-transfer objective, we must still explain why mechanisms are observed that result in deadweight loss. Just (1985) argues that nondistorting transfers are simply not possible, and that the government is constrained to policies which have proven implementable in the past. That is, although many of the policies we observe are inefficient as income-transfer mechanisms, this is only because practical considerations limit the use of more efficient policies. An alternative explanation—the one adopted in this paper—is that existing policy mechanisms are not constraints in policy formation, but rather the result of a particular policy objective.

Specifically, taking a mechanism-design approach, I consider three alternative governmental objectives, and analyze optimal policies that would result under each. The analysis thus extends the work of Chambers (1988, 1992) to objectives other than weighted utilitarian. One important consequence is an explanation for a widely used farm policy: production subsidies with total payment caps. This mechanism can be viewed as a stylized version of U.S. target-price policy that, until very

\footnote{Although this type of policy has not proven implementable in the more distant past, recent policy experience suggests that times may be changing. Specifically, the U.S. Department of Agriculture's recent decision to restrict cost sharing of waste management activities in "large" confined livestock operations is an example of explicit discrimination based on an observable characteristic of farms (U.S. Department of Agriculture, 1997). However, it is also noteworthy that this decision was made in the context of an environmental concern.}
recently, was the primary policy tool used in major program crops, and which has included a total payment cap since the early 1970s (Kimpton, 1984). Results in this paper suggest that such a mechanism is consistent with the government attaching a social benefit to the production of high-cost farmers beyond market benefits. For example, this might occur if the government wishes to adopt a policy that not only transfers income to agriculture, but also alters its structure by insuring that there are a greater number of relatively high-cost producers, each producing more than they would in the absence of intervention.

The theoretical analysis in this paper borrows from the literature on mechanism design (e.g., Fudenberg and Tirole, 1992, Chapter 7). An important advantage of this approach is that one is able to allow for heterogeneity across farmers, in contrast to more traditional models of agricultural policy where all farmers are treated equally. Recent contributions in the agricultural economics literature that use a similar approach include Lewis et al. (1989), Smith (1995), and Bourgoen et al. (1995). In the model that follows, I suppose the government designs a “farm policy” that specifies the payment or transfer that each farmer receives, and her associated production. Transfers come from consumers in the form of market revenues, and from taxpayers. The model is consistent with other models of agricultural policy in treating consumers and taxpayers as a single group (e.g., Gardner, 1983; Alston and Hurd, 1990), and there is only a single commodity. Finally, optimal policy is analyzed under three alternative assumptions about the motivations underlying the government’s desire to intervene in agricultural markets.

2. THE MODEL

Producers are indexed by a single parameter, \( \theta \), that represents their farm type. For output \( q \), variable production costs are given by the continuously differentiable function \( C(q, \theta) \) with \( C(0, \theta) = 0 \), \( C_1(q, \theta) > 0 \), and \( C_{11}(q, \theta) > 0 \). Thus, there are no fixed costs, and variable production costs are assumed strictly increasing and strictly convex in \( q \). Furthermore, farmers with larger \( \theta \) are assumed less efficient in the sense of having strictly higher total and marginal production costs: \( C_2(q, \theta) > 0 \), and \( C_{12}(q, \theta) > 0 \).

The distribution of farm types is weighted with mass \( N \), and is assumed continuous on the interval \( \Theta = [\underline{\theta}, \overline{\theta}] \) with distribution function \( G(\theta) \) known to the government, and associated density \( g(\theta) \) strictly positive on \( \Theta \). A farmer’s type is assumed to be private information. Thus, although the government knows the distribution of farm types \( G(\theta) \) it does not know the type of a given farmer. For example, the government might have reasonably good information on the distribution of production costs for some crop and county combination, and yet not know the exact costs of an individual farmer. Alternatively, \( \theta \) might represent an observable characteristic (e.g., farm size) that, perhaps for political reasons, cannot be the explicit basis of policy.

We also make the following simplifying assumptions: (A1) \( \partial / \partial \theta (G(\theta) / g(\theta)) \geq 0 \), (A2) \( C_{12}(q, \theta) \geq 0 \), and (A3) \( C_{22}(q, \theta) \geq 0 \). These assumptions simplify our analysis in two ways: First, in two of the cases analyzed below, assumption (A2) is a sufficient condition for an optimum, and second, (A1)-(A3) rule out pooling (where a nondegenerate interval of farm types produce the same amount) that

\[ \text{Subscripts on functions always indicate partial derivatives with respect to the argument indicated.} \]
arises solely from the structure of $G(\theta)$ or $C(q, \theta)$. Because part of the analysis examines pooling that derives from the government’s objective, these assumptions help isolate cause and effect.

In each of the governmental policy objectives analyzed below, farmers receive the market revenues from their production, $pq$, where $p > 0$ represents the market price, plus a transfer, $t$, from the government. To avoid imposing any artificial structure on the government’s policy mechanism, we suppose the government uses a revelation mechanism: Each farmer is asked to report their type, say $\hat{\theta}$, in exchange for a pair of functions $q(\hat{\theta})$ and $t(\hat{\theta})$, and these functions are chosen to induce truthful revelation of $\theta$. The allocation of a type-$\theta$ farmer is then given by $(q(\theta), t(\theta))$. Because the functions $q(\theta)$ and $t(\theta)$ are allowed to depend in an arbitrary way on each farmers’ report of $\theta$, this particular mechanism can mimic the outcome of any arbitrary mechanism the government might use.

This insight, which is a consequence of the revelation principle, is useful in this analysis because it allows us to focus attention on the government’s policy objective, and on the outcomes $(q(\theta), t(\theta))$, without considering the choice of a specific policy instrument. Once we choose an optimal allocation from the set of allocations that can be implemented via a truthful revelation mechanism, we can then consider the design of a specific instrument to achieve the desired outcome. Thus, our analysis endogenizes the government’s choice of policy instrument, and we can ask the question: “For a given policy objective, what policy instrument (other than a truthful revelation mechanism) would be optimal?”

In a revelation mechanism, farmers report their type truthfully if and only if the following incentive compatibility condition is satisfied:

$$t(\theta) + pq(\theta) - C(q(\theta), \theta) \geq t(\bar{\theta}) + pq(\bar{\theta}) - C(q(\bar{\theta}), \theta) \quad \forall \theta, \bar{\theta} \in \Theta \times \Theta$$

That is, farmers report truthfully if a truthful report at least weakly dominates any other report. A standard result from the mechanism-design literature is:

**Lemma 2.1.** (Guesnerie and Laffont) An incentive-compatible production profile $q(\hat{\theta})$ is nonincreasing, and for all $\theta$ where $q(\theta)$ is differentiable, necessary and sufficient conditions for (2.1) are:

$$\frac{dt(\hat{\theta})}{d\theta} \bigg|_{\hat{\theta} = \theta} + p \frac{dq(\hat{\theta})}{d\theta} \bigg|_{\hat{\theta} = \theta} - C_1(q(\hat{\theta}), \theta) \frac{dq(\hat{\theta})}{d\theta} \bigg|_{\hat{\theta} = \theta} = 0,$$

and

$$\frac{dq(\theta)}{d\theta} \leq 0.$$
Having defined the set of incentive compatible allocations, we now consider
the choice of an optimal allocation under three alternative governmental
objectives. As a point of reference, we first define the free-market outcome as
the quantity that maximizes producer profit, taking price as given:

\[ q^m(\theta) = \arg \max \{pq - C(q, \theta)\} \]

Let \( \theta^c \), if it exists, be the unique solution to \( p - C_1(0, \theta^c) = 0 \). That is, producers
with \( \theta \geq \theta^c \) do not produce. The free-market production profile is then given by 0
for \( \theta \geq \theta^c \), and by the solution to \( p - C_1(q, \theta) = 0 \) for \( \theta < \theta^c \).

2.1. Income-Support Objective. To begin, suppose that the government’s
agricultural policy objective is to ensure, at least cost, a minimum level of net income
for all farmers. This objective is consistent with the view that the purpose of agri-
cultural policy is to ensure a level of net income considered reasonable relative to
earnings in other sectors of the economy. The total cost of the program to the
government is \( N \int_0^\theta t(\theta) dG(\theta) \), which we assume to be positive. Denoting \( \pi \) as the
minimum net-income target for all farm types, the government’s problem is stated as

\[
\min_{q(\theta), t(\theta)} \quad N \int_{\theta} t(\theta) dG(\theta) \quad s.t.
\]

\[ \Pi(\theta) = t(\theta) + pq(\theta) - C(q(\theta), \theta) \geq \pi \quad \text{for all } \theta \]

(2.2) and (2.3) for all \( \theta \)

This problem can be solved using methods familiar from the literature on hidden-
information agency problems. First, note that when (2.2) is satisfied, \( \Pi'(\theta) =
- C_2(q(\theta), \theta) < 0 \). Thus, in an incentive-compatible mechanism, if the income-
support constraint is satisfied at \( \bar{\theta} \), it is also satisfied for all \( \theta < \bar{\theta} \). Intuitively, a
farm type \( \theta_l < \bar{\theta} \) can always earn more than the highest cost farmer by reporting \( \bar{\theta} \)
in the revelation mechanism. This is easily verified by noting that

\[ t(\theta) + pq(\theta) - C(q(\theta), \theta_l) > t(\theta) + pq(\bar{\theta}) - C(q(\bar{\theta}), \bar{\theta}) = \Pi(\bar{\theta}) \]

Thus, in an incentive-compatible mechanism, meeting the minimum-income target
for all types only requires that it be met for the highest cost type. Since we only
consider incentive-compatible mechanisms, we therefore replace the government’s
income support constraint with \( \Pi(\bar{\theta}) \geq \pi \).

Next, \( t(\theta) \) must be chosen to satisfy (2.2) if the contract schedule \((q(\theta), t(\theta))\) is
to be incentive compatible. This fact can be used to derive an explicit formula for
\( t(\theta) \) that is equivalent to condition (2.2). To see this, note that integration of \( \Pi(\theta) \)
over \([\theta, \bar{\theta}]\) yields \( \Pi(\theta) = \Pi(\bar{\theta}) + \int_\theta^{\bar{\theta}} C_2(q(c), c) dc \). Using the definition of \( \Pi(\theta) \), we
can then derive the following expression for \( t(\theta) \):

(2.4)

\[ t(\theta) = \Pi(\bar{\theta}) + \int_\theta^{\bar{\theta}} C_2(q(c), c) dc - pq(\theta) + C(q(\theta), \theta) \]

Given an equilibrium \( q(\theta) \) satisfying (2.3), condition (2.4) represents the payment
required to induce truthful revelation. When \( \theta = \bar{\theta} \), the payment is equal to the
difference between \( \Pi(\bar{\theta}) \), the amount earned by the highest cost type, and market
returns. For all \( \theta < \bar{\theta} \), \( t(\theta) \) also includes a component representing the returns
above \( \Pi(\bar{\theta}) \) that must be paid in order to ensure incentive compatibility.
Substituting expression (2.4) into the government’s objective, integrating by parts, and defining \( \nu(\theta) \equiv g(\theta)/g(\theta) \) yields the following:

\[
\min_{q(\theta), \Pi(\overline{\theta})} N \Pi(\overline{\theta}) + N \int_{\theta}^{\overline{\theta}} \left( C(q(\theta), \theta) + C_2(q(\theta), \theta) \nu(\theta) - pq(\theta) \right) dG(\theta) \quad \text{s.t.}
\]

\[
\Pi(\overline{\theta}) \geq \Pi
\]

(2.3) for all \( \theta \).

The government’s objective is strictly increasing in \( \Pi(\overline{\theta}) \), indicating that it would never be optimal to choose \( \Pi(\overline{\theta}) > \Pi \). Thus, in the solution to this problem, \( \Pi(\overline{\theta}) = \Pi \). The government never pays the highest cost farmer more than is necessary to ensure that all farm types earn at least \( \Pi \).

In solving the problem that remains we first ignore (2.3) and then verify that it is satisfied in the solution. Choosing \( q(\theta) \) as a pointwise optimum then yields the following condition for an interior solution:

(2.5)

\[
p - C_1(q(\theta), \theta) = C_{12}(q(\theta), \theta) \nu(\theta).
\]

If for some \( \theta \), say \( \theta_* \),

\[
p - C_1(0, \theta_*) = C_{12}(0, \theta_*) \nu(\theta_*) < 0,
\]

then \( q(\theta) = 0 \) for all \( \theta \geq \theta_* \). Using assumptions (A1)-(A3), one can easily verify that (2.5) is both necessary and sufficient for an optimum, and that the solution satisfies (2.3).

Since the right hand side of (2.5) is strictly positive for all \( \theta > \theta_* \), the solution involves underproduction relative to the free-market equilibrium for all but the least-cost producer. This occurs because it is cheaper for relatively high-cost producers to cut back on production than for lower cost producers. Each loses the same marginal revenue, but since marginal costs are strictly increasing in \( \theta \), relatively high-cost producers achieve greater cost savings for the same reduction in output. The government can therefore separate high from low-cost producers by offering allocations that involve underproduction relative to what would occur in a free market.

Expression (2.5) also implies that the transfer is increasing in \( \theta \). Thus, as one would expect, higher cost producers receive greater support from the government. This can be verified by totally differentiating the expression in (2.4):

\[
\frac{dt(\theta)}{d\theta} = \frac{dq(\theta)}{d\theta} \cdot (C_1(q(\theta), \theta) - p) \geq 0.
\]

Furthermore, if for some \( \theta > \theta_* \),

\[
pq(\theta) - C(q(\theta), \theta) > \Pi + \int_{\theta}^{\overline{\theta}} C_2(q(a), a) da,
\]

then relatively low-cost producers are taxed. The right hand side of this expression represents the minimum total surplus that a type-\( \theta \) producer must earn if \( (q(\theta), t(\theta)) \) is to be incentive compatible. Thus, any surplus above this amount which is earned from sale of \( q(\theta) \) in the marketplace is taxed away and redistributed to higher cost farm types.

In summary, if the intent of agricultural policy is to ensure a minimum level of net income for all agricultural producers at least cost to taxpayers, then production
will generally be less than would occur in the free market, and relatively low cost producers may be taxed to help finance support of higher cost producers. This appears to have been the motivation underlying the 1986 dairy buyout program that paid willing dairy farmers to slaughter their herds with fees assessed on the sale of milk by producing dairies. Chambers (1988) succinctly summarizes the rational for such a policy by noting that “sometimes it is cheaper to kill cows than let them produce surplus milk.”

2.2. Income-Transfer Objective. Next, suppose that the government’s agricultural-policy objective is to transfer income to the farm sector. The total money available for transfer is assumed exogenous, and is denoted by $\overline{B}$. The assumption implicit in a fixed budget is that it is determined independent of the objectives of agricultural policy. Stated formally, the government maximizes total net income in the farm sector subject to the budget constraint and incentive compatibility:

$$(IT) \quad \max_{q(\theta), t(\theta)} \quad N \int_{\Theta} (t(\theta) + pq(\theta) - C(q(\theta), \theta)) \, dG(\theta) \quad s.t.,$$

$$N \int_{\Theta} t(\theta) dG(\theta) \leq \overline{B}$$

(2.2) and (2.3) for all $\theta$.

The following proposition is easily verified:

**Proposition 2.2.** The solution to (IT) is given by $q^m(\theta)$ and $t(\theta) = \overline{B}/N$ for all $\theta$.


Another way of stating Proposition 2.2 is to say that the solution to (IT) is first best, or is the same that would occur in the absence of asymmetric information. Because the market mechanism is Pareto efficient, no other mechanism can generate greater total surplus. Furthermore, the government is unconcerned with the distribution of resources within the farm sector, so that one incentive-compatible way to distribute the budget is to offer each farmer the same lump-sum payment.

The recently enacted Federal Agriculture Improvement and Reform Act of 1996 offers essentially this mechanism as a means of transferring roughly 35 billion dollars over 7 years to producers of wheat, feed grains, upland cotton, and rice. However, in this program lump-sum payments are distributed according to historical levels of production and acreage planted (for details see Nelson and Schertz, 1996, pg. 6), reflecting sensitivity to political issues like the distribution of program benefits within the agricultural sector. Nevertheless, this mechanism appears to be more consistent with a strict income-transfer objective than mechanisms used in previous years.

2.3. Augmented Income-Transfer Objective. Now suppose that the government also derives “non-market” benefits from the production of relatively high-cost farms represented by

$$V(q, \theta) = \begin{cases} v(q), & \text{for } \theta \geq \hat{\theta} \\ 0, & \text{for } \theta < \hat{\theta} \end{cases}$$

where $v(q) \geq 0$ is assumed continuously differentiable, strictly increasing, and concave.
One rational for this type of function is that it represents, in reduced-form, the
general populace’s desire to preserve relatively inefficient or small farms. Political
rhetoric associated with U.S. agricultural policy often appeals for preservation of
the “family farm” and rural communities generally. If family-run farms produce
at higher cost than industrial or corporate farms, then $V(q, \theta)$ might represent a
reduced-form version of this desire. In this section, we suppose that the government
wishes to transfer income to the farm sector, in addition to valuing the output of
high-cost producers in the form of $v(q)$.

Under the income-transfer objective, the government’s and farmers’ objectives
were perfectly aligned. In such a setting, it would never be optimal for some farm
type to earn negative returns. Here, however, the government values production
differently than farmers, and an allocation that generates negative farm returns,
may simultaneously generate positive social returns. To be consistent with actual
policy, we suppose that farmer participation is voluntary, and hence rule out the
possibility of negative farm returns:

\[
(2.6) \quad t(\theta) + pq(\theta) - C(q(\theta), \theta) \geq 0 \quad \text{for all } \theta.
\]

Augmenting the government’s income-transfer problem with $V(q, \theta)$ and (2.6),
the government now solves

\[
(\text{AIT}) \quad \max_{q(\theta), t(\theta)} N \int_{\Omega} (V(q, \theta) + t(\theta) + pq(\theta) - C(q(\theta), \theta))dG(\theta) \quad \text{s.t.}
\]
\[
N \int_{\Omega} t(\theta)dG(\theta) \leq B
\]

(2.2), (2.3), (2.6) for all $\theta$.

The first thing to note about this problem is that the government’s budget con-
straint must bind. If this were not true, the government could distribute the
surplus evenly across all farm types without affecting any of the other constraints, and
thereby increase the value of its objective. Using arguments identical to those used
in analysis of the government’s income-support objective, we can then replace (2.6)
with $\Pi(\theta) \geq 0$, and eliminate (2.2) and $t(\theta)$ with expression (2.4).

Substituting the expression from (2.4) for $t(\theta)$ into the binding budget constraint
(and integrating by parts) yields

\[
(2.7) \quad \Pi(\theta) = B/N - \int_{\Omega} (C_2(q(\theta), \theta)\nu(\theta) - pq(\theta) + C(q(\theta), \theta))dG(\theta).
\]

Thus, returns of the highest cost type are given by $1/N$ times the budget surplus
available after subtracting the minimum total budget necessary to implement $q(\theta)$.

Replacing $t(\theta)$ and (2.2) with expression (2.4), and using the proceeding ex-
pressing for $\Pi(\theta)$, (AIT) can equivalently be written as

\[
\max_{q(\theta)} B + N \int_{\Omega} (V(q(\theta), \theta) + pq(\theta) - C(q(\theta), \theta))dG(\theta) \quad \text{s.t.}
\]
\[
B/N - \int_{\Omega} (C_2(q(\theta), \theta)\nu(\theta) - pq(\theta) + C(q(\theta), \theta))dG(\theta) \geq 0
\]
\[
\frac{dq(\theta)}{d\theta} \leq 0 \quad \text{for all } \theta.
\]
Letting $g(\theta) = dq(\theta)/d\theta$ be a piecewise continuous control variable, and $q(\theta)$ be a piecewise differentiable state variable, (AIT) can be viewed as an optimal-control problem with an isoperimetric constraint and a non-positivity restriction on the control.

To solve this problem, we first construct a state variable
\[
z(\theta) = \int_\theta^\theta (pq(\tau) - C(q(\tau), \tau) - C_2(q(\tau), \tau)\nu(\tau)) g(\tau) d\tau.
\]
Letting initial and terminal conditions for $z(\theta)$ be given by $z(\theta) = 0$ and $z(\theta) \geq -\overline{B}/N$, we can then replace constraint (2.7) with the expression
\[
\frac{dz(\theta)}{d\theta} = (pq(\theta) - C(q(\theta), \theta) - C_2(q(\theta), \theta)\nu(\theta)) \frac{g(\theta)}{\theta}.
\]
The Hamiltonian for this problem is then
\[
H(y, q, \delta, \theta, \lambda) = (V(q, \theta) + pq - C(q, \theta)) g(\theta) + \\
\lambda (pq - C(q, \theta) - C_2(q, \theta)\nu(\theta)) g(\theta) + \delta y
\]
\[
= (V(q, \theta) + (1 + \lambda) (pq - C(q, \theta)) - \lambda C_2(q, \theta)\nu(\theta)) g(\theta) + \delta y
\]
where $\delta(\theta)$ and $\lambda$ are the costate variables for $q(\theta)$ and $z(\theta)$, respectively.\(^3\)

Necessary and sufficient conditions for an optimum are given by
\[
(2.8) \frac{d\delta(\theta)}{d\theta} = -V_1(q(\theta), \theta) \frac{g(\theta)}{\theta}
\]
\[
- (1 + \lambda) (p - C_1(q(\theta), \theta) \frac{g(\theta)}{\theta}) = \lambda C_2(q(\theta), \theta)\nu(\theta) \frac{g(\theta)}{\theta}
\]
for all $\theta$
\[
\frac{dq(\theta)}{d\theta} = y(\theta) \quad \text{for all } \theta
\]
\[
(2.9)
\]
\[
\delta(\theta) \leq 0, \frac{\delta(y)}{\theta} \leq 0, \delta(\theta) \frac{y(\theta)}{\theta} = 0 \quad \text{for all } \theta
\]
\[
(2.10)
\]
\[
\frac{dz(\theta)}{d\theta} = (pq(\theta) - C(q(\theta), \theta) - C_2(q(\theta), \theta)\nu(\theta)) g(\theta) \quad \text{for all } \theta
\]
\[
(2.11)
\]
\[
z(\theta) = 0, z(\theta) \geq -\overline{B}/N
\]
with transversality conditions $\delta(\theta) = \delta(\theta) = 0, \lambda \geq 0,$ and $\lambda \cdot (z(\theta) + \overline{B}/N) = 0$.\(^4\)

Expressions (2.8) and (2.9) are the equations of motion for $\delta(\theta)$ and $q(\theta)$, respectively, and (2.10) is the first-order condition and complementary slackness for the control $y(\theta)$. The last two conditions just repeat the definition of our constructed state variable $z(\theta)$ and its boundary conditions. The state variable $q(\theta)$ has no initial or terminal conditions so its costate variable must be zero at both boundaries. Similarly, the transversality conditions for $z(\theta)$ are standard for a state variable with fixed initial and inequality terminal conditions.

Figure 1 displays the production profile $q(\theta)$ satisfying (2.8) when $\delta(\theta) = 0$ for all $\theta$. Because $q(\theta)$ must be continuous (piecewise differentiability implies continuity) and nonincreasing, the outcome in Figure 1 clearly cannot be a solution. The government would like to increase $q(\theta)$ discontinuously at $\theta$, but doing so violates (2.3). The solution to a similar problem is discussed in Guesnerie and Laffont (1984,\(^5\))

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\(^3\)When constructing a state variable to accommodate an isoperimetric constraint, it is always the case that its associated costate variable, $\lambda$ in this case, is a constant.

\(^4\) $V_1(q, \theta)$ is given by $v'(q)$ for $\theta \geq \theta$ and 0 otherwise.
Figure 1. Production profile under (AIT) with $\delta(\theta) = 0$ for all $\theta$.

pg. 344). Generally, an optimal response to this type of nonmonotonicity involves a patched or blocked region where the production profile $q(\theta)$ is constant.

In what follows, we construct a solution satisfying conditions (2.8)-(2.12) and the associated transversality conditions. The solution involves an interval $[\theta^1, \theta^2]$ around $\hat{\theta}$ where $q(\theta)$ is constant. To do so, first note that in any interval where $y(\theta) < 0$ for all $\theta$ in the interval, $\delta(\theta)$ must equal zero. In such an interval, $d\delta(\theta)/d\theta = 0$ for all $\theta$ in the interval and (2.8) implies

$$\frac{\nu'(q(\theta))}{1 + \lambda} + p - C_1(q(\theta), \theta) - \frac{\lambda}{1 + \lambda} C_{12}(q(\theta), \theta) \nu(\theta) = 0$$  \hspace{1cm} (2.13)

if the interval lies above $\hat{\theta}$, or

$$p - C_1(q(\theta), \theta) - \frac{\lambda}{1 + \lambda} C_{12}(q(\theta), \theta) \nu(\theta) = 0$$  \hspace{1cm} (2.14)

if the interval lies below $\hat{\theta}$. To simplify the analysis, we assume that an interior solution exists in each of these equations. We therefore rule out the possibility of zero production for any farm type.

If $q(\theta)$ is constant on an interval $[\theta^1, \theta^2]$, then because we assume an interior solution for all $\theta$ outside this interval, $y(\theta) > 0$ for all $\theta \notin [\theta^1, \theta^2]$. By the continuity of $\delta(\theta)$, it is then true that $\delta(\theta^1) = \delta(\theta^2) = 0$. Denoting $\overline{\theta}$ as the value of $q(\theta)$ for $\theta \in [\theta^1, \theta^2]$, integrating (2.8) yields

$$\int_{\theta^1}^{\theta^2} \left( V_1(\overline{\theta}, \theta) + (1 + \lambda)(p - C_1(\overline{\theta}, \theta)) - \lambda C_{12}(\overline{\theta}, \theta) \nu(\theta) \right) dG(\theta) = 0.$$  \hspace{1cm} (2.15)

Equations (2.13)-(2.15) constitute three equations in three unknowns: $\theta^1, \theta^2$, and $\overline{\theta}$. Let $\theta^2(\overline{\theta})$ and $\theta^1(\overline{\theta})$ be the solutions to (2.13) and (2.14), respectively, when $q(\theta) = \overline{\theta}$. It is easily verified that these solutions are unique. We have therefore constructed a solution satisfying (2.8)-(2.12) and the associated transversality conditions. Because these conditions are necessary and sufficient for an optimum, we do not need to consider any other potential solutions.
The optimal solution expressed in terms of \( q(\theta) \) is depicted graphically in Figure 2 for the case where \( \Pi(\theta) > 0 \), and hence where \( \lambda = 0 \). In this case, there is enough money in the budget to ensure that all farm types earn strictly positive returns. Relative to the free-market equilibrium, high-cost farmers \( (\theta > \theta^1) \) overproduce, whereas low-cost farmers \( (\theta \leq \theta^1) \) produce efficiently. Furthermore, the interval of types that produce positive optimal output is at least as large as in the free market. This can be verified by noting that, from (2.13) and the definition of \( q^m(\theta) \), it is possible that \( q(\theta) > 0 \) for \( \lambda = 0 \), while \( q^m(\theta) = 0 \). Thus, because the government values the production of relatively high-cost farmers, it prefers that there be a larger number of them, each producing more output than they would in the absence of government intervention.

Up to this point, we have only considered the shape of the optimal production profile. The blocked portion of \( q(\theta) \) has an interesting counterpart in terms of the optimal payment \( t(\theta) \). By Lemma 2.1

\[
\frac{dt(\theta)}{d\theta} = (C_1(q(\theta), \theta) - p) \frac{dq(\theta)}{d\theta}.
\]

This expression indicates that when \( \lambda = 0 \), the producer’s payment schedule is constant for \( \theta \leq \theta^1 \). Relatively efficient farm types therefore receive a lump-sum payment. We can also represent the payment as a function of \( q(\theta) \) by defining \( t_q(q(\theta), \theta) \) as the value of \( t(\theta) \) from (2.4) for any \( \theta \), evaluated at the optimal \( q(\theta) \). Figure 3 displays the optimal payment profile as a function of \( q(\theta) \).

This type of payment structure is similar to a stylized version of a specific policy mechanism often used in agriculture: a production subsidy with a total payment cap. That is, suppose that the government offers the following payment schedule:

\[
s(q) = \begin{cases} 
\alpha q, & \alpha q < \sigma \\
\sigma, & \alpha q \geq \sigma
\end{cases},
\]

where \( \alpha \) is a per-unit production subsidy, and \( \sigma \) a total-payment cap. Clearly, this payment structure is qualitatively similar to that presented in Figure 3. In fact, when \( \lambda = 0 \) we can state the following proposition:
TARGET PRICES AND PAYMENT LIMITATIONS

![Graph showing payment profile under (AIT) as a function of q(θ).]

**Figure 3.** Payment profile under (AIT) as a function of q(θ).

**Proposition 2.3.** Assume an interior solution exists for all θ ≥ \( \bar{\theta} \) in (2.13) and for all θ < \( \bar{\theta} \) in (2.14) with \( \lambda = 0 \) and \( v'(q) = \alpha \). Then a production-subsidy mechanism with a total payment limit of \( \pi = \alpha \bar{\pi} \) implements the solution to (AIT).


Thus, so long as there is sufficient money in the government’s budget, and marginal off-farm benefits are constant, a production subsidy mechanism that includes a total payment cap can implement the government’s optimal allocation.

So far we have discussed the solution to (AIT) when \( \lambda = 0 \), or when the budget is sufficiently large to ensure that all farm types earn a strictly positive return. Equation (2.13) indicates that the degree of overproduction by relatively high-cost producers is reduced as the budget shrinks and \( \lambda \) becomes positive. This simply reflects the inability of the government to finance the distortion it desires: It must now choose a production distortion that reflects both its desire to have high-cost farmers overproduce, and its limited capacity for financing overproduction.

The relationship between actual policy and the optimal policy under (AIT) may even be closer, although more difficult to characterize analytically, when \( \lambda > 0 \). This is because U.S. target price mechanisms were normally coupled with acreage restrictions. That is, producers wishing to participate in U.S. farm programs were forced to idle a fraction of their acreage. Thus, relatively low cost farmers might actually have produced less than their free-market output in order to participate in the program (and hence receive the payment limit), while the incentive for relatively high-cost farmers to overproduce as a result of the production subsidy was somewhat mitigated by the acreage restriction. Introduction of a strictly positive \( \lambda \) in equations (2.13) and (2.14) achieves exactly this type of effect.

A production subsidy-type mechanism, coupled with a total payment limitation, is therefore consistent with a government that attaches an additional social benefit to the production of relatively high-cost farmers beyond market benefits. In a production-subsidy mechanism with a total payment limit, low-cost farmers produce where price equals marginal cost, whereas high-cost farmers overproduce.
relative to the market equilibrium. If the government only wanted to transfer income to the farm sector, it would never have high-cost farmers overproduce because this wastes resources that could otherwise be transferred.

3. Conclusion

This paper examines the motivations underlying the government’s choice of particular policy mechanisms for subsidizing agriculture. The work extends the analysis in Chambers (1988, 1992) to objectives other than weighted utilitarian, and provides an explanation for observing overproduction by high-cost producers relative to a market equilibrium. The analysis suggests that such a distortion might arise from the existence of non-market values for the production of relatively high-cost farmers in the government’s objective. One plausible reason for the existence of such values is that the government perceives a connection between the existence of relatively high-cost farm operations, and the preservation or sustainability of rural communities. If many relatively high-cost farms are perceived to be more conducive to the survival of rural areas than a few low-cost farms, and if the government wishes to support rural communities, it would prefer that more production come from high-cost farms.
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