

# Increasing at an Increasing Rate: The Potential Convexity of Discrete-Choice Welfare Measures\*

by

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## Abstract

When environmental quality enters random utility models linearly, welfare measures are convex in quality. The convexity is partly due to site substitution, and it has implications for whether changes in quality should be concentrated or diffuse. The effects of functional form are illustrated in a model of Great Lakes fishing.

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In the environmental economics literature environmental benefit functions are commonly used in optimization frameworks. To facilitate solutions, environmental benefits are usually assumed to be increasing concave functions of environmental quality (or environmental damages are assumed to be increasing convex functions of pollutants). In practice, one way to derive an environmental benefit function is to use the travel cost method to relate the demand for recreation to environmental quality at specific recreation sites. Use-values for changes in environmental quality can then be obtained from the recreation demand model. Discrete-choice models such as the random utility model (RUM) dominate the modern literature on recreation demand. However, one point that has not to have received much attention is that welfare measures for changes in site quality from discrete-choice models are often convex in site quality.

Here, it is shown that when the indirect utility of recreation sites is specified using the linear form, individual and aggregate welfare measures based on RUM site-choice models *must* be convex in site quality. Hence, environmental benefit functions derived from RUM-based models will be convex in site quality whenever site utility is linear in quality. The convexity of the benefit function has nothing to do with the specification of the RUM error terms or the researcher's uncertainty about the individual RUM welfare measures. Rather, the convexity is a result of the underlying discrete nature of the choices. The rationale is straightforward: increases in quality at a site yield utility to current users *and* to new users who are induced to switch sites. It is the site-substitution effect that leads to the convexity of the welfare measure. Moreover, it is shown that even if individual utility functions for sites are concave in site quality, the aggregate welfare measure will be locally convex whenever the incremental utility to new users offsets the diminishing incremental utility to existing

users. For site utility functions that are logarithmic in quality, the range of parameters that ensure concavity is demonstrated.

An empirical example illustrates these effects by specifying indirect utility as linear and non-linear functions of site quality. Each of the functional forms is applied to data on the site choices for Great Lakes trout and salmon anglers in Michigan. The catch rate for trout and salmon is used as the environmental quality variable, and each of the models is estimated as a multinomial logit. Non-nested statistical tests of model fit provide weak evidence that a logarithmic form (log of catch rates) is preferred for the example considered. The illustration demonstrates that the resulting welfare measures for increases and decreases in catch rates are remarkably different depending on the linear or non-linear forms.

While the model specification issues addressed by the paper have received little attention in the recreation demand literature, the paper is not just about which functional form fits the data best. The ultimate point is that the differing functional forms support opposite policy rules for the general management of environmental quality, namely whether to concentrate or spread out changes in environmental quality (Helfand and Rubin).

## The Random Utility Model (RUM)

The literature contains several expositions of the theory of discrete-choice known as the random utility model (McFadden, 1981 and 1997; Small and Rosen; Hanemann 1982 and 1983; Train; Ben-Akiva and Lerman; Morey), and reviews in the recreational demand context are given by Freeman; and Bockstael et al., 1984 and 1992. The RUM choice problem can be stated as choosing the alternative  $k$  that maximizes the conditional indirect utility  $U_k$ , that is,

$$\text{Max}_{\{k\}} U_k(Y-p_k, q_k). \quad (1)$$

By utility maximization, observing the choice of  $k'$  implies the following utility inequalities:  $U_{k'} > U_k$

for all  $k \neq k'$ . This becomes a "random" utility model by recognizing that not all attributes that affect utility can be observed by the researcher. Thus, researchers must predict the probability that any alternative is the best in the choice set as follows:

$$\pi_k = \Pr [ U_k > U_0, \dots, U_k > U_{k-1}, U_k > U_{k+1}, \dots, U_k > U_K ]. \quad (2)$$

The choice probabilities serve as the *expected* demand functions for alternatives in the choice set.

The conditional utility function is typically assumed to be

$$U_k = V_k + \epsilon_k, \quad (3)$$

where  $V$  is the deterministic portion of utility to be estimated, and  $\epsilon$  is a stochastic portion known to individuals but not to researchers. In practice, researchers commonly adopt the linear form,  $V_k = \mu(Y - p_k) + \beta q_k$ , where  $\mu$  is the marginal utility of income and  $\beta$  is a vector of marginal utilities associated with a vector of characteristics  $q_k$ .

To derive an econometric model, one assumes a distribution for the error terms in (3). When the errors are assumed to be i.i.d. from a type 1 extreme value (EV) distribution, the choice probabilities will have the familiar multinomial logit (MNL) form

$$\pi_j = \frac{e^{V_j}}{\sum_{j=1}^J e^{V_j}}. \quad (5)$$

Thus, the  $\pi_j$  (expected demands) depend on the prices and qualities of all alternatives.

## Welfare Measures in RUMs

Recall that in the RUM, individuals are assumed to choose the site yielding the highest utility (the best site). The nature of the problem implies that individuals only get utility from changes in quality when they actually use the site where quality changes, i.e., only *use values* matter. For any individual, the value of a change in site quality depends on the difference between the indirect utility

of the old best site and the indirect utility of the new best site. Thus, changes in site quality only have value when they occur at the best site or when they are large enough so that the site where quality changes becomes the best site. This insight will be useful in establishing the shape of the aggregate benefits function derived from the RUMs.

Deriving the RUM welfare measures is complicated by the fact that the error terms,  $\epsilon$ , are unknown to the researcher. We begin by ignoring this uncertainty and focussing on the logic of the welfare measure. Under certainty and full knowledge of the error terms for each individual, compensating variation for a change in prices and quality from  $(p^0, q^0)$  to  $(p^1, q^1)$  would be:

$$\text{Max}_{j \in J^0} U(Y - p_j^0, q_j^0, \epsilon_j) = \text{Max}_{j \in J^1} U(Y - p_j^1 - CV, q_j^1, \epsilon_j) \quad (6)$$

From this expression, it is clear that if a policy change does not affect the best site before or the best site after a change, then the CV for that change will be zero. That is, if a policy change does not affect sites that are being used, then the policy does not generate use value.

Of course, the preceding assumes the  $\epsilon$ 's are known, yet the essence of the RUM is that the  $\epsilon$ 's are not known. Since the utilities in (6) are random variables, the maximum utility is also a random variable, so the CV itself is a random variable (Hanemann, 1982). One way to deal with this is to calculate the expected CV -- typically a complex matter when the errors follow anything other than extreme values distributions. Here we focus on EV errors, though the results also apply to GEV. With EV distributions, the expected maximum conditional indirect utility is given by the "inclusive value,"  $IV = \ln(\sum_i e^{V_i})$  (Johnson and Kotz; McFadden, 1978). The IV function has two convenient properties: (1) the first derivatives with respect to the deterministic indirect utilities are the estimated probabilities (McFadden 1981; Hanemann, 1983), and (2) the second derivatives are the marginal effects on the probabilities; that is

$$\frac{\partial IV}{\partial V_j} = \frac{e^{V_j}}{\sum_{m=1}^J e^{V_m}} = \pi_j \quad \text{and} \quad \frac{\partial^2 IV}{\partial V_j^2} = \frac{\partial \pi_j}{\partial V_j} = \pi_j(1 - \pi_j). \quad (7)$$

Using the inclusive value notion, if the error terms are EV and income enters V linearly, then expected CV is given by

$$E[CV] = \frac{IV^1 - IV^0}{\mu} = \frac{\ln\left(\sum_{m=1}^{J^1} e^{V_m^1}\right) - \ln\left(\sum_{m=1}^{J^0} e^{V_m^0}\right)}{\mu}. \quad (8)$$

The above equation has long been utilized as a benefits measure for recreational demand models based on the RUM (Hanemann, 1982; Bockstael et al., 1984). The expression provides a means of evaluating complex arrays of changes in site quality and prices as well as site closures and additions.

Using (7) and (8), with the linear utility function we have

$$\frac{\partial IV}{\partial p_j} = -\mu \pi_j, \quad \frac{\partial IV}{\partial q_j} = \beta_q \pi_j, \quad \text{and} \quad \frac{\partial CV}{\partial q_j} = \frac{\beta_q}{\mu} \pi_j \quad (9)$$

where  $\mu$  is the negative of the price parameter and  $\beta_q$  is the utility parameter on quality characteristics.

The later term is the marginal welfare measures for a change in resource quality at site j,  $q_j$ , (Hanemann 1983; McFadden 1981).

## The Shape of the Benefits Function

In the literature on recreational site choices, there is little discussion of the basic properties (the shape) of the RUM welfare measure (Hanemann, 1983, is an exception). The shape is important in the use of the benefits function for benefit cost analysis and for general policy prescriptions. In this section, it is demonstrated that the benefits functions derived from discrete choice models are convex in improvements in quality when the site utility functions are linear in quality. This has obvious

implications for the use of these models to derive "optimal" levels of environmental quality.

To establish the shape of B, we make use of the properties of the IV function given in (9).

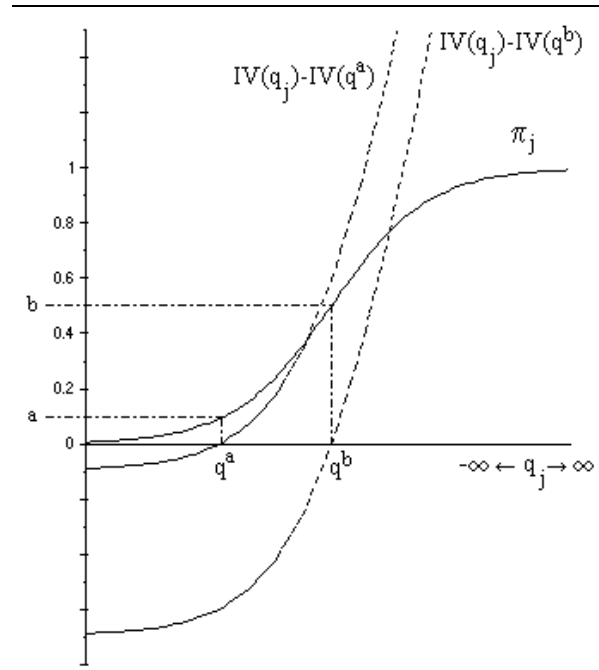
Again assuming a linear V, these properties imply the following

$$\frac{\partial B(\cdot)}{\partial q_j} = \frac{\beta}{\mu} \pi_j > 0 \text{ if } \beta > 0 \quad \text{and} \quad \frac{\partial^2 B(\cdot)}{\partial q_j^2} = \frac{\beta^2}{\mu} \pi_j (1 - \pi_j) > 0. \quad (10)$$

Since the probabilities must be non-negative, it is clear that the IV function is increasing in  $q_j$  if  $\beta > 0$ . Moreover, the benefits function is convex in changes in site quality. The first point, that the marginal benefit of a change in quality is dependent on the probability of a site, has been discussed by Hanemann (1983). This fundamental insight results in policy prescriptions such as an improvement in quality is more valuable at *more* popular sites than at *less* popular sites. Alternatively, when benefits of environmental quality at various sites are specified as increasing at a decreasing rate, then the policy prescription would be to allocate given increments of quality to the sites with the lowest quality (provided the marginal costs are the same).

Figure 1 illustrates the shape of the benefits function for some hypothetical sites and numbers. Quality is graphed on the horizontal axis. The site probability function is also shown (labeled  $\pi_j$ ). Two alternative functions are shown from two alternative initial quality levels,  $q^a$  and  $q^b$ . For simplicity, assume that  $\mu=1$ , so that the two benefits functions can be labeled  $IV(q_j)-IV(q^i)$  for  $i=a,b$ . The first thing to note is that the benefits functions equal zero where  $\Delta q_j=0$ . These points are highlighted by the dashed lines which show the

Figure 1: Benefits Function with Linear V.



initial probabilities associated with the two initial quality levels,  $q^a$  and  $q^b$ . Since the slope of the IV function is equal to  $\pi$  times the marginal implicit price of quality, the slope of the IV function for  $a$  is less for all increments in quality than is the slope for  $b$ . As the increment of quality change goes to  $\pm$  infinity, the slopes of the two benefit functions become the same, and approach 0 and  $\beta_q/\mu$ , respectively. From the diagram, one can see that for "large" changes one would expect an improvement in quality to be much more valuable than the absolute value of an equivalent decrease in quality.

Of course, for policy purposes we are interested in the aggregate benefits function, not the individual benefit functions. Aggregate benefits are commonly given by the sum of the individuals benefits functions (a weighted sum could also be used). Since each of the individual benefits functions is convex in  $q_j$ , any aggregate benefits function that is a sum of the individual benefits (or a weighted sum) will also be convex. The next two sections discuss the extent to which the convexity of  $B$  depends on the error terms and the linear indirect utility function.

*A Brief Return to Certainty:* Is the convexity of the benefits function an underlying feature of the discrete-choice framework, or is it simply an outcome of using expected welfare measures (because of the stochastic terms in the indirect utility functions)? Consider the welfare measure under knowledge of the error terms, when the indirect utility function is linear in the site characteristics. For all individuals who initially choose  $j$  as best,  $CV = \Delta q_j \beta/\mu$ . Thus, for initial changes in quality, the slope of the *aggregate* benefits function is the sum of marginal implicit prices for those individuals that actually choose site  $j$ ; i.e.,  $S_j \beta/\mu$  where  $S_j$  is the number of individuals choosing alternative  $j$ . As  $q_j$  increases, there will be individuals for whom alternative  $j$  becomes the best site. Let the total number of individuals choosing site  $j$  under conditions  $q_j^1$  be given by  $S_j^1$ . For each of these additional individuals, the marginal benefit is also  $\beta/\mu$ . The slope of the aggregate benefits function is therefore  $S_j^1 \times \beta/\mu$  at  $q_j^1$ . With sufficient smoothness in  $S_j^1$ , the aggregate benefits function would be increasing



at an increasing rate if  $S_j^1$  is increasing in  $q_j^1$ . This is true even though each individual's marginal benefit is constant. When quality improves, it is the increase in the number of individuals choosing  $j$  that leads to the convexity in the aggregate benefits function. This is essentially the same as the results when one accounts for the uncertainty regarding the error terms because the choice probabilities,  $\pi_j^1$ , should approximate  $S_j^1$ . Thus, the potential for a convex benefits function is a due to the underlying discrete choices. In particular, for any other specification of errors (e.g., normal), the convexity result would remain if the resulting site choice probabilities are increasing in quality. Therefore, the convexity result is not an artifact of the assumed error distribution.

*Relaxing Linear Utility:* The convexity of aggregate benefits depends in part on the linear specification of site utility. However, even if the individual utility functions are all assumed to be concave in site quality, the aggregate benefit function is not necessarily globally concave. To see this, consider the case where the individual utility functions are logarithmic in  $q_j$ . Here,

$$\frac{\partial^2 B}{\partial q_j^2} = \frac{\beta \pi_j}{\mu q_j^2} (\beta(1 - \pi_j) - 1). \quad (11)$$

Thus, the benefits function will be globally concave if  $\beta \leq 1$ . However, for any initial level of  $\pi_j^0$ , there will be some value of  $\beta$  large enough so that the benefits function is locally convex. With  $\beta > 1$ , for benefits to be locally concave, one must have  $\pi_j > (\beta - 1)/\beta$  which leaves little leeway for local concavity. For example, if  $\beta = 1.25, 1.5,$  and  $2$ , respectively, then for benefits to be locally concave,  $\pi_j^0$  must exceed  $0.2, 0.33,$  and  $0.5$ , respectively. Of course, as  $q_j$  goes to infinity, the probability goes to 1. So the benefits function would look like the site choice probabilities, initially increasing at an increasing rate and ultimately increasing at a decreasing rate.

For the cases where indirect utility function are non-linear with respect to  $q$ , some simple intuition can be provided regarding the shape of the benefits function. The second derivative of the

benefits function is composed of two effects. The first is the effect of a change in  $q_j$  on the site choice probability  $\pi_j$ . The second is the effect of changes in  $q_j$  on the marginal utility derived from visits to site  $j$  which is zero in the linear model and negative if the site utility is concave in  $q_j$ . Thus, whether the aggregate benefits function is convex or concave for increases in  $q_j$  depends on whether the diminishing effect on utility for site users (the second effect) is enough to offset the increases in utility accruing to the new users (the first effect). Consequently, concavity of  $V(q)$  is not sufficient to insure global concavity of the aggregate benefits function.

*Empirical Evidence in Literature:* There are some investigations of the effects of functional form that have appeared in the contingent valuation literature (e.g., Boyle). There is also a literature on the effects of functional form in single site travel cost studies. However, the issue has received less attention in the context of the RUM travel cost method. Moreover, the extent to which increments of improvements in quality are larger than equivalent decrements in quality is largely an empirical question. There are some studies that report valuation results for a range of changes in an environmental quality variable where one can see evidence of convexity in the estimated environmental benefits. Space precludes a complete review, but some examples include Jones and Lupi, and Lupi et al. In another example, Chen et al. compare multinomial probit and logit recreation demand models and examine a range of quality changes using each model. The welfare measures for the both models exhibit similar degrees of convexity in the benefits for changes in site quality.

## Illustrative Example

Here, a multinomial logit model is estimated using an indirect utility function where quality enters in a linear and logarithmic form. The model is a simplified version of one of the site choice models of Jones and Sung (1993). The illustration uses data for trips that were single-day trout and salmon fishing trips to Great Lakes sites in Michigan. Sites are defined as counties in Michigan with

Great Lakes shore line that are within 250 miles of an angler's home. The sample consists of 363 anglers, and the average angler's choice set contains 21 sites. Site utility is a function of travel costs for individual  $i$  to each site  $j$ ,  $p_{ij}$ , and a total catch

Table 1: Multinomial Logit Parameters

Variable	Linear	Log
Travel cost, $-\mu$	-0.0636 (-18.9)	-0.0648 (-18.9)
$f(\text{catch rate}), \beta$	3.37 (2.70)	0.458 (3.59)
LogL value	-555.07	-551.87

rate for trout and salmon species at each site  $j$ ,  $q_j$ . Site utility takes the linear-in-parameters form,  $V_j = -\mu p_{ij} + \beta f(q_j)$ . For comparison, the model was estimated with  $f(q_j) = q_j$  and  $f(q_j) = \ln(q_j)$ . The model estimation results are listed in Table 1. The  $\ln(q)$  model has a larger likelihood value. Vuong's test for non-nested models was also used to compare the linear to the logarithmic model (Vuong). The results imply that the  $\ln(q)$  model provides a significantly better fit than the linear model for  $\alpha > 0.065$ . Thus, there is some very weak support for the log form relative to the linear form.

Two benefit functions are derived from the above models in order to illustrate the effect of functional forms on the shape of the benefit function. For the first, catch rates are varied at a single Lake Michigan county (Allegan). For the second, catch rates are varied at all sites bordering Lake Michigan (about half of the sites). For each of these, the sample average per-trip values and sample average choice probabilities are calculated for the range of catch rate changes. These results are presented in Table 2 and are graphed in the accompanying figures.

Notice that unlike the linear model, the  $\ln(q)$  model has the property that as  $q_j$  goes to zero,  $\pi_j$  goes to zero. For catch rates, this is a sensible property since catch rates "near" zero becomes the same as eliminating the site. Notice from the graphs that there is a striking difference between how "fast" the  $\pi_j$  goes to zero for the single county compare to the probability of going to the entire lake. For the lakewide policy, the choice probabilities do not tail off until the reductions in catch rates become very close to zero. At a 95% reduction in catch rates at the entire lake, the choice

probabilities have only fallen by 9%. Of course, the linear model performs much worse in this regard. As shown in the first rows of Table 2, reducing catch rates to zero in the linear model bears little resemblance to a site closure. For Allegan Co., reducing catch rates to zero results in a loss about 1/3 that of the site closure. Strikingly, for Lake Michigan, reducing catch rates to zero results in a loss that is only 5% of the site closure.

## Discussion

Space constraints preclude a complete discussion of the results, but a few observations can be squeezed in. The above results illustrate the effect that functional form can have on estimated environmental values. While the linear models examined here do not exhibit a large degree of convexity, these are "per-trip" results that are conditioned on the number of Great Lakes trout and salmon fishing trips being held constant. With a participation level that models changes in trips, the values for the linear model would show "more" convexity, and the values for the log model would show "less" concavity. The reason is that total trip changes would mitigate some of the losses and enhance the gains. For example, in Lupi et al. a 50% increase in catch rates is over twice as valuable as a 50% decrease. As the theory shows, this is a direct result of the substitution effects inherent in the discrete choice framework. The empirical examples also illustrate the potential hazards of using the linear form to value extreme changes in site quality.

The possible convexity of the benefits function is a potential concern for using RUM type models as part of a cost benefit analysis. As demonstrated above, while the use of concave site utility function can result in an aggregate benefits function that is concave, concave site utility functions are not sufficient to guarantee global concavity for the aggregate benefits function. The implications that the shape of the benefits functions has for environmental policy are discussed more fully in Helfand and Rubin.

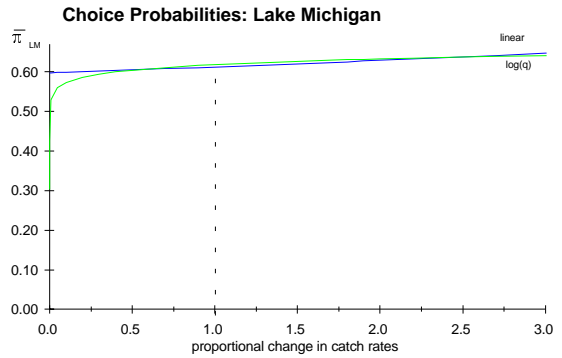
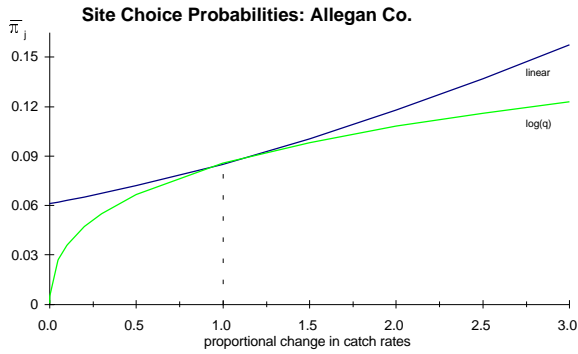
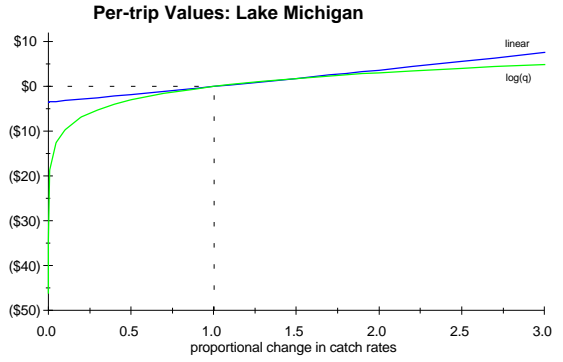
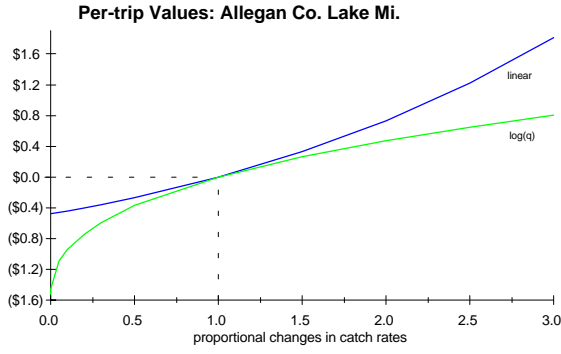


Table 2: Some values used to plot the above benefit functions for proportional changes in catch rates.\*

	Vary catch rates at <i>Allegan Co.</i>					
	Avg. per-trip value		Avg. site probabilities		% of baseline prob.	
	<i>linear</i>	<i>log(q)</i>	<i>linear</i>	<i>log(q)</i>	<i>linear</i>	<i>log(q)</i>
<b>Site elim.</b>	(\$1.55)	(\$1.53)	0	0	0	0
<b>0.0001</b>	(\$0.48)	(\$1.51)	0.061	0.002	72%	2%
<b>0.05</b>	(\$0.46)	(\$1.09)	0.062	0.027	73%	32%
<b>0.1</b>	(\$0.44)	(\$0.94)	0.063	0.036	74%	42%
<b>0.5</b>	(\$0.27)	(\$0.37)	0.072	0.067	85%	78%
<b>1.0</b>	\$0.00	\$0.00	0.085	0.086	100%	100%
<b>1.5</b>	\$0.33	\$0.26	0.100	0.098	118%	115%
<b>2.0</b>	\$0.73	\$0.47	0.118	0.108	138%	126%
<b>3.0</b>	\$1.81	\$0.80	0.158	0.123	185%	144%
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	Vary catch rates at <i>all Lake Michigan counties</i>					
	Avg. per-trip value		Avg. lake probabilities		% of baseline prob.	
	<i>linear</i>	<i>log(q)</i>	<i>linear</i>	<i>log(q)</i>	<i>linear</i>	<i>log(q)</i>
<b>Site elim.</b>	(\$62.07)	(\$63.38)	0	0	0	0
<b>0.0001</b>	(\$3.47)	(\$34.33)	0.598	0.426	98%	69%
<b>0.05</b>	(\$3.30)	(\$12.48)	0.599	0.561	98%	91%
<b>0.1</b>	(\$3.13)	(\$9.70)	0.600	0.574	98%	93%
<b>0.5</b>	(\$1.76)	(\$3.00)	0.605	0.605	99%	98%
<b>1.0</b>	\$0.00	\$0.00	0.613	0.619	100%	100%
<b>1.5</b>	\$1.81	\$1.79	0.621	0.627	101%	101%
<b>2.0</b>	\$3.67	\$3.07	0.630	0.633	103%	102%
<b>3.0</b>	\$5.60	\$4.07	0.638	0.638	104%	103%

\* Results are in 1984 dollars and are for site-choice only, i.e., total GL trout & salmon day-trips are held constant at initial level.

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