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The Price Responsiveness of Farmers in Latin America: An Empirical Test for Cereal and Potato Producers in Chile

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The question of the responsiveness of the producers of agricultural products to alterations in product prices has generated considerable discussion at both the academic and policy-making levels. One of the foremost supporters of the positive response hypothesis is Professor T. W. Schultz, who has observed that[1]

> The doctrine that farmers in poor countries either are indifferent or respond perversely to changes in prices . . . is patently false and harmful. Price policies based on it always impair the efficiency of agriculture.

In a recent study of Latin American agriculture, Dr. Montague Yudelman comments that[2]

> Low prices, exchange rate policy and export taxes reduce the incentive to producers and help explain the poor performance of the agricultural sector in some otherwise well-endowed countries.

At the other extreme of this question of the price elasticity of supply of farm products is the now well-enshrined position taken by the structuralist inflation school in Latin America. In this model it is observed that high population growth rates plus even higher rates of expansion of urban populations induce a rapid growth in the demand for foodstuffs. This demand increase provokes a rise in the prices of foodstuffs, but the price-elasticity of supply is low (or nil) so that output does not significantly respond. This supply inelasticity is largely owed, in the view of the adherents, to the socio-economic structure prevailing in the countryside which is allegedly dominated by the large non-capitalistic or non-profit maximising latifundio or the subsistence level, non-market oriented minifundio. Given the low supply

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1 T. W. Schultz, *Economic Crisis in World Agriculture*, Ann Arbor: University of Michigan Press, 1965, p.49.

2 M. Yudelman, *Agricultural Development in Latin America*, Washington: Inter-American Development Bank, 1966.

responsiveness of farmers, demand increases result in rising prices for foodstuffs, which make up most of the urban worker's expenditures and thus lead him to demand higher wages, provoking increases in manufacturing costs. The resulting relatively high prices of domestic farm and manufactured products will encourage imports and discourage exports, conducing to adverse trade balances and/or devaluation. At the same time, the increased prices for foodstuffs will tend to transfer income from the lower-income urban groups to rural landlords or latifundistas.[3]

The debate is not merely of academic significance. The economic policies which are consistent with or called by the structuralist position are typically quite unlike those which would be indicated by the positive and significant price responsiveness hypothesis. Among the former are:

(1) The setting of maximum retail prices on foodstuffs, usually in the major urban centers, hoping to avoid sharp increases in food costs and therefore in cost of living, especially as measured through official price indices.

(2) Heavy importation of foodstuffs to supplement local production, thus alleviating supply shortages which may provoke price increases.

(3) The implantation, formally or informally, of rationing, e.g., meatless days, the occasional disappearance of products from the market, lower quality products replacing higher quality ones, etc. A companion is often the appearance of black markets and dual price systems, illegal of course, and probably tending to favor distributors rather than producers.

Citation of the specific instances in Latin America where these measures have been invoked would exceed the space we have available here.

The question of the price responsiveness of the supply of agricultural products is an empirical one which may be subjected to verification using econometric techniques. Unfortunately there have been very few attempts to organize data and utilize them to provide an empirical test of either the price responsiveness or the structural hypothesis in Latin America. There have been a number of studies in other regions which suggest that farmers in both developed and underdeveloped economies are remarkably similar in their price-responsiveness.[4] In part, we suspect that the persistence with which the structuralist position has been propagated and taken root in Latin America is related to this paucity of empirical studies. We hope this short paper examining the Chilean experience will contribute to filling this void.

The major impulse to price-responsiveness studies derives from the work of Dr. Marc Nerlove in this field in the mid-1950's and the greater part of the

3 For a detailed review of the literature with respect to the structuralist model, see Werner Baer, "The inflation controversy in Latin America: a survey", *Latin American Research Review*, Vol.II, No.2 (1967), pp.3–26.

4 For a review of price-response research, see Raj Krishna, "Agricultural price policy and economic development", in H. M. Southworth and B. F. Johnston, Ed., *Agricultural Development and Economic Growth*, Ithaca: Cornell University Press, 1967, pp.497–540.

empirical studies since then have been patterned after his research.[5] We will utilize an "adjustment lag" model such as that proposed by Raj Krishna in his study of price-responsiveness in India and Pakistan, as this estimation form does not have the problems of serial correlation in the error term which characterize the "price-expectations" model of Nerlove.[6]

In this study we will test four different forms of the regression equation to estimate the supply response of wheat, corn, oats, barley, rye, rice and potatoes. Models I and II will utilize price as the major independent variable; the latter will be of the autoregressive form. Models III and IV will utilize gross returns per hectare as the major independent variable; again the latter will be of the autoregressive type. In every case the dependent variable (Y) represents a given year's cultivated area in the crop in question. The price variable is the price of the crop under study in the preceding year, (P) deflated by the price index (Q) of all seven crops.[7] The weights for the price index were calculated for 1960–61 prices and output, as these were years of relative price stability. The index base is 1930–1932.

It would seem reasonable for farmers to make cropping decisions not merely on the basis of the relative price movements of crops, but rather by comparing the relative profitability of crops. We do not have sufficient information available to enable us to estimate directly the profitability of the crops under study here. We will, however, use as an approximation of this profitability the average gross income per hectare of each crop examined. This is simply the reported physical yield (R) times the relevant year's price. For each crop this is deflated by an index of gross returns per hectare (S), constructed in the same manner as the price index.[8] This variable is also lagged one year with respect to cultivated area. The trend variable (Z), included to take account of "exogenous" movements in cultivated area or unexplained factors, consists of the total cultivated area for all seven crops for the relevant year, less the area in that year of the crop under study.

The three variables listed above are generally the standard ones used in price responsiveness studies.[9] This is not the case with the one which follows: only recently have they been brought under consideration.[10] While the prevalence and rationality of some degree of risk aversion appear to have become widely accepted in economic theory, there have not been numerous instances where quantitative estimates of its impact have been made in

5 Marc Nerlove, *The Dynamics of Supply: Estimation of farmers' response to price*, Baltimore: the Johns Hopkins Press, 1958.

6 Raj Krishna, "Farm supply response in India-Pakistan: a case study of the Punjab Region", *Economic Journal*, Vol.73 (Sept., 1963), pp.477–487.

7 When deflating wheat and potato prices, the deflator index excludes their respective prices, as these two crops had relatively heavy weights in the index.

8 As in the case of the price deflator, wheat and potatoes were excluded from their respective gross income deflators, owing to their relatively heavy weight in the index.

9 Actually, the use of the gross income (or profitability) variable in lieu of the price variable was not noted in any of the studies we consulted.

10 cf. Jere Behrman, *Supply response in underdeveloped agriculture: a case study of four major annual crops in Thailand, 1937–1963*, Ph.D. Thesis, Massachusetts Institute of Technology, 1966, pp.182–187.

empirical studies. In the context of this paper, we should consider that among the risks faced by a cultivator, price and yield variability would be the major ones. We have included two measures of the first factor and one of the second. The first measure of price variability consists of the standard deviation of the deflated price of the crop in question during the three years preceding the year whose cultivated area is under consideration. The second measure is the ratio formed by dividing the standard deviation of the nominal (i.e. undeflated) price of the crop in the three preceding years by the price deflator's standard deviation in the preceding three years.[11] The measure of yield variability consists of the ratio of the standard deviation of this crop's yield (K), in the preceding three years to the standard deviation of an index of crop yields (W) in the same period.[12] The final independent variable, included in the autoregressive models, is the area cultivated in the relevant crop in the preceding year (Y').

Thus the four estimating equations may be written:

$$\text{(I)} \quad Y = b_0 + b_1 \frac{P}{Q} + b_2 Z + b_3 \frac{\sigma P}{Q} + b_4 \frac{\sigma P}{\sigma Q} + b_5 \frac{\sigma K}{\sigma W}$$

$$\text{(II)} \quad Y = b_0 + b_1 \frac{P}{Q} + b_2 Z + b_3 \frac{\sigma P}{Q} + b_4 \frac{\sigma P}{\sigma Q} + b_5 \frac{\sigma K}{\sigma W} + b_6 Y'$$

$$\text{(III)} \quad Y = b_0 + b_1 \frac{R \cdot P}{S} + b_2 Z + b_3 \frac{\sigma P}{Q} + b_4 \frac{\sigma P}{\sigma Q} + b_5 \frac{\sigma K}{\sigma W}$$

$$\text{(IV)} \quad Y = b_0 + b_1 \frac{R \cdot P}{S} + b_2 Z + b_3 \frac{\sigma P}{Q} + b_4 \frac{\sigma P}{\sigma Q} + b_5 \frac{\sigma K}{\sigma W} + b_6 Y'$$

The data we have available for the application of the estimating equations are of varying quality. The Dirección de Estadística y Censos of Chile has for several decades made a serious effort to accumulate data on crop area, output and prices. For the first few years of the decade of the thirties, coverage was not complete, while the sample designed from the 1955 agricultural census to develop subsequent estimates appears to have over- or underestimated movements of the data; fortunately adjustments based on the 1965 census lessen this problem. As far as prices are concerned, these are wholesale prices and not those received by farmers, although they do refer to the harvest period in each year.[13]

The computational results for models I and III are reproduced in Table I. Examining the squared coefficient of multiple correlation (corrected for

11 The inclusion of this variable was suggested to the author by Dr. Behrman in a personal communication.

12 The choice of a three-year period for these variability measures was an arbitrary one.

13 Considerably greater discussion of the Chilean statistics is contained in the author's *Essays in the Chilean Agricultural Economy* to be published (in Spanish) by the Instituto de Economía, Universidad de Chile.

degrees of freedom) we observe that in every case except rye and rice the gross income model has more explanatory power than the price model. The price variable is statistically significant in five out of the seven cases. The exceptions are corn and oats. Only in the case of rice is the price variable significantly negative. The gross returns variable is statistically significant in six out of the seven cases, and is consistently positive. An examination of the table indicates that the remaining variables demonstrate moderate to poor performance.

The computational results for Models II and IV are summarized in Table II. There we observe that the squared coefficient of multiple correlation (corrected for degrees of freedom) is consistently higher than in the case of the preceding models. Moreover, the gross income model demonstrates more explanatory power than the price model for each crop. The price variable is statistically significant (and with positive sign) for wheat, corn, barley and rye. For the short-run model wheat and rye were also significant. Unlike the results of the latter model, in the autoregressive model the price variable is not significant for rice and potatoes. In the gross income model this variable is statistically significant (and positive) in every instance. The performance of the remaining variables is again diverse.

One of the basic assumptions to be met in order that ordinary least squares estimating procedures be efficient is that the errors relating to successive observations be independent. This condition may be particularly difficult to satisfy when models involving time series data are to be investigated. Even in the presence of autocorrelated errors, the use of least squares techniques entails no bias in the estimation of regression parameters. Nevertheless, this method is no longer the most efficient and the usual formulae for calculating the standard errors of the regression coefficients result in their underestimation.[14]

In autoregressive models the application of least squares techniques results in parameter estimates which are asymptotically unbiased, i.e., estimates which approach true values as the sample size increases.[15] In the presence of autocorrelated residuals, however, the bias affecting the results obtained by least squares estimating procedures may be considerable, and it does not generally tend to zero as the number of observations tends to infinity and may even tend to increase.[16] More seriously, it is not possible to use the autocorrelation coefficient of the residuals to assess correctly the degree of their relationship as this coefficient is generally calculated as being near zero even if the correlation is fairly considerable. In this situation the Durbin-Watson statistic, traditionally used to detect such relationships, is biased toward the value indicating the absence of serial correlation. Hence the test can be used only for non-autoregressive models.[17]

In a paper scheduled to be published in *Econometrica,* J. Durbin proposes

14 E. Malinvaud, *Statistical Methods of Econometrics*, Chicago: Rand McNally, 1966, pp.420–421.

15 *Ibid.* pp.454–456.

16 *Ibid.* p.459.

17 *Ibid.* pp.462–465.

a new test for serial correlation in autoregressive models.[18] This new variable -H- is asymptotically distributed as a normal standard deviate and permits a large sample test of the null hypothesis with respect to the absence of significant first order serial correlation of the residuals for autoregressive equations. In Table III are reproduced the results of the calculations of the "H" statistic. There we observe that the null hypothesis is rejected only in the case of oats.

In Table IV are presented the price and income elasticities of supply for these crops according to the four models utilized here. The results are considerably disparate; nevertheless we observe only one instance of a significantly negative price responsiveness, in the short-run price model for rice. It is interesting to note the relatively low supply elasticity estimates for wheat. This cereal typically accounts for between 60% to 70% of the total area in annual crops. Given its magnitude, proportionate increases in wheat acreage would have a considerably greater impact on factor markets than would be the case for the other crops. Thus factor supply inelasticity could in part restrict output responsiveness for this crop.

Compared to results for studies in the United Kingdom or the U.S.A., the results for wheat in Chile are relatively low. The corn results compare favorably to those in the U.S. Studies of barley in the U.K. have given elasticity estimates similar to our results for Chile. Studies of the price responsiveness of oats in the U.K. have given estimates inferior to our results for this crop, but the problem of serial correlation introduces uncertainty with respect to their accuracy. With respect to studies in other underdeveloped areas, Raj Krishna indicates that the price elasticity of supply of barley is typically low (0.0 to 0.1), considerably below the results for Chile, while estimates for wheat, corn and rice range from low (0.0–0.1) to medium (0.1–0.4); thus the Chilean results tend to match or surpass the typical experience of other underdeveloped areas.[19]

We will terminate with a final *caveat*. In no way are we proposing that the problem of increasing farm output in the less developed countries is merely a question of product price policy. Few proponents of the price-responsiveness hypothesis would support such a point of view. Rather we wish to point out that coupled with programs promoting the use of improved inputs and management practices, the empirical evidence indicates that an appropriate price policy may be very rewarding in terms of output increases. On the other hand, promotion programs plus an output price policy which ignores or misconceives the response of output to prices, such as suggested by the structuralist model, will result in output considerably below what we would have otherwise achieved.

18 J. Durbin, "Testing for serial correlation in least squares regression when some of the regressors are lagged dependent variables".

19 Raj Krishna, "Agricultural Price Policy and Economic Development", in Southworth and Johnston, *op.cit.*, p.504.

TABLE 1

Calculation Results for Models I and III

Crops	Model	R^{2x}	Constant Term	Real Price	Real Gross Income	Trend	Std. Dev. Real Price	Std. Dev. Price / Std. Dev. Gral. Price Index	Std. Dev. Yield / Std. Dev. Gral. Yield Index	Durbin-Watson Statistic
Wheat	I	0.330	506.958	2951.479		0.697	−9,907.039	196.540	−0.016	1.289[f]
				(1.972)[c]		(2.361)[b]	(3.029)[a]	(0.380)	(0.042)	
	III	0.501	481.064		391.938	0.691	−10,684.004	−433.304	−0.249	1.258[f]
					(3.808)[a]	(3.278)[a]	(4.041)[a]	(0.914)	(0.714)	
Corn	I		75.835	591.512		−0.018	−669.069	−100.591	−0.213	0.214[g]
				(0.645)		(0.397)	(90.429)	(0.494)	(90.856)	
	III	0.211	38.320		99.720	−0.005	−131.786	−191.178	−0.278	0.518[g]
					(3.294)[a]	(0.121)	(0.103)	(1.272)	(1.381)[d]	
Oats	I	0.103	−10.361	−22.285		0.094	1,745.946	−126.016	0.234	0.934[g]
				(0.030)		(1.777)[c]	(0.877)	(0.554)	(1.160)	
	III	0.189	−13.134		122.353	0.077	986.436	−287.096	0.297	0.895[g]
					(1.688)[d]	(1.517)[d]	(0.512)	(1.295)	(1.703)[c]	
Barley	I	0.399	12.880	1315.056		−0.001	274.636	−156.001	0.527	1.236[f]
				(2.195)[b]		(0.033)	(0.204)	(0.998)	(4.864)[a]	
	III	0.516	−12.754		107.100	0.020	−256.296	−145.512	0.467	1.158[f]
					(3.534)[a]	(0.704)	(0.209)	(1.098)	(5.03)[a]	
Rye	I	0.093	−2.548	268.463		0.006	2.604	−86.250	−0.048	0.587[g]
				(1.518)[d]		(0.992)	(0.265)	(1.888)[c]	(1.338)[d]	
	III	0.079	−2.186		19.159	0.008	7.316	−55.981	−0.045	0.477[g]
					(1.365)[d]	(1.364)[d]	(0.734)	(1.461)[d]	(1.250)	
Rice	I	0.151	30.767	−2063.946		0.057	−1,857.581	382.922	−0.016	1.655[e]
				(1.836)[c]		(1.382)[d]	(1.473)[d]	(1.434)[d]	(0.641)	
	III	0.003	14.963		7.586	0.015	−1,269.855	−60.336	−0.004	0.727[g]
					(0.805)	(0.400)	(0.911)	(0.413)	(0.158)	
Potatoes	I	0.480	−27.500	1119.285		0.070	2,120.789	−30.246	−0.023	0.991[f]
				(3.996)[a]		(2.087)[b]	(2.480)[b]	(0.597)	(1.776)[c]	
	III	0.629	−59.755		23.683	0.086	2,525.438	−42.258	−0.028	1.060[f]
					(5.760)[a]	(3.022)[a]	(3.574)[a]	(0.994)	(2.565)[b]	

a. Significant at 99% level
b. Significant at 95% level
c. Significant at 90% level
d. Significant at 80% level

x Corrected for degrees of freedom

e. No serial correlation at 99% level
f. Test inconclusive at 99% level
g. Serial correlation at 99% level

TABLE II

Calculation Results for Models II and IV

Crop	Model	R^{2x}	Constant Term	Real Price	Real Gross Income	Trend	Std. Dev. Real Price	St Dev. Price / Std. Dev. Gral. Price Index	Std. Dev. Yield / Std. Dev. Gral. Yield Index	Lagged Cul'd Area	Durbin-Watson Statistic
Wheat	II	0.489	127.402	4,380.430		0.909	–9594.406	–229.266	–0.096	0.371	2.172
				(3.160)[a]		(3.409)[a]	(3.359)[a]	(0.486)	(0.278)	(3.075)[a]	
	IV	0.563	300.539		385.739	0.695	–9402.008	–457.606	–0.277	0.231	1.660
					(4.001)[a]	(3.521)[a]	(3.701)[a]	(1.031)	(0.848)	(2.193)[b]	
Corn	II	0.812	27.289	751.469		–0.040	–390.622	–77.361	0.045	0.933	2.386
				(1.972)[c]		(2.091)[b]	(0.603)	(0.914)	(0.427)	(11.436)[a]	
	IV	0.845	28.387		45.787	–0.033	32.374	–61.855	–0.051	0.837	2.642
					(3.188)[a]	(1.863)[c]	(0.057)	(0.914)	(0.552)	(10.556)[a]	
Oats	II	0.249	–1.258	460.667		0.029	3,178.481	–199.322	0.020	0.513	1.695
				(0.646)		(0.533)	(1.664)[d]	(0.948)	(0.096)	(2.500)[b]	
	IV	0.380	–0.350		157.505	0.009	2,515.458	–353.155	0.016	0.557	1.781
					(2.445)[b]	(0.172)	(1.430)[d]	(1.810)[c]	(0.092)	(3.052)[a]	
Barley	II	0.558	5.848	1,800.090		–0.027	–791.384	–152.010	0.340	0.515	2.116
				(3.366)[a]		(0.949)	(0.660)	(1.134)	(3.118)[a]	(3.272)[a]	
	IV	0.640	–15.614		117.121	0.001	–1,080.014	–108.353	(0.286)	0.441	2.011
					(4.447)[a]	(0.044)	(0.993)	(0.943)	(2.912)[a]	(3.205)[a]	
Rye	II	0.570	–2.675	266.568		0.001	3.956	–61.760	–0.030	0.625	1.576
				(2.189)[b]		(0.248)	(0.584)	(1.945)[c]	(1.193)	(5.566)[a]	
	IV	0.638	–2.754		29.017	0.002	10.046	–34.428	–0.026	0.683	1.589
					(3.251)[a]	(0.387)	(1.605)[d]	(1.420)[d]	(1.119)	(6.534)[a]	
Rice	II	0.455	–23.027	–869.507		0.057	–1384.511	156.571	0.013	0.623	2.404
				(0.869)		(1.674)[d]	(1.333)[d]	(0.680)	(0.569)	(2.943)[b]	
	IV	0.509	–44.786		10.409	0.044	–833.438	–44.488	0.018	0.728	2.004
					(1.542)[d]	(1.613)[d]	(0.834)	(0.427)	(0.916)	(3.941)[a]	
Potatoes	II	0.888	13.164	144.069		–0.004	823.164	–10.617	–0.013	0.838	2.555
				(0.883)		(0.257)	(1.966)[c]	(0.449)	(2.058)[b]	(9.940)[a]	
	IV	0.899	1.000		5.868	0.006	1,011.513	–18.379	–0.015	0.762	2.364
					(1.965)[c]	(0.351)	(2.476)[b]	(0.823)	(2.495)[b]	(8.566)[a]	

a. Significant at 99% level
b. Significant at 95% level
c. Significant at 90% level
d. Significant at 80% level

x Corrected for degrees of freedom

TABLE III

Test for the Presence of Serial Correlation in the Autoregressive Models

CROP	MODEL	H STATISTIC
Wheat	II	–0.685 *
	IV	1.226 *
Corn	II	–1.255 *
	IV	–2.070 *
Oats	II	**
	IV	**
Barley	II	–0.777 *
	IV	–0.576 *
Rye	II	1.594 *
	IV	1.481 *
Rice	II	–4.828 *
	IV	–0.022 *
Potatoes	II	–1.826 *
	IV	–0.022 *

* No serial correlation at 95% level of confidence.

** Null hypothesis of absence of serial correlation rejected at 95% level of confidence.

TABLE IV

Price and Income Elasticity of Supply

Crop	Short Run				Long Run		United Kingdom * * *		United States * * *	
							Short Run	Long Run	Short Run	Long Run
	I (Price)	II (Price)	III (Income)	IV (Income)	II (Price)	IV (Income)				
Wheat	0.104	0.155	0.171	0.168	0.246	0.218	0.33	0.46	0.48	0.93
Corn	x	0.349	0.669	0.307	5.229	1.888			0.10	0.18
Oats	x	0.120[xx]	0.294	0.380[xx]	0.247[xx]	0.860[xx]	0.11 0.18	0.16 0.24		
Barley	0.658	0.901	0.765	0.837	1.857	1.496	0.63	1.75		
Rye	0.887	0.881	0.417	0.632	2.351	1.996				
Rice	2.038	x	x	0.276	x	0.994				
Potatoes	0.321	x	0.555	0.138	x	0.579				

x Regression coefficient not statistically significant.

xx Null hypothesis with respect to serial correlation of the residuals in the autoregressive model cannot be rejected, thus these elasticity estimates may be biased.

* * * Raj Krishna, "Agricultural Price Policy and Economic Development", in H.M. Southworth and B.F. Johnston, eds., *Agricultural Development and Economic Growth*, Ithaca, Cornell University Press, 1967, p.508.

Maximization with Several Objective Functions

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1. *Introduction*

The usual technique of farm planning by linear (or non-linear) programming assumes that there is a single objective function to be maximized subject to constraints given by the available resources. The result is an optimum plan. Usually there are some factors not taken into account in this procedure which may affect the decision of the farm operator. Such factors may lead to a selection of a program other than the optimal one. We may view such a situation as an evidence that the definition of optimum used in programming is not identical with that used by the decision maker. By this it is not meant that the decision maker does not want to maximize profit but rather that he gives some weight to other factors as well. For instance, he may wish to forego some income in order to diminish uncertainty. In this case his objective function consists of two components, income and say risk. On the surface if the weights that the decision maker attaches to the two components were known, the two components could be aggregated and the problem could be reduced into a standard form. However, this is only true if the weights are constants, or in other words if the indifference curves in the space of the two components (say income and risk) are linear for all their relevant combinations. If this is not so, then such an aggregation does not lead to an optimal plan. The analytic approach to such a problem requires to draw first the efficient frontier (transformation curve) between the various components of the objective function. Having this information, the decision maker can apply his subjective evaluation (utility function) to select that plan that maximizes his utility. This paper deals with the construction of the efficient frontier.

While the technique itself is rather simple its applications are of prime importance and of wide scope. Two implications are discussed briefly. One deals with extension work and the other with an entirely different subject—that of empirical analysis of cross section data.

2. *The Framework*

(a) The initial problem. Let us start with a usual linear programming scheme:

$$\max_{\chi} \quad \pi = c'\chi \tag{1}$$

$$\text{subject to} \quad A\chi \leqslant b \; ; \chi \geqslant 0 \tag{2}$$

where A is the matrix of resource requirements, b is a vector of available resources, χ is the solution vector representing levels of operation of the

1 This research has been financed by a grant made by the United States Department of Agriculture, under P.L. 480. I am indebted to Yigal Danin for skilful performance of the calculations and for being helpfully responsive in discussing the work.

various activities and c is the vector of, say, incomes of the various activities, π is the income associated with the solution χ.

Let the optimal plan of this problem be χ^0 and the resulting income is

$$\pi^0 = c'\chi^0. \quad (3)$$

(b) The suboptimal set. We now want to describe all possible plans (solutions) which produce income not smaller than $\lambda\pi^0$ where $0 \leq \lambda \leq 1$. For instance, if we set $\lambda = .95$ we find all plans which produce income not smaller than 95% of that obtained by the optimal plan. Formally, we want

$$S_\lambda = \{ \chi | \, c'\chi \geq \lambda\pi^0 = c'\chi^0 \, ; A\chi \leq b \, ; \chi \geq 0 \} \quad (4)$$

This set is given simply by all possible solutions of the system:

$$\begin{pmatrix} A \\ -c' \end{pmatrix} \chi \geq \begin{pmatrix} b \\ -\lambda\pi^o \end{pmatrix} \quad ; \chi \geq 0 \quad (5)$$

which consists of (2) and the additional constraint written in (4):

$$-c'\underline{\chi} \leq -\lambda\pi^0 \quad (6)$$

By varying λ parametrically we can generate the sets S_λ for different levels of λ.

The set S_λ is illustrated in Figure 1 for the case of two activities χ_1 and χ_2. The initial feasible set is given by OABCDE.

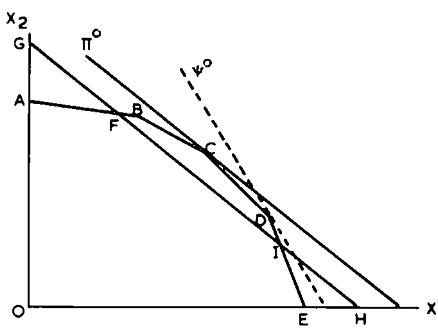

Figure 1

In terms of our notation it can be denoted as $S_{\lambda=0}$. Under the given prices, π is maximized at point C where it assumes the value π^0. The line GH represent $\pi = 0{\cdot}9\pi^0$. The suboptimal set $S_{\lambda=\cdot9}$ is given by FBCDI. Obviously this is a convex set. Let P_1 and P_2 be two activities in this set, then $P = \Theta P_1 + (1-\Theta)P_2$ for $0 \leq \Theta \leq 1$ is also in this set. Since P_1 and P_2 are in $S_{\lambda=0}$, so is P. So all we have to show is that $c'P \geq \cdot9\pi_0$. But $c'P = c'[\Theta P_1 + (1-\Theta)P_2]$ and since P_1 and P_2 are in $S_{\lambda=\cdot9}$, it follows that $c'P \geq \cdot9\pi_0$.

(c) Optimization according to a second objective function. Suppose now, that among all possible solutions in S_λ we want to select that χ which maximizes (or minimizes) a new objective function. Let the new objective function be

$$\psi = e'\chi \quad (7)$$

The linear programming problem is now

$$\max_{\chi} \psi \text{ subject to } (5)$$

The solution to this problem is illustrated in Figure 1 where ψ is maximized at point D.

(d) The two criteria frontier. By repeating parametrically on the procedure described in the previous section we can generate the efficient frontier of the two criteria. Given the frontier, the decision maker can now apply his subjective evaluation (utility function) for selecting the particular point on the frontier. For instance, for the indifference curves drawn in Figure 2, point W represents the optimum solution. To every point on the frontier corresponds a particular production program.

The resulting frontier is invariant to the order of the maximization. That is, we could start with maximizing (7) subject to (2) and then follow it by maximizing (1) subject to (2) and $-e'\chi \leqslant -\lambda\ \psi 0$ where $\psi 0$ is the value of (7) obtained in the first maximization. All we have to show in order to prove it is that an exterior point in the first problem is also exterior in the second problem and similarly for interior points. Referring to Figure 2, point N is an exterior one according to the first problem. Now suppose that it is not exterior to the second problem. Note that N and R are at the same level of π. Hence, N was not reached in the first problem not as a result of the income constraint, (6), but rather due to the resource constraint given by (2). Hence, it must also be an exterior point in the second problem. A similar argument holds for interior points such as T.

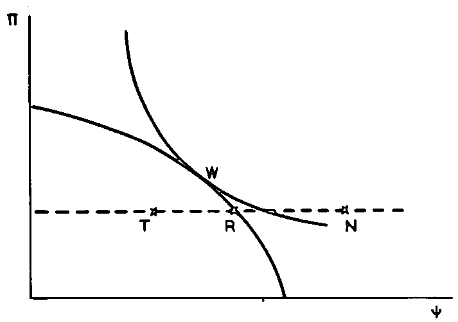

Figure 2

3. *Illustration.*

The method is applied in a short run planning of an individual (family) farm in Moshav in the southern end of the costal area of Israel. The activities and constraints are summarized in Table 1. In the first stage, the objective function is gross income. The optimal plan which maximizes this function is selected. In the second stage the objective function is the working capital.

Table 1

CONSTRAINTS				ACTIV											
												Silage feeding			Pulp feed-ing
Name	Units	Type	Quantity	Farm unit	Cow head	Oats 10 d.	Beet d.	Corn d.	Rhodes d.	Corn silage d.	Barley silage d.	IX-XII 00'f.u.	I-IV 00'f.u.	V-VIII 00'f.u.	ton
Working capital	IL	F			250	374	137.4	91.5	89.6	170.3	72.8				22.5
Gross income	IL	C			2605	-374	-137.4	-91.5	-89.6	-170.3	-72.8				-22.5
Labour	h.		1												
annual		L		-3480	162	214	79	26.5	70	14.5	7	.3	.3	.3	.75
IX-XII		L		-1390	54		30	14.0	15	14.5	5	.3			
I-IV		L		-1390	54	214		12.5	20				.3		.75
V-VIII		L		-1390	54		49		35					.3	
Farm units		E	1	1											
Food requirements	f.u.														
IX-XII		G			-2062			560	320			100			
I-IV		G			-2062	3500			40				100		153
V-VIII		G			-2062		1600		840					100	
Celulose	kg.														-
IX-XII		G			-319			252	192			50			
I-IV		G			-319	1580			24				50		
V-VIII		G			-319				504					50	
IX-XII		L			-471			252	192			50			
I-IV		L			-471	1580			24				50		
V-VIII		L			-471				504					50	
Silage	f.u.														
IX-XII		L			-504							100			
I-IV		L			-504								100		
V-VIII		L			-504									100	
Citrus pulp	f.u.	L			-527										1
Cotton seeds	ton	L			-.25										
Roughage	f.u.														
IX-XII		G			-480			560	320			100			
I-IV		G			-480	3500			40				100		
V-VIII		G			-480				840					100	
Silage store	f.u.	L								-642	-427	100	100	100	
Hay store	f.u.	L													
Beet rotation	d.	L				-8.0	4	-.4	-1	-.4	-.8				
Land	d.														
IX-XII		L	30			10	1	1	1		1				
I-IV		L	30			10	1		1		1				
V-VIII		L	30				1		1	1					
Water	000'c.m.														
annual		L	12.0			.50	.63	.55	1.06	.5					
peak period		L	7.2				.20	.38	.57	.1					
Existing citrus	d.	L	10												

T I E S

Hay feeding			Straw feeding			Concentrate feeding			Cotton-seed feeding			Hay pur-chasing	Exist-ing Citrus	New Citrus	Apple	Pear	Peach	Apri-cot	Pecan	Avo-cado
IX-XII	I-IV	V-VIII	IX-XII	I-IV	V-VII	IX-XII	I-IV	V-VIII	IX-XII	I-IV	V-VIII									
00'f.u.	00'f.u.	00'f.u.	00'f.u.	00'f.u.	00'f.u.	ton	ton	ton	ton	ton	ton	ton	d.	d.	d.	d.	d.	d.	d.	d.
2	3		60	60	60	292	292	292	237	237	237	160	150		400	330	320	270	200	215
-2	-3		-60	-60	-60	-292	-292	-292	-237	-237	-237	-160	428	-196	292	899	331	317	649	649
.1	.1	.1	.2	.2	.2	.2	.2	.2	.2	.2	.2		72		148	130	176	78	76	80
.1			.2			.2			.2				3		8	.9	10	4	60	30
	.1			.2			.2			.2			40		10	25	10	4	6	40
		.1			.2			.2			.2		29		130	96	156	70	10	10
100			222			910			1000											
	100			222			910			1000										
		100			222			910			1000									
75			350																	
	75			350																
		75			350															
75			350																	
	75			350																
		75			350															
									1	1	1									
100			222																	
	100			222																
		100			222															
100	100	100										-330								
														1	1	1	1	1	1	1
														1	1	1	1	1	1	1
														1	1	1	1	1	1	1
													.75		.60	.40	.50	.40	1.00	.80
													.36		.36	.24	.20	.20	.85	.36
													1	-1						

This function is to be minimized. Although the problem is used for purpose of illustration, it should be noted that the second objective function has some substantive meaning. A plan which produces the same amount of income with less working capital is less risky than one which requires more working capital. So, if a choice is available, it stands to reason that the low working capital plan will be selected. The question is what price in terms of income will the farm operator be willing to pay for reducing the amount of working capital to be tied up in his plan. The decision, of course, is left to the operator but the information necessary for such a choice is provided here.

Following the discussion of the model we generate the sets of suboptimal plans as functions of the parameter λ. For each of these sets we select the optimal plan which minimizes the amount of working capital. The results are reported in Table 2. The column headed 1 gives the optimal plan for the first objective function which in this case is gross income. Accordingly, the maximum achievable income for the present problem is IL 21,300. The plan includes 8.6 cows. The feeding scheme consists of 10.8 dunams of barley silage etc. In addition the plan contains 8.4 dunams of avocados.

The next column, headed 2, gives the plan which minimizes the second objective function, working capital, subject to the original resource constraints and the additional constraint that income will not be less than 96% of 21,300. The plan now consists of 5.2 cows etc. Similarly we can trace the remaining columns.

The first two rows in the table give the values of the two objective functions which are obtained as a result of changing λ. Those values are plotted in Figure 3. Since working capital is to be minimized, its axis in the Figure is reversed. Consequently we obtain a transformation curve with the usual shape. The farm operator can now select the optimum plan according to

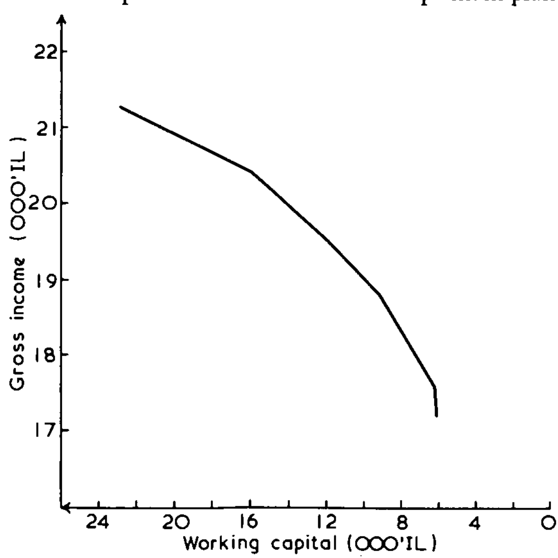

Figure 3 Two Objective Functions – Frontier.

Table 2: *Summary Results* (rounded figures) (*)

	No. of Plan / unit	1	2	3	4	5	6	7	8	9
Working capital	IL.	22,900	15,900	13,200	11,700	10,700	9,200	8,900	6,300	6,100
Gross income	IL.	21,300	20,400	19,800	19,400	19,200	18,800	18,700	17,600	17,200
Shadow price of gross income		7.2	4.3	4.3	4.2	3.9	2.8	2.4	1.6	0.4
Level of activities included in plans:		1	.96	.93	.90	.88	.88	.88	.83	.81
Farm units		1	1	1	1	1	1	1	1	1
Cows	head	8.6	5.2	3.9	3.1	2.5	1.8	1.7	0	0
Fodder crops:										
Oats	d.	0	0	0	0	0	2.5	5.0	0	0
Barley silage	d.	10.8	9.8	9.4	9.1	9.0	6.4	4.0	0	0
Feeding activities:										
Silage IX-XII	f.u.	2,280	1,390	1,020	1,540	1,280	910	850	0	0
I-IX	f.u.	2,280	1,390	1,950	1,540	1,280	910	0	0	0
V-VIII	f.u.	40	1,990	1,030	810	1,280	910	850	0	0
Citrus pulp I-IV	ton	29.6	18.1	13.3	10.5	8.7	6.2	5.8	0	0
Hay V-VIII	f.u.	2,900	0	0	0	0	0	0	0	0
Straw IX-XII	f.u.	830	510	370	190	160	110	110	0	0
I-IV	f.u.	830	510	240	190	160	0	0	0	0
V-VIII	f.u.	530	510	370	300	160	110	110	0	0
Concentrates IX-XII	ton	15.0	9.1	6.7	4.8	3.9	2.8	2.6	0	0
I-IV	ton	10.0	6.1	3.8	3.0	2.5	1.1	0.9	0	0
V-VIII	ton	12.6	7.7	5.7	4.5	3.3	2.3	2.2	0	0
Cotton seed V-VIII	ton	2.1	1.3	1.0	0.8	0.6	0.5	0.4	0	0
Hay purchase	ton	8.9	0	0	0	0	0	0	0	0
Pear	d.	8.4	10.4	11.3	11.8	12.1	12.5	12.6	13.6	13.7
Avocado	d.	10.8	9.8	9.4	9.1	9.0	8.6	8.4	8.2	5.1
Pecan	d.	0	0	0	0	0	0	0	0	2.5

Abbreviation used in Tables 1 & 2 : IL - Israeli Pound; h - man hours; f.u. - feed units; d - dunam = ¼ Acre; c.m. - cubic meter; 000' - one thousand.
For types of constraints: F - second objective function; C - first objective function; L - lower than; G - greater than; E - equality.

his subjective preference. Schematically, the solution can be shown by imposing on the Figure the indifference curves of the operator.

The values of the λ constraint are given in the fourth row. Those values were not actually preassigned but rather obtained by using parametric programing technique.[2] That is, every time the basis was changed, a value of λ was obtained. In fact, Table 2 does not report all solutions. In the computation, the computer was instructed to print only those plans which change the objective function by more than a preassigned value. The fact that plans 5–7 have the same value of λ is merely a result of rounding.

The third row in Table 2 gives the shadow price of the λ constraint given in terms of 1 IL. That is, a value of 7.2 in col. 1 means that a reduction of income by 1 IL reduces working capital by IL 7.2. As λ declines the shadow price declines as well. That is, the marginal rate of transformation of working capital for income declines as working capital increases, as expected.

4. *Implications for Extension Work*

In the foregoing discussion we have demonstrated how to obtain the efficient frontier of the two objective functions. The implications of this method are far reaching. It is of prime interest to note that a small change in λ may lead to substantial changes in the actural farm plan. For instance, a comparison of columns 1 and 2 of Table 2 indicates that the number of cows declines from 8.6 to 5.2. That is, a change of 4% in income reduces the size of the herd by 40%. The variations in the number of cows and in some other activities with λ are plotted in Figure 4.[3] Some more extreme changes occur, in the feeding activities. For instance hay feeding in V-VIII declines from 2900 f.u. to zero. This may not be an important change in the feeding scheme but it serves as an illustration for the possibility that a particular activity may completely vanish due to a small change in λ. To generalize this observation it is concluded that plans may be very sensitive to small changes in the objective function. Since no programming model takes into account all the considerations to which the farm operator gives weights, it is rather desirable to report programming results not as a plan but instead as a set of suboptimal solutions.

All this is important for extension work. For it has to be kept in mind that a recommended plan for a given situation can only be defended within the framework of the problem whereas the framework of application may be much wider in scope. Suppose that a particular farmer does not like to engage in the production of a particular crop and willing to do without it at some sacrifice in income; he may be maximizing his utility by doing without it. The suggested scheme tells him the resulting sacrifice in income. This is of course, only a simple case that could be handled directly by solving two linear pro-

2 The computations were performed by parametric programming on λ. An alternative is to define and aggregate objective function $\pi + \Theta\psi$ and to solve the system by parametric programming on Θ. Dorfman, R. Samuelson, P. A. and Solow, R. M. *Linear programming and Economic Analysis* McGraw Hill (1958) pp. 304-306.

3 The levels of activities are reported in Figure 4 as percentage of those obtained in the first optimal solution.

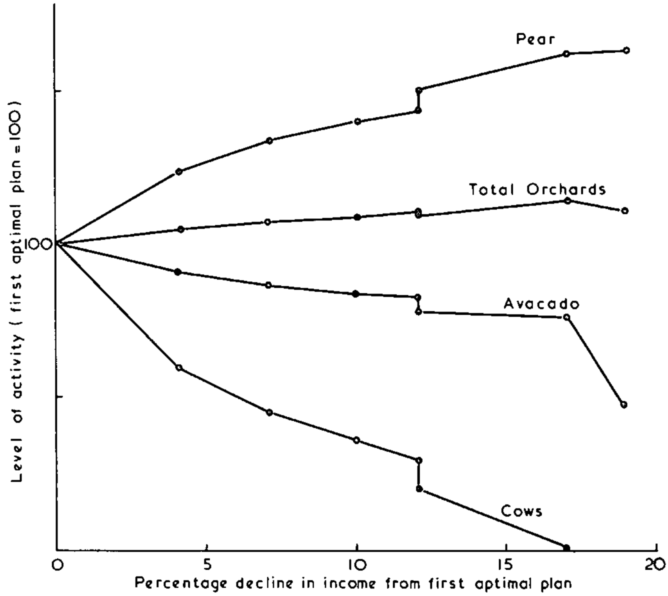

Figure 4

gramming problems with and without the particular crop. It is brought here only as an illustration for the type of considerations which may affect actual decisions. No one can, and in fact should, go into all such considerations. Instead, reporting the set of all suboptimal solutions provides the pertinent information that could be prepared for the decision maker so that he knows the price of any suboptimal choice.

5. *Implications for Empirical Analysis*

Theory tells us that firms operating under competitive conditions facing the same prices and technology should have the same equilibrium solution or simply should be of the same size and composition. Agriculture should be a good industry for examining empirically this proposition. But it is well known that farms, even in a given region which is relatively homogeneous, differ and sometimes widely, in terms of their production plan. Such a spread is accounted for in various ways which are certainly relevant: (1) Differences in management. (2) Differences in resources which are considered fixed in the short run. (3) Prices, after all, are not exactly the same. Nevertheless, those factors still leave some more to be explained. Differences in management are likely to contribute more to differences in scale rather than to differences in composition. To some extent this is also true for differences in resources. The fixed resources, such as land, water, equipment can be used for producing a whole range of products and as such do not determine composition in a unique fashion. Prices may be important in differentiating among products. Yet empirical analysis of behavioral functions, such as product supply or factor demand based on cross-section data often fail to detect the role of prices. The price variables in such analysis are either not significant or at best explain only a small proportion of the total variance.

Whatever is the explanatory power of the forementioned factors, we can now add another one which bears directly on the question of spread in the cross-section. This is simply a recognition of the fact that there are other considerations in determining the optimal plan. This by itself is not novel and in fact it is a standard excuse when the economic variables fail to do the work. What, however, remains for elaboration is to indicate the scope for this explanation.[4] Doing this, we can then also attempt to quantify in terms of any particular problem the admissible spread. We examine this effect for a particular firm. For illustration we reproduce parts of Figure 1 in Figure 5. We note that for $\lambda = 0.9$, the admissible range of x_i is $[\chi_i(.), \bar{x}_i\ i()]$ where $x_i\ (.)\ \text{-}\ x_i\ (\lambda = 0.9)$

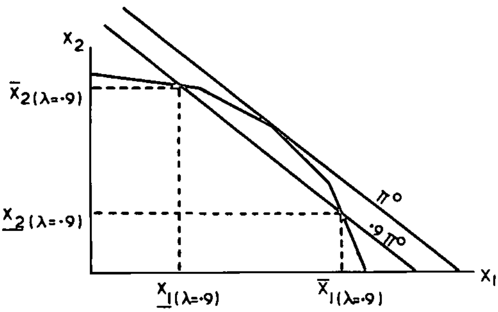

Figure 5

consequently, we don't have a supply function but rather a correspondence.[5] We can express the supply correspondence for the i-th product as

(8) $x_i = S^i\,(p, b, \lambda)$

In the conventional formulation $\lambda = 1$ and we then refer to (8) as the supply function. The generalization of (8) is in that λ is not necessarily restricted to 1. Strictly speaking, (8) is also a correspondence when $\lambda = 1$. For if the profit line π_0 had the slope of say the segment BC in Figure 1 then again there would be multiple solutions. So the main effect of admitting values $\lambda \neq 1$ is to destroy the uniqueness of the solution whenever it exists under $\lambda = 1$.

A selection of a particular plan in the suboptimal set requires a second objective function and perhaps there is more than one. Consequently, even if all producers have the same prices, p, resources b, and the *same* λ, their outputs are likely to differ. This explains why there are wide variations in

4 See also Mundlak, Y. 'Cross-Section Analysis' in the *International Encyclopedia of the Social Sciences*, MacMillan (1968)3:pp. 522-526.

5 For the definition and properties of correspondence, see Debreu, G. *Theory of Value*, Wiley (1959).

outputs of different products among farms which operate under similar technology, prices and resources. When one attempts to estimate a supply function for a group of such farms with prices and resources being the explanatory variables it often occurs that prices appear to be relatively unimportant. That is, their partial correlation coefficients with outputs are low and often statistically non-significant. The situation is different in time-series analysis and particularly so when the unit of observation is an aggregate over individuals. The possible variations in the objective functions to be taken into consideration and the weights given to them are much smaller for two points in time than for two individuals. It is for this reason that the economic variables and particularly prices are more prominent in time series analysis.

Comparability of Positivistic and Normative Supply Elasticities for Agricultural Commodities

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U.S.A.

Supply elasticity, defined as the percentage change in output of a commodity associated with a one per cent change in the commodity price, is an important parameter for public policy. In developed countries, for example, it can show how much additional, perhaps superfluous, output is generated by a government price support policy in the absence of production controls. In developing countries, it can indicate the degree to which producers act as 'economic men' in response to higher output prices, and can give planners a basis for setting output prices to meet production targets. In general, the supply elasticity measures ability of producers to adjust production to changing economic conditions continually confronting them in a dynamic economy. Elasticities are used *historically* to explain what has happened, but are more frequently used to *predict* the response at some point in the future of farm output to a price change.

Two basic approaches are used to estimate supply elasticities. Traditionally, economists estimated supply parameters using least squares techniques and time series data on output, prices, and time or technology. Because the approach is a somewhat objective means to find an average relationship between past output and price, and to quantify 'what is' the price-quantity relationship without necessarily saying why the particular relationship exists, we call this the *positivistic* approach.

In the 1950's, farm management specialists began to compute whole-farm

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plans using linear programming (LP) techniques. The plans showed the optimal organization of the farm to maximize net income, subject to constraints or conditions affecting prices, resource availability and production processes. By systematically varying prices and computing the optimal farming plan at each price, a supply curve for a commodity could be generated for a particular farm. Economists soon reasoned that by carefully choosing a farm to represent a homogenous area, and by handling heterogenous conditions by setting up a representative farm for each different resource situation, a few farms could depict virtually the entire national production of a commodity such as cotton.

A number of regional LP studies were made in the late 1950's and 1960's. One objective was to generate commodity supply schedules. Programming studies are generally based on some variant of the goal of maximizing net income. While this does not carry the normative imperative of 'what ought to be', it is based on the norm of 'what would be' if producers followed the profit norm. So we say the technique is *conditionally normative.*

Time series or other data are often, especially in developing countries, unavailable or too crude to make positivistic estimates of supply response. But supply response data may be available from linear programming models used in farm management studies; and supply response estimates are a low cost complement or by-product of farm management analysis. Not only are LP estimates of supply from representative farms a low cost data source, but the technique also allows flexibility in exploring the implications of price changes and other policy variables not experienced in the past–hence not available from positivistic models.

Will the conditionally normative estimates from LP models provide estimates useful for public policy of producers' actual supply response? This report helps answer this question. Our procedure is to compare normative estimates of supply elasticities with positivistic, least squares estimates, the latter presumably a measure of producers' actual supply response. Supply elasticities are examined for cotton, wheat, feed grain, hog and beef production.

Supply Elasticities for Cotton

Positivistic estimates of cotton supply elasticity are taken from studies published from 1959 to 1966 (Table 1). Data from earlier years were frequently used in the analysis because government programs obscured the positivistic price-quantity relationship in recent years.

Most estimates are for acreage rather than for production. If acreage does not influence yield, then the elasticity of supply or output is the simple sum of the elasticities with respect to acreage and yield [cf. 12, Table 3]. The yield elasticity is low, approximately .1 in the short run, but may be over unity in the long run [cf. 12]. Thus, based on data in Table 1, the supply elasticity of cotton output in the U.S. with respect to cotton price is actually .3 to .4 in the short run (1–2 years) and more than one in the long run.

The elasticity of cotton acreage in the world, excluding the U.S. and

TABLE 1 *Selected Positivistic and Normative Estimates of the Price Elasticity of Cotton Supply*

Source	Data	Elasticity	Length of Run
Positivistic Estimates (Annual Time Series)			
Cromarty, 1959 [6]	U. S., 1929–53	.36 (output)	Not stated
Nerlove, 1958 [8]	U. S., 1909–32	.27 (acreage)	Short run
		.67 (acreage)	Long run
Blakley, 1962 [3]	U. S., 1921–32	.27 (acreage)	Short run
		.41 (acreage)	Long run
Cathcart and Donald, 1966 [4]	World, except U. S. and communist countries, 1948–63	.20 (acreage)	Not stated
Normative Estimates			*Cotton Price, Lint* (c/lb.)
S–42, 1966 [1]	Representative farms, U. S., South, West linear programming	5.05 (output)	15–20
		8.76	20–21
		3.28	21–22
		2.32	22–23
		4.42	23–24
		2.23	24–25
		.93	25–30
		.12	30–35

Communist countries, is .20 according to an estimate by Cathcart and Donald [4]. While they did not label the length of run, it probably applies to a period of 1–4 years. If the yield elasticity is .1, then the 'world' cotton supply elasticity is approximately .3 in this length of run.

It is apparent from Table 1 that normative cotton supply elasticities generally overestimate the actual response of farmers to price changes. This is especially apparent at low cotton prices. The U.S. cotton price is currently approximately 20 cents per pound, but the supply elasticity of 8.76 vastly overestimates farmers actual response to price change around that price.

It is notable, however, that the elasticity of supply is near unity in the 25–30 cent price range, a very typical range of actual prices during the period used to estimate the positivistic supply elasticities. The positivistic long-run elasticities are near unitary and appear to be somewhat in line with the normative elasticities in the upper price range. Because LP solutions are timeless and assume instantaneous adjustments that in fact would take years to accomplish, it is not surprising that the normative estimates are at least remotely in line with the long-run, but not with the short-run, positivistic elasticity estimates.

The actual level of production predicted at the various price levels by the normative study is not of much interest, because researchers normalized the base quantity supplied at a level that coincided with actual current production.

Supply Elasticities for Wheat

We shall assume for wheat as for cotton that the price elasticity of yield with respect to price is .1 and that the supply (output) elasticity is the sum of the acreage and yield elasticities. The positivistic estimate of the supply elasticity for wheat appears to be .5 in the short run of 1–2 years and approximately unity in the long-run. Foreign supply appears to be more inelastic according to estimates by Tweeten [11].

As with cotton, normative elasticities are much higher than positivistic estimates at low product prices (Table 2). But at wheat prices of $1.25 to $1.75 per bushel, a fairly typical price range in historic perspective, the normative elasticity ranges from .85 to .96. Again there is some similarity between long-run positivistic estimates for the U.S. and the normative estimates. Given enough time to adjust to changing prices, wheat farmers appear to make the adjustment the income maximization norm suggests they should make.

TABLE 2 *Selected Positivistic and Normative Estimates of the Price Elasticity of Wheat Supply*

Source	Data	Elasticity	Length of Run
Positivistic Estimates (Annual Time Series)			
Cromarty, 1959 [6]	U. S., 1929–53	.27 (output)	Short run
Nerlove, 1958 [8]	U. S., 1909–32	.48 (acreage)	Short run
		.93 (acreage)	Long run
Tweeten, 1965 [11]	World, except U. S. and communist countries, 1901–38	.14 (output)	Short run
		.28 (output)	Long run
Normative Estimates			*Wheat Price* ($/bu.)
GP–5, W–54 [2]	Representative farms, U. S. Great Plains and Northwest, linear programming	3.78 (output)	1.00–1.25
		.96	1.25–1.50
		.85	1.50–1.75
		.39	1.75–2.00
		.12	2.00–2.25
		.06	2.25–2.50
		.02	2.50–3.00

Supply Elasticities for the Feed Grain-Livestock Sector

Feed grain and livestock sectors of an agricultural economy involve complex technical and economic interrelationships. Pork and beef are substitutes both in production and consumption. Corn and other feed grains are sold as products or used as inputs in livestock production, but at extreme relative prices can become substitutes in production for livestock.[1] In the United States, most efforts to unravel the economics of supply in the feed grain-livestock sector have been econometric analysis of varying complexity and involving time series. Marc Nerlove's classic treatment of supply response

1 Feed grain production fluctuates because of weather, and livestock production moves up and down the pig, hog, beef and other cycles.

via distributed lags, published in 1958, gave considerable attention to the U.S. feed grain-livestock sector [8]. William Cromarty singled out the feed grain-livestock sector for special attention in his 1957 study of the structure of agriculture [6]. More recent time series analysis of the U.S. feed grain-livestock sector were conducted by Petit at Michigan [9] and Van de Wetering at Iowa [12]. And Harlow in the USDA completed an extensive study of factors affecting the price and supply of hogs in 1962 [7].

But during the 1960's, the supply response in the feed grain-livestock sector was analysed with linear programming. The major effort was under regional research project NC-54, Supply Response and Adjustments for Hog and Beef Cattle Production in the Corn Belt, initiated July 1, 1961 [5].

Estimates of supply elasticities for feed grain, pork, and beef cattle from the above studies are summarized in Table 3. Positivistic estimates in the short run fall in the .10 to .36 range for feed grain; in the .04 to .82 range for pork; and in the .04 to .12 range for beef. Nerlove estimated a long-run supply elasticity of .18 for feed grains. Petit found a .48 intermediate-run supply elasticity for pork and a .34 intermediate-run supply elasticity for beef.

Since livestock can be sold immediately or fed to heavier weights, we might expect livestock, especially pork, to show a greater response to price than feed grain. Estimates from both Cromarty and Petit confirm this. It is interesting to note that Nerlove, studying the 1909–32 period and Petit, studying the 1939–62 period, essentially agree that the short-run supply elasticity of feed grain is .1–indicating that the elasticity not only was low but did not increase over time.

All the positivistic supply elasticity estimates summarized in Table 3 for feed grain, pork, and beef indicate a low but decidedly positive supply response. And if we eliminate the .82 supply elasticity found by Harlow using spring farrowing as the dependent variable, the positivistic estimates of short-run supply elasticities fall in the .10 to .36 range, certainly not a wide range in total perspective.

In contrast to the positivistic estimates, the normative estimates in Table 3 not only are widely dispersed, but those for feed grain are negative. As the commodity price is increased from low to medium to high levels, the normative supply elasticities change from -.8 to -1.4 for feed grain, 5.2 to .9 for pork, and 7.7 to 4.0 for beef. The negative elasticities for feed grain are not adequately explained, and may arise from inadequate specification of the model.

We would expect the normative estimates, being long-run, to be larger than the short-run and intermediate-run positivistic estimates. But the very large supply response indicated by the normative elasticities are unreasonable. The NC-54 researchers concluded that the quantities of livestock indicated were so large that they clearly could not be absorbed by U.S. consumers at any reasonable price levels. For example, hog production for the Region at most price levels would be several times the annual average output for the U.S. in the early 1960's [5, p.62]. The NC-54 researchers accordingly adjusted their supply response estimates downward to more reasonable levels. In recent

TABLE 3 *Selected Positivistic and Normative Estimates of the Price Elasticities of Feed Grain, Hog and Beef Cattle Supply*

Source	Data	Elasticity	Length of Run
Positivistic Estimates (Annual Time Series)			
Feed Grain			
Petit, 1965 [9]	U. S., 1929–62	.11 (output)	Short run
Cromarty, 1959 [6]	U. S., 1929–53	.36 (output)	Short run
Nerlove, 1958 [8]	U. S., 1909–32	.10 (acreage)	Short run
		.18 (acreage)	Long run
Pork			
Van de Wetering, 1964 [13]	U. S., unavailable	.044 (output)	Short run
Petit, 1965 [9]	U. S., 1929–62	.32 (output)	Short run
		.48 (output)	Intermediate run
Cromarty, 1959 [6]	U. S., 1929–53	.13 (output)	Short run
Harlow, 1962 [7]	U. S., 1949–60	.82 (spring farrowings)	Short run
Beef			
Van de Wetering, 1964 [13]	U. S., unavailable	.07 (output)	Short run
Petit, 1965 [9]	U. S., 1929–62	.12 (output)	Short run
		.34 (output)	Intermediate run
Cromarty, 1959 [6]	U. S., 1929–53	.037 (output)	Short run
Normative Estimates (North Central U. S.)			*Price*
Feed Grain			
NC–54, 1967 [5]	Aggregated from LP models of representative farms	–.84	Low to medium corn prices[1]
		–1.42	Medium to high corn prices[1]
Pork			
NC–54, 1967 [5]	Aggregated from LP models of representative farms	5.22	Low to medium pork prices[2]
		.90	Medium to high pork prices[2]
Beef			
NC–54, 1967 [5]	Aggregated from LP models of representative farms	7.70	Low to medium beef prices[3]
		5.05	Medium to high beef prices[3]

[1] Beef and hog prices held fixed at medium levels.

[2] Beef and corn prices held fixed at medium levels.

[3] Pork and corn prices held fixed at medium levels.

normative LP models of representative farms used to determine aggregate supply response, such as the national model constructed by the U.S. Department of Agriculture, flexibility constraints are imposed to keep aggregate adjustment in proper perspective [10].

Summary and Conclusions

Much can be learned about the behaviour of producers both from positivistic supply elasticities generated by least squares and from the conditionally normative supply elasticities generated by linear programming. The LP supply curves rise steeply at very low prices because the commodity is not profitable. With higher prices, this inelastic portion of the curve gives way to a somewhat horizontal, elastic portion as the commodity becomes competitive with other crops and occupies more acreage. With even higher prices, the supply curve becomes more steeply sloped and inelastic—the acreage suited for it has been used and marginal inputs experience sharply diminishing return. This phenomenon, apparent in LP supply curves, suggests that the elasticity varies measurably over the curve, yet most positivistic curves are characterized by a *constant* slope or elasticity. The observations in time series give rise to a short segment of the supply curve that can be approximated by a straight line. But unless the 'inverted lazy S' of normative estimates averages out over many producers to nearly a straight line, the constant slope or elasticity coefficient in least squares estimates should be interpreted with caution, especially when examining the impact of policies that fall outside the range of experience reflected in past data. Furthermore, none of the supply estimates in this study account for the irreversibility of the supply curve arising from asset fixity.

The LP models provide somewhat realistic long-term elasticity estimates for commodities characterized by well defined resource restraints and cash markets. The model predicts fairly well for wheat, less well for cotton, and poorly for the feed grain-livestock economy. Wheat is heavily dependent on land resources and traditionally has been sold for cash as a food grain, so its economic structure is comparatively simple and predictable. On the other extreme, the livestock economy is not so clearly restrained by land resources and its complex economic structure allows numerous substitutions in production and marketing. An LP model is unlikely to depict all the necessary restraints and substitutions. The LP models are more realistic for showing regional shares of production (based on comparative advantage) than for showing the absolute level and elasticity of supply.

Positivistic estimates have certain advantages. Where available and where the structure of economy has not markedly changed, they almost surely provide a more realistic prediction of supply response than do normative estimates. The data in this paper suggest, however, that linear programming estimates can sometimes be used to gauge the long-term impact of price changes on the output of a commodity. Where flexibility, timeliness and the need to consider policies outside the range of historic experience are important, linear programming appears to be of some value for analyzing the impact of public policies.

Where LP estimates are highly suspect, especially where absolute as well as relative production potentials are important, and predictions of short-run response are desired, one solution might be to use positivistic estimates of flexibility constraints in representative farm LP models at the local and regional level. This approach is similar to the procedure used by the U.S.

Department of Agriculture in its recursive programming model of the U.S. farm economy. It is too early to judge the results. It is hoped that introduction of positivistic flexibility restraints into the normative LP models will combine the *best* rather than the *worst* features of the two basic approaches.

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SPECIAL GROUP I REPORT

(A) Paper by D. A. Fitchett (Argentina) 'Price responsiveness of Latin American farmers'.

There is a difficulty in defining an appropriate price deflator for each of the crops, particularly for wheat. This is connected with the geographical distribution of the crops and the fact that substantial sectors of the crops are irrigated, and to treat each as a homogeneous entity is rather unrealistic. Likewise, it is somewhat simplistic to hope that the interdependencies between crops can be adequately captured through focussing only on prices. In such work there are always imperfections and inadequacies in the data available, but it is nevertheless important to try to build a sound base for policy decisions. The present study was primarily directed to measuring price responsiveness and accounting for the role of risk in this, and it is believed

fair progress was made. No problems of multicollinearity were encountered in the regression analyses.
A. Valdes *Chile* and Y. Maruyama *Japan* participated in the discussion.

(B) Paper by Y. Mundlak (Israel) 'Maximisation with several objective functions'.

The author was gratified to lean that the approaches similar to that developed in the paper have been used elsewhere in the world – particularly for national planning models in Sweden and Hungary. For centrally planned economies there is the possiblility of varying prices porometrically to trace out frontiers of 'efficient' plans but this approach raises some further difficulties. In the present method, the frontier is a set function of the assumed prices.

It was agreed that farmers' constraints and possibly goals can be explicity incorporated within the planning matrix. The essence of the presented approach was to avoid as much as possible the necessity to define individual preference or utility functions. Presentation of a (possibly reduced) set of efficient form plans (or range of solutions) was suggested as being a superior and more workable procedure in practice. Thus there are some conceptual similarities between the presented approach and the Monte Carlo programming work of groups in Sweden and elsewhere. However, some concern was expressed about the difficulty of presenting and comprehending sets of plans where there are more than two dimensions to the criterion or objective functions. Another aspect of this problem is where (as in a collective form) there are several or many decision makers involved, the computational burden of accounting for the preferences of all these people may become very heavy.
S. S. Johl *India*, M. Carleson *Sweden*, D. A. Fitchett *Argentina*, K. T. Khan *Thailand*, J. Sebestyen *Hungary* and L. J. A. Folkesson *Sweden* took part in the discussion.

(C) Paper by L. Quance and L. G. Tweenten (U.S.A.) 'Positivistic and normative supply elasticities'.

The present experience corresponded closely with some Indonesian work. The discussion centered around the appropriatness of the normative and positivistic estimates under various circumstances. Since the positivistic estimates are based on historical price fluctuations they can relate to only a very restricted segment of the long-run supply curve. For short-run predictions the positivistic estimates will probably prove most reliable. However, in the long run where previously unexperienced prices may be met, the normative estimates will be more relevant.

Some experiences of the U.S.S.R. Central Planning Branch were related – particularly concerning the rational distribution of factors of production, such as fertilizer, between the producing units. Problems of choosing and interpolating between alternative criteria, such as maximising output and minimising the land inputs, were discussed.
A. Birowa *Indonesia* and V. Miloserolov *U.S.S.R.* participated in the discussion.