

**A Dynamic Analysis of Price Determination
Under Joint Profit Maximization in Bilateral Monopoly**

by

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Abstract

This study provides a dynamic mathematical treatment of equilibrium price determination under bilateral monopoly in which the seller and buyer maximize their joint profits. The mathematical treatment of this problem gives rise to the following key results: a) a quantitative solution for the equilibrium price is obtained; b) profits of the buyer and the seller are shown to be equal; c) the dynamic stability of the equilibrium price is established; and d) for the larger value of the intermediate product the bargaining process achieves the equilibrium price at a faster pace.

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I. Introduction

Bilateral monopoly involves a single seller and a single buyer. The seller produces an intermediate product and sells it to the buyer who uses it as an input in producing a final output. In such a market, four possibilities arise in the determination of equilibrium price and quantity of the intermediate product: (i) monopoly case in which the seller may dominate the market and make the buyer to accept his/her price and quantity decisions; (ii) monopsony case in which the buyer may dominate the market and force the seller to follow his/her price and quantity decisions; (iii) the seller and buyer can collude and determine the equilibrium quantity and price by maximizing their joint profits; and (iv) noncooperation by the seller and buyer can result in market failure and nonexistence of both parties.

The treatment of bilateral monopoly can be found in selected text books. Blair, Kaserman, and Romano (BKR) provide a list of microeconomics text books and highlight the confusions and incorrect solutions that abound in these texts.¹ Furthermore, they note that the optimal solution in a correct analysis of bilateral monopoly calls for a joint profit maximization by the seller and buyer. BKR conclude that, in such a solution, the equilibrium quantity of intermediate product is determinate but the equilibrium price of this product is indeterminate. BKR point out that only two texts (Henderson and Quandt and Layard and Walters) confer such a correct solution and present a detailed discussion of indeterminacy of price using the contract curve approach.

Truett and Truett employing the same contract curve approach and graphical analysis, however, establish that equilibrium price of the intermediate product is in fact determined through a bargaining process between the seller and buyer. It should be noted that the indeterminacy of

price in Henderson and Quandt and Layard and Walters is because their analysis is based on static optimization and does not allow for the bargaining process to take place. In contrast, the analyses by BKR, and Truett and Truett do employ the bargaining process. The reason for the indeterminacy of price in BKR is because they did not complete the bargaining process as carried out by Truett and Truett.

Though the bilateral monopoly is mainly treated in the literature as a text book example, notable real world examples can be found. In the agricultural industry, examples of bilateral monopoly include farmer cooperatives and processing companies (sugarbeet farmers cooperative and sugar manufacturer; cattle farmers cooperative and beef processing firm; farmers cooperative and fertilizer company). More general examples are labor union vs. management in the auto industry and professional players union vs. owners.

The purpose of this paper is to provide a simple dynamic mathematical treatment of equilibrium price determination under bilateral monopoly in which the seller and buyer maximize their joint profits. The mathematical treatment, in lieu of the graphical analysis, of this problem gives rise to the following key results: a) a quantitative solution for the equilibrium price is obtained; b) profits of the buyer and the seller are shown to be equal; c) the dynamic stability of the equilibrium price is established; and d) for the larger value of the intermediate product, the bargaining process achieves the equilibrium price at a faster pace.

II. Model and Analysis

The seller (upstream monopolist) produces the intermediate product (q) and sells it to the buyer (downstream monopsonist) who uses it as an input in producing the output (y). The per unit prices of q and y are, respectively, p and r . The seller's profit (Π_s) and the buyer's profit

(Π_B) are given by

$$\Pi_S = pq - c(q), \quad (1)$$

$$\Pi_B = rf(q) - pq. \quad (2)$$

where $c(q)$ is the seller's cost function and $f(q)$ is the buyer's production function. Since the focus of this study is on price and quantity determination in a bilateral monopoly and to simplify the analysis, we assume that the external markets are competitive markets. That is, the seller purchases inputs from competitive markets and the buyer sells the output y in a competitive market.²

The conventional analysis of bilateral monopoly is to examine the domination of one party and then the other (i.e., the first two outcomes noted in the introduction), which is followed by the case where two parties do not dominate each other (the third outcome). Henderson and Quandt present a clear exposition of equilibrium price and quantity determination under (i) seller's dominance (monopoly case) with buyer as a price taker and (ii) buyer's dominance (monopsony case) with seller as a price taker.

If neither the seller nor the buyer entity is willing to behave as a price taker, the market mechanism may break down and both may go out of business (the fourth outcome).³ One possibility for avoiding the nonexistence is for both the seller and buyer to recognize their mutual interdependence and act in unison by bargaining for mutually favorable quantity and price for the intermediate product. As noted in Henderson and Quandt, this collusion process can be accomplished through a two-step process: first, determine the quantity that maximizes the joint profits of the seller and buyer; second bargain for a satisfactory price to distribute the profits among them (the third outcome). The joint profits are given by

$$\begin{aligned}\Pi &= \Pi_B + \Pi_S = [rf(q) - pq] + [pq - c(q)] \\ &= rf(q) - c(q).\end{aligned}\tag{3}$$

The first order condition for joint profit maximization is

$$\frac{d\Pi}{dq} = rf(q) - c'(q) = 0.\tag{4}$$

Thus, the optimum collusive output (q^*) is determined by equating seller's marginal cost to buyer's marginal value product.⁴ Since the joint profit maximization provides only the optimum quantity, the price has to be determined through negotiation. Henderson and Quandt observe that the price is not unique; rather it is bounded by an upper and lower limit as given below:

$$\frac{rf(q^*)}{q^*} \geq p \geq \frac{c(q^*)}{q^*}.\tag{5}$$

The upper (lower) bound ensures that buyer's (seller's) profit is nonnegative.

The indeterminacy of price in the above model is the result of a static model which does not allow for the bargaining process to take place. In this study, we use a dynamic price adjustment mechanism which allows for the continuous bargaining and negotiation process between the seller and buyer.

The price adjustment mechanism is modeled as follows. We assume that, because of the close interaction between the both parties, each party has some knowledge of the nominal profits of the other party. Consequently, if $\Pi_B > \Pi_S$, the seller is likely to enter into the negotiation process to obtain a higher price for q to increase his/her profits. Thus p will adjust upwards if $\Pi_B > \Pi_S$. On the other hand, if $\Pi_B < \Pi_S$, the buyer is likely to enter into the negotiation process to obtain a lower price to increase his/her profits. Thus p will adjust downwards if $\Pi_B < \Pi_S$. If $\Pi_B = \Pi_S$, further change in price will not occur, and the equilibrium price will be obtained. This price adjustment mechanism can be captured succinctly by the following equation:

$$p'(t) = \frac{dp}{dt} = \theta[\Pi_B - \Pi_S] \quad \theta > 0, \quad (6)$$

where the parameter θ captures the speed of price adjustments or the intensity and effectiveness of the bargaining process in moving the price toward the equilibrium price.

This price adjustment mechanism is similar to the one used by Truett and Truett in their graphical analysis. It should be noted that the disagreement profits for each party will be zero if they did not enter into the bargaining process and fail to resolve the differences. Thus, either party, if it is rational, cannot afford not to enter into the bargaining process. By substituting for $\Pi_B - \Pi_S = \{rf(q) - pq - [pq - c(q)]\}$ in the above price adjustment mechanism, equation (6) can be written as:

$$p'(t) = \frac{dp}{dt} = \theta[rf(q) + c(q) - 2pq]. \quad (7)$$

The equilibrium price (\bar{p}) is obtained by setting $p'(t) = 0$ in the above equation and solving for price:

$$\bar{p} = \frac{rf(q) + c(q)}{2q} \quad (8)$$

Thus the equilibrium price is half of per unit revenue of the buyer plus half of per unit cost of the seller. Since the buyer's cost (pq) and seller's revenue (pq) cancel out each other in the transaction of q between them, the equilibrium price depends only on the buyer's revenue and seller's cost. Furthermore, this equilibrium price is at the midpoint of the upper and lower bounds of the price range given in equation (5). Once the equilibrium price is arbitrated, the profits of the buyer and seller can be determined by substituting (\bar{p}) into their respective profit functions. This substitution results in

$$\Pi_S = \Pi_B = \frac{1}{2}[rf(q) - c(q)] \quad (9)$$

Thus, the buyer's profit and seller's profit are equal, and are also equal to half of the joint profits.

An implication of equilibrium price being the half of per unit revenue of the buyer plus half of per unit cost of the seller, and buyer's and seller's profit being equal is that under this price adjustment mechanism the seller and the buyer have equal bargaining power.

Next, we examine the stability of the equilibrium price by using the phase diagram analysis of the first-order differential equation (7) and quantitatively solving this equation. The phase diagram of this equation is drawn in Figure 1 with the negatively sloped phase line as the slope of $p'(t)$ with respect to p is negative $(-2\theta q)$. Since $p'(t)$ is positive for points above the horizontal axis, p is increasing over time, as shown by the arrowheads on the upper part of the phase line approaching toward \bar{p} . Anywhere below the horizontal axis, $p'(t)$ is negative and p is decreasing over time, as shown by the arrowheads on the lower part of the phase line tending toward \bar{p} . Thus, phase line converges toward the equilibrium price \bar{p} (where $p'(t)$ is zero), indicating that it is a stable equilibrium. The time paths of $p(t)$ generated by the phase line can be drawn by plotting $p(t)$ against time t , as shown in Figure 2. For any initial price below (above) \bar{p} , the time path of $p(t)$ approaches the equilibrium price from below (above) implying that the equilibrium is dynamic stable.

A quantitative solution of the first-order differential equation (7) further illustrates this dynamic stability of price. This first-order differential equation is linear with a constant coefficient and a constant term. The solution to this equation is (see Chiang):

$$\begin{aligned}
 p(t) &= [p(0) - \frac{rf(q) + c(q)}{2q}]e^{-2\theta qt} + \frac{rf(q) + c(q)}{2q}, \\
 &= [p(0) - \bar{p}]e^{-2\theta qt} + \bar{p}
 \end{aligned}
 \tag{10}$$

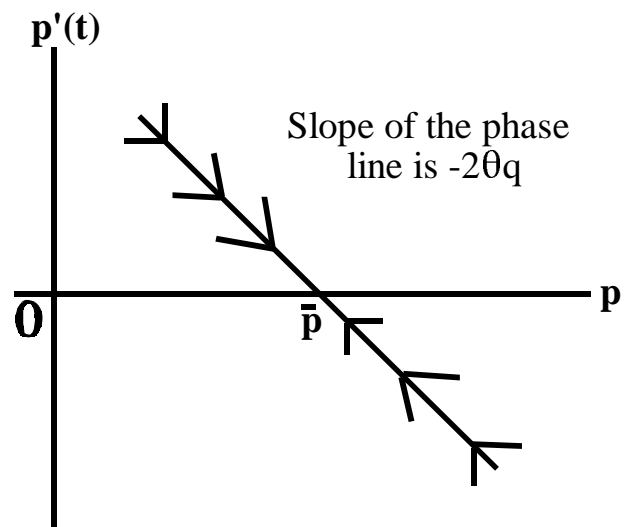


Figure 1. Phase Diagram

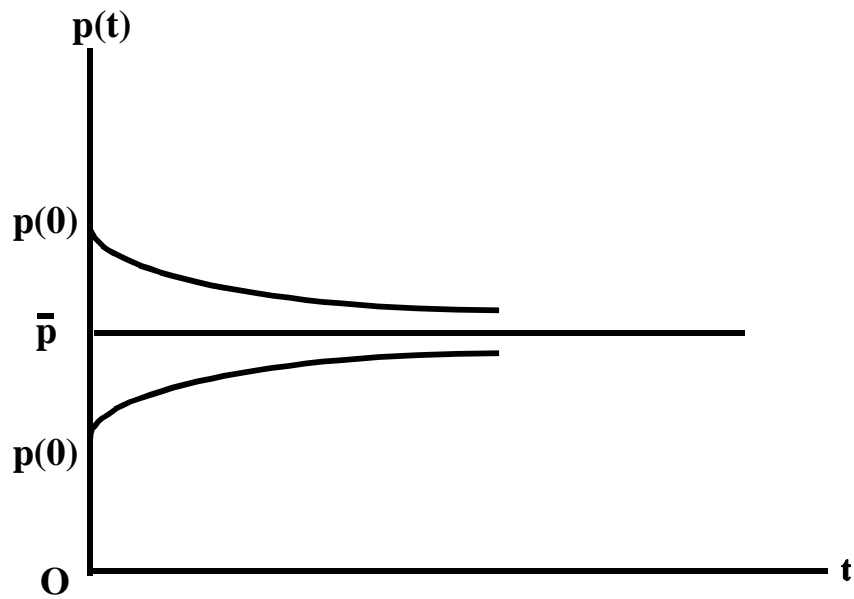


Figure 2. Convergence of Price to its Equilibrium Level

where $p(0)$ is the initial price. Since θ and q are positive, as t increases, $e^{-2\theta qt}$ tends to zero and the difference between $p(0)$ and \bar{p} (irrespective of whether $p(0)$ is below or above \bar{p}) shrinks, and thus, $p(t)$ approaches \bar{p} as depicted in Figure 2. Also, the parameter θ , which influences the price adjustments through the intensity and effectiveness of the bargaining process, determines the rate at which $p(t)$ approaches \bar{p} . For a larger (smaller) value of θ , $p(t)$ approaches \bar{p} faster (slower). Furthermore, the magnitude of q also plays a key role in determining the pace at which $p(t)$ approaches \bar{p} . The larger (smaller) the magnitude of q , the faster (slower) the $p(t)$ move toward \bar{p} . The economic significance of this latter result is that the larger the value of q the greater is the stake for both the seller and buyer, and thus, both parties take keen interest in the negotiations to achieve a mutually favorable price at a faster pace.

The above results are summarized in the following proposition.

Proposition In a bilateral monopoly with joint profit maximization, the price adjustment mechanism $\{p'(t) = \theta[\Pi_B - \Pi_S], \theta > 0\}$ lead to the following results:

- a) the equilibrium price is equal to half of the per unit revenue of the buyer plus half of the per unit cost of the seller, i.e., $\bar{p} = \frac{rf(q) + c(q)}{2q}$;
- b) buyer's and seller's profits are equal, i.e., $\Pi_B = \Pi_S = \frac{1}{2}[rf(q) - c(q)]$ and also equal to half of their collusive profits;
- c) equal bargaining power for the seller and the buyer;
- d) the equilibrium price is dynamically stable;
- e) $p(t)$ approaches \bar{p} faster for larger values of q and θ .

Conclusion

The controversial issue in the literature surrounding the bilateral monopoly is the determinacy of equilibrium price when the seller and buyer collude and maximize their joint profits. This study employs a dynamic price adjustment mechanism in resolving this issue. The findings of this study are also useful in resolving the real-world problem surrounding the impasse between the parties of the bilateral monopoly (e.g., the labor union and management, the farmers cooperative and a commodity processor) in determining a mutually agreeable price. In such cases, the optimal price of the intermediate product is the average of the per unit revenue of the buyer and the per unit cost of the seller, and this price results in equal profits for both parties.

Endnotes

1. Earlier work on this topic include studies by Bowley, Fellner, and Henderson.
2. BKR and Truett and Truett assume that the downstream buyer is a monopolist in selling its output y . This assumption has no bearing on the key results of this paper.
3. If both parties are rational economic agents, this outcome will not occur.
4. The optimal collusive output (q^*) is equal to the equilibrium quantity under perfect competition because the first order conditions under perfect competition for the buyer and seller are, respectively, $rf'(q) = p$ and $p = c'(q)$, and thus, $rf'(q) = c'(q)$ as in equation (4).

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