# Some Implications of the Two-Constraint Joint Recreational Choice Demand Model 

Daniel K. Lew<br>Department of Agricultural and Resource Economics<br>University of California, Davis

May 1998

For presentation at the AAEA Annual Meeting
Salt Lake City, UT August 1998
(C) 1998 by Daniel K. Lew. All rights reserved. Readers may make verbatim copies of this document for noncommercial purposes by any means, provided that this copyright notice appears on all such copies.

## Some Implications of the Two-Constraint Joint Recreational Choice Demand Model*

## Introduction

The recreation demand literature has grown extensively over the years as researchers have explored ways to generalize the popular travel cost method (TCM), both within the model and by proposing new ones. The impetus for such research is the rather restrictive assumptions of the travel cost model, such as assuming that on-site time is fixed (Randall; Freeman). McConnell relaxed the fixed on-site time assumption of the TCM to derive the theoretically consistent demand function when on-site time is endogenous using a model in which recreationists value both trip-making and on-site time. This two-constraint joint recreational choice demand model has also been used by Bockstael et al. and Larson to provide a more general framework for analyzing value of time issues.

The purpose of this paper is to explore the implications of the two-constraint joint recreational choice demand framework as it relates to the estimation of the demand for and benefits of a recreation site when on-site time and the number of trips demanded are endogenous. Recent contributions by Larson and Shaikh to the value of time literature have implications for the structure of demand in two-constraint models and will be applied here. The model differs from the traditional TCM in that it divides up the recreational experience into two separate activities, travel and on-site time, and models them as functions of own- and cross- prices instead of lumping travel and on-site time together as a single own-price. This permits the researcher to analyze a recreation site where both on-site time and the number of trips are endogenous. In this case, it is possible to parametrically define the average days on-site (a). In addition, since two demand functions can be estimated there is a question about the proper approach to welfare measurement.

[^0]Hof and King argue that due to the complementarity between recreational activities (travel, lodging, etc.), the consumer surplus of the recreational experience can be found by integrating over the demand for any of the components of that experience.

In this paper, the appropriate structure of the demand functions for total on-site time and total trips are derived from the joint recreational choice model and used to estimate the demand for recreation at an Alaskan salmon sportfishery. Since both total days on-site and the number of trips are endogenous, average days on-site is parametrically determined and comparative statics of significant variables are determined. In addition, consumer surplus values are calculated.

## Theoretical Framework for Joint Recreational Choices

Larson provides the general modeling framework for the joint recreational choice model. In the model, recreationists are assumed to value recreation trips over some period of time (e.g., a year) to sites $\mathrm{j}\left(\mathrm{r}_{\mathrm{j}}\right)$, total days on-site during that period of time at sites $\mathrm{j}\left(\mathrm{d}_{\mathrm{j}}\right)$, and all other goods represented by a numeraire good $(\mathrm{z})$, where $\mathrm{j}=1, \ldots, \mathrm{n}$. McConnell presents a similar model, but substitutes total days on-site with average on-site time at $j\left(a_{j}\right)$. The two models are linked by the identity

$$
\begin{equation*}
\mathrm{a}_{\mathrm{j}} \equiv \mathrm{~d}_{\mathrm{j}} / \mathrm{r}_{\mathrm{j}} \quad \forall \mathrm{j}=1, \ldots, \mathrm{n} . \tag{1}
\end{equation*}
$$

A consequence of this relationship and the fact that days on-site and trip money and time prices are assumed exogenous is that the linear budgets in the following formulation become non-linear in McConnell.

Construction of the two-constraint joint recreational choice demand model begins by assuming that each recreationist has a finite money budget M and time budget T with which to
allocate among scarce recreational goods defined above ${ }^{1}$. Each recreational trip to site j has associated with it a money price ( $\$ /$ trip) and a time price (how long it takes), denoted $\gamma_{j}$ and $\alpha_{j}$, respectively. To consume the days on-site at any site $\mathrm{j}\left(\mathrm{d}_{\mathrm{j}}\right)$, individuals must pay a money price per day on-site $\left(\delta_{\mathrm{j}}\right)$. Since time prices are measured in days, the time cost of consuming one day on-site is one unit of time. For simplicity, assume that the numeraire good $(z)$ is measured in days such that the time cost of consuming one unit of z is also one unit of time and denote the price of the numeraire good $p^{2}$. Therefore, each individual recreationist seeks to maximize their utility by choosing $\mathrm{r}_{\mathrm{j}}$ and $\mathrm{d}_{\mathrm{j}}$ for all $\mathrm{j}=1, \ldots, \mathrm{n}$ sites subject to a time constraint ${ }^{3}$

$$
\begin{equation*}
\alpha^{\prime} \mathbf{r}+\mathbf{e}^{\prime} \mathbf{d}+\mathrm{z} \equiv \mathrm{~T}, \tag{2}
\end{equation*}
$$

where $\mathbf{r}=$ vector of trips, $\mathbf{d}=$ vector of total days on-site, $\alpha=$ vector of trip travel time, and $\mathbf{e}=$ vector of ones, and a money constraint

$$
\begin{equation*}
\gamma^{\prime} \mathbf{r}+\delta^{\prime} \mathbf{d}+\mathrm{pz} \leq \mathrm{M} \tag{3}
\end{equation*}
$$

where $\gamma=$ vector of trip money prices and $\delta=$ vector of on-site money prices. Assuming utility is quasi-concave, the individual will maximize the following Lagrangian function ${ }^{4}$ :

$$
\begin{aligned}
\operatorname{Max} L & =U(\mathbf{r}, \mathbf{d}, \mathrm{z})-\lambda\left[\gamma^{\prime} \mathbf{r}+\delta^{\prime} \mathbf{d}+\mathrm{pz}-\mathrm{M}\right]-\mu\left[\alpha^{\prime} \mathbf{r}+\mathbf{e}^{\prime} \mathbf{d}+\mathrm{z}-\mathrm{T}\right] \\
& \text { subject to } \mathrm{r}_{\mathrm{j}} \geq 1, \mathrm{~d}_{\mathrm{j}} \geq 0, \mathrm{z} \geq 0 \forall \mathrm{j} .
\end{aligned}
$$

[^1]The Lagrangian multipliers, $\lambda$ and $\mu$, are the marginal utility of income and the marginal utility of time, respectively. For an interior solution ${ }^{5}$ (where the money constraint is binding), the firstorder conditions (FOC) are, after dividing through by $\lambda$,

$$
\begin{align*}
& \mathrm{U}_{\mathrm{rj}} / \lambda=\gamma_{\mathrm{j}}+(\mu / \lambda) \alpha_{\mathrm{j}}, \quad \forall \mathrm{j}=1, \ldots, \mathrm{n}  \tag{4}\\
& \mathrm{U}_{\mathrm{dj}} / \lambda=\delta_{\mathrm{j}}+\mu / \lambda, \quad \forall \mathrm{j}=1, \ldots, \mathrm{n}  \tag{5}\\
& \mathrm{U}_{\mathrm{z}} / \lambda=\mathrm{p}+\mu / \lambda, \tag{6}
\end{align*}
$$

where the expression $\mu / \lambda$ is the marginal (scarcity) value of time and $U_{x}$ denotes the partial derivative of the utility function with respect to the variable x (where x represents the choice variables, $\mathrm{r}_{\mathrm{j}}$, $\mathrm{d}_{\mathrm{j}}$, or z ). (4) - (6) represent the key results of Larson. In general, they state that individuals will consume until the marginal value of consuming another unit equals the explicit marginal cost of another unit plus the value of time necessary to consume the additional unit.

Since the marginal time value of consuming each good is added to the money value, the righthand side of (4) - (6) can be interpreted as the full prices faced by individuals. Moreover, recent work by Larson and Shaikh has shown that for this two-constraint model, a general consequence of duality restricts the structure of the arguments of demand functions derived in a two-constraint problem to be expressed in full prices $^{6}\left(\gamma_{j}+\rho \alpha_{j}, \delta_{j}+\rho, p+\rho\right)$ and full incomes $(M+\rho T)$. Together with (2) and (3), these conditions imply that it is possible to estimate two Marshallian demand equations for each site j :

$$
\begin{align*}
& \mathrm{r}_{\mathrm{j}}=\mathrm{r}_{\mathrm{j}}(\gamma+\rho \alpha, \delta+\rho \mathbf{e}, \mathrm{p}+\rho, \mathrm{M}+\rho \mathrm{T}), \forall \mathrm{j}=1, \ldots, \mathrm{n}  \tag{7}\\
& \mathrm{~d}_{\mathrm{j}}=\mathrm{d}_{\mathrm{j}}(\gamma+\rho \alpha, \delta+\rho \mathbf{e}, \mathrm{p}+\rho, \mathrm{M}+\rho \mathrm{T}), \forall j=1, \ldots, \mathrm{n} \tag{8}
\end{align*}
$$

[^2]where we let $\rho$ represent the scarcity value of time, $\mu / \lambda$. If we started with the dual problemwhere the recreationist minimizes expenditure (based on full prices) subject to a fixed level of utility, $\mathrm{U}_{0}$-we would arrive at two Hicksian, or income-compensated, demand functions.

Several points should be made at this point. First, the manner in which $\rho$, the scarcity value of time, is incorporated in the estimation is a matter of considerable debate. Some contend that $\rho$ is an exogenous parameter which can be represented as the average wage rate or some fraction of it (Cesario; Smith et al.; McConnell and Strand) which could be arbitrarily assigned or estimated. Others argue that the value of time at the margin (when individuals trade time for money) is the appropriate realization of $\rho$ and set forth empirical methods to elicit this value (Bockstael et al.; Larson; Larson et al.).

Second, parametrically estimating total days on-site and total trips from the incomplete demand system allows the researcher to parametrically define the average on-site time, $\mathrm{a}_{\mathrm{j}}$, by dividing (8) by (7). This allows average on-site time to be expressed as

$$
\begin{equation*}
\mathrm{a}_{\mathrm{j}}=\mathrm{a}_{\mathrm{j}}(\gamma, \delta, \alpha, \mathrm{p}, \rho, \mathrm{M}, \mathrm{~T}) \quad \forall \mathrm{j}=1, \ldots, \mathrm{n} . \tag{9}
\end{equation*}
$$

This equation can be used to provide insight to park managers and other recreation managers who wish to know what would happen to average lengths of stay on-site if, for example, gas prices, entrance fees, or on-site snack bar prices were to be raised.

A third point relates to welfare measurement. In some cases, researchers are interested in the value of a recreational experience (Bell and Leeworthy; Hof and King). In a model with two choice variables and a single site $(\mathrm{n}=1)$, the total compensating variation $(\mathrm{CV})$ is obtained by integrating over each Hicksian demand function (denoted $\mathrm{D}(\cdot)$ and $\mathrm{R}(\cdot)$ ) with respect to own prices from the initial prices to the price at which Hicksian demand goes to 0 . Two possible price paths for the calculation of total recreation CV are the following:

$$
\begin{align*}
& \mathrm{CV}\left(\gamma_{0} \rightarrow \gamma_{\mathrm{c}}, \delta_{0} \rightarrow \delta_{\mathrm{c}}\right)=\int_{\gamma_{0}}^{\gamma_{\mathrm{c}}} \mathrm{R}\left(\gamma, \delta_{0}, \mathrm{p}, \mathrm{U}_{0}\right) \mathrm{d} \gamma+\int_{\delta_{0}}^{\delta_{\mathrm{c}}}  \tag{10}\\
& \mathrm{D}\left(\gamma_{\mathrm{c}}, \delta, \mathrm{p}, \mathrm{U}_{0}\right) \mathrm{d} \delta  \tag{11}\\
& \mathrm{CV}\left(\delta_{0} \rightarrow \delta_{\mathrm{c}}, \gamma_{0} \rightarrow \gamma_{\mathrm{c}}\right)=\int_{\delta_{0}}^{\delta_{\mathrm{c}}} \mathrm{D}\left(\gamma_{0}, \delta, \mathrm{p}, \mathrm{U}_{0}\right) \mathrm{d} \delta+\int_{\gamma_{0}}^{\gamma_{\mathrm{c}}} \\
& \mathrm{R}\left(\gamma, \delta_{\mathrm{c}}, \mathrm{p}, \mathrm{U}_{0}\right) \mathrm{d} \gamma
\end{align*}
$$

where $\gamma_{0}$ and $\delta_{0}$ represent the initial prices and $\gamma_{c}$ and $\delta_{c}$ represent the choke prices such that $\mathrm{R}\left(\gamma_{\mathrm{c}}\right.$, $\left.\delta, \mathrm{p}, \mathrm{U}_{0}\right) \equiv 0$ and $\mathrm{D}\left(\gamma, \delta_{\mathrm{c}}, \mathrm{p}, \mathrm{U}_{0}\right) \equiv 0 . \gamma, \delta$, and p are full prices and $\mathrm{U}_{0}$ is the reference utility level.

Bowes and Loomis argue that the second term in (10) should always be 0 when it is too expensive to travel $\left(\gamma=\gamma_{c}\right)$ because no time is spent on-site (since recreationists cannot reach the site). Therefore, the demand for on-site time must necessarily be 0 for any on-site price ( $D\left(\gamma_{c}, \delta\right.$, $\mathrm{p}, \mathrm{M}) \equiv 0$ ). Hof and King claim that if it becomes too expensive to spend any time at a recreation site $\left(\delta=\delta_{c}\right)$, then recreationists do not get any utility from making the trip there, and therefore would not make the trip in the first place, implying $R\left(\gamma, \delta_{c}, p, M\right) \equiv 0$. According to this joint complementarity argument, the total CV of the recreational experience can be derived from either the Hicksian trips or days demand. Since Hicksian demands are unobservable, CV is approximated by CS using either (8) or (9) (Willig; Randall and Stoll).

There are several potential problems with this approach. If recreationists derive utility (or disutility) from travel (a reasonable assumption), then the Hof and King argument cannot hold ${ }^{7}$. In addition, imposing the complementarity suggested above may require highly restrictive constraints on estimated parameters of the system of demand equations (depending on the functional form chosen). Moreover, as Just et al. point out, whenever there are income effects accompanying price changes, the path-dependent CS values used to approximate CV are not unique and can have substantial discrepancies. These caveats imply that, in general, CS estimates
found by integrating over estimated demand equations in the joint recreational choice model should be interpreted as the CS of the activity in question and not as indicators of the value of the total recreation experience unless the appropriate parameter restrictions are imposed on the estimated system of demand functions.

## Data and Estimation Results

To parametrically define average on-site time and calculate the CS derived from the two demand functions, several common functional forms of demand were estimated for a single recreational site (therefore no substitute prices). The data consisted of recreational fishing data collected through a mail survey of anglers who frequented Willow Creek in Alaska during the summer of $1980^{8}$. The survey elicited detailed trip-specific information related to salmon fishing activities on-site as well as travel cost-related information. Of 324 returned questionnaires, 221 were used to estimate the demand functions in the analysis ${ }^{9}$. Travel costs $(\gamma)$ included lodging and travel operation expenses, but omitted food and drink expenses since it was assumed not to be exclusive to the travel experience. For the same reason, food and drink expenses were omitted from on-site costs $(\delta)$ too. For consistency with the joint recreational demand model, prices and income were augmented to account for the value of time thus creating full prices and income (using both one-third and the full wage rate to represent $\rho$ ). This approach differs from Larson et al. since $\rho$ is imposed on the system instead of being endogenous to it. The numeraire price was determined by dividing the residual income by the residual time. Table 1 presents summary statistics of the 221 observations.

[^3]Table 1: Summary Statistics of Willow Creek Angler Sample

| Variable | Variable Name | Mean | Standard Deviation | Minimum | Maximum |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Total trips | r | 3.8597 | 2.916 | 1 | 20 |
| Total on-site time (in hours) | d | 144.98 | 155.46 | 24 | 1080 |
| Income | M | 29174 | 18899 | 2500 | 80000 |
| On-site cost (\$/hr) | $\delta$ | 0.95 | 2.5451 | 0 | 25 |
| Travel cost (\$/trip) | $\gamma$ | 20.481 | 48.532 | 1.227 | 708 |
| Average trip time (hours) | $\alpha$ | 3.267 | 2.9242 | 0 | 35 |
| $\rho=1 / 3$ wage rate assumption |  |  |  |  |  |
| Full income (in \$/year) | M | 31687 | 20513 | 2660 | 88960 |
| Full on-site cost (\$/hour) | $\delta$ | 4.9101 | 3.1422 | 0.42 | 13.39 |
| Full travel cost per trips (\$/trip) | $\gamma$ | 36.456 | 52.002 | 2.5 | 726.3 |
| Full numeraire good cost (\$/hour) | p | 23.182 | 17.728 | 1.394 | 151.1 |

$\rho=$ wage rate assumption

| Full income (in <br> $\$ / y e a r)$ | M | 36713 | 23755 | 2980 | 106900 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Full on-site cost <br> $(\$ /$ hour | $\delta$ | 14.635 | 9.4317 | 1.25 | 40.06 |
| Full travel cost per <br> trips $(\$ /$ trip $)$ | $\gamma$ | 68.406 | 71.022 | 2.5 | 763 |
| Full numeraire good <br> cost (\$/hour) | p | 25.426 | 17.761 | 1.658 | 116.4 |

## Table 2: Estimated Coefficients of Total Trips and On-site Days Demand

 ( $t$-values in parentheses)| Estimated <br> Coefficient | Linear <br> $(\mathbf{1 / 3}$ wage) | Linear <br> (full wage) | Semi-log <br> $(\mathbf{1 / 3}$ wage) | Semi-log <br> (full wage) | Log-linear <br> $(\mathbf{1 / 3}$ wage) | Log-linear <br> (full wage) |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| $\alpha_{1}$ | 4.8198 | 4.8297 | 1.3823 | 1.3789 | 16.276 | 12.170 |
|  | $(13.49)$ | $(13.70)$ | $(15.81)$ | $(16.01)$ | $(3.73)$ | $(1.602)$ |
| $\beta_{11}$ | -.0081828 | -.0076022 | -.0028785 | -.0026048 | -.32353 | -.28536 |
|  | $(-2.204)$ | $(-2.50)$ | $(-3.166)$ | $(-3.506)$ | $(-4.624)$ | $(-3.753)$ |
| $\beta_{12}$ | .62036 | .13026 | .10265 | .00012433 | .42488 | -.043392 |
|  | $(1.034)$ | $(.3145)$ | $(.6989)$ | $(.001229)$ | $(.8553)$ | $(-.03945)$ |
| $\beta_{13}$ | .058988 | .072278 | .016604 | .021502 | 1.3308 | 1.3702 |
|  | $(2.864)$ | $(2.178)$ | $(3.293)$ | $(2.652)$ | $(6.254)$ | $(4.393)$ |
| $\beta_{\mathrm{M} 1}$ | -.00016017 | -.00011424 | -.000033787 | -.000017799 | -1.8274 | -1.3554 |
|  | $(-1.745)$ | $(-.7442)$ | $(-1.504)$ | $(-.4746)$ | $(-3.617)$ | $(-1.408)$ |
| $\alpha_{2}$ | 164.41 | 167.97 | 4.8141 | 4.827 | 20.657 | -1.0477 |
|  | $(11.10)$ | $(11.25)$ | $(46.55)$ | $(47.07)$ | $(4.018)$ | $(-.1137)$ |
| $\beta_{21}$ | -.25920 | -.22983 | -.0023284 | -.0021747 | -.25298 | -.19673 |
|  | $(-1.683)$ | $(-1.784)$ | $(-2.166)$ | $(-2.461)$ | $(-3.069)$ | $(-2.134)$ |
| $\beta_{22}$ | -16.331 | -86.664 | -.038175 | -.29362 | -.31611 | -3.3638 |
|  | $(-.6564)$ | $(-4.94)$ | $(-.2198)$ | $(-2.44)$ | $(-.5401)$ | $(-2.522)$ |
| $\beta_{23}$ | 11.256 | 17.294 | .03985 | .059691 | 2.3137 | 2.5838 |
|  | $(13.18)$ | $(12.30)$ | $(6.683)$ | $(6.19)$ | $(9.228)$ | $(6.832)$ |
| $\beta_{\mathrm{M} 2}$ | -.006018 | .022371 | -.000028469 | .000072699 | -2.1199 | .67424 |
|  | $(-1.581)$ | $(3.441)$ | $(-1.071)$ | $(1.63)$ | $(-3.561)$ | $(.5778)$ |

Total days ${ }^{10}$ (d) and total trips (r) were estimated using Zellner's seemingly-unrelated regression approach since it is postulated that the error terms of (7) and (8) are correlated. Thus, (12) and (13), (14) and (15), and (16) and (17) were estimated as incomplete demand systems. Table 3 contains the estimated coefficients under both the full and partial wage assumptions.

$$
\begin{align*}
& \mathrm{r}(\gamma, \delta, \mathrm{p}, \mathrm{M})=\alpha_{1}+\beta_{11} \cdot \gamma+\beta_{12} \cdot \delta+\beta_{13} \cdot \mathrm{p}+\beta_{\mathrm{M} 1} \cdot \mathrm{M}  \tag{12}\\
& \mathrm{~d}(\gamma, \delta, \mathrm{p}, \mathrm{M})=\alpha_{2}+\beta_{21} \cdot \gamma+\beta_{22} \cdot \delta+\beta_{23} \cdot \mathrm{p}+\beta_{\mathrm{M} 2} \cdot \mathrm{M}  \tag{13}\\
& \ln \mathrm{r}(\gamma, \delta, \mathrm{p}, \mathrm{M})=\alpha_{1}+\beta_{11} \cdot \gamma+\beta_{12} \cdot \delta+\beta_{13} \cdot \mathrm{p}+\beta_{\mathrm{M} 1} \cdot \mathrm{M}  \tag{14}\\
& \ln \mathrm{~d}(\gamma, \delta, \mathrm{p}, \mathrm{M})=\alpha_{2}+\beta_{21} \cdot \gamma+\beta_{22} \cdot \delta+\beta_{23} \cdot \mathrm{p}+\beta_{\mathrm{M} 2} \cdot \mathrm{M}  \tag{15}\\
& \ln \mathrm{r}(\gamma, \delta, \mathrm{p}, \mathrm{M})=\alpha_{1}+\beta_{11} \cdot \ln \gamma+\beta_{12} \cdot \ln \delta+\beta_{13} \cdot \ln \mathrm{p}+\beta_{\mathrm{M} 1} \cdot \operatorname{lnM}  \tag{16}\\
& \ln \mathrm{~d}(\gamma, \delta, \mathrm{p}, \mathrm{M})=\alpha_{1}+\beta_{21} \cdot \ln \gamma+\beta_{22} \cdot \ln \delta+\beta_{23} \cdot \ln \mathrm{p}+\beta_{\mathrm{M} 2} \cdot \ln \mathrm{M} \tag{17}
\end{align*}
$$

where the error terms are suppressed for the discussion.
The results show that the own-price effect on trips and the constant terms are highly significant for all models, whereas the own-price coefficient on days is highly significant only for the full wage models and statistically non-significant under the $1 / 3$ wage rate models. This implies that the value of time is the crucial factor for determining the effects of on-site prices in this model. The effect of a change in on-site price generally does not affect the number of trips taken as indicated by the $t$-values on the cross price coefficient, $\beta_{12}$. On the other hand, in each of the models except one, the travel price does significantly negatively affect the days variable (gross complements). In addition, the cross-price effect of the numeraire price is not surprisingly highly significant and positive (since it is constructed using the other prices). However, the income coefficient is ambiguous in sign and significance across the models.

From the estimated equations, the average on-site time can be written from (1) as

[^4]\[

$$
\begin{align*}
& \mathrm{a}(\gamma, \delta, \mathrm{p}, \mathrm{M})=\frac{\alpha_{2}+\beta_{21} \cdot \gamma+\beta_{22} \cdot \delta+\beta_{23} \cdot \mathrm{p}+\beta_{\mathrm{M} 2} \cdot \mathrm{M}}{\alpha_{1}+\beta_{11} \cdot \gamma+\beta_{12} \cdot \delta+\beta_{13} \cdot \mathrm{p}+\beta_{\mathrm{M} 1} \cdot \mathrm{M}}  \tag{18}\\
& \mathrm{a}(\gamma, \delta, \mathrm{p}, \mathrm{M})=\mathrm{e}^{\left(\alpha_{2}-\alpha_{1}\right)+\left(\beta_{21}-\beta_{11}\right) \cdot \gamma+\left(\beta_{22}-\beta_{12}\right) \cdot \delta+\left(\beta_{23}-\beta_{13}\right) \cdot \mathrm{p}+\left(\beta_{\mathrm{M} 2}-\beta_{\mathrm{M} 1}\right) \cdot \mathrm{M}}  \tag{19}\\
& \mathrm{a}(\gamma, \delta, \mathrm{p}, \mathrm{M})=\mathrm{e}^{\alpha_{2}-\alpha_{1} \cdot \gamma^{\beta_{21}-\beta_{11}} \cdot \delta^{\beta_{22}-\beta_{12}} \cdot \mathrm{p}^{\beta_{23}-\beta_{13}} \cdot \mathrm{M}^{\beta_{\mathrm{M} 2}-\beta_{\mathrm{M} 1}} \quad \text { (semi-log) }} \quad \text { (log-linear) } \tag{20}
\end{align*}
$$
\]

The effect of changing prices or income can be determined by using (21), (22), or (23) ${ }^{11}$

$$
\begin{align*}
& \partial \mathrm{a}(\gamma, \delta, \mathrm{p}, \mathrm{M}) / \partial \mathrm{x}=\left(\beta_{2 \mathrm{x}}-\beta_{1 \mathrm{x}} \mathrm{a}\right) / \mathrm{r} \\
& \partial \mathrm{a}(\gamma, \delta, \mathrm{p}, \mathrm{M}) / \partial \mathrm{x}=\left(\beta_{2 \mathrm{x}}-\beta_{1 \mathrm{x}}\right) \mathrm{a} \\
& \partial \mathrm{a}(\gamma, \delta, \mathrm{p}, \mathrm{M}) / \partial \mathrm{x}=\left(\beta_{2 \mathrm{x}}-\beta_{1 \mathrm{x}}\right) \mathrm{a} / \mathrm{x}
\end{align*}
$$

where x represents any of the exogenous parameters (the full prices and income). Thus, for the linear case, the sign of the effect of an exogenous price or income change on the average length of stay depends both on the effect of the exogenous change on both days and trips and on the average on-site time at which it is evaluated. However, for both the semi-log and log-linear forms, the sign of the change depends solely on the difference between the estimated coefficients. It is important to note that for each functional form, the average on-site value at which the comparative statics are done is critical for determining the magnitude of the expected change brought about by a change in prices or income. In the linear case, the number of trips taken is also an important determinant affecting the magnitude.

For illustration, Table 3 shows the estimated change in average length of stay (in hours) at Willow Creek with an incremental increase in on-site costs, trip costs, and income. Asymptotic standard errors and $t$-values were calculated using the estimated variance-covariance matrices. As expected, when on-site costs increase the models unambiguously predict that average time will decrease. Each model yields different levels of change, however, with the linear model predicting

[^5]the largest changes for changing on-site costs. Since on-site costs are primarily driven by time spent on-site, it is not surprising to see larger predictions for the full wage models. The full wage models all have significant on-site cost and income variables, while travel costs are not significant in any model. McConnell showed how the trip cost has an ambiguous effect on average length of stay. Here, trip costs appear to have little, if any, effect on changing a. Similarly, small income changes do not appear to have much impact on average time spent on-site. This small income effect is not surprising since the budget share of these activities is small.

| Table 3: Estimated Change in Average On-Site Time (in hours) with a Change in On-site Costs, Trip Costs, and Income (evaluated at mean days, trips, and prices; asymptotic t-values in parentheses) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Variable | Linear (1/3 wage) | $\begin{gathered} \text { Linear } \\ \text { (full wage) } \end{gathered}$ | $\begin{aligned} & \text { Semi-log } \\ & (1 / 3 \text { wage }) \\ & \hline \end{aligned}$ | $\begin{gathered} \text { Semi-log } \\ \text { (full wage) } \end{gathered}$ | Log-linear <br> (1/3 wage) | Log-linear (full wage) |
| On-Site Costs | $\begin{aligned} & \hline-10.27 \\ & (-5.669) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline-23.72 \\ & (-6.085) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline-5.28 \\ & (-1.315) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline-11.03 \\ & (-3.999) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline-5.67 \\ & (-1.864) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline-8.52 \\ & (-3.909) \\ & \hline \end{aligned}$ |
| Trip Cost | $\begin{aligned} & \hline .0125 \\ & (.356) \end{aligned}$ | $\begin{aligned} & \hline-.0144 \\ & (.504) \end{aligned}$ | $\begin{aligned} & \hline .0021 \\ & (.830) \end{aligned}$ | $\begin{aligned} & \hline .017 \\ & (.797) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline .0727 \\ & (1.259) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline .050 \\ & (1.510) \end{aligned}$ |
| Income | $\begin{aligned} & \hline-.00000063 \\ & (-.001) \\ & \hline \end{aligned}$ | $\begin{aligned} & .0069 \\ & (4.781) \end{aligned}$ | $\begin{aligned} & \hline .0002 \\ & (.325) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline .0034 \\ & (3.324) \end{aligned}$ | $\begin{aligned} & \hline-.00035 \\ & (-.723) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline .0021 \\ & (2.731) \end{aligned}$ |

The consumer surplus values for the average individual associated with the estimated
linear and semi-log demand functions were calculated using the following equations (with all cross prices and income evaluated at the mean values and suppressed for convenience):

$$
\begin{array}{ll}
\mathrm{CS}_{\mathrm{r}}=.5\left(\gamma_{\mathrm{c}}-\gamma_{0}\right) \mathrm{r}\left(\gamma_{0}\right) & \text { (linear) } \\
\mathrm{CS}_{\mathrm{d}}=.5\left(\delta_{\mathrm{c}}-\delta_{0}\right) \mathrm{d}\left(\delta_{0}\right) & \text { (linear) } \\
\mathrm{CS}_{\mathrm{r}}=-\mathrm{r} / \beta_{11} & \text { (semi-log) } \\
\mathrm{CS}_{\mathrm{d}}=-\mathrm{d} / \beta_{22} & \text { (semi-log) } \tag{27}
\end{array}
$$

where r and d are evaluated at their mean values. The results are listed in Table 4. ${ }^{12}$
The CS values in Table 4 are the consumer's surplus of the trip experience and on-site experience. The results show that all the models except one yielded significant CS values ${ }^{13}$. The large magnitude of these estimates can partially be attributed to the fact that they embody both

[^6]time and money prices. A natural next step would be to estimate the exact welfare measures by integrating back to find the quasi-preferences and compare these true welfare measures with the approximate ones obtained here (LaFrance; LaFrance and Hanemann). However, note that integrating the functional forms in this paper would require considerably more structure to be imposed upon the estimated system to get theoretically consistent quasi-expenditure functions ${ }^{14}$.

| Table 4: Consumer Surplus Values for the Average Individual Associated with Total Trips and Total On-Site Demand for Linear and Semi-log Cases (calculated asymptotic t-values are in parentheses) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| CS of activity | Linear (1/3 wage) | $\begin{gathered} \text { Linear } \\ \text { (full wage) } \end{gathered}$ | $\begin{gathered} \hline \text { Semi-log } \\ (1 / 3 \text { wage }) \end{gathered}$ | Semi-log <br> (full wage) |
| Total Trips | $\begin{aligned} & \hline \$ 910.27 \\ & (20.758) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \$ 979.79 \\ & (20.702) \\ & \hline \end{aligned}$ | $\begin{array}{\|l} \hline \$ 1340.87 \\ (3.203) \\ \hline \end{array}$ | $\begin{aligned} & \hline \$ 1481.76 \\ & (3.546) \\ & \hline \end{aligned}$ |
| Total On-Site Time | $\begin{aligned} & \hline \$ 643.71 \\ & (18.799) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \$ 121.24 \\ & (18.357) \\ & \hline \end{aligned}$ | $\begin{aligned} & \$ 3797.77 \\ & (.222) \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \$ 493.77 \\ & (2.468) \\ & \hline \end{aligned}$ |

## Conclusion

In this paper, the joint recreational choice demand framework typically used to analyze value of time issues provided the framework to parametrically define average on-site time and calculate CS values for the average angler at Willow Creek, Alaska. Using common functional forms with full prices and income, the demand for total trips to Willow Creek and total on-site time were estimated. Comparative statics of the resulting parameterization of average on-site time suggested that both trip costs and income do not significantly affect the average angler's decision to stay longer on average.

A natural extension would be to use a preference-based approach with a theoreticallyconsistent flexible functional form. Additionally, determining the total use value of a recreational resource in this framework needs to be explored. Although joint complementarity appears to provide a potentially fruitful approach to welfare estimation in this regard, the restrictions necessary to impose this condition on the functional forms in this paper appear to be quite severe and were not pursued here.

[^7]
## References

Bell, F.W. and V.R. Leeworthy. "Recreational Demand by Tourists for Saltwater Beach Days." Journal of Environmental Economics and Management 18 (1990): 189-205.

Bockstael, N.E., I.E. Strand, and W.M. Hanemann. "Time and the Recreational Demand Model." American Journal of Agricultural Economics (May 1987): 293-302.

Bowes, M.D. and J.B. Loomis. "A Note on the Use of Travel Cost Models with Unequal Zonal Populations." Land Economics 56 (November 1980): 465-470.

Cesario, F.J. "Value of Time in Recreation Benefit Studies." Land Economics 52 (February 1976): 32-41.

Freeman, A.M. The Measurement of Environmental and Resource Values: Theory and Methods. Washington, D.C.: Resources for the Future, 1993.

Hof, J.G. and D.A. King. "Recreational Demand by Tourists for Saltwater Beach Days: Comment." Journal of Environmental Economics and Management 22 (1992): 281-291.

Just, R.E., D.L. Hueth, and A. Schmitz. Applied Welfare Economics and Public Policy. Englewood Cliffs, New Jersey: Prentice-Hall, 1982.

LaFrance, J. "Incomplete Demand Systems and Semilogarithmic Demand Models." Australian Journal of Agricultural Economics 34(August 1990): 118-131.

LaFrance, J. and W.M. Hanemann. "The Dual Structure of Incomplete Demand Systems." American Journal of Agricultural Economics 71 (May 1989): 262-74.

Larson, D.M. "Joint Recreation Choices and Implied Values of Time." Land Economics, 69 (August 1993): 270-286.

Larson, D.M. and S.L. Shaikh. "Empirical Specification Considerations for Two-Constraint Models of Recreation Demand." Working paper, Department of Agricultural and Resource Economics, University of California, Davis, 1997.

Larson, D.M., S.L. Shaikh, and J.B. Loomis. "A Two-Constraint AIDS Model of Recreation Demand and the Value of Leisure Time." Working paper, Department of Agricultural and Resource Economics, University of California, Davis, 1997.

McConnell, K.E. "On-Site Time in the Demand for Recreation." American Journal of Agricultural Economics (November 1992): 918-925.

McConnell, K.E. and I. Strand. "Measuring the Cost of Time in Recreation Demand Analysis: An Analysis to Sportfishing." American Journal of Agricultural Economics (February 1981): 153-156.

Randall, A. and J.R. Stoll. "Consumer's Surplus in Commodity Space." American Economic Review 70 (June 1980): 449-455.

Randall, A. "A Difficulty with the Travel Cost Method." Land Economics 70 (February 1994): 88-96.

Smith, V. Kerry, William H. Desvousges, and Matthew P. McGivney. "The Opportunity Cost of Travel Time in Recreation Demand Models." Land Economics, 59 (August 1983): 259-78.

Willig, R.D. "Consumer's Surplus Without Apology." American Economic Review 66 (September 1976): 589-597.

Wilman, E.A. "The Value of Time in Recreation Benefit Studies." Journal of Environmental Economics and Management 7 (September 1980): 272-286.


[^0]:    * This paper has benefited from several discussions with Doug Larson. His encouragement is gratefully acknowledged. All remaining errors are my own.

[^1]:    ${ }^{1}$ We assume that the individual has already made an optimal decision by choosing the number of hours to work (W) and to spend in leisure activities (T). That is, recreationists are assumed to have maximized $\mathrm{U}(\mathrm{T}, \mathrm{W}$ ) subject to both a money and total time budget. This maximization leads to an optimal leisure activity time, T. Bockstael et al. (1987) analyze this first-stage by including both discretionary and non-discretionary work time.
    ${ }^{2}$ Wilman (1980) includes two numeraire goods, one that is time costly and another that is not.
    ${ }^{3}$ Since the time constraint is an identity, the sign on the Lagrangian multiplier will be ambiguous whereas for the money constraint, we would expect the multiplier to be non-negative. It follows that the scarcity value of time can be either positive or negative. Note that although the individual can spend less money than is available, it is impossible for the person not to spend all of his/her time in some activity.
    ${ }^{4}$ Note that this specification will lead to an incomplete demand system since non-recreational goods are subsumed into a numeraire good.

[^2]:    ${ }^{5}$ Larson also discusses the Kuhn-Tucker conditions associated with the corner solutions corresponding to $\mathrm{r}=1$ and $\mathrm{d}=0$.
    ${ }^{6}$ Larson and Shaikh use the two dual expenditure functions associated with the primal two-constraint problem and Roy's Identity to derive relationships between the coefficients on time and money prices and the marginal value of time, $\rho$.

[^3]:    ${ }^{7}$ This does not preclude the possibility of imposing complementarity of the sort discussed by Bowes and Loomis.
    ${ }^{8}$ I thank Doug Larson for providing access to the data.
    ${ }^{9}$ In addition to the observations used, a couple observations were dropped that contained all of the necessary information but were deemed outliers. These were individuals with extremely high incomes and no on-site expenses.

[^4]:    ${ }^{10}$ Total days was actually measured in hours to avoid single-day logged terms being thrown out.

[^5]:    ${ }^{11}$ The effect of longer trip times on average on-site time can also be derived by noting that $\partial \gamma / \partial \alpha$ is $\rho$.

[^6]:    ${ }^{12} \mathrm{CS}$ values were not calculated for the log-linear case since calculations require arbitrarily choosing a choke price.
    ${ }^{13}$ Again, asymptotic t-values were calculated based on variance-covariance matrix of estimated demand functions.

[^7]:    ${ }^{14}$ Tests showed that the estimated coefficients did not conform to integrability restrictions.

