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POLICIES, PLANNING
AND MANAGEMENT
FOR AGRICULTURAL DEVELOPMENT

PAPERS AND REPORTS

FOURTEENTH
INTERNATIONAL CONFERENCE
OF AGRICULTURAL ECONOMISTS

*Held at the Byelorussian
State University, Minsk, U.S.S.R.*

AUGUST 23rd—SEPTEMBER 2nd 1970

OXFORD
INSTITUTE OF AGRARIAN AFFAIRS
FOR
INTERNATIONAL ASSOCIATION OF AGRICULTURAL
ECONOMISTS
1971

SPECIAL GROUP F

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The Mathematics of the Economy of the Individual Farm Business

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We, as applied economists, are interested in both mathematical models of economic relationships and mathematical tools of analysis.

In recent years, a large number of different types of mathematical models and associated solution procedures have become available for solving individual firm problems. This mushrooming of mathematical models and techniques has made it difficult to keep abreast of developments in terms of technical knowledge regarding the use of these decision-aiding techniques. The task of staying abreast of developments was made even more difficult by the apparent lack of an underlying common theoretical and/or mathematical structure. It is, therefore, fitting to examine the development, generality, and relevance of the mathematics of the individual farm business with a view toward the recent developments of mathematical models for use by applied economists.

This examination of the mathematics of the individual farm business will proceed in four major steps: First, the differences in emphasis between a market-oriented and a management-oriented theory of the firm will be established. Second, firm theory and models and the associated mathematics will be discussed. Third, various categories of operational subsystems will be delineated. Finally, the paper will end with conclusions and challenges for the profession of applied economists.

II. Market vs. Management Orientation

The agricultural economist's interest in the mathematics of the economy of the farm firm arises either from his task of explaining macro relationships of agriculture (e.g. policy operations), or his occupation with management of individual farm businesses. While this paper is concerned with the latter, it will first be shown that relevant theories (and associated mathematics) can (and in general will) differ, depending on which of the above orientations prevails.

Theory of the firm as we teach it to our students today dates back to Ricardo's discussion of the problems of English agriculture a century and a half ago. A century of development and refinement of this theory culminated

in the formulation of the modern theory of the firm as expressed in Alfred Marshall's work. The latter work was further refined by Chamberlain, Robinson and Viner in the 1930's, but 'the marginalist analysis of the behavior of the firm and the allocation of productive resources has enjoyed practically complete acceptance for the past seventy-five years, so that it is fair to say that in spite of more than a century of refinement, the analysis of production to this very day is based upon the original Ricardian-Malthusian concepts.¹

This theory suited Ricardo's purpose well, since his analysis was one of long-run economic equilibrium. Thus, the theory of the firm was designed to help explain the behaviour of the *market* (in the long run) and not how a firm is to be organized and managed in the short run. This market-oriented theory of the firm was axiomatized (by Marshall and his followers) with the differential calculus. This is still the theory which is first and foremost taught as useful for applied economists.

While the theory of the firm was developed for purposes of explaining market behavior, attempts were made to use it to explain the forces which govern an entrepreneur's decisions, both in the short and in the long run. In the early fifties, a reaction to this refind application of the marginal approach developed. This reaction was fueled by two forces: First, empirical studies became available which showed that business decision making is based on considerations quite different from the concepts of marginal analysis and perhaps inconsistent with it. Second, developments of the technique of linear programming offered the possibility for an improved axiomatized theory. Its teaching and numerous applications in every corner of the globe are well known and require no further elaboration. What does deserve mention, however, are those aspects in which the theory which is axiomatized by linear programming is believed to be superior to that theory which is axiomatized by differential calculus. These are as follows:

- (a) Fixed factors of production are considered explicitly;
- (b) The production function does not have to be continuously differentiable;
- (c) The need for simultaneous variation of complementary factors of production is recognized, and
- (d) Several different techniques of production can be employed simultaneously.

In spite of its advantages over differential calculus, criticism was leveled at the linear programming approach. This criticism² essentially states that a theory of the firm as axiomatized by linear programming is still basically a market-oriented theory, a theory which is appropriate for a different set of questions, than one which has a 'micro' or management orientation. To be appropriate for the latter, a theory needs to recognize that

1 Dorfman, R., *Application of Linear Programming to the Theory of the Firm*, Berkeley, 1951, p.2.

2 For a fine discussion of this question see Cyert, R. M. and March, J. G., *A Behavioral Theory of the Firm*, Englewood Cliffs, N. J., 1963.

- (a) firms operate in an environment which is characterized by imperfect knowledge;
- (b) a firm is not solely concerned with profits, but recognizes a multi-dimensional objective;
- (c) firms may not maximize but satisfy;
- (d) firms are dynamic organizations (with social and psychological aspects) which change over time (this is contrary to the market-oriented theory of the firm where the structure of the firm stayed the same and market conditions change).

Theories which incorporate the above characteristics have come to be known as 'behavioral theories'.³

While any brief presentation of theories and their development is likely to suffer from distortion dictated by condensation, we felt such a brief presentation necessary in order to ascertain the current state of consensus (or disagreement) and to provide a basis for the orientation of the following discussion.

III. Models and Mathematics of the Economy of the Individual Farm Business⁴

Two points need to be made at the very outset:

- (a) What we shall say here as regards the mathematics of the farm business applies equally well to any business; and
- (b) we are not preparing to present a 'universal mathematics' for the farm business. Indeed, the main point of this section will be to show that this is infeasible.

The agricultural economist's desire for a 'mathematics of the business' stems from his wish to have a chain of rules of manipulation, a *calculus*, which—when appropriately applied—will result in optimal management of the business. The chain of rules of manipulation must (or should) be consistent with a theory, which, in turn, must (or should) be isomorphic to the reality being modeled. The question then is: What does the mathematics of the business (the chain of rules of manipulation, the calculus) which contains these properties look like?

Following Hart⁵ we shall accept that a (business) firm can be viewed as an organization with a production space of finite size at any point in time. Over time, the size of this space may change. These changes in size may occur through either external changes in technical or marketing relationships, or, following Penrose,⁶ through internal changes brought on by either correct or

3 c.f. Cyert, R. M. and March, J. G., op. cit.

4 This section is heavily influenced by Lee, G. E., *Exploitation of Information for Capital Formation*, Ph.D. dissertation, Purdue University, June 1970; and Menges, G., *Vorentscheidungen*, in *Operations Research Verfahren* (R. Henn, ed.), Anton Hain, Meisenheim, 1965.

5 Hart, A. G., *Anticipations, Uncertainty and Dynamic Planning*, Augustus M. Kelley, New York, 1951.

6 Penrose, E., *The Theory of Growth of the Firm*, New York 1959.

incorrect entrepreneurial moves in the history of the firm. This universe of reality of the firm can mathematically be expressed as:⁷

$$b_{iugt} \cdot s_{iugt} c_{iugt} = m(a_{iut} \cdot s_{iut} \cdot d_{iut}) \quad (1)$$

Where

- (b:beB) = outcomes of actions taken
- (s:seS) = possible states of nature
- (c:ceC) = unique internal structure of the firm brought forth by previous decisions and states of nature
- (a:aeA) = actions open to the firm
- (d:deD) = behavioral characteristics of the entrepreneur, and where the subscripts⁸
- (i:ieI) = items in a set
- (l:leL) = level of item
- (g:geG) = level of desirability
- (t:teT) = time period

Equation (1) expresses the universe of reality for a business firm in symbolic form. This is not meant to imply that the universe is known, but it does imply that it exists, and it is further accepted that attempts are made to know more about it. This is accomplished by constructing a model (or models) which parallel(s) the structure of reality. Conceptually this is easily done, namely

$$\beta_{i\lambda\gamma\tau} \cdot \sigma_{i\lambda\gamma\tau} \cdot \psi_{i\lambda\gamma\tau} = \mu(\alpha_{i\gamma\tau} \cdot \sigma_{i\gamma\tau} \cdot \delta_{i\gamma\tau}) \quad (2)$$

where the variables correspond to those in equation (1), except that the variables in (2) now need to be linked not only to the universe of reality but also to observable phenomena. Thus, if the sets α , σ , δ , β , and ψ were completely specified as well as the relationship, μ , between them complete prediction would be possible.

If fact, such complete specification is not possible if we wish to have an operational calculus for the (farm) business.⁹ The only realistic alternative, then, is reduction of the decision space. This requires a sequence of complicated decisions,¹⁰ which are occasionally arbitrary and subjective, which are not amenable to treatment by a chain of rules of manipulation, and cannot efficiently be made by a computer. These predecisions eventually lead to operational subsystems. But they also force us to accept more than one 'mathematics of the business.' Which of these 'mathematics of the business' is appropriate depends on the type of problem and circumstances. How these various 'mathematics of the business' relate to the universe of reality in (1)

7 Notation and mathematical expressions are those by Lee, op. cit. A different notation and a similar mathematical expression are found in Menges, op. cit.

8 The same subscripts do not need to range over the same values for all variables.

9 c.f. both Lee, op. cit., and Menges, op. cit.

10 Referred to as 'predecisions' by Menges, op. cit.

i.e., how they represent various circumstances, will be discussed for each of several operational subsystems.

IV. Operational Subsystems

The operational subsystems discussed here are the differential calculus, mathematical programming, and simulation.¹¹

1. *Differential Calculus*: The classical model of the firm under pure and perfect competition—as we teach it to students of agricultural economics today—relies heavily on differential calculus. To make the differential calculus applicable, the decision space of equation (2) is drastically reduced by making predecisions regarding time span, entrepreneurial knowledge, and goal dimensions. These predecisions reduce (2) to

$$\beta_i\lambda = \mu(\alpha_i\lambda) \quad (3)$$

Also, assumptions are made regarding μ . One of these assumptions is that the relationship between α and β can be deduced from the law of variable propositions and that the rational producer will not select alternatives falling below the surface so defined. Further, the assumption of continuous differentiability between α and β is made. Given this reduction in the decision space and if the above assumptions hold, differential calculus will permit the ascertainment of the one and only set of optimal values of the variables.

Differential calculus is easily understood and used, and numerous applications can be cited.¹² But the operational models are simple and generally more 'market oriented' than 'management oriented.' Large (and more isomorphic) problems become impossible to answer.

2. *Mathematical Programming*: Conceptually the major difference between differential calculus models and mathematical programming models lies in the assumptions regarding μ . In mathematical programming models the relationship between the variables is assumed to be linear and additive, variable levels are assumed to be continuous, and the number of variables is assumed to be finite. Symbolically the model may be written as

$$\text{Maximize } \beta_i\lambda, \quad (4)$$

$$\text{subject to } \beta_i\lambda = \mu(\alpha_i\lambda) \quad (5)$$

The 'chain of rules of manipulation' in mathematical programming utilize simplicial decompositions and matrix algebra. The decision space so analyzed

11 Major techniques, such as dynamic programming, are omitted deliberately, since it is believed that the operational subsystems considered fully demonstrate the point to be made here.

12 c.f. Heady, E. O., 'Data Needs and Interdisciplinary Cooperation' *Computer Use in Farm Management Analyses and Production Decisions (Proceedings)*, USDA, Washington, D.C. 1968.

can have substantial dimensions, and numerous applications are available and generally known.¹³

The standard mathematical programming model can be expanded (towards the conceptual model in (2) above) in a variety of ways. For instance, multi-period models are possible and can be expressed as follows:

$$\text{Maximize } \beta_{i\lambda\tau} \quad (6)$$

$$\text{Subject to } \beta_{i\lambda\tau} \cdot \psi_{i\lambda\tau} = \mu(d_{i\lambda\tau}) \quad (7)$$

where

$$\psi_{i\lambda\tau} = \alpha_{i\lambda} (\tau + 1) \quad (8)$$

Various approaches have been developed to alleviate the onus of the assumptions of linearity, continuity, and one dimensional objective function (i.e. $\beta_{i\lambda} \rightarrow \beta_{i\lambda\tau}$). However, the chains of the rules of manipulation for models of this type are currently only feasible for the exploration of decision spaces possessing small and few dimensions.

3. *Simulation*: The 'calculus of simulation' is not dependent on particular conditions (differentiability, continuity, linearity, etc.) within the decision space. As far as the structure of the decision space is concerned, the structure of a 'simulation model' can, therefore, approach that of equation (2).

The calculus of simulation relies upon experimental techniques. The best known of these is the Monte Carlo technique, which attempts to find the optimal point in the decision space by sampling the decision space. This sampling can be entirely random, but variance reduction techniques, sequential sampling schemes, and learning devices of some sort are generally used in order to reduce the domain of search on some or all the variables.

Application of the calculus of simulation to farm business problems are of more recent vintage, but substantial experience has accumulated.¹⁴ Results are highly promising. More research is required to assess the trade-off between model isomorphism, cost of model building, cost of solution, and value of increased model solution precision.

13 For comprehensive treatises in this area see Heady, E. O. and Candler, W., *Linear Programming Methods* Ames, 1960, and Reisch, E., *Die lineare Programmierung in der landwirtschaftlichen Betriebswirtschaft*, BLV, Muenchen 1962.

14 Ambrosious, J. E., *Optimal Tractor and Combine Replacement Policies on Corn Farms under Alternative Firm Growth Patterns*, Ph.D. dissertation, Purdue University, January 1970; Carlson, M. Hormark, B. and Lindgren, Inger, *Studies of Farm Planning Problems by a Monte Carlo Method*, Lantbrukshogskolan, Institutionen for ekonomisk statistik, Uppsala; Donaldson, G. F., and Webster, J. P. G., *An Operating Procedure for Simulation Farm Planning—Monte Carlo Method*, Wye College, Ashford, Kent, 1968. Eidman, V. R., Dean, G. W., and Carter, H. O., *An Application of statistical Decision Theory to Commercial Turkey Production*, Am. Journal of Agr. Econ., November, 1967. Hesselbach, J. L., and Eisgruber, L. M., *Betriebliche Entscheidungen Mittels Simulation*, Berlin, 1967, Lee, G., op. cit., Thompson, S. C., *An approach to Monte Carlo Programming*, Study No. 3, Univer. of Reading, Nov. 1967; Zusman, P., and Amiad, A., *Simulation—A Tool for Farm Planning under Weather Uncertainty*, Journal of Farm Economics, 1965, pp.574-594.

V. Conclusions and Challenges

In the foregoing discussion we have attempted to make the following points: (1) that a market-oriented theory is unlikely to be universally applicable or most useful for business decision making, (2) that one universal operational model and one calculus (mathematics) for the farm business is infeasible from a practical viewpoint, and (3) that operational subsystems of the universe of reality of the farm firm can be constructed, but that these subsystems reduce the universal decision space in their own particular way, make their own unique assumptions regarding the structure of the decision space, and depend on different calculi.

This leads to the conclusion that our profession needs to be concerned not with one theory, one general model, and one type of calculus but instead with theories, models, and calculi. We acknowledge that monism is most attractive to the reflective thinking mind. But we also maintain that pluralism is the more operational philosophy.¹⁵ One need only consider the discussion of operational subsystems of the previous section to recognize that these operational subsystems are a poor approximation of a universal model. This is not expected to change drastically in the foreseeable future. Thus, the question is not which is the best model and calculus, but rather which is the best model and calculus *for a given problem* 'Which model?' may be a more important question than whether or not the exact solution (in a purely mathematical way) has been obtained once a model has been chosen. This is so because selection of a particular model reduces the decision space in a particular way and, thus, eliminates search for solutions in much of the domain described by (2). Consequently, our profession is well advised to spend relatively more time to analyse the nature of the problem to be solved and less on calculi (which Menges calls the 'Scheinproblem').¹⁶

Reorientation of the type suggested above would have several desirable consequences. First, it would result in an attitude which will recognize models and calculi as means rather than ends, an attitude which suggests that we do what is needed rather than what is 'methodologically possible'.¹⁷ Secondly, efforts on the part of the applied economist may not decrease in mathematical content, but they could increase their level of relevance.¹⁸ Third, the rift between the 'practioners' and the 'methodologists' in the profession would decrease. The result can only be higher productivity on the part of both.

Such reorientation should also be reflected in the curricula of agricultural economics, both at the undergraduate and the graduate levels. At present, these curricula tend to be narrowly oriented towards the market-oriented

15 c.f. Churchman, W., 'The Nature of Simulation', Baldevston and Hoggat (eds.). *Symposium on Simulation Models*, Cincinnati, pp. 1-12, 1964.

16 Menges, op. cit.

17 c.f. Bereano, L., *The Scientific Community and the Crisis of Belief*, American Scientist, Winter 1969.

18 c.f. Johnson, G. L., 'Stress on Production Economics.' *Australian Journal of Agricultural Economics*, 7, 1, June 1963.

theory of the firm, the associated differentiated calculus, and linear programming.

Finally, it is not our intent to propose that it is feasible to expect agricultural economists interested in the economy of the individual farm business to possess expertise in all theories, all calculi, and all problem formulation aspects. If any degree of competence is achieved in any of these, it will be achieved only with specialists in the respective areas. Thus, significant problems will likely be solved by teams of specialists and the teams are likely to be of different compositions for different problems. We make a case for specialists in the face of our profession's, call-out for generalists, since we believe that something can be learned from history. After all, 'the physical science made slow progress so long as the brilliant but impatient Greek genius insisted on searching after a simple basis for the explanation of all physical phenomena: and their varied progress in the modern age is due to a breaking up of broad problems into their component parts. Doubtless there is a unity underlying all the forces of nature; but whatever progress has been made towards discovering it, has depended on knowledge obtained by persistent specialized study, no less than on occasional broad surveys of the field of nature as a whole. . . . But it is the duty of those who are giving their chief work to a limited field to keep up a close and constant correspondence with those who are engaged in neighboring fields. Specialists who never look beyond their own domain are apt to see things out of true proportion; . . . they work away at the details of old problems which have lost most of their significance and have been supplemented by new questions rising out of new points of view.'¹⁹

SPECIAL GROUP F REPORT

The scope of the paper presented was narrow compared to the generality of the title since the author discussed only the operational subsystems of differential calculus, mathematical programming and simulation, leaving ample opportunity for participants to introduce, for example, the use of econometric simultaneous moves and utility analysis as well as other operational sub-systems. In spite of this the discussion directly concerned with the mathematics of the economy of the individual farm business was restricted to those subsystems included by the author. It was, perhaps, unfortunate that the attempts of a participant to clarify the current situation concerning the usefulness of simulation techniques restricted the discussion on operational subsystems still further, in so far that methods of differential calculus and mathematical programming received scant attention. The remainder of the discussion was concerned with the philosophical differences of socialist and non-socialist countries and the effects of these differences on the application of mathematical techniques to study problems of the individual firm.

19 Marshall, A., *Principles of Economics*, 9th edition, Camdan, 1961, pp.770-771.

It was generally agreed that the calculus of simulation was a more flexible tool than those of the differential calculus and mathematical programming, and suggested that this flexibility led to the simplification of problems being studied. Indeed, in the use of simulation techniques we are led to make assumptions which are themselves simplifications of reality. Since the processes and results of simulation techniques were not a substitute for the general theory of the firm a plea was made for more research into the relationships between them. Participants were reminded that whether or not we used differential calculus, mathematical programming and simulation depends on specific situations. Research work at present was concerned more with the theory of the firm and its relationship to the various kinds of models and the processes by which the models could be handled, than with the application of such models to real situations although there was a direct connection between theory and reality.

The opening discussant said that the paper may give a new stimulus to research efforts and economists in the socialist sphere of the world in a field where they were beginning to face a gap between practice and economic theory. Until recently the planning system had now expressly required the development of a socialist theory of the firm but the present policies for economic reform had produced a situation in which such a theory, preferably a mathematically formalised one, was sadly needed.

Perhaps to some the most important feature to emerge from the discussion was the enlightenment that in spite of the ideological differences between the socialist and non-socialist countries there *does* exist a common platform for the study and application of mathematical techniques to problems of the individual firm, no matter whether that firm is privately owned and acting in terms of self-interest in a non-socialist society, or is a production unit acting directly in the national interest in a socialist one. This realization led to the suggestion that the list of attributes given on page 389 as being appropriate to questions of 'micro' a management orientation, should also include some consideration of group decision-making. Even on a family farm in a non-socialist country families make decisions and, indeed, there was no real functional difference between a board of directors of a capitalist company and a committee composed of a manager and his specialist advisers in a peoples enterprise. The aim to maximize profits was, more often than not, consistent with the desire to maximise output.

But the opening discussant had already pointed out that the large-scale farms existing in most socialist countries are complex organisations within which can be recognised a number of dynamic systems of different interests, with a sector of zones associated with them, as well as a number of different spheres of decision making. The example specifically mentioned was the case of the members of cooperatives who, in their capacity as owners, are placed above the managers to whom they are responsible as workers. The need was for a development of a theory of the firm suited to a socialist planned economy which offers much more freedom for the decision makers who were faced with a much greater responsibility than they had previously been accustomed to.

A number of speakers from socialist countries stressed that there were a number of levels of decision-making, and that an important problem facing planners was to find the means of identifying the farming systems which were the most important so that optimal results could be obtained on a national scale. A considerable experience had been built up in the U.S.S.R. in the formulation and use of mathematical models for state and collective farms to solve their problems.

No list of participants provided.