

**Productivity and Efficiency of Agricultural and Non Agricultural Banks in the United  
States: DEA Approach.<sup>1</sup>**

**by**

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# **Productivity and Efficiency of Agricultural and Non Agricultural Banks in the United States: DEA Approach**

## **Introduction**

Farm producers rely heavily on credit to finance their capital base, to modernize farming operations, and to serve as a source of liquidity in responding to risk. Significant long-term changes in the farm sector such as larger farm size, greater capital intensity and adoption of new technology have been facilitated by availability of credit. In addition, the survival of family farms can also depend on the availability of agricultural credit. Since the passage of the Deposit Institution Deregulation and Monetary Control Act of 1980 (DIDMCA), agricultural banks have come under intense competition from non-agricultural banks and other non-traditional financial intermediaries. While an efficient credit market is important for most producers, it is also of interest to policy makers to determine the effects of deregulation on the efficiency and survival of financial institutions.

Performance evaluation of the financial sector has shifted from the measurement of scale and product mix economies to measurement of efficiency. The approach of financial institutions has been one in which an efficient frontier (or best-practice frontier) is estimated. Efficiency is measured as the difference between those firms that are on the frontier and those that are below it.

A substantial number of studies have found large inefficiencies on the order of 20 percent or more of total bank industry costs and very minute or negative total factor productivity growth. Consequently there is little or no consensus on the sources of measured productivity differences (Berger, 1997, p 851). The potential sources lie in the choice of methodology, the data used, the conceptual differences of efficiency and other exogenous factors related to efficiency measures such as market and regulatory characteristics. Most bank cost studies have been biased toward scale and product mix economies until the late 1980s (Bauer et al 1993, p.385).

Meaningful as these are, they provide only one facet of economic performance. In contrast, efficiency analysis provides more important measures when decomposed into its components.

The objective of this study is to analyze total factor productivity (TFP) growth in the banking industry in an attempt to understand the sources of productivity changes in the banking sector and the robustness of results to the choice of methodology. This has been an area of extensive research since banks are trying to gain cost efficiencies with the onset of competition after deregulation. The majority of previous research have used a parametric approach to efficiency analysis through dual cost and profit functions. Although data envelopment analysis (DEA) efficiency measures do not allow statistical inferences they provide significant checks to their parametric counterparts. The flexibility involved allows the DEA approach to meaningfully evaluate productivity changes and decompose TFP into technical efficiency change and technical change by using the generalized Malmquist TFP index. The methodology was first developed by Fare, Grosskopf, Lindgren and Roos in 1995. This was applied by Griffel-Tatje and Lovell (1994) in their analysis of total factor productivity of the Spanish Banking industry. This study differs from Griffel-Tatje and Lovell's with respect to the choice of input and output variables. The variables considered here are ratios of interest expenses to total expenses, employee salary and benefit expenses to total expenses, total capital, demand deposits to total deposits and total loans. In addition, instead of pooling all the banks this analysis uses a stratified sample of banks based on the magnitude of asset holdings along with a second subdivision between agricultural banks and non-agricultural banks enabling the comparison of productivity and efficiencies between those two types of banks.

## **Credit Market Performance**

The traditional approach to credit market analysis and its role in economic development considered credit as an input into the production process. Reduction of the interest rates lowers the cost of this input and provides the much needed incentive to productive capital formation. Consequently the industry was regulated and interest rate ceilings were imposed. Implicit in the assumption of credit as an input in the production process is that development hinges on capital accumulation. Additional capital, via the credit market, would either promote or facilitate a more rapid rate of economic development.

Many previous studies have used one of four different approaches to estimate inefficiencies; the econometric frontier approach (EFA), the thick frontier approach (TFA), data envelopment analysis (DEA), and the distribution free approach (DFA) (Akhavain et. al 1997). In econometric approaches the problem of estimating inefficiencies is defined as one of distinguishing between two component error terms added to a cost or profit function in the case of dual, and to a production function in the case of the primal approach. Though it is limiting with respect to choice of a functional form it can be approximated by using the translog flexible functional form. One component of the error term by assumption represents x-inefficiencies, and the other represents random disturbance with a mean of zero. It is likely that efficiency estimates will suffer due to specification errors, by the effects of explanatory variables excluded from cost or profit functions, or by inconsistencies of the error term added to the cost (profit) or share function. Further, such analysis requires price data of outputs and inputs. DEA neither encounters specification errors nor does it require prices to be known. But the drawback is that it does not allow any statistical inferences with respect to significance of estimated values of TFP growth or efficiencies.

Bauer et al (1993) analyzed the efficiency and productivity growth in U.S. banking industry using a stochastic econometric frontier and a thick frontier approach. They estimated the translog cost function for a sample of 683 U.S. banks for the period of 1977-1988. Their results show a decrease in TFP growth in the range of 0.57 to 2.14 percent which is a technical regression for the study period. The scale economies are positive and in the range of 0.997 to 1.08 for banks of assets sizes of \$100 million to \$10 billion respectively. Stochastic econometric inefficiencies ranged from 18.4 to 27.7 percent. A recent study by Akhavein et al (1997), used a profit function approach to estimate the inefficiencies of the U.S. commercial banking sector for the period of 1984-1989. It shows overall lower inefficiency figures compared to that of Bauer et al. The estimated total inefficiencies are on the order of 1.25 %, 0.45%, and 0.37 % for what this study categorized as UNIT banks, LIMITED branching banks and STATE wide branching banks respectively. Moreover, it found larger banks to be both more allocative and technically efficient than small and medium size banks.

### **Malmquist Productivity Indexes**

Let  $x^t = (x_1^t, \dots, x_n^t) \in \mathbb{R}_+^N$  and  $y^t = (y_1^t, \dots, y_m^t) \in \mathbb{R}_+^M$  denote an input and output vector in period  $t$ ,  $t = 1, \dots, T$ . The graph of the production technology

$$\Gamma^t = \{ (x^t, y^t) : x^t \text{ can produce } y^t \} \quad t = 1, \dots, T \quad (1)$$

is the set of all feasible input-output vectors. The output sets are defined in terms of  $\Gamma^t$  as

$$P^t(x^t) = \{ y^t : (y^t, x^t) \in \Gamma^t \}, \quad t = 1, \dots, T \quad (2)$$

The output sets are assumed to be closed, bounded and convex, and to satisfy strong disposability

of outputs. A functional representation of the production technology is provided by Shephard's (1970) output distance function.

$$D_o^t(x^t, y^t) = \inf [\theta : (y^t/\theta) \in P^t(x^t, y^t)], \quad t=1, \dots, T \quad (3)$$

This output distance function also satisfies the inequality  $D_o^t(x^t, y^t) \leq 1$ , with  $D_o^t(x^t, y^t) = 1$  if and only if,  $y^t \in \text{Isoq } P^t(x^t) = [y^t : y^t \in P^t(x^t), \theta y^t \notin P^t(x^t), \theta > 1]$

Distance function  $D_o^t(x^t, y^t)$  is referred to as the within period output distance function (Fare, Griffel, Grosskopf and Lovell 1995). The adjacent period output distance functions can be defined the same way as:  $D_o^t(x^{t+1}, y^{t+1})$  and  $D_o^{t+1}(x^t, y^t)$ . Within period and adjacent period distance functions are then used in the definition and decomposition of the output oriented Malmquist productivity index.

The output oriented Malmquist productivity index is calculated as:

$$M_o^t(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)} \quad (4)$$

$M_o^t(x^t, y^t, x^{t+1}, y^{t+1})$  compares  $(x^{t+1}, y^{t+1})$  to  $(x^t, y^t)$  by scaling  $y^{t+1}$  to the Isoq  $P^t(x^{t+1})$ , that is by using period  $t$  technology as reference. Although  $D_o^t(x^t, y^t) \leq 1$ , it is possible that  $D_o^t(x^t, y^t) > 1$ , since period  $t+1$  output may not be feasible with period  $t$  technology. Thus  $M_o^t(x^t, y^t, x^{t+1}, y^{t+1})$  greater than, less than or equal to 1, represents positive, negative and zero productivity change respectively between periods  $t$  and  $t+1$ , from the perspective of period  $t$  technology.

The period  $t+1$  output oriented Malmquist productivity index can be calculated as

$$M_o^{t+1}(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^t, y^t)} \quad (5)$$

Since different time reference points represent different technologies, these measures can generate

qualitatively and quantitatively different empirical results concerning productivity change.

To avoid such ambiguities in empirical results Fare, Grosskopf, Lindgren and Roos (1995) suggested using the geometric mean of  $M_o^t(x^t, y^t, x^{t+1}, y^{t+1})$  and  $M_o^{t+1}(x^t, y^t, x^{t+1}, y^{t+1})$  as the productivity index which can be decomposed to give a technical efficiency change term and a technical change term.

$$M_o^G(x^t, y^t, x^{t+1}, y^{t+1}) = [M_o^t(x^t, y^t, x^{t+1}, y^{t+1}) \bullet M_o^{t+1}(x^t, y^t, x^{t+1}, y^{t+1})]^{1/2} \quad (6)$$

$$M_o^G(x^t, y^t, x^{t+1}, y^{t+1}) = \Delta TE(x^t, y^t, x^{t+1}, y^{t+1}) \bullet \Delta T^G(x^t, y^t, x^{t+1}, y^{t+1}) \quad (7)$$

$$= \frac{D_o^{t+1}(x^{t+1}, y^{t+1})}{D_o^t(x^t, y^t)} \left[ \frac{D_o^t(x^{t+1}, y^{t+1})}{D_o^{t+1}(x^{t+1}, y^{t+1})} \frac{D_o^t(x^t, y^t)}{D_o^{t+1}(x^t, y^t)} \right]^{1/2} \quad (8)$$

This represents the productivity of the production point  $(x^{t+1}, y^{t+1})$  relative to the production point  $(x^t, y^t)$ . A value greater than one will indicate positive total factor productivity growth from period  $t$  to period  $t+1$ . To empirically estimate equation (7) we must calculate the four component distance functions for each pair of years for each bank. These involve solving of four linear programming problems (LP) problems as given in Fare et al (1994).

### Data and Results.

Panel data for six banking categories for 1980-1991 were obtained from the USDA database for Banking Operating Statistics. The data were already stratified based on size of asset holdings. The different categories were less than \$25 million, \$25m-\$50m, \$50m-100m, \$100-300m, \$300-\$500m, and greater than 500 million dollars in asset holdings.

These banks were further subdivided into agricultural banks (Agbank) and non-agricultural banks (Non-Agbank) based on their operational characteristics. The categorization of Agbank and Non-agbank is based on a mean ratio of agricultural loans to total loans (MELRATIO) for all banks. If the MELRATIO is higher than individual bank's ratio of agricultural loans to total loans, the bank is categorized as agricultural bank and if it is less than MELRATIO it is categorized as non-agricultural bank. Aggregate data for a eleven year period of 1980-1991 were obtained from the USDA database for all of these categories. Three inputs and two outputs were selected based on their relative importance in the bank balance sheets and income statements for multi-input, multi-output analysis. Inputs selected were total capital, ratio of employee salaries and benefit expense to total expenses, and the ratio of interest expenses to total expenses. Outputs examined were the ratio of demand deposits to total deposits and total loans outstanding. The output oriented TFP indexes were calculated using *OnFront* efficiency and productivity measurement program developed by Roos (1997). Table 1.0 shows the Malmquist index summary for agricultural and non-agricultural banks for asset sizes of less than \$25m and \$25m-\$50m. Given in Table 2.0 are the Malmquist index summary for asset sizes of \$50m-\$100m and \$100m-\$300m, whereas Table 3.0 shows the Malmquist index summary for asset sizes of \$300m-\$500m and for asset size of greater than \$500 million.

Results presented in Table 1.0 show a summary of Malmquist indices for agricultural and non agricultural banks of asset sizes of less than \$25m and \$25-\$50m. As TFP measures indicates, productivity of small agricultural banks have varied from a contraction in growth of 20% just after the DIDMCA (1980) to a 2% growth rate in 1987. After 1987 these banks have remained close to the frontier (Fig.1). Productivity growth of small non-agricultural banks has been very similar.



One difference is that non-agricultural banks have been on the frontier for most of the post deregulation period compared to small agricultural banks. Medium sized agricultural banks (\$50-\$300m) show a negative productivity growth rate of 5% on the average for the study period and non agricultural banks show a productivity decrease of 4% for the same period. Nevertheless, the rate of efficiency change for medium sized non agricultural banks in the post deregulation period has been impressive compared to that of agricultural banks. Non agricultural banks have shown efficiency increases ranging from 1% to 15% for the period. Agricultural banks on the other hand shown 2 % to 8% increase in efficiency for four out of eleven years of the study period. Large (greater than \$500m, Table 3) non agricultural banks have shown an impressive performance with increasing productivity growth ranging from 3% to 6% for a number of years in the post deregulation period. In contrast, large agricultural banks do not show any noticeable growth except for 1986, 1987 and 1991. While both types of banks have remained on the frontier for the entire period ( Fig. 3) non agricultural banks have been the innovators. This may be due to increased competition which was brought about by deregulation. As our results indicate, the small and medium size banks of both categories have experienced efficiency gains after deregulation.

## **Conclusions**

The smaller banks of both types have experienced efficiency increases indicating that those banks remained competitive after deregulation due to efficiency gains. Larger banks, on the other hand, needed adoption of new technology to remain competitive in the industry and they emerge as innovators. These results are consistent with Akhavein et al (1997) who indicates inefficiencies ranging from 1.25 percent to 0.37 percent. But they are well below the inefficiencies found by

Bauer et al in their 1993 study in which inefficiency figures ranged from 18 percent to 27 percent. Nevertheless, while the differences in study period and data does not allow any direct comparison, these DEA results are closer to the more refined parametric studies done by Akhavein et al (1997). Negative TFP growth which represents technological regression in an era of high technological advances and wide spread adoption of innovations in the banking industry make these findings less defensible. These results might be explained away by other exogenous factors such as market and regulatory characteristics once the data and methodological differences are accounted for. Despite results of negative TFP growth there has been an increase in the quality of services provided by the banks such as automatic teller machines, rapid wire transfer of funds and speedy check clearing services (Bauer et al, 1993). These factors have not been adequately captured in the analyzes. Moreover, changes in the regulatory environment affect competition in credit markets. The recent deregulation of depository institutions, including current focus on interstate banking and the restructuring of the farm credit systems have increased the competitiveness in the market and are expected to affect performance significantly. No definitive conclusion can be reached before addressing these factors properly.

**Table 1. Malmquist Index Summary for Agricultural and Non agricultural Banks of asset sizes <\$25m and \$25m-\$50m (1981-1991).**

	<\$25m						\$25m-\$50m					
	AgBanks			NonAgBanks			AgBanks			NonAgBanks		
	M	EC	TC	M	EC	TC	M	EC	TC	M	EC	TC
1981	0.80	0.88	0.91	0.79	0.99	0.79	0.89	0.91	0.99	0.84	0.97	0.86
1982	0.92	1.00	0.92	0.90	1.01	0.89	0.95	0.95	1.00	0.92	1.02	0.90
1983	0.94	0.91	1.03	0.97	0.90	1.08	0.97	0.98	0.99	0.97	0.89	1.08
1984	0.94	1.09	0.86	0.86	1.11	0.78	0.96	1.03	0.94	0.95	1.06	0.90
1985	0.96	1.00	0.96	0.96	1.00	0.96	0.89	0.96	0.94	0.98	1.05	0.93
1986	1.00	0.95	1.06	1.06	1.00	1.06	0.92	0.95	0.97	1.03	0.99	1.04
1987	1.02	1.02	1.00	1.01	1.00	1.01	0.93	0.96	0.97	0.98	1.03	0.95
1988	0.99	0.86	1.15	0.96	0.76	1.27	1.00	0.98	1.01	0.98	0.91	1.07
1989	0.99	1.26	0.79	0.92	1.32	0.70	0.99	1.08	0.91	0.95	1.14	0.84
1990	1.00	1.02	0.98	0.98	1.00	0.98	0.99	1.02	0.97	0.96	1.00	0.95
1991	0.98	0.85	1.14	1.02	0.86	1.19	0.98	1.00	0.99	1.01	0.92	1.10
Mean	0.96	0.98	0.98	0.95	1.00	0.97	0.95	0.98	0.97	0.96	1.00	0.97

M=Malmquist Index, EC=Efficiency Change, TC=Technical Change.

**Table 2. Malmquist Index Summary for Agricultural and Non-agricultural Banks of asset sizes \$50m-\$100m and \$100m-\$300m (1981-1991).**

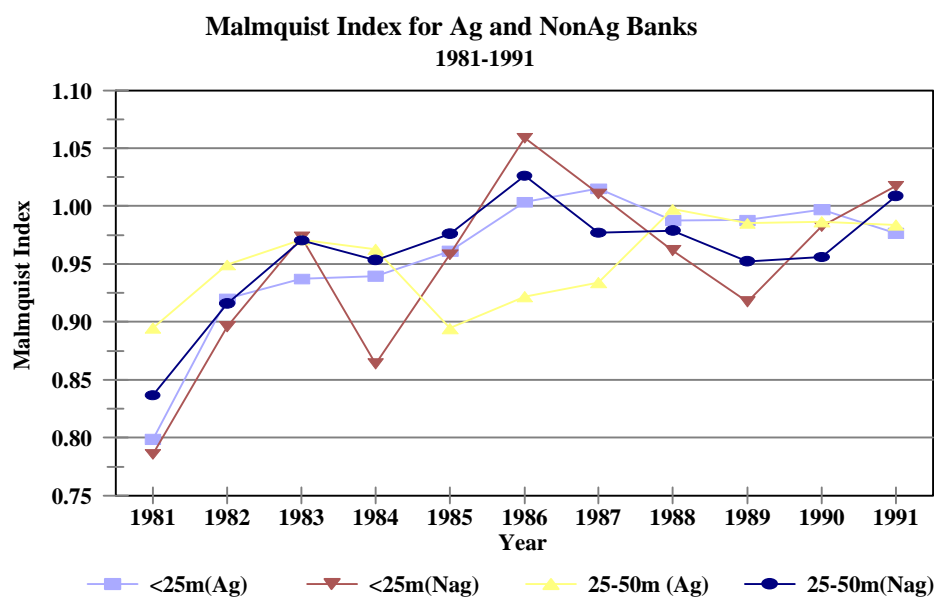
	\$50m-\$100m						\$100m-\$300m					
	AgBanks			NonAgBanks			AgBanks			NonAgBanks		
	M	EC	TC	M	EC	TC	M	EC	TC	M	EC	TC
1981	0.94	0.93	1.00	0.86	0.96	0.90	0.85	0.87	0.98	0.88	0.97	0.91
1982	0.94	0.93	1.01	0.93	1.00	0.93	0.98	0.99	0.99	0.93	0.99	0.94
1983	0.97	0.98	0.99	0.98	0.93	1.06	1.03	1.04	0.99	0.97	0.94	1.03
1984	0.97	1.04	0.94	1.01	1.10	0.92	0.94	1.03	0.91	1.07	1.15	0.93
1985	0.89	0.93	0.96	0.97	1.00	0.97	0.92	0.92	0.99	1.01	1.03	0.98
1986	0.94	0.95	0.99	0.99	0.96	1.03	0.98	0.93	1.05	0.97	0.95	1.02
1987	0.93	0.97	0.96	0.99	1.06	0.94	0.98	0.97	1.01	1.00	1.05	0.96
1988	0.99	0.98	1.02	0.98	0.97	1.01	0.99	0.99	1.01	1.02	1.00	1.02
1989	0.97	1.03	0.93	0.95	1.02	0.93	0.97	1.06	0.92	0.99	0.99	1.00
1990	0.97	1.02	0.94	0.96	1.01	0.95	0.99	1.00	0.99	0.99	1.01	0.98
1991	0.94	0.99	0.95	0.98	0.98	1.00	0.96	1.01	0.95	0.94	1.01	0.94
Mean	0.95	0.98	0.97	0.96	1.00	0.97	0.96	0.98	0.98	0.98	1.01	0.97

M=Malmquist Index, EC= Efficiency Change, TC=Technical Change.

**Table 3. Malmquist Index Summary for Agricultural and Non-agricultural Banks of asset sizes \$300m-\$500m and >\$500m (1981-1991).**

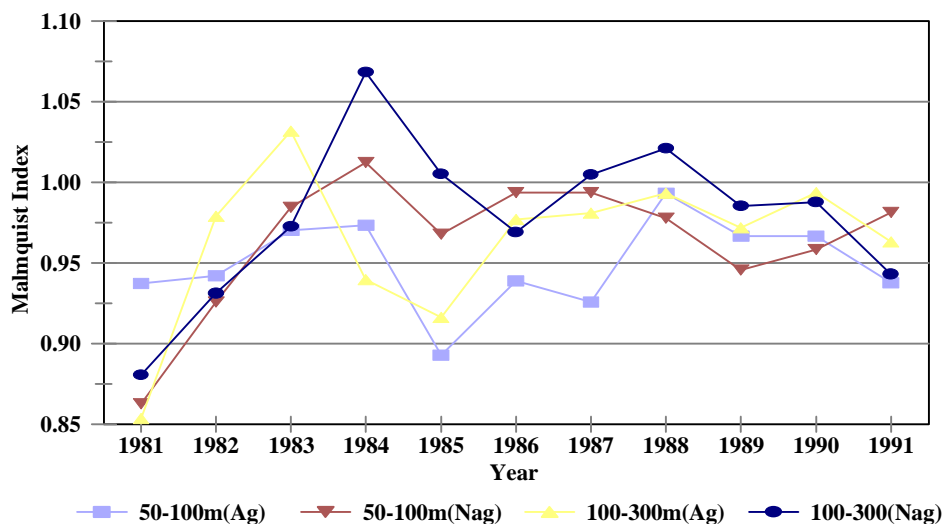
	\$300m-\$500m						>\$500m					
	AgBanks			NonAgBanks			AgBanks			NonAgBanks		
	M	EC	TC	M	EC	TC	M	EC	TC	M	EC	TC
1981	0.99	1.00	0.99	0.88	1.00	0.88	0.79	1.00	0.79	1.04	1.00	1.04
1982	0.94	1.00	0.94	0.89	1.00	0.89	1.01	1.00	1.01	0.97	1.00	0.97
1983	0.59	1.00	0.59	1.00	0.92	1.09	0.88	1.00	0.88	1.00	1.00	1.00
1984	0.93	1.00	0.93	0.98	1.09	0.91	0.79	1.00	0.79	1.03	1.00	1.03
1985	0.98	1.00	0.98	0.94	1.00	0.94	0.99	1.00	0.99	1.04	1.00	1.04
1986	1.13	1.00	1.13	1.01	1.00	1.01	1.12	1.00	1.12	1.06	1.00	1.06
1987	1.16	0.90	1.29	0.93	1.00	0.93	1.32	1.00	1.32	0.96	1.00	0.96
1988	0.92	1.11	0.82	1.02	1.00	1.02	0.90	1.00	0.90	1.03	1.00	1.03
1989	0.86	1.00	0.86	0.96	1.00	0.96	0.87	1.00	0.87	1.05	1.00	1.05
1990	1.00	1.00	1.00	0.97	1.00	0.97	0.91	1.00	0.91	0.98	1.00	0.98
1991	0.58	0.80	0.72	0.94	1.00	0.94	1.02	1.00	1.02	0.96	1.00	0.96
Mean	0.92	0.98	0.93	0.96	1.00	0.96	0.96	1.00	0.96	1.01	1.00	1.01

M=Malmquist Index , EC=Efficiency Change, TC=Technical Change.



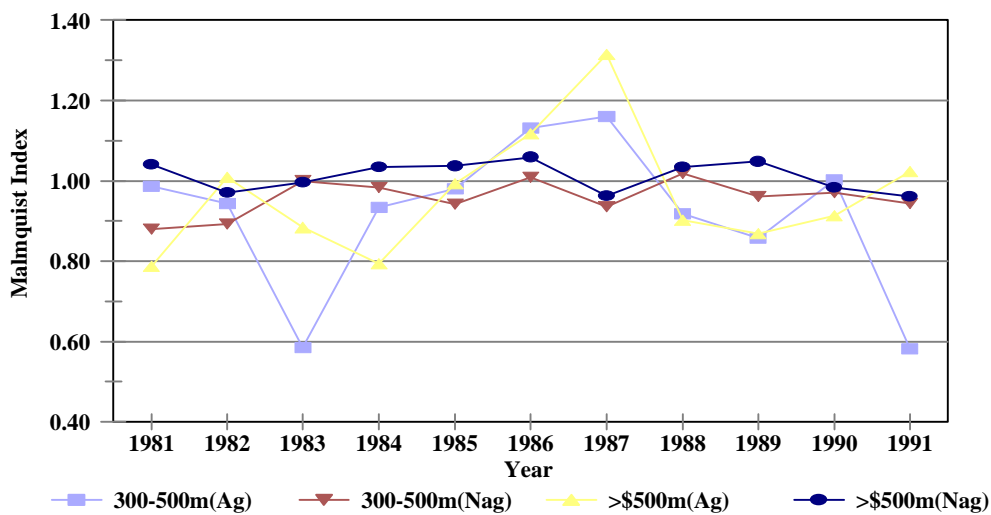
**Fig.1 Malmquist Index Comparison between Ag. and Non-Ag. Banks of Asset Sizes <\$25m and \$25m-\$50m.**

**Malmquist Index for Ag and NonAg Banks  
1981-1991**



**Fig.2. Malmquist Index Comparison between Ag and Non-Ag Banks of Asset Sizes \$50m-100m and \$100-\$300m.**

**Malmquist Index for Ag and NonAg Banks  
1981-1991**



**Fig. 3 Malmquist Index Comparison between Ag. and Non Ag. Banks of Asset Sizes \$300m-\$500m and >\$500m**

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