Nonparametric Analysis of Cost Minimization and Efficiency

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For presentation at the AAEA Annual Meeting
August 2-5, 1998
Salt Lake City, Utah

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Abstract

Abstract: This study examines cost minimizing behavior and efficiency measures for a sample of

Kansas wheat producers, using nonparametric methods. As such, it verifies the applicability of

nonparametrics within a production economic framework.

Key words: nonparametrics, WACM, production

Nonparametric Analysis of Cost Minimization and Efficiency

The assumption of cost minimization is often adopted when modeling economic behavior of farm operators. If a farm demonstrates cost minimization behavior, the input bundle it employs produces its output at lowest cost, given all feasible input/output possibilities for the farm. Parametric models of cost minimization must assume a functional form for the underlying technology, cost function, or profit function. Choice of functional form may influence results from parametric tests due to the imposition of relationships between variables inherent in any specified form. Failure of hypothesis tests involving parametric models is necessarily a failure of a joint hypothesis, one half of which is that the functional form chosen is the correct one (Lim and Shumway). Because the true functional form is unknown, relying on the assumptions of a parametric model is limiting (Chavas and Cox). Nonparametric analysis allows determination of the bounds of firm behavior consistent with cost minimization, without assuming any functional form. It utilizes a minimum number of restrictions on the technology, such as concavity, and monotonicity in prices, but does not specify exact technological relations between inputs and outputs (Ray and Bhadra).

Several measures of efficiency can also be tested using nonparametric analysis, such as technical, allocative, and cost efficiencies. Technical efficiency is defined either in an "output-anchored" way, as the use of the smallest possible input set to produce a given level of output, or in an "input-anchored" manner, as the production of the largest possible output for a given level of inputs. Allocative efficiency includes prices in the analysis, and is defined as the use of inputs and production of outputs such that the firm is operating in a cost minimizing way, given the prices it faces. Cost efficiency is measured by looking at the ratio of minimum cost to actual cost.

This study uses nonparametric methods to analyze the validity of an assumption of cost minimization behavior, as well as efficiency measures, for a sample of Kansas wheat producers.

Nonparametric analysis

There are often two parts to a nonparametric analysis. The deterministic part of the analysis is the most commonly utilized nonparametric test. This consists of an exhaustive search over all observed outputs and inputs to check for violation of a behavioral objective, which in this study is the objective of cost minimization. The Weak Axiom of Cost Minimization (WACM) states cost minimization behavior is present, given the technology, if the observed input bundle minimizes costs over all bundles that can produce at least the observed output, y^i (Varian 1984). Therefore, the deterministic test is a search over all observed outputs and inputs to check for violation of WACM. In equivalent terms, it is a test for the following condition:

$$\text{If } y^j \leq y^i \text{, then } w^j x^j \leq w^j x^i,$$

where y is observed output, w is input price, x is input level, and i and j are observations. When the above condition holds, the production function is known to be continuous, monotonic, and quasiconcave. When it does not hold, then observation j violates WACM.

The stochastic part of the analysis allows for the possibility violations may be due to measurement error in the observations. In the case where violations are found, they are tested for statistical significance. An acceptable level of error due to measurement is decided upon, and the standard error of the sample is found. If the standard error is at or below the acceptable measurement error level, the possibility is allowed that violations found may be due to measurement error in the data. Varian (1985) introduced this widely accepted method for stochastic analysis. As an example, the following steps are used to carry out Varian's method for a study of cost minimization behavior.

A quadratic programming problem must first be solved, such that,

$$R = \min \sum_{j=1}^{n} \sum_{k=1}^{m} (\phi_{jk} / x_{jk} - 1)^2$$

subject to

$$\sum_{k=1}^{m} w_{jk} \phi_{ik} \geq \sum_{k=1}^{m} w_{jk} \phi_{jk}, \ \forall i = 1,2,...,n, and \ y^j \leq y^i,$$

where R is the minimum sum of squares of the deviations of quantities that satisfy WACM, relative to observed quantities (Hseu and Buongiorno), w_{jk} is the cost of input k for the jth firm, x_{jk} is the actual observed quantity of input k for the jth firm, and ϕ_{jk} is the solution to the above problem, representing ideal (with respect to cost minimization) quantity of input k for firm j. If the variance of measurement error (σ^2) in the data set were known, then the hypothesis of adherence to cost minimization in the sample would be rejected if $R / \sigma^2 > \chi^2_{\alpha,nxm}$, or $\sigma^2 < R / \chi^2_{\alpha,nxm}$, where n is number of observations, m is number of inputs, and χ^2 is a chi-square statistic with α and (n×m) degrees of freedom. Therefore $\bar{\sigma}$ equals $100\sqrt{R / \chi^2_{\alpha,nxm}}$. The hypothesis of cost minimization is rejected at the α-level of significance when $\bar{\sigma}$ obtained through this method is greater than the percent of measurement error expected (or acceptable) in the data. In other words, $\bar{\sigma}$ is the percent of measurement error that must be accepted in order not to reject the hypothesis of cost minimization.

Efficiency measures

The scale for measuring technical, allocative, and cost efficiency is between zero and one, with a measure of one implying that the firm in question is efficient. For any one firm,

multiplication of its technical efficiency measure by its allocative efficiency measure yields the measure of cost efficiency. A cost efficiency rating of one implies that a firm is also both technically and allocatively efficient.

The following is derived from Paris' presentation. Farrell first theorized about measurement of efficiency among firms through tests of assumptions modeled with linear programming techniques. He reasoned if all individual firms within a given sample are assumed to follow the same production function, linearly homogeneous in inputs, then a unit-output isoquant can be defined for the technology, representing the set of input-output combinations that are characterized by perfect technical efficiency. Any firm with input/output ratios represented by a point strictly above this isoquant is technically inefficient. The ratio of the distance along a ray, from the origin to the unit isoquant and from the origin to the new isoquant is a measure of technical efficiency. That is, in Figure 1, x_i = inputs (normalized by output); i=1, 2, and **gf** = the unit-output isoquant. The point **S** represents the input-output ratios for firm Z - a function of Z's input usage. Since all points on **gf** represent technically efficient input-output ratios, firm Z is inefficient; it uses more input per unit output than is technically efficient. The index of firm Z's technical efficiency can be defined as $TE = \dfrac{OB}{OS}$.
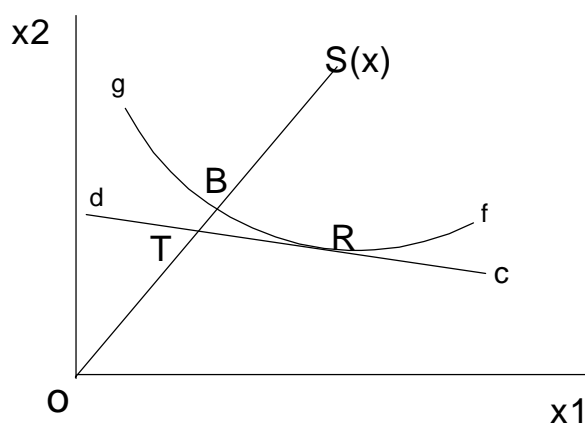
Figure 1. Technical and allocative efficiency.

Allocative efficiency is also shown in Figure 1, through use of a unit-output isocost curve, **dc**. All points on **dc** represent efficient price combinations for inputs producing a unit output (which says nothing about technical efficiency). The tangency between **gf** and **dc**, **R**, is a point of allocative efficiency, where both price and technical efficiency exist. The index of allocative efficiency can be defined as $AE = \dfrac{OT}{OB}$. Cost efficiency, being the combination of technical and allocative efficiency, is represented as $CE = \dfrac{OB}{OS} x \dfrac{OT}{OB} = \dfrac{OT}{OS}$.

Farrell saw a distinct linear programming (LP) problem for each firm in a given sample could be formulated to obtain efficiency indices for each firm. The primary task is to define the unit-output isoquant. For example, in a two input, three firm sample, there would be three

separate LP problems to solve. As in Paris's demonstration, let firms be numbered by $j = 1,2,3$.

Then the primal representation of the LP problem is:

$$\max V_{oj} = v_1 + v_2 + v_3$$
$$s.t.$$
$$a_{11}v_1 + a_{12}v_2 + a_{13}v_3 \leq a_{1j}$$
$$a_{21}v_1 + a_{22}v_2 + a_{23}v_3 \leq a_{2j}$$
$$v_k \geq 0, k = 1,2,3$$

where v_k = the producible output quantity for the kth firm, given the inputs available to the jth firm;

a_{ij} = use level of the ith input by firm j, normalized by output for firm j. The input quantities of

firm j produce a unit-output isoquant, due to the normalization of input quantities by output. If j

= 1, and firm 1 is efficient, then the LP problem will be optimized at $v_1 = 1$; v_2, $v_3 = 0$, and the

objective function value will be $V_{01} = 1$. However, if firms 2 and 3 are more efficient than firm 1,

then the solution will be $v_1 = 0$; $v_2 + v_3 > 1$, with the objective function value, $V_{01} > 1$. In this case,

combining the technical processes of firms 2 and 3 with the input usage levels of firm 1 allows

production of output greater than the unit quantity. This problem can also be solved from the

dual side, represented by:

$$\max W_{oj} = a_{1j}w_1 + a_{2j}w_2$$
$$s.t.$$
$$a_{11}w_1 + a_{21}w_2 \geq 1$$
$$a_{12}w_1 + a_{22}w_2 \geq 1$$
$$a_{13}w_1 + a_{23}w_2 \geq 1$$
$$w_1, w_2 \geq 0$$

In the dual representation, minimum usage of inputs to produce the unit-output is sought.

As in the primal, when the jth firm is technically efficient , then $W_{0j} = 1$; when it is inefficient, then

$W_{0j} > 1$.

Thus the unit-output isoquant of Farrell's problem is defined by all points representing normalized-input output ratios for firms within the current sample that are technically efficient, as determined through the above LP problem.

Problem Setting and Analysis

A sample of 107 observations from the Farm Management Data Bank was analyzed for adherence to a cost minimization objective, as well as cost efficiency. The data were collected from 1973 to 1990, on single-output Kansas wheat producers, ranging in harvested acreage from 121 to 3545 acres, and include output level, input levels and input prices for each farm observation. Selection criteria for observations in the sample were 95% or more of row crop acreage in wheat production, and 90% or more of farm income resulting from wheat sales. Land, fertilizer, pesticide, machinery, and miscellaneous factors are the five inputs used in the analysis. Available technology is assumed to be the same for all farms in the sample, over the span of years represented. Analysis is carried out for both constant and variable returns to scale.

To test the assumption of cost minimization, violations of WACM are sought among all observations. A cost matrix is constructed by multiplying a matrix of observed input levels from each farm by a matrix of observed input prices faced by those farms. Each row corresponds to one farm, with its observed cost being the element on the diagonal of the matrix. Since observations are ordered by increasing level of output, then by construction, when violations of WACM occur, they appear to the right of the diagonal element in the row, as an element smaller than the diagonal element itself.

The study draws on the work of Fare, Grosskopf, and Lovell (1985, 1994) and Chavas and Aliber, who expanded on Farrell's original thesis. They use the input requirement set to represent technology (under constant returns to scale) as follows:

$$T = \left\{ (y,-x): y \le \sum_{i=1}^{n} \lambda_i y^i , x \ge \sum_{i=1}^{n} \lambda_i x^i , \lambda_i \in \Re^+ , \forall i \right\},$$

where x is a vector of inputs, y is a vector of outputs, and λ is a scaling factor.

The set T (technology) is closed, convex, and negative monotonic. Chavas and Aliber use it to obtain the Farrell technical efficiency index for each firm through the linear programming problem:

$$TE(y^i, x^i, T) = \min_{k,\lambda} \left\{ k_i : y^i \le \sum_{j=1}^{n} \lambda_j y^j , k_i x^i \ge \sum_{j=1}^{n} \lambda_j x^j , \lambda_j \in \Re^+ , \forall j \right\}$$

A measure for allocative efficiency can be derived from the above measure, using minimized cost:

$$AE(r, y, T) = C(r, y, T) / (r^{'}(TE)x),$$

where $C(r, y, T)$ is minimum cost, obtained from the following linear programming problem:

$$C(r, y, T) = \min_{x,\lambda} \left\{ r^{'}x : y^i \le \sum_{j=1}^{n} \lambda_j y^j , x \ge \sum_{j=1}^{n} \lambda_j x^j , \lambda_j \in \Re^+ , \forall j \right\}$$

Results

The joint hypothesis of a concave, monotonic technology and adherence to a cost minimization objective for the entire sample of farmers is overwhelmingly rejected through the deterministic test. Eighty-three percent of the observations demonstrated one or more violations of cost minimization. This is further clarified by stating that for all 107 observations there were 5671 possible violations (the total number of elements to the right of all diagonal elements in the cost matrix), and 812 actual violations, or approximately 14.3%. Average size of violation is approximately 14.7%. In other words, where there is a violation in this study, the firm's cost is on average about 15% higher than minimum cost, as defined through WACM.

After finding WACM violations in the sample through the deterministic test, a stochastic test was conducted to determine whether these could be due to measurement error. A level of 10% was chosen as the maximum acceptable for possible measurement error in the data, based on historically accepted levels of measurement error in other nonparametric analyses involving agricultural data (Lim and Shumway, Williams). A critical value for standard error was then determined in the data, following the method of Varian (1985). This is a lower bound for the true standard error in the data. Having set the acceptable level of measurement error at 10%, if the lower bound is calculated from the data to be above 10%, then the standard error in the data is greater than what can be attributable purely to measurement error. For the Kansas wheat farm sample, the critical value (the lower bound) was calculated to be approximately .3529 (at $\alpha = .05$), suggesting that a minimum standard error of 35% would have to be accepted in order to attribute violations found to measurement error. With acceptable level set at 10%, though, it is unjustified to believe measurement error alone could account for the violations of WACM in this sample. The violations must be deemed truly behavioral in nature.
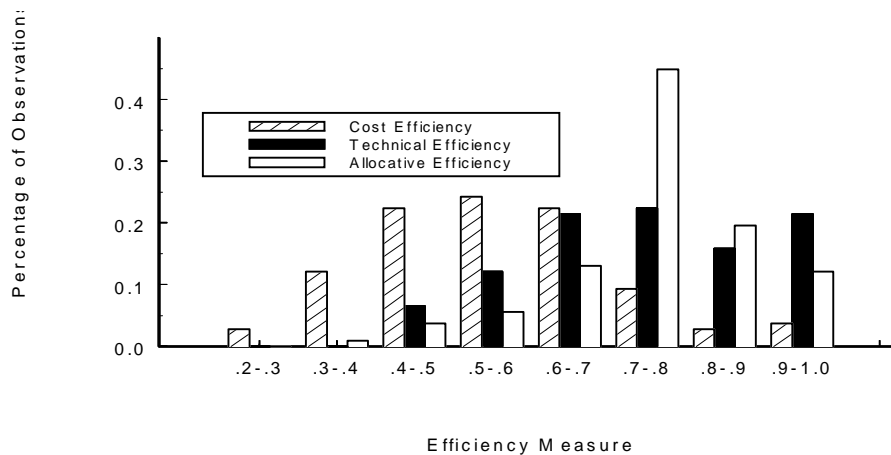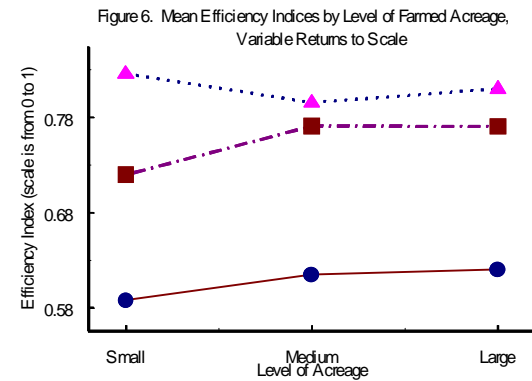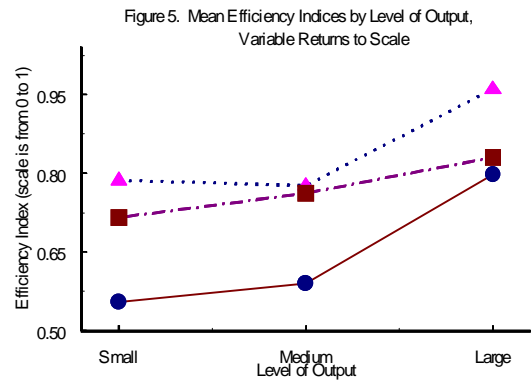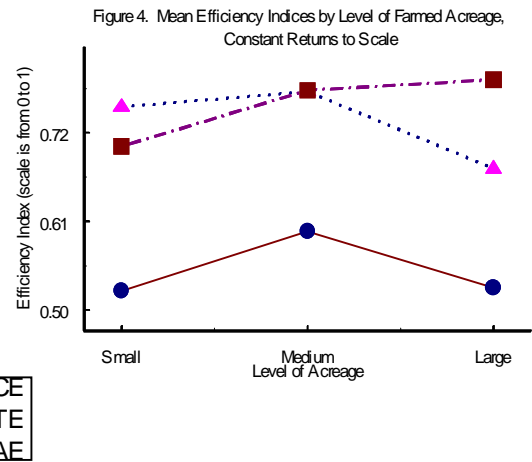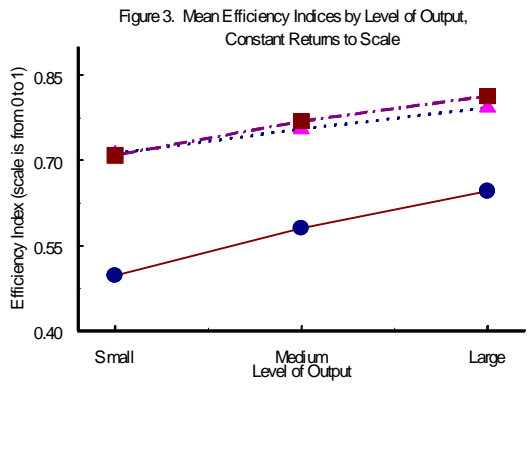
Figure 2. Efficiency measures found for Kansas wheat farm sample.

The efficiency indices found for each farm are summarized in Figure 2. Sixteen of the 107 observations demonstrated technical efficiency. One example of cost efficiency was found - in the earlier graphical analysis, this could be represented by point **R** in Figure 1: where allocative and technical efficiency both equal 1. This observation was for a farm of 1354 acres, with annual output of approximately 165,747 bushels of wheat, and relatively low usage levels for fertilizer, machinery, pesticides and miscellaneous inputs. As evident from Figure 2, the majority of farms are not technically nor allocatively efficient, suggesting that improvement may be possible in this regard at the farm level. However, 48% of all observations showed between 70% and 80% allocative efficiency, and another 21% demonstrated 80%-90% allocative efficiency.

In Figure 3, output level is divided into three categories. "Small" refers to farms with output from 10,000 to 100,000 bushels, "medium" defines farms producing between 100,000 bu. and 200,000 bu., and "large" refers to farms with production between 200,000 bu. and 415,000 bu. Mean efficiency levels are calculated for each category, for each index. This graphic infers that, in general, under the assumption of constant returns to scale, efficiency levels rise as the level of output increases. Results are shown in Figure 4 with respect to farm size, and divisions of small (up to 1000 acres), medium (from 1000 to 1500 acres), and large (over 1500 acres), are utilized. Here allocative efficiency rises as farmed acreage increases. Technical efficiency, however, rises slightly as acreage goes from small to medium, then decreases again as acreage goes from medium to large. The resulting cost efficiency measure, a product of the above two efficiencies, also rises between the small and medium categories, then falls as acreage goes from medium to large. Under variable returns to scale, all efficiencies rise with level of output, as seen in Figure 5. Technical efficiency falls between small and medium land size, but then rises slightly from medium to large land size. Both cost and allocative efficiencies appear to rise with size of

acreage under variable returns to scale. Econometric analysis showed no statistically significant relationship between technical efficiency and output level, under both constant and variable returns to scale. Land size, however, was a statistically significant determinant of technical efficiency when entered quadratically or in log form.

Figure 3. Mean Efficiency Indices by Level of Output, Constant Returns to Scale

Figure 4. Mean Efficiency Indices by Level of Farmed Acreage, Constant Returns to Scale

Figure 5. Mean Efficiency Indices by Level of Output, Variable Returns to Scale

Figure 6. Mean Efficiency Indices by Level of Farmed Acreage, Variable Returns to Scale

Conclusions

While certain farms in the Kansas wheat farm sample did not violate WACM, 83% of observations demonstrated one or more violations. This suggests that behavior of Kansas wheat farmers in the sample is not consistent with a primary objective of cost minimization. Efficiency measures were variable among observations.

Conclusions are tempered by the knowledge that some problems are present in this study. First, no data was available for a labor input variable. Its absence in the data is a major limitation

to the applicability of this research. Secondly, the 107 observations are treated in the study as independent from each other, however, some of the observations included are for farms already included in the sample, but at a different year. Treating all observations as independent may cause some results to be overstated.

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