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RECENT APPLICATIONS OF QUANTITATIVE RESEARCH IN AGRICULTURAL ECONOMICS

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THE nature of quantitative research in agricultural economics has completely changed during the last ten or twenty years. Admittedly the conventional methods like budgeting or time series analysis of the simplest kind have kept their place and their meaning. But they have been supplemented by a number of more refined methods of which the application requires at least some basic knowledge of mathematics and of mathematical statistics. The application of these methods has increased so rapidly during the last decade that in many cases it is not possible to recognize from the title of a paper which of the modern methods has been used or whether one of them has been used at all. At least some of the standard procedures of modern methods have become common tools, of which the application is no longer subject to methodological discussion. It is impossible to give a complete bibliography of the quantitative work which has been done; I can only try to show some of the basic lines of development, mainly in the fields of production and supply analysis. In these fields the rapid increase in the use of mathematical and mathematical statistical methods in the last decade has been mainly due to the following factors:

(1) The discovery of the mathematical core of the theory of economic equilibrium.

(2) The rapid improvement of computers which made possible the solution of numerical problems which nobody would have dared to consider fifteen years ago.

Production function Analysis

The mathematical core of the equilibrium theory was formulated by Walras, Jevons, Menger, and others at the end of the nineteenth century. Production function analysis is based on the mathematical formulation of the theory of the firm as it was developed in their work. Although production function analysis has lost importance for the formation of decision models on the micro-economic level with the development of more powerful calculation methods like linear programming, considerable work has been devoted to its application even in very recent years. However, the range of application has changed. The tool which was developed originally to formulate decision models has been changed into a tool for determining input–output relations between factors. Here two fields of application can be distinguished: (1) the establishment of input–output relations from farm data, (2) the establishment of input–output relations from experiments.

Production function from cross-section data. The derivation of production-functions from cross-section data encounters mainly three problems:

(1) The choice of the function and the way a chosen function is to be estimated.
(2) Selecting a meaningful disaggregation of inputs, in other words, the choice of the variables and the problems arising from multicollinearity.[1]
(3) The problem of inter-farm and intra-farm relationships.

Almost none of these problems has been satisfactorily solved. For the estimation of the function, the ordinary least squares method is applied in most cases, although recent discussion indicates that the selection of the estimation procedure has considerable influence on the bias and the stability of the function and that probably no single estimation procedure is satisfactory in all circumstances.[2]

The choice of function and the choice of variables are usually governed by the statistical possibilities rather than by the technical conditions of the production process analysed. When a Cobb–Douglas function is fitted, this is more likely to be done not so much because one can expect the input–output relation to be of that kind as because the application of this function simplifies the calculation procedure. However, the principal problem is that of inter-farm and intra-farm relationship. The danger of arriving at a 'hybrid production function' is well known to everybody who has worked in this field.

[1] Knud Rasmussen with M. M. Sandilands, *Production Function Analyses of British and Irish Farm Accounts*, University of Nottingham, June 1962. Chr. Beringer, 'Estimating Enterprise Production Functions from Input–Output Data on Multiple Enterprise Farms', *Journal of Farm Economics*, vol. xxxviii (1956), pp. 923 ff. R. M. Parish and J. L. Dillon, 'Recent Applications of the Production Function in Farm Management Research', *Review of Marketing and Agricultural Economics*, Dep. of Agr. N.S.W., Australia, (pp. 215 ff.), Dec. 1955.

[2] J. Kmenta and M. E. Joseph, 'A Monte Carlo Study of Alternative Estimates of the Cobb–Douglas Production Function', *Econometrica*, vol. xxxi (1963), pp. 363 ff.

It can be eliminated or at least reduced by certain statistical techniques, which I am not going to discuss here.[1]

However, the admission that each farm operates on its own production function, of which the shape or at least the coefficients are influenced by management factors, calls in question the suitability of the whole concept for the purpose of deriving input–output relationships to be used in construction of decision models on the individual farm basis. The determination of useful functions requires the measurement of managerial abilities in terms which can be applied to the individual farm. Much effort has been invested in the attempt to isolate the managerial factor.[2] The results of these investigations show that the managerial factor is of such a complex structure that it seems impossible to express it in measurable units in such a way that it can be handled as an independent variable, like the capital or labour input, in production function analysis. The conclusion is: The determination of input–output relationships as a basis for individual decision models is not the field of application for production functions derived from cross-section data.

However, if this type of analysis is used as a method of farm comparison, it may well help to formulate an adequate agricultural policy which aims at the improvement of farm organization and farm income. It does not aim, then, at the determination of a more or less elegant-looking production function, but it may well give valuable insights into the possible effects of an investment policy, of a change

[1] Y. Mundlak, 'Empirical Production Function Free of Management Bias', *Journal of Farm Economics*, vol. xliii (1961), pp. 44–56. J. F. Stollsteimer, R. G. Bressler and J. N. Boles, 'Costs Functions from Cross-Section Data, Fact or Fantasy?', *Agricultural Economics Research*, vol. xiii (1961), pp. 79–88. M. Bronfenbrenner, 'Production Functions Cobb–Douglas, Interfirm, Intrafirm', *Econometrica*, vol. xii (1944), pp. 35 ff. Knud Rasmussen and M. M. Sandilands, op. cit., pp. 5 ff. and pp. 16 ff. Zvi Griliches, 'Specification Bias in Estimates of Production Functions', *Journal of Farm Economics*, vol. xxxix (1957), pp. 8–20.

[2] L. M. Eisgruber, 'The Human Factor in Farm Management', *Journal of the American Society of Farm Managers and Rural Appraisers*, vol. xxiii, pp. 67 ff. W. E. Henry, 'Personality Factors in Managerial Reactions to Uncertainty', in *Social Science Research Council*, New York, 1958 (ed. by M. J. Bowman). J. Hesselbach, 'Quantifizierung des Betriebsleitereinflusses durch Vergleich mit einem Optimal-Modell', *Berichte über Landwirtschaft*, Bd. xl (1962), pp. 73 ff. J. Hesselbach, *Measurement of Managerial Ability; Present Status and Future Possibilities*. Paper presented to the 3rd Meeting of Experts on Farm Rationalization at the E.C.E. 1963. F. Schneppe and E. Walter, 'Der Einfluß objektiver und subjektiver Faktoren auf den Betriebserfolg', *Agrarwirtschaft*, 9. Jahrgang (1960), pp. 110 ff. N. Westermarck, 'Management and Success in Farming, Part III, Influence of Individual Advisory Services', *Acta Agriculturae Scandinavica*, vol. x, 4 (1960). P. Blanckenburg, 'Die Persönlichkeit des Landwirtschaftlichen Betriebsleiters in der ökonomischen Theorie und der sozialen Wirklichkeit', *Berichte über Landwirtschaft*, Bd. xxxv (1957), pp. 308 ff. Glenn L. Johnson, A. N. Halter, R. Jensen, and D. W. Thomas (editors), *A Study of Managerial Processes of Midwestern Farmers*, Ames (Iowa) 1961.

in size structure, of an improvement of education, of information programmes, of a change in the system of values of farmers, &c.

In most cases regression analysis is applied for this purpose. Some researchers very recently have used factor analysis also.[1]

Production-functions from experiments. The derivation of production function from experiments encounters mainly three problems:

(1) The choice of the function.
(2) The instability of other uncontrolled input data such as moisture and temperature.
(3) The problem of transmission of results gained from experiments to practical farming.

The choice of the function should be determined by the nature of the technical relations which are involved on the one hand, and by the goodness of the fit on the other. The first requirement limits the application of production-function analysis more or less strictly to the crop sector. The choice of the function does not create serious problems here. Several functions with one or more independent variables[2] (Spillman, Mitscherlich, Cobb–Douglas, quadratic square root transcendental), which describe the general relation between inputs of fertilizer, water, &c., and output per land unit have been developed. The choice between these functions is governed by the goodness of the fit. The recent work, therefore, is mainly concerned with the problems of the instability of uncontrolled input data like weather and the transmission of the results from experiments to practical farming. The necessary information on how fertilizer

[1] G. Hamming and A. H. J. Liberg, 'Aspecten vav de bedrijfsvoering van gemengde bedrijven of zandgrond', in *Bedrijfseconomische mededelingen,* Nr. 31, März 1960, Landbouw-Economish Institut, Den Haag. G. Hamming, *Een bedrijfsvergelijkend streekonderzoek op de zandgronden.* Rapport Nr. 384, Landbouw-Economish Institut, Den Haag. G. H. Mac Eachern, D. W. Thomas, L. M. Eisgruber, *Analysis of Human Attributes and their Relationships to the Level of Performance of Farm Tenants Production,* Economic Paper no. 6108, Purdue University, Nov. 1961.

[2] E. O. Heady, J. T. Pesek, and W. G. Brown, *Crop Response Surfaces and Economic Optima in Fertilizer Use,* Agricultural Experiment Station, Iowa State College, Research Bulletin 424, Ames (Iowa), Mar. 1955. E. O. Heady and J. L. Dillon, *Agricultural Production Functions,* Ames (Iowa), 1961. W. G. Brown, E. O. Heady, J. T. Pesek, and J. A. Stritzel, *Production Functions, Isoquants, Isoclines and Economic Optima in Corn Fertilization for Experiments with two and three Variable Nutrients,* Agricultural Experiment Station Iowa State College, Research Bulletin 441, Ames (Iowa), Aug. 1956. E. L. Baum, E. O. Heady, J. T. Pesek, and C. G. Hildreth, *Economic and Technical Analysis of Fertilizer Innovations and Resource Use,* Ames, Iowa State College Press, 1957. E. W. Paasch, 'Methodische Untersuchungen über den Einsatz der Produktionsmittel in der Landwirtschaft'. *Dargestellt am Beispiel der mineralischen Düngung.* Halle, 1958. H. Ruthenberg, *Die Bestimmung der optimalen Aufwandshöhe und Aufwandszusammensetzung bei der Mineraldüngung.* Bul. 36 (1958), pp. 69 ff.

response differs with different conditions of the uncontrolled factors requires a considerable increase in the number of experiments.[1]

However, the consideration of the uncontrolled input factors is not only a question of a sufficient number of experiments. Two further problems are involved: (1) the expression of the uncontrolled input factors as independent variables in the estimation equation, (2) the determination of the probability distribution of the quantitative value of these variables.

From a purely theoretical point of view a possible solution of these problems might be described as follows: (1) the construction of an index of uncontrolled input factors, which meets the problems of critical seasons, of relation between moisture and temperature, &c., (2) the establishment of a production function which includes these indices as independent variables, (3) the calculation of a (joint) probability function for the value of the index (indices).

The optimum input is then determined by selecting the optimum input rate for each value of the index of uncontrolled input factors within a given level of probability of occurrence for a given time period and summing up the returns for each input rate of all index values multiplied by the probability of occurrence.

Under simple conditions, under which the coefficients of controlled inputs are influenced mainly by the variability of one uncontrolled input factor (e.g. rainfall) this procedure has been followed successfully.[2] However, it is doubtful whether its application can be extended if two or more uncontrolled factors occur. The construction of a weather index under conditions in which temperature and rainfall are unstable factors, of which the level in different critical seasons influences the relations between output and controlled input, is a difficult problem which has not yet been solved. In these cases the optimum rate of input of controlled factors has to be determined directly from the observed production functions of a given observation period. The determination of the optimum input rate becomes a 'Game against nature', in which the chances of profit and loss at least for extreme input rates are widely unknown.[3]

Furthermore, it is very likely that not only the coefficients of a

[1] C. G. Hildreth, *Possible Models for Agronomic Research in Fertilizer Innovation and Resource Use*, ed. by E. L. Baum, E. O. Heady, J. T. Pesek, and C. G. Hildreth, Iowa State College Press, Ames (Iowa), 1957.

[2] J. L. Knetsch, 'Moisture Uncertainties and Fertility Response Studies', *Journal of Farm Economics*, vol. xli (1959), pp. 70 ff. R. J. Hildreth, 'Influence of Rainfall on Fertilizer Profits', *Journal of Farm Economics*, vol. xxxix (1957), pp. 522 ff.

[3] E. R. Swanson, 'Problems of Applying Experimental Results to Commercial Practice', *Journal of Farm Economics*, vol. xxxix (1957), pp. 382 ff. John Milnor, 'Games Against Nature', *Decision Processes* (ed. by R. M. Thall), New York, 1954.

selected function but the choice of the function itself is influenced by various levels of uncontrolled input factors. The question arises, therefore, whether it is worth while to derive a production function under these conditions for the preparation of decisions on the micro-economic level.

The point estimates, which are needed for decision models of the linear programming type, could be taken directly from experiments. Each amount of input could be associated with a certain range of output observed in the experiments. Mathematical elegance certainly will be missed in this procedure. However, it does not pretend accuracy and certainty where the situation is characterized by inaccuracy and uncertainty.

Decision Models on the Micro-Economic Level

Within the restrictions mentioned in the introduction, linear programming procedures are the most frequently used methods for the formulation of decision models on the micro-economic level. In most cases the standard simplex procedure with 'fixed' input–output coefficients and given capacities is applied. This model has static character and assumes perfect knowledge of future prices and of input–output relations. But there is growing evidence that model builders, and users of models, are aware that farm planning is essentially planning under 'non-certainty', and that many of the decision problems are dynamic in character.

In order to meet the implications arising from the absence of certainty, Knight's distinction of risk and uncertainty seems useful.[1] The practical difference between the two categories is that in the former the distribution of the outcome in a group of instances is known (either through calculation, *a priori*, or from statistics of past experiences), while in the case of uncertainty this is not true, the reason being, in general, that it is impossible to form a group of instances because the situation dealt with is in a high degree unique. In short: *risk* is used for the measurable non-certainty while *uncertainty* is defined as non-measurable non-certainty. In addition to these categories the concept of *subjective uncertainty* might be useful,[2] by which the opinion of the decision maker as to the probability of the occurrence of a future event is taken into account.

[1] F. H. Knight, *Risk, Uncertainty and Profit*, New York, 1921.

[2] G. Tintner, 'The Theory of Choice under Subjective Risk and Uncertainty', *Econometrica*, vol. ix (1941), pp. 298 ff.

The income variability from a risk situation can be calculated by stochastic programming models.[1]

In order to overcome the implications arising from uncertainty for the formulation of the optimum plan, Renborg has introduced the distinction between stable and unstable elements of the farm plan.[2] The stable elements are defined as the activities or, more precisely, as the levels of activities which remain in the optimum plan over the whole range of variation of the factors of which the precise numerical value is unknown. The unstable elements are the activities or the level of the activities which are included in the optimum plan only if certain constellations of the uncertain numerical values occur. Of course, there is no clear distinction between the two kinds of element since any level of stability is possible. However, empirical investigations show that this definition gives valuable insight into what the long run optimum plan of a farm would be under unstable conditions.[3] Renborg separates the stable and unstable factors by an historical test in asking what would have been the optimum plan for a given farm in past years. This includes the assumption that economic history is a repeating process. Since at least some trend movement is contained in the change of economic events the results of this procedure are questionable. However, it seems possible to combine the stability test with price forecasting.

The calculation of income variability by stochastic models or the separation of stable and unstable factors might be described as a passive approach to non-certainty.[4] In order to make allowance for different levels of risk aversion this approach must be combined with the theory of choice. Such theories have been developed by Wald, Savage, Laplace, Simon, Shackle, and others.[5]

[1] G. Tintner, C. Millhalm, and J. K. Sengupta, 'A Weak Duality Theorem for Stochastic Linear Programming', *Unternehmensforschung*, Bd. xii (1963), pp. 1 ff. G. Tintner, 'Stochastic Linear Programming with Application to Agriculture', *Second Symposium in Linear Programming Proceeding* 1, 1955. M. M. McFarquhar, 'Rational Decision Making and Risk in Farm Planning', *Journal of Agricultural Economics*, vol. xiv, no. 4 (1961). B. M. Camm, 'Risk in Vegetable Production on a Fen Farm', *The Farm Economist*, vol. x (1962), pp. 89 ff. E. O. Heady, and W. Candler, *Linear Programming Methods*, Ames, Iowa State College Press 1963, pp. 559 ff. R. J. Freund, *The Introduction of Risk into a Linear Programming Model*, Diss. North Carolina State College, Raleigh 1955. R. J. Freund, 'The Introduction of Risk into a Programming Model', *Econometrica*, vol. xxiv (1956), pp. 253 ff.

[2] U. Renborg, *Studies on the Planning Environment of the Agricultural Firm*, Uppsala, 1962.

[3] U. Renborg, op. cit. G. Scheller, *Der Einfluß der wirtschaftlichen Entwicklung auf Organisation und Ertragslage mittel- und großbäuerlicher Betriebe*, Diss. Göttingen, 1962.

[4] E. O. Heady, W. Candler, op. cit., pp. 555.

[5] A. Wald, *Statistical Decision Functions*, Wiley & Sons, New York, 1950. L. J. Savage, 'The Theory of Statistical Decisions', *Journal American Statistical Association*, vol. xlviii, pp. 238–48. L. Hurwicz, *Optimality Criteria for Decision Making under Ignorance*, Cowles

The combination of choice theory with optimum calculation can be shortly described as follows. A pay-off matrix can be constructed from the optimal plans calculated with price simulation within a certain range, treating at least the most important input–output coefficients as stochastic variables. Such a pay-off matrix makes possible the choice of the optimum plan related to a given attitude to risk. Since non-certainty always contains a certain amount of 'subjective uncertainty' and since his attitude to risk is usually unknown to the decision-maker himself the result is that the farmer should be confronted with a number of optimum plans under assumptions differing within a certain range.

The possibility of calculating a number of optimum plans for the most likely 'environment' of the farm and of confronting the farmer with the consequences of his possible choices seems to be the most promising approach for the use of decision models on the micro-economic level. Some encouraging work has been done. However, most of the published papers deal with only one aspect of non-certainty. A model remains to be constructed which combines the technical input–output coefficients subject to risk with price expectations subject to uncertainty. The construction of such models would create no methodological or theoretical problems but numerical problems, arising from storage problems and rounding errors during the calculation of the optimum plan. If one can call the attempts to overcome the implications arising from non-certainty one of the front lines of quantitative research at the micro-economic level on the production side the attempts to dynamize static models in order to solve investment problems must be called a second front line. The conceptual and analytical aspects have been explored by Bellman, Arrow, Karlin, and others.[1] The problem of finding an effective algorithm for a characteristic functional equation such as

$$f_{n+1}(p) = \max \quad [g(p, q)+f_n(T(p, q))] \qquad (1)$$

remains to be solved.[2] The solution is mainly a question of storage.

Belmann describes the problem as follows:

If p is a point in an n dimensional space ($p = p_1\, p_2 \ldots p_n$) we face the

Commission Paper, no. 355, Chicago (mimeo), 1951. H. A. Simon, *Models of man*, Wiley & Sons, New York, 1957. G. L. S. Shackle, *Expectation in Economics*, Cambridge University Press, London, 1949. J. L. Dillon, and E. O. Heady, *Theories of Choice in Relation to Farmer Decisions*, Iowa State University, Agricultural and Home Economics Experiment Station, Research Bulletin 485 (1960).

[1] R. Bellman, *Dynamic Programming*, Princeton, 1957. K. J. Arrow, S. Karlin, and H. Scarf, *Studies on the Mathematical Theory of Inventory and Production*, Stanford, 1958.

[2] R. Bellman, 'Some Directions of Research in Dynamic Programming', *Unternehmensforschung*, Bd. vii (1963), pp. 97 ff.

problem of storing three functions of N variables, $f_n(p)$, $f_{n+1}(p)$ and $q_n(p)$, the maximizing value of q in (1), when we turn to this formula as a computational algorithm. Proceeding in direct fashion, which is to say storing a function as the set of values it assumes, we see that if each component is allowed k different values, then a total of $3 \times k^N$ values must be stored to determine the function $f_n(p)$ in sequence starting with

$$f_1(p) = \max g(p, q) \tag{2}$$

The storage problem becomes even more complicated if one has to handle risk and uncertainty which are usually included in investment problems. Bellman[1] supposes that some progress can be made by approximation in function space, in policy space, and in structure space. However, very little has been done so far. Furthermore, in order to approach the structure of the real world the calculation model has not to be only dynamic but integer also for the solution of investment problems on the farm level, since it is not practicable to build a stable for one cow in the year t and for the second cow in the year $t+1$.

Although intensive research is under way in many places in the world, no solution which is satisfactory from the methodological point of view as well as in application has become known to the author. A few attempts at using integer programmes for the solution of farm problems have been made.[2] The first approaches to the use of dynamic models for the solution of simple problems have also been published.[3] The problems of combining dynamic and integer programming have not yet been solved. At least no solution is known to the author. As long as these problems remain unsolved, researchers who are confronted with investment problems will have to use such pragmatic approaches as parametric programming in combination with the comparison of different optimum programmes.

In spite of a number of problems remaining unsolved considerable progress has been made in applying quantitative methods on the micro-economic level. One might say, we have arrived at the beginning of a period in which the newly developed tools will be available to facilitate decision making even at the farm level. Two ways

[1] R. Bellman, 'Some Directions in Dynamic Programming', op. cit.

[2] R. M. Peat, *The Use of Integer Solutions to Linear Programming for Optimizing a Materials Handling System*. Purdue University, Lafayette Mimeo, 1959. C. Edwards, 'Using Discrete Programming', *Agricultural Economics Research*, vol. xv (1963), pp. 49 ff.

[3] W. G. Smith, *Dynamic Linear Programming of Conservation Alternatives*, Ph.D. Diss. Iowa State University, 1958. W. G. Smith and E. O. Heady, *Use of a Dynamic Model in Programming Optimum Conservation Farm Plans on Ida Monona Soils*, Iowa State University, Agricultural and Home Economics Experiment Station, Research Bulletin 475 (1960). O. R. Burt and J. R. Allison, 'Farm Management Decisions with Dynamic Programming', *Journal of Farm Economics*, vol. xlv (1963), pp. 121 ff.

of making modern achievement available to farmers can be recognized:

(1) The simplification of the calculation procedure by making available programme planning methods which can be handled with desk calculators at least by advisors.

(2) The extension of the calculation procedures, in the way described in this paper, and division of labour between advisers, farmers, and a regional computing centre.

Problems of Application on the Aggregate Level

Modern quantitative methods have been applied on the aggregate level of the production side in supply analysis, in regional analysis, in projections, and in formulating decision models for policy purposes. Among these fields, supply analysis undoubtedly is the key problem since the problems in the other fields cannot be solved without adequate knowledge of the supply functions.

Supply analysis. In spite of much effort the problem of measuring the quantitative effect of a change of prices on production has not been solved yet. Bucholz, Judge, and West recently published 'A Summary of Selected Estimated Behaviour Relationships for Agricultural Products in the United states'.[1] Looking at the examples presented, which could easily be extended by examples from other countries one is inclined to say that, with some notable exceptions, most of the work is measurement without a sufficient theoretical background. Far from being solved the main problems are often rather ignored than attacked.

The aggregation problem. Admittedly, macro-economic supply functions are generally based upon the assumption that supply is homogeneous of zero order with respect to prices; that is to say, supply is subject to changes of price relations and not of prices. The specific form of the supply function, however, is almost always arbitrarily selected.

The determination of the shape of the supply curve is a problem of aggregation. The following ways have been proposed to solve it:

(1) Direct deduction of empirical aggregate supply functions from the individual functions of which they consist. This is impossible in general owing to the lack of suitable observations.

[1] H. E. Bucholz, G. G. Judge, V. I. West, *A summary of Selected Estimated Behaviour Relationships for Agricultural Products In the United States*, Res. Rep. A.E.R.R.—57, Urbana, Ill. 1962.

(2) Deriving supply functions from marginal cost curves. The marginal cost curves are either determined by cross-section analysis[1] or they are derived from a representative programming model.[2]

These supply functions have the following qualities:

(1) They have normative character and are the equivalent of empirical functions only if economic principles are properly observed and the optimum output is reached at each price.
(2) The supply functions are short-run curves. That is to say, in accordance with the assumptions of the economic models used they are valid only if the amount of quasi-fixed production factors is independent of changes in the price relations.
(3) The aggregate supply function can only be directly derived from the average or representative firm if the firms in the various groups are absolutely homogeneous with regard to the data, which determine the parameters of the marginal cost function.

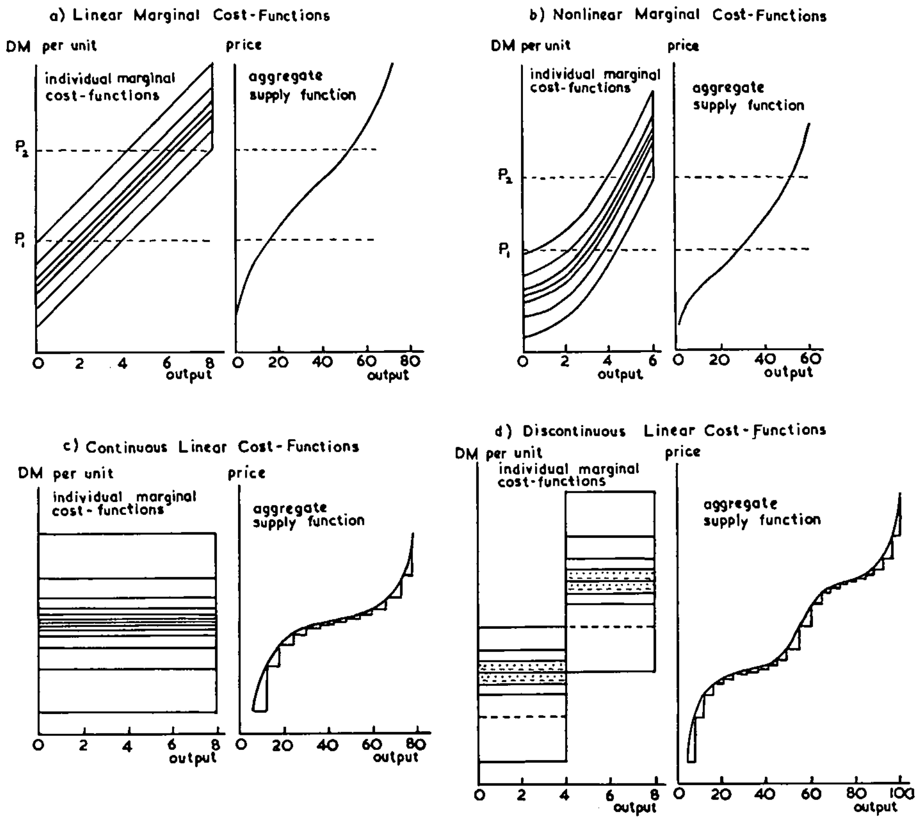
Since the condition under (3) does not usually occur in the real world, the variance of the variables which determine the individual marginal cost curves has to be taken into account in order to derive the normative aggregate supply curve. This requires, in general, a laborious computational process in which the shape of the individual marginal cost curves, and the distribution function of the factors which determine marginal costs at a given output per farm, have to be taken into account. For special cases, however, considerable simplification is possible.

Figure 1 indicates that, under neo-classical assumptions, the shape

[1] E. W. Kehrberg, 'Determination of Supply Functions from Cost and Production Functions' in E. O. Heady, C. B. Baker, H. G. Diesslin, E. W. Kehrberg, S. Staniforth, *Agricultural Supply Functions*, Ames Iowa State University Press, 1961. E. W. Kehrberg, *An Example of Estimating a Supply Function through the Use of Data Obtained from a Cross Section of Individual Farm Cost Relations*. Paper delivered at the annual meeting of the Gesellschaft für Wirtschafts- und Sozialwissenschaften des Landbaues, Stuttgart (Okt. 1961).

[2] D. E. Mckee and L. D. Loftsgard, 'Programming Intra-Farm Normative Supply Functions' in E. O. Heady, C. B. Baker, H. G. Diesslin, E. W. Kehrberg, S. Staniforth, op. cit. G. W. Ladd and E. V. Easley, *An Application of Linear Programming to the Study of Supply Responses in Dairying*, Agricultural and Home Economic Experiment Station, Iowa State College, Research Bulletin 467 (1959). D. Krenz, E. O. Heady, and R. V. Baumann, *Profit-Maximizing Plans and Static Supply Schedules for Fluid Milk in the Des Moines Milkshed*, Iowa State University of Science and Technology, Agricultural and Homes Economics Experiment Station, Research Bulletin 486 (Okt. 1960). G. L. Johnson, 'Budgeting and Engineering Analyses of Normative Supply Functions' in E. O. Heady, C. B. Baker, H. G. Diesslin, E. W. Kehrberg, S. Staniforth, op. cit., pp. 171 ff.

I. Form of individual and corresponding aggregate supply functions



of the aggregate supply function corresponds to the shape of the individual functions from which it is derived if no new firm is entering the market and if a further increase of production is possible in every firm which has already entered the market.

Furthermore, Day has proved that linear programming models require the following assumptions about the aggregates:[1]

(1) Proportional variation in the constraint vectors (including fixed, quasi-fixed, behavioural and policy bounds).

[1] R. H. Day, 'On Aggregating Linear Programming Models of Production', *Journal of Farm Economics*, vol. xlv (1963), pp. 797 ff. H. Theil, *Linear Aggregation of Economic Relationship*, Amsterdam, 1954. N. Georgescu-Roegen, 'The Aggregate Linear Production Function and its Application to the Neumann's Economic Model' in Koopmans (ed.), *Activity Analysis of Production and Allocation*, New York, 1951.

(2) Proportional variation of net return expectations (including proportional variation of input–output relations and price expectations).

Since allowance for these assumptions permits about as much freedom in stratification as is enjoyed by 'men in a chain gang (who) are not altogether constrained since they can wiggle their toes at will'[1] it seems necessary in order to meet these assumptions, at least, that farms will need to be stratified into much smaller aggregates than they are now.

The problem of the use of quasi-fixed factors. In most cases regression analysis or other related methods form the core of the econometric models which have been evolved for supply analysis. It is assumed in that case that: (*a*) changes in supply which are due to price changes are reversible and that (*b*) the price supply function is a continuously rising function.

However, Cassels stated as early as 1933 in his famous paper 'The Nature of Statistical Supply Curves':

> Capital once fixed in a specialized form cannot quickly be withdrawn, and entrepreneurs committed to a particular line of production will commonly continue to produce even when the price they receive does little more than cover the direct costs of operation. If the producers have alternative products to which they can turn, or if they are sufficiently conscious of their common interests to restrain production 'for fear of spoiling the market', the supply will be more sensitive to price declines but even in these circumstances there is no reason to suppose that the process of contraction will be an exact reversal of the process of expansion. It seems to me, therefore, that each supply curve must be regarded as relating to an established level of output and should be recognized to have two distinct parts, one representing expansion beyond that output and the other representing contraction below it.[2]

In other words: If quasi-fixed factors form part of the input, the supply function is irreversible in the sense that a given constellation of parameters, which cause an increase in production, do not cause an equal decrease of production when effective in the opposite direction with the same intensity. Using regression analysis one has to distinguish between the effect of an increase of prices on supply and the effect of a decrease of prices on supply.[3]

[1] R. H. Day, op. cit.

[2] J. M. Cassels, 'The Nature of Statistical Supply Curves', *Journal of Farm Economics*, vol. xv (1933), no. 2, pp. 384.

[3] H. W. Halvorson, 'The Supply Elasticity for Milk in the Short Run', Ibid., vol. xxxvii (1955), pp. 1186 ff.

The Problem of the uncertainty of price expectations. The problem of the uncertainty of price expectations originates in the time necessary for production. It is evident that farmers are influenced in their decisions not by the price at the beginning of the production period, but by the price expectations for the end of it.

As regards empirical analysis in this connexion there now arises the problem of finding the price which determines the decisions of farmers. There exist three different methods for the determination of this price.[1]

(1) *The extrapolative expectation model.* This represents the most common approach in agricultural supply analysis. It is founded upon the assumption that expectational variables can be directly identified with some past actual value of these variables for instance:

$$P^*_t = P_{t-1} \quad (3)$$

or

$$P^*_t = P_{t-1} + \alpha(P_{t-1} - P_{t-2}) \quad (4)$$

(2) *The adaptive expectation model.* In the extrapolative model the values of the variables for the year t are derived from the values of these variables during the past one or two years. In the adaptive model, however, the value of the variables are derived from a weighted mean of *all* prices of the past. According to the adaptive model, expectations are revised periodically by some portion of the error between expectations of the last periods and what actually occurred.

$$P^*_t - P^*_{t-1} = \beta\,(\bar{P}_{t-1} - P^*_{t-1}) \quad (5)$$

It can be shown that this model leads to a representation of the expected price as the geometrically weighted moving average of past prices.[2] Muth[3] has shown that adaptive expectations are optimal, in the sense of being good forecasts, only if the time series to be forecasted is the result of two kinds of random components, one lasting a single period and the other lasting through all the subsequent time periods. Since this condition is generally not fulfilled, Muth proposes an estimation model, which he calls 'the rational expectation hypothesis'.

[1] I summarize here chiefly Marc Nerlove's statements in his article 'Time-Series Analysis of the Supply of Agricultural Products' in E. O. Heady, C. B. Baker, H. G. Diesslin, E. W. Kehrberg, S. Staniforth, op. cit.

[2] The problems of estimation of β are broadly discussed in Marc Nerlove, 'Distributed Lags and Demand Analysis; U.S. Department of Agriculture', *Agricultural Handbook* no. 141, 1958.

[3] J. F. Muth, *Optimal properties of exponentially weighted forecasts of time series with permanent and transistory components*; Carnegie Inst. of Tech. O.N.R. Res. Mem. no. 64, 1959. *Rational expectations and the theory of price movements*; Carnegie Inst. of Tech. O.N.R. Res. Mem. no. 65, 1959.

(3) *The rational expectations hypothesis.* The rational expectations hypothesis is that 'expectations, being informed predictions of future events, are essentially the same as the prediction of the relevant economic theory'. The rational expectations hypothesis has faced the test of application only to a very limited extent. Attractive as its application is from the point of view of economic theory, the determination of the coefficients of expectation in the individual case is difficult, unless the rational expectations hypothesis coincides with the adaptive or extrapolative hypothesis.

The problem of the influence of uncontrolled input factors. From the existence of uncontrolled input factors such as weather or diseases result two problems in supply analysis:

(1) The existence of risk, which may influence farmers' decisions in a way similar to the way uncertain price expectations do.
(2) The existence of a causal relationship between the value of these inputs in a given year and the output (yield, acreage, yield and acreage).

While the first difficulty can be easily overcome by using averages or estimation models similar to those mentioned in the foregoing section the solution of the second requires measurement of these inputs in units which make it possible to use them as independent variables in estimation equations. Several attempts have been made. They use either a special weather index, like, e.g. Stallings, or they take into consideration rainfall or temperature in certain critical seasons.[1]

The problem of technological change. The problem arising from changes in technology are similar in nature to the problems arising from the existence of uncontrolled input factors. Their solution requires the measurement of technological changes in units which can be used as independent variables in estimation equations. Several attempts have been made to isolate the effect of technological progress.[2]

[1] James L. Stallings, *Indexes of the Influence of Weather on Agricultural Output.* Unpublished Ph.D. thesis, Michigan State University, East Lansing, Michigan, 1959. C. W. Thornthwaite, 'An Approach Toward a Rational Classification of Climate', *The Geographical Review*, vol. xxxviii, no. 1, (Jan. 1948). M. Ezekiel and Karl A. Fox, *Methods of Correlation and Regression Analysis*, 3rd ed., J. Wiley & Sons, New York, 1959. William A. Cromarty, *An Econometric Model for U.S. Agriculture.* Unpublished Ph.D. dissertation, Michigan State University, East Lansing, Michigan, 1959. B. Oury, *A Tentative Production Model For Wheat and Feedgrains In France (1946–1961)*, Ph.D. dissertation, Madison 1963, pp. 58 ff.

[2] L. B. Lave, 'Empirical Estimates of Technological Change in United States Agriculture 1850–1958', *Journal of Farm Economics*, vol. xliv (1962), pp. 941 ff. Vernon W. Ruttan, 'The Contribution of Technological Progress to Farm Output: 1950–1955', *Review of Economics and Statistics*, vol. xxxviii (1956), pp. 64–69. E. Domar, 'On the Measurement of Technological Change', *Economic Journal*, vol. lxxi (1961), pp. 709 ff.

In most of these investigations technological progress is treated as a more or less homogeneous factor. However, for the purposes of supply analysis it seems useful to distinguish between:

(*a*) *biologically technical* progress. This is a kind of progress through which primarily the yield per ha. or the yield per animal (kg. milk per cow, number of eggs per hen, &c.) is increased. Its introduction is generally connected with the use of new varieties, new breeds or new animals of the same stock. Biological technical progress is usually taken into account by introducing a trend variable into the estimation equation.

(*b*) *Mechanically technical* progress. Primarily this kind of progress renders possible an increase in labour productivity at a given or reduced yield per ha. or per animal. This progress is usually effected by the introduction of new machines or by the improvement of buildings. Thus, it is usually accompanied by an increased investment in quasi-fixed production factors. Therefore it cannot be handled in the *conventional* estimation models.

The following conclusion can be derived from the existence of quasi-fixed factors, of uncertainty of price expectation and of technological change. There exist non-price factors which are difficult to measure but which have a specific influence on the change of output. Their influence can be taken into account by introducing the level of output of one or more of the preceding years as an independent variable in the estimation model.

Recursive programming. On this idea the application of recursive programming is based.[1] Recursive programming may be described as an attempted synthesis of linear programming analysis and time-series analysis. It rests on the specification of *flexibility contraints*, which specify that only a limited change from the production of the preceding year can be expected. The fact that production response is, or might be, irreversible in the sense mentioned above, could be considered by introducing lower and upper limits of change or flexibility coefficients separately for an increase and for a decrease of output. If one combines the recursive programming model with one

Zvi. Griliches, '"Hybrid Corn", An Exploration in the Economics of Technical Change', *Econometrica*, vol. xxv (1957), pp. 501–22. A. W. G. Kopejan, *Growth of Arable Productivity, especially by Plant Breeding*. Paper read at the E.C.E. conference on Methods of projecting agricultural production, Geneva, Oct. 1961.

[1] R. H. Day, 'Recursive Programming', in E. O. Heady, C. B. Baker, H. G. Diesslin, E. W. Kehrberg, and S. Staniforth, op. cit., pp. 108 ff. R. H. Day, 'An Approach to Production Response', *Agricultural Economics Research*, vol. xiv (1962), pp. 134 ff. J. M. Henderson, 'The Utilization of Agricultural Land, A Theoretical and Empirical Inquiry', *Review of Economics and Statistics*, vol. xli (1959), pp. 242–59.

of the price expectation models mentioned previously one could develop a dynamic solution for acreage and price which might be useful to predict the time-path of output and prices for certain commodities. The recursive programming model seems to be as difficult in its empirical application as it is attractive from a theoretical point of view. Two principal problems are encountered in the application:

(1) The estimation of flexibility coefficients. It is doubtful whether they can be estimated from time series in general as Day and Henderson proposed.

(2) The requirements relating to the homogeneity of the material mentioned above. The usual position is this: either the aggregate consists of a homogeneous group and no data are available or the data are available and the aggregate consists of a non-homogeneous group.

Even with an optimistic view of the possibility of solving these problems, one must admit that recursive programming is but one of the possible approaches to the solution of the supply problem. Others remain to be considered.

The Nerlove approach. In order to estimate the short run effect of a change of prices Nerlove proposed the following equation:

$$x_t = a + b\, P_{t-1} y + c x_{t-1} + M_t \qquad (6)$$

Though based on different theoretical assumptions the effect of quasi-fixed factors, uncertainty and the like is taken into account by introducing the output of the previous year as an independent variable. A difference equation of higher order which describes the time-path of the output over a certain period could be easily derived from equation (6). It is supposed, however, that both the influence of a change in output and the influence of a price change are reversible. Therefore the use of such an estimation equation probably will lead to good results with respect to the occurrence of the predicted output only if prices have changed mainly in the same direction.

The combination of normative micro-economic farm models with statistical expectation models. The above-mentioned theory which treats farm organization consisting of stable and unstable elements suggests the combination of normative micro-economic farm models with statistical expectation models, perhaps of the Markov-Chain-type. Investigations of farms in Lower Saxony by the Institute for Farm Management of the University of Göttingen, have had the following results:[1]

[1] G. Scheller, *Der Einfluß der wirtschaftlichen Entwicklung der Organisation und Ertragslage mittel- und großbäuerlicher Betriebe* (Untersuchung in 242 Betrieben Niedersachsens von 1950/51–1959/60), Göttingen, 1961.

(*a*) For a number of crops the optimal acreage at the prices and the technical knowledge in 1950 equalled the optimal acreage at the prices and the technical knowledge in 1960.

(*b*) The actual acreage gradually approached the optimal acreage of the normative model within these ten years.

In such cases the probability that the aggregate acreage of a certain crop will reach the value x in a certain year, can be determined by means of statistical expectation models. The material available is too limited to prove the hypothesis. Further investigations are under way. This also suggests another approach to the estimation of rates of adjustments on farms. If aggregation is possible without loss of information, it should be possible to find a relation between the calculated *normative* time-path of the production of a region and its observed time-path.

Conclusion. Further intensive investigation is needed in order to determine the usefulness of the different approaches and the range of application of each of them. However, from the growing criticism of conventional estimation methods it seems evident that (1) results of empirical analysis which are based on such broad aggregates as the production of the United States, or the production of France, the United Kingdom, Germany, or the like are very likely to be false;[1] and (2) the neo-classical approach which underlies the use of the conventional estimation methods by simple regression analysis or related methods, will certainly keep its place, but it has to be supplemented by other methods. In each case, one has to determine which approach will be the best according to the given conditions.

With this opinion I am in a poor position to describe the application of modern quantitative methods in other fields of macro-economics, since almost any other application requires an adequate knowledge of the supply function.

Macro-economic Decision Models

Macro-economic decision models aim at the estimation of the effects of a given (intended or expected) change of the economic structure in order to form a quantitative basis for existing policy alternatives. The models of the Cowles Commission type are probably the most ambitious of this kind. The general structure, the underlying assumption, and the statistical problems involved in the application of such models are assumed to be known. Among

[1] See e.g. T. W. Gardner, 'The Farm Price and the Supply of Milk', *Journal of Agricultural Economics*, vol. xv (1962), pp. 58 ff.

the applications, the statistical study of livestock production and marketing by Hildreth and Jarret and Cromarty's attempt to predict the impact of alternative government programmes on the wheat and feed-livestock economies may be mentioned.[1] Since both models include the estimation of supply functions from aggregate data of the U.S. economy our doubts on the empirical results have been expressed already. However, an evaluation merely on this basis would be inappropriate. The authors are conscious of the shortcomings themselves.[2] In our opinion, not the results but the sharpening of the theoretical discussions which follows from the precise formulation of the assumptions must be considered as the most important gain. It is pioneer work which shows the direction in which further research will be needed. In this respect, one of the more important insights is that the level of aggregation in space has to be broken down. Changes of structure happen in space as well as in time. Therefore, at least regional models and probably, in most cases, regional models with distinct treatment of size groups within regions, must be applied in order to gain results on which policy decisions may be based.

Regional analysis. The introduction of space in economic theory has certain parallels to the introduction of time. Transportation costs are introduced instead of interests ('transportation costs in time') and activities in different regions must be distinguished instead of activities in different time periods.[3] The computational burden of quantitative regional analysis grows rapidly in gradually approaching the structure of the real world. Since a general formulation of the problems involved includes the consideration of space and time simultaneously the development of quantitative regional analysis is seriously restricted by computational limitations (storage capacity, &c.) and by the lack of sufficient data.

Adjusting the formulation of the problem to the available data and computational facilities requires in each case a compromise with the theoretical principles and the relevance of possible generalizations. The extent of the necessary compromises is indicated by the following problems which must be faced: (1) the definition of the region, (2) the consideration of the non-homogeneous structure of the region, (3) the introduction of opportunity costs for the use of

[1] C. Hildreth and F. G. Jarrett, 'A Statistical Study of Livestock Production and Marketing', in *Cowles Commission Monograph* No. 15, New York and London, 1955. W. A. Cromarty, *Predicting the Impact of Alternative Government Programs on the Wheat and Feed-Livestock Economies*, Michigan State University, Agricultural Experiment Station, Technical Bulletin.

[2] C. Hildreth and F. G. Jarrett, op. cit., p. 3.

[3] E. v. Böventer, *Theorie des räumlichen Gleichgewichts*, Tübingen, 1962, p. 12 ff.

fixed (and sometimes quasi-fixed) factors, (4) the possible non-linearity of the production functions, (5) the introduction of technological and structural changes, (6) the problems of determination of meaningful supply functions which have already been discussed in detail, (7) stability problems arising from discrete producing units and the irreversibility of the regional supply functions.

The definition of the region. The theoretical spatial equilibrium can be separated into continuous and discrete spatial models. The former (Thünen on the production side, Weber, and Isard) assume an infinite, the latter (Lefeber) a finite number of production (and consumption) points.

In agricultural production the continuous model has to be considered as the best approximation of the structure of the real world. However, the quantitative models usually must assume a finite number of production and consumption points for computational reasons each representing one predetermined region. The predetermination of regions may lead to possible misinterpretation of results. In continuous spatial models, the separation of surplus and deficit regions, and thus the flow of products is determined by the solution of the continuous equilibrium model. In a discrete spatial model the separation of surplus and deficit regions and thus the flow of products can be heavily influenced by the delineation of the regions. Thus, the usefulness of results with respect to practical application depends on an adequate predetermination of regions. (It is to be noted that there exist solutions for the calculation of continuous spatial models.[1] However, they require data on the spatial distribution of production and consumption which are usually not available in detail.) While the problems 1 and 2 are encountered in the application of all known regional models the problems mentioned under 3, 4, and 5 are handled differently in the various regional models.

Transportation Models

The logical structure of problems which can be solved by the application of transportation models have been described by Enke:

> There are three (or more) regions trading a homogeneous good. Each region constitutes a single and distinct market. The regions of each possible pair of regions are separated—but not isolated—by a transportation cost per physical unit which is independent of volume. There are no legal restrictions to limit the actions of profit-seeking traders in each region. For each region the functions which relate local production and local use

[1] M. Beckmann, 'A Continuous Model of Transportation', *Econometrica*, vol. xx, no. 4 (1952), pp. 643.

to local price are known, and consequently the magnitude of the difference which will be exported or imported at each local price is also known. Given these trade functions and transportation costs, we wish to ascertain: (1) the net price in each region, (2) the quantity of exports or imports for each region, (3) which regions export, import, or do neither, (4) the aggregate trade in the commodity, (5) the volume and direction of trade between each possible pair of regions.[1]

Transportation models are best suited to short-run problems in which the supply and the demand of the various regions are given. The input of all factors is predetermined; only the flow of goods which minimize total transportation costs is to be determined. Since the transportation model makes no allowance for opportunity costs or, more precisely, since they assume opportunity costs to be the same in all regions, only single commodity problems can be handled. The modified transportation model of the Enke–Samuelson, Beckman type, in which total consumption (or supply) within regions is endogenously determined, given predetermined production (or demand) along with a demand function (supply function) for each region, has been applied in agriculture in most cases.[2]

Input–Output models. Input–output models may be considered as the opposite extreme with respect to fixity of resources. However, the conventional input–output models can provide only a description of interrelationships among geographic sectors or among the agricultural and non-agricultural sectors of the economy. The main limitation of the application of the models in interregional competition is the impossibility of measuring how changes in supply relations of one region will affect the output of another. Instead of reviewing the competitive relations between regions it stresses the complementarity between them with respect to the satisfaction of a given demand. Furthermore, its application is limited because of the difficulties of taking into consideration the effects of a structural or technological change and of approximating the existence of non-linear input–output relations. It may be because of these reasons that only a few pilot studies of the application of input–output models are known.[3]

[1] S. Enke, 'Equilibrium among Spatially Separated Markets; Solution by Electric Analogue', *Econometrica*, vol. xix (1959), p. 41.

[2] K. A. Fox, 'A Spatial Equilibrium Model of the Livestock-Feed Economy in the United States', Ibid., vol. xxi, no. 4 (1953), pp. 547. T. D. Wallace and G. G. Judge, *Econometric Analysis of the Beef and Pork Sectors of the Economy*, Oklahoma State University, Technical Bulletin, T—57, 1958. G. G. Judge and T. D. Wallace, *Spatial Price Equilibrium Analyses of the Livestock Economy*, Oklahoma State University, Technical Bulletin TB—78, 1959.

[3] G. A. Peterson and E. O. Heady, *Application of Input–Output Analysis to a Simple Model Emphasizing Agriculture*, Iowa State College, Agricultural Experiment Station,

A multiple product approach with linear programming. The description of the logical structure of spatial equilibrium problems mentioned above could be modified as follows:

There are three or more regions capable of producing a certain set of goods. Each region constitutes a single decision-making unit and a single distinct market for each of the goods produced. The regions of each possible pair of regions are separated but not isolated by a transportation cost per physical unit which is independent of volume and which may or may not differ for each good. For each region the input–output relation for each good produced, the prices of variable factors and the amount of fixed resources are known. The production costs of each good may or may not differ between regions. Furthermore, the demand for each region is assumed to be given at the first stage. Later on a demand function may be introduced.

Given these data we wish to ascertain: (1) the quantity produced of each good in each region, (2) the net price of each good, (3) the quantity of imports and exports of each good, (4) the volume and direction of trade between each possible pair of regions with each good.

The solution is to minimize total costs (transportation and production costs).

This structure differs from the structure which underlies the use of the transportation model mainly in one point; instead of a fixed production, a fixed set of resources is assumed for each region. The model for the determination of the solution is a general linear programming model which combines the properties of the transportation model with the properties of a production model.

A model of this general type could be used as follows:

A spatial equilibrium model could be computed which maximizes the returns to fixed factors of production or minimizes total production costs using such a model; the effects of changes of demand on output and prices could be derived. The effect of changing production restrictions could be tested. Furthermore the effects of structural or technological changes in all or some regions could be computed by parametric methods.[1] However, the answers have a strictly normative

Research Bulletin 427, 1955. H. O. Carter and E. O. Heady, *An Input–Output Analysis Emphasizing Regional and Commodity Sectors of Agriculture*, Agricultural and Home Economics Experiment Station, Iowa State University, Research Bulletin 469, 1959. J. H. F. Schilderinck, *De Betekenis Van De Pluimsveehouderij Voor De Nederlandse Economie in 1958*, Landbouw-Economisch-Instituut Den Haag, Studies no. 4, 1963.

[1] A. C. Egbert and E. O. Heady, 'Interregional Competition or Spatial Equilibrium Models in Farm Supply Analysis', *Agricultural Supply Functions*, A. C. Egbert and E. O. Heady, *Regional Adjustments in Grain Production*. U.S. Department of Agriculture,

character according to the assumptions of the underlying model. The problems mentioned above are implied by the assumptions under which the regional supply functions are derived. In order to take into account at least the implications which arise from the existence of quasi-fixed factors of production, one could possibly introduce a recursive programming model in which the amount of change in a certain time period is limited separately for an increase and a decrease of supply in each region. The assumptions with respect to the homogeneity of the structure of the regions which are necessary in the application of such a model have been discussed earlier in this paper. It seems evident that they are not valid in general. Therefore other principles of dis-aggregation have to be taken into account. It is very likely that there exist comparative advantages with respect to size and to managerial abilities which are of the same order of importance as the comparative advantages with respect to location, at least under certain conditions. The simultaneous consideration of more than one principle of dis-aggregation creates serious theoretical and computational problems which are still far from being solved. Therefore one might say, recent quantitative research shows that economists have become aware of the importance of the problem of dis-aggregation, but only some preliminary steps have been made towards solving it. Much theoretical and empirical work remains to be done before we arrive at satisfactory solutions. It is almost certain that a considerable amount of the future quantitative work will be devoted to the investigation of ways in which these solutions may be obtained.

E. O. HEADY, *Iowa State University, U.S.A.*

Professor Weinschenck has provided an excellent review of the mathematical tools now available to agricultural economists, and of the research realms in which they are being applied. One can hardly criticize this general coverage since it is well done. Hence, my statements will serve largely as a supplement to the presentation of Weinschenck; although I do wish to raise some questions with respect to points he has included or excluded.

His paper includes (1) a review of recent mathematical approaches being used by, or available to, the agricultural economist and (2) some evaluation of the conditions under which they are, or can be, applied. Since a paper covering these two broad areas tends to become over long, he might have given a somewhat more penetrating insight

Technical Bulletin no. 1241, 1961. E. O. Heady and A. C. Egbert, *Mathematical Linear Programming of Regional Production Patterns*', Ames, Mimeograph.

had he restricted his paper, either to a detailed explanation of the models being employed, or to a criticism of the conditions under which they are applied. Also, it would seem that he has left aside some fairly important or promising approaches which stand on a level with those reviewed. One such is composed of models broadly grouped under operations research. Although operations research is partly represented by the linear programming models which Professor Weinschenck discusses, additional techniques such as queueing are finding some application and have promise for certain problems of agriculture. Simulation models may also present some potential for problems and analyses falling in both the micro and macro realms. While they are implied in his discussion of inserting demand functions into interregional programming models, some rather extensive formulations and early applications of non-linear programming models for interregional competition are now under way. These would appear to have great utility under such institutional and structural changes as those implied in the European Economic Community and in the developmental plans of various countries. His discussion of regression models centres largely on estimation of production and supply functions, where *least-squares* methods are either most applicable or conventional. Techniques which Professor Weinschenck somewhat neglects, are the alternatives to least-squares estimates for demand functions, and similar market relationships. I should like to have heard an evaluation of our present position in respect to simultaneous equations and related procedures for these estimational problems. Given the international character of this Association and the members attending this conference, an inventory and evaluation of investment and developmental models could have proved useful and interesting. Finally, some of the models and approaches which he discusses have now found much wider application and refinement than is implied in his presentation. I must hasten to state, however, that Professor Weinschenck was asked only to write a paper. The points that I have raised could have been handled only had he written a book. Only thus could a complete description of all available models, with their realms of applicability and limitations, have been sufficiently treated.

I wish to emphasize, along with Professor Weinschenck, that so-called mathematical methods in agricultural economics are here to stay. Not only has there been a tremendous mushrooming in the application of these techniques over the past two decades, but there will be an equally, or more, rapid advance in the next twenty years. It is likely that the number of new mathematical tools developed in the next period, will not be as great as that of the past twenty years. Yet certainly the breadth of application and the degree of refinement of those available will exceed those of the past. This prospect has important meaning to the members attending this conference. The mature members of the profession, and especially those at this conference who guide and train young entrants, will and should affect this development through the guidance they provide to students. It is absolutely necessary that both undergraduate and advanced training incorporate the proper amount and appropriate distribution of mathematics and statistics in order that the profession can advance as rapidly as possible along these lines.

Quantitative analysis is not something new to agricultural economics. Of all fields of specialization in the overall discipline of economic science, agricultural economics has always rested largely on quantitative analysis, and more so than other specialized economic fields. The profession has become one of the wider users of econometric methods, and this will continue to be true because problems in agricultural economics are dominant in their demand for quantitative analyses and solutions. The use of modern quantitative methods will grow rapidly because of the power of modern computing facilities, allowing the attack on large scale problems which could not previously be considered. Interregional analysis is an example. Whereas we could only talk qualitatively in previous decades, we can now undertake extremely large-scale models; as in some of our models in Iowa with up to 150 regions, 1,000 equations, 6,000 variables and non-linear objective functions. The capacity of computers also makes possible more realistic and applicable analyses for certain problems and objectives. For example, the linear programming of a farm can be accomplished on a computational scale, which allows consideration of detail and physical and institutional restraints far exceeding that of farm budgeting. We can expect the future to open up new and larger opportunities in this realm. The development of new and refined economic models will lead to an extension of computor programmes to facilitate their application, just as larger-scale computors and computor programmes will open up the need and opportunity for developing more detailed and powerful economic models.

A point of caution still needs to be added to Professor Weinschenck's discussion, however. I wish that he had elaborated on the opportunities and limitations that exist in applying modern quantitative methods or mathematical models in terms of the data available. I believe that the power of models and the capacity of computors, has far outrun the quantity and quality of data to be processed by them. Perhaps we are now at a point in time where we should give much greater weight to the collection of appropriate data, relative to the weight given to tool refinement, so that greater use and utility can be had from the available quantitative models and methods. The opportunity in interregional and developmental analysis, for example, is tremendous and exciting. The models and computor programmes and capacities are available, but appropriate data generally are not. Systematically, we must begin organizing data collection so that it comes abreast, in quality and quantity, of modern quantitative methods. Otherwise, the latter have too little power and meaning for us. The day, perhaps, is past when agricultural economists should be impressed with the simple ability of a person to illustrate the application of a mathematical model to simple illustrative data which have no content in respect to data and problems of the real world.

We have gone through a period in which mathematical models and methods have come to the fore, often overshadowing important quantitative analyses completed with less powerful techniques. Perhaps, there have been too many styles and fads which centre around the question, 'Here is a new mathematical model or tool. Where can I use it?', rather than around the question, 'Here is an important problem, what is the most appropriate model or tool with which to solve it?' I would guess that future research and emphasis will revolve around the merging of priority problems and modern quantitative methods. When this is true, we shall have become a more mature applied science. Certainly it is true that modern quantitative methods and mathematical models are, among other sciences, raising the stature of agricultural economics as a science and improving its ability to serve agricultures and societies. These opportunities will progress most rapidly if concurrently we move ahead on the three fronts: of refining models to fit the problems of agriculture, of improving the data available for their use, and of applying them to the problems which are most relevant and pressing.

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This subject is very large, if not too large. This has already been indicated both by Professor Weinschenck and Professor Heady. However, I want to add more: the numerous studies on the *demand* side. Further, within the treatment of the macro-economic decision models there could have been included a general statement on the necessity to take into account, in one way or another, other sectors of the economy. As an example, I am here thinking of the experiments going on within the 'Rural Areas Development Program' in the U.S.A. to include all the relevant economic sectors in growth models for these areas. These remarks are not so much directed to Professor Weinschenck. They are an appeal to the programme committees of our future conferences to divide up the field of quantitative research into its many subgroups. Quantitative research has reached such a stage of development that the opportunities for this Association to promote future progress should preferably proceed by means of such a subdivision.

Professor Weinschenck gives two factors which have influenced the rapid increases of quantitative methods in agricultural economics. I am hesitant about the first factor, which is called the *discovery* of the mathematical core of the theory of economic equilibrium. I am more inclined to think of this as the formulation in mathematical terms of the economic man's way of seeing his production problem within available scarce resources. Moreover, I hesitate to say that this factor, introduced in 1870, is of specific importance for the *recent* development of quantitative research. Of more importance here is Professor Weinschenck's point on the development of electronic computers. I want also to add the following factors of importance for the recent development of quantitative research in agricultural economics: (*a*) the extensive quantitative econometric work going on within general economics (Econometrics) and business economics (Management Science), and (*b*) the new schooling of agricultural economists (*Heady*, 1953).

An important part of the paper deals with the problems of estimating production functions from experiments. This section will be treated here as covering the problem of estimating input–output data for individual products or branches of production. Understood in this way I should like to add to Professor Weinschenck's list of important problems the following two points: (which logically come before his three points): (i) the choice of basic material; and (ii) the choice of quantitative method. So far as (i) is concerned, economists have turned from farm accounts to experimental data to get better information. Experimental data are, as a rule, inadequate but in a different way from farm accounts. In the procedure of randomizing factors which have to be excluded from influence in the biological experiments, many factors, important for the economists, remain uncontrolled (unobserved). This is true for example for meteorological factors, variations in soil conditions on the farm, effects of the edges of the fields, variations in time and methods of harvesting, &c.

The choice of basic material for estimation of input–output data for individual products has, therefore, to contain a discussion of how these factors can be included. It may very well be profitable to include in the production function data not only experimental data but also data from farm accounts, soil maps, meteorological statistics, and official yield statistics. An important initiative for action in this field has lately been taken by the O.E.C.D. On the second point, up to now the methods to choose from in estimating input–output relationships are regression analysis (production functions) or analysis of variance (point estimates). Professor Weinschenck discusses extensively the production function calculations but finally calls these calculations in question and advocates the use of point estimates, these being more useful for planning purposes. Being not yet convinced, I still prefer the production function approach as being the cheapest way to get information on the beginning and the end of the rational 'Part II' of the production function, and on input–output relations which can easily be adjusted to variations in price relations. Moreover, point estimates are also influenced by uncontrolled variables. Thus, they cannot in 'a simple way be taken from experiments directly', unless the majority of the problems discussed by Professor Weinschenck for production function analysis are already solved.

Professor Weinschenck states, and I agree, that the choice of the function should be determined by the nature of the technical relations involved. However, he says thereafter that this limits (more or less strictly) the application of production function analysis to the crop sector. This, then, means a discrimination against the livestock sector which I do not understand and should be very glad to have clarified.

The paper also indicates that the choice of the *type* of function is comparatively easy and that it is governed by the goodness of the fit. I think this is to give in to the statistical viewpoint a little too early. As we all know, different functions have different characteristics as to number of turning points, the shape of the isoclines and the isoquants and assume constant or variable production elasticities. This should be taken into account before the dubious goodness of the fit approach is used.

Professor Weinschenck also indicates the important problem of transmitting to practical farming the results gained from experiments. I understand this not as an extension service question but as the problem of transforming information *from* the experimental plot or group (where many factors are nicely randomized according to

Fisher's principles, but unfortunately uncontrolled) *to* the farm as a whole, and preferably to other farms. Understood in this way I think that this is one of the more important fields for future work in extending our knowledge of input–output relations, no matter what methods are used to systematize these relations. The gap between what is happening on the experimental plot and on the farm as a whole is very wide indeed.

Among the applications of quantitative methods on the *macro* level Professor Weinschenck first discusses supply analysis. In his treatment of the aggregation problem various ways of constructing supply functions are discussed as well as the requirements that the basic material has to fulfil to give results free of objections. I will not go into a discussion on these points. I want to state only that it is time to go from words to things in the aggregation problem. This means going down from the etherial sphere of theory to laborious empirical studies. In them the structure of individual farms is examined and the farms are thereafter aggregated in proper ways. A start on this line is now being made in Hungary by Mr. Joseph Sebestyen at the Agricultural Economics Research Institute of the Hungarian Academy of Sciences.

Professor Weinschenck summarizes his discussion on quasi-fixed factors, uncertainty of price expectations and technological change by mentioning that non-price factors influence the level of output and that the influence of these non-price factors can be taken into account by introducing the level of output of some preceeding year or years in the estimation model. The factors mentioned may be of a non-price character but they can all be looked upon as economic factors, i.e. factors which can be expressed in economic terms, in some kind of calculation of incomes minus costs. Personally I think this is to stop a little too early in the analysis when measuring the effects of these factors by introducing some earlier level of output.

In this connexion I should like to ask the specialists on supply analysis how they include in their models a phenomenon that we in Sweden call the 'Horndal-effect' (named after the factory where it was first shown in a study on productivity changes). This effect is the rise in total output (supply) due to reorganization within available resources, all other factors remaining unchanged. The result of this is that one gets an increase in output without being able to register any change in the volume of inputs.

The last part of Professor Weinschenck's paper contains a discussion of macro-economic decision models (regional analysis, transportation models and input–output models). The reason why only few studies have been made as yet, lies probably on a much less sophisticated level than the one he mentioned, namely in the mass of data to be collected and handled and in the lack of relevant basic data material, especially for regional analysis within countries, as trade statistics between intra-country regions seldom exist.

Finally, I should like to indicate some problems in regional analysis beyond those mentioned by Professor Weinschenck. These are problems that occupy us in Sweden in connexion with a study on location of our agricultural production carried out with a linear programming model. In this study Dr. Birowo of Indonesia is carrying the heaviest load of work and responsibility. He may intervene on the subject.

The problems I should like to mention are the following:

(*a*) Handling of the interaction between prices and supplies of various goods. Here I mean the effects that the changes in total supply and in regional distribution of this supply will have on the prices used in the model to calculate profit and cost elements.

(*b*) The linking up of the model, which is normative in character, with the actual development. This is partly a problem of combining it with Day's recursive programming, but is also a dynamic problem, a problem over time, of estimating, in micro and macro, the economically most profitable speed with which development in indicated directions should and will go.

(*c*) The checking of results of the regional analysis with other studies (actual development, local micro-studies).

(*d*) The proper way of analysing the results of the model—for example, to what extent is a difference between actual location or development trends and the results of the study of a real need to change the regional production pattern, or from errors in the basic data or the assumptions, or does it arise from the structure of the model as such?

This last point may be looked upon, however, not as a difficulty or a possible reason for rejecting the approach as such, but as an important possibility and starting-point for acquiring a better understanding of the factors influencing the agricultural production. It would thus indicate that quantitative research could increase our chances of recommending realistic means of supporting a profitable adjustment of agriculture in various regions. On this point I want once again to stress Professor Weinschenck's opinion that quantitative studies are of special importance owing to the sharpening of the discussion that the precise formulation of the assumptions and models leads to.

FRANCESCO LECHI, *Institute of Economic and Political Affairs, Padua, Italy*

Professor Weinschenck has informed us of the most recent progress on the quantitative models at the micro- and macro-economic levels. On this subject it may be useful to add some minor remarks and to stress some perplexities to which he has drawn attention.

First, as to production functions, one of the more difficult problems is that of the managerial factor. Few definitive efforts have been made to solve it, and these have met with little success. Since the definition of this variable is connected with elements of a sociological as well as economic order it is suggested that closer co-operation between agricultural economists and rural sociologists would be beneficial. Greater attention should be given to the so-called institutional factors in the construction of econometric models, particularly where the maximization of profit is restricted by severe limitations of an extra-economic nature.

Again, many perplexities and difficulties arise when elements of uncertainty are introduced into the decision models. Perhaps it would be useful to improve the definitions distinguishing uncertainty, risk, and subjective uncertainty, both in micro- and macro-economics, and to give proper attention to those definitions. It is unusual indeed to seek the solution of problems concerning uncertainty, when that uncertainty is by definition not measurable.

The review of the studies that attempt to render static models dynamic in order to evaluate long-run investments was of great interest. It may be useful to remember that in a recent contribution the time element was introduced into the models by discount formulae, so as to make possible the development of programmes for three cultivations with poly-annual cycles in Southern Italy.[1]

Also, let us discuss briefly the relationships between research and extension of knowledge to farmers. The introduction into the models of elements relating to uncertainty may allow the entrepreneurs to express more precise evaluations, giving them a larger freedom of choice. Yet at the same time it may give the impression of turning into certainty what is no more than an effort to evaluate the probabilities of uncertainty. It is highly advisable therefore that those who report the results to the farmers be fully aware of the limits of the research and do not narrow, more or less consciously, the freedom of choice of the entrepreneurs. It is they who will pay for any pos-

[1] G. W. Dean, M. De Benedictis, 'A Model of Economic Development for Peasant Farms in Southern Italy', *Journal of Farm Economics* vol. xlvi, no. 2, p. 295.

sible rashness of the advice. Furthermore, one can easily see that, while research has made much progress in the formulation of theoretical models, little has been done in the field of extension. Agricultural economists dealing with these subjects should be aware of the necessity to extend the research for each type of model to a wider number of cases, besides deepening scientific research to improve such models. This is necessary both to give more help to farmers and also to test the effectiveness and profitability of the application of the models.

Finally, as to the decision models of linear programming at the aggregate level, it has to be pointed out that Day's hypotheses very rarely apply in reality, as Day himself suggested. Therefore, the optimal solution of a farm aggregate very rarely corresponds to the sum of the optimal solutions of single farms, as demonstrated in a recent study on a group of similar farms in Italy.[1] This mainly occurs because the factors cannot move easily and rapidly from one farm to another. On the basis of these results, particular care is needed in drawing conclusions from models of aggregates so as not to recommend advice for single farms which might turn out to be unrealistic.

M. BANDINI, *Rome, Italy*

Time does not allow me to comment fully, so I shall speak shortly and certainly dogmatically on four points.

As a result of studies and of many years' experience, I am quite clear that the mathematico-quantitative methods in our studies have a scientific value only when they are applied to the recognition and interpretation of actual economic reality. For instance, I would mention the theory of economic equilibrium (Marshall, Walras, Pareto and, more recently, Schumpeter and Hicks), or the two Leontieff schemes, or the comparative studies of economic efficiency of enterprises. Secondly, I do not agree to the *scientific* value of the mathematico-quantitative method application in *normative* function (whether individual, collective, regional, or political). Professor Weinschenck's paper reinforces my convictions. During the last ten years I have written a great deal about this argument but, for the reasons so well explained by Professor Westermarck, I cannot affirm that my contributions are well known. Thirdly, the examples of mathematical application in a normative function, so largely followed in the world,

[1] O. Ferro, *Di alcuni problemi derivanti da applicazioni di programmazione lineare in agricoltura* (mimeo).

have created a kind of fantasy in agriculture, like the world which Alice in Wonderland found through the looking glass. It is an attractive and suggestive world of which the only fault is that it does not correspond either to agricultural reality or to its static and dynamic problems and which, further, does not give a proper interpretation of the process of economic development, or gives only a formal interpretation. These are in effect mathematical spells which always seem attractive to students of this science. Fourthly, the importance of these questions is so great that I would very much like our Association to organize meetings, or exchanges of opinions, devoted to these problems of method and economic logic. These are problems which have to be taken into consideration in order to avoid many years of useless work because people are not prepared to spend a few months in establishing the bases for their arguments.

R. P. SINHA, *Manchester University, U.K.*

In order to get some of my own ideas clarified, I have a question concerning demand analysis. Can we take goodness of fit as one of the main criteria of the choice of function? We have tried to fit six different functions to consumption data from India. The functions were linear, double log, semi-log, log-inverse, log-log inverse, and simple quadratic. In each case the goodness of fit was nearly the same. The value of $\bar{R}^2$ was somewhere between 0·90 and 0·95. But the income elasticity of demand for food calculated from the different functions varied from 0·45 to 0·75. When we are projecting the demand for food for a future year, the differences in the estimates of income elasticity of this magnitude make a good deal of difference to the estimates of the projected demand. What objective criteria would Professor Weinschenck suggest for the selection of the function?

A. BIROWO, *University of Uppsala, Sweden*

Based on my experiences in interregional programming studies in Swedish agriculture, I should like to make the following comments. Professor Weinschenck stated that multiple-product programming technique would specify, among other things, the *net price* of each good. His concept of net price is confusing, because it could be misunderstood as the *net return* of each activity to be optimized. Since his intention is to state the *shadow price* or the *dual solution*, I would suggest that he use the term *marginal value* of the parameters. For the cost minimizing models, the marginal value of each good represents *marginal-cost-supply* prices, whereas in the profit-maximizing model it is the *marginal-revenue* supply prices. Secondly, Professor Weinschenck states that there is *one* main point of difference between the transportation and the multi-product model. That is, in the multi-product, instead of a fixed production, a fixed set of resources is assumed for each region. This is a misleading statement. Even in multi-product models production can be fixed and resources determined by the model, by using a cost-minimizing approach. The general difference is that in the transportation technique only one commodity or group of commodities can be considered, and that the flexibility of changing the parameters is less than with multi-product models. Thirdly, for the developing countries, the application of economic methods for solving our problems is indeed very challenging. How would Professor Weinschenck estimate the prospects and opportunities which are open and the limitations which are imposed upon the application of modern quantitative approaches in solving agricultural economic problems in low-income countries?

G. ROTTIER, *C.R.E.D.O., Paris, France*

Some very brief remarks, first of all to thank Professor Weinschenck for an outstanding paper, whose text we should like to see elaborated so full is it of riches. I must, however, make a fundamental objection to the title of this paper: the term 'quantitative methods'. The subject of our discussion here is in fact nothing other than economic theory in the form it has now assumed, and any references to the course of it—I speak primarily as a specialist in demand, since I am much less acquainted with the problems of supply and production—couched in older terminology and in particular in the classic terminology of Marshall's partial equilibrium, are just about as fertile as the decadent scholasticism of the sixteenth century.

I have reservations on the use of the term 'quantitative', for as Professor Weinschenck has pointed out, the quality of the basic data is such—and Professor Klatzmann will insist upon this point at the beginning of his exposition, if he follows the text which we have received—that we are rarely in a position to quantify the results obtained by the application of this modern economic theory which Professor Weinschenck has so remarkably summed up. The principal effort demanded of applied economists (and this we were taught years ago by the first texts on the subject by the Cowles Foundation), is to dedicate themselves to the boring, tedious, and lengthy task of improving the basic data from which we work.

In fact the core of my remarks is that this 'new' economic theory is not really so very new. It is in a more powerful form, the economic theory we have always known even if adapting ourselves to it involves a painful intellectual effort for those many of us who are over thirty. This theory is being applied, then, to problems which we have not sufficiently *defined*—and I speak as a practitioner called upon to assist planners. What seems to me to be one of the most important problems to examine now is the precise definition of the questions we should put to the theorist and to the econometrician.

Let us take a very simple example, since I do not wish to speak for long. There has been mention of 'decision models', and this occupies several pages of Professor Weinschenck's text, but we are never told what decision is meant, who takes it, and to what purpose. In our studies we never distinguish sufficiently between the different levels of decision, the different optimum levels, and the possible conflicts between the decisions corresponding to these different optimum levels. In particular—to be as brief as possible—in a linear programming scheme, I wonder whether we give enough attention to the definition of the economic function we seek to optimize, or to the interpretation, in terms of the categories of the real world, of this economic function. Professor Koopmans has written some notable things on this subject in one of his 'Three Essays on the State of Economic Science' pointing out the difference which exists between the conceptual categories of the theorists and the categories of the real world. In the same way when we go on to the dual form of a linear programme, we do not give sufficient care to the interpretation, in terms of the real world, of the prices of efficiency obtained. I speak as a practitioner. For this point of view I think that what Professor Weinschenck has described is simply economic theory as it should only now exist. But the effort to quantify it is a very long-term effort which should have priority. This is not of course the job for all of us; only some of us are public statisticians. On the other hand it is the duty of all of us to define in terms of policy decision, at whatever level, the problems we shall put to the theorist, and to interpret the answers received from him.

GÜNTHER WEINSCHENCK (*in reply*)

Discussion of all the points which have been raised in the very useful comments would require a new paper, probably more extensive than the one I have already given. I will therefore only mention those points which appear most important.

I will begin with Professor Bandini's comment because of its general importance. In this context it might be useful to distinguish between mathematical economics and econometrics. The purpose of mathematical economics is to explain the structure of the economic world or more precisely to explain the most important characteristics of the economic world. The purpose of econometrics is either to describe the most important characteristics of economic problems of a (usually) limited scope or to formulate decision models in order to improve or to facilitate the selection of alternatives with respect to an 'optimal solution' of special economic problems. In the latter sense Professor Bandini is right, when he casts heavy doubts on the value of econometric models using normative functions. (To avoid misunderstanding normative is used here as a short cut for assumed behaviour of economic variables, which have been neither proved nor tested in reality.) However, as a first step it might be useful to apply normative functions, perhaps more than one function, each assuming a different behaviour of the variables. The work which has been done in supply analysis and in regional analysis is a first step and everybody should realize this. As Professor Renborg has already said, the most important gain is not the empirical result but the sharpening of the theoretical discussion which follows from the precise formulation of the assumptions.

The limitation of production function analysis to the crop sector does not of course mean a discrimination against the livestock sector. One must hesitate before using production function analysis here because of the technical conditions which have been reported from investigations in animal nutrition. Due to their findings we have linear relations between the output and the input of protein, net energy, and other nutrients of minor importance. The capacity of each animal to produce a certain output per time unit and the capacity to consume a certain input per time unit are limited. Furthermore, the input of protein, net energy, and other nutrients is a linear function of the feed input of a given composition. Finally, there is no substitution possible between protein and net energy or any other essentials of the feed ration. In my opinion these are not the conditions to be analysed by production function analysis. Of course the results which have been gained in animal nutrition may be wrong. However, I think it is beyond the competence of any normal economist to prove that they are wrong. Point estimates of input–output relations are, of course, influenced by uncontrolled variables. However, one could take care of it by randomising the output point estimates within a certain range. This method is neither elegant nor precise, but it does not pretend to accuracy in a situation characterized by inaccuracy.

I agree with Mr. Lechi's point about precise definitions for situations with incomplete information. Perhaps the following is acceptable: Risk defines a situation where a probability function can be derived from the available data and where the requirement of the law of great numbers are met. Uncertainty defines a situation where only a certain range can be defined which separates the possible events from those of which the occurrence is considered to be very unlikely, if not impossible. Within the 'uncertainty-range' defined in this way the probability of occurrence of the events cannot be defined by objective estimation. Only subjective estimations are possible.

It is one of the basic characteristics of economic analysis that a model which assumes a fixed set of resources asking for the optimum level and composition of production can be used to analyse a situation where a fixed level and composition of production is assumed and the level and composition of inputs at the minimum cost level is under investigation and all resources are assumed to be variable. But with respect to the interregional immobility of important production factors in agriculture this seems to me like looking at the tail wagging the dog.

In conclusion, I think that the logic of economic problems is the same in under-developed countries as elsewhere in the world. The main problems in applying quantitative methods are the lack of appropriate data, and perhaps especially the consideration of different behaviour or objective functions. This is no theoretical problem but a question of empirical research which has to be done in the countries themselves.