THE ROLE OF VARIOUS PLAYERS IN THE PORT INDUSTRY – THEORY AND PRACTICE

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ABSTRACT

This paper examines the role of the various players in the port industry and their interactions, and determines the impact of their actions in port operations. A mathematical approach has been developed to assist port authorities in decision making for infrastructure investment. The model examines the port investment decisions within the context of a multimodal transportation system. Model results are used to answer questions regarding the optimal investment strategy of a port authority in order to maximize the net social benefit; the impact of this strategy to the terminal operators and users; the effect of competition or cooperation between carriers; and the shippers' behavior in terms of quantity and price of goods shipped over the intermodal network. The paper concludes with a practical interpretation of the results of the theoretical models. Further improvements that would capture real world issues that are not adequately treated by the current models are discussed.

INTRODUCTION

Steamship lines, railroads, motor carriers, brokers, shippers, forwarders, port terminal operators, port authorities and other regulatory agencies are all players in the complex port industry environment. Interaction among these players is influenced by whether the decisions they are making are long- or short-term decisions and by whether the market in which they operate can be described as a monopoly, oligopoly, or perfect competition. Despite the industry's dynamic nature and volatility of conditions prevailing, the long-term decision, once committed, is difficult to change in the short term. Hence, the interaction between the long-term decision-maker and the short-term decision-maker has a sequential nature. The market conditions also influence the interaction by bestowing the decision-maker with different levels of market power under different market conditions. For example, the monopoly supplier or the monopoly consumer has strong control over the market price. On the contrary, under the market condition of perfect competition the supplier or the consumer acts as the price taker. Understanding the short-term or long-term nature of each player's decision and the market conditions is very important to the understanding of the interaction among the players and the formulation and implementation of long term strategies for the port business environment.

This paper examines the role of the various players in the port industry and their interactions, and determines the impact of their actions in port operations. The impact of port activity to the greater area served by the port is also examined. Due to the existence of different transportation modes and related complicated interactions between components of the freight system within the port terminal, the analysis of port operations is considered within the framework of a multimodal freight system. To represent this complex environment, a three level model is used. The first level describes the behavior of the Port Authority in choosing the best investment strategy to maximize net social benefit. The second level describes the behavior of the carriers in choosing optimal service charge and routing pattern to maximize their profits. The third level formulates the behavior of the shippers, which is to determine the supply and demand of each commodity at each market and the distribution pattern of each commodity on the network. The interaction between oligopolistic private port terminal operators and several shippers is formulated using the Stackelberg equilibrium. The optimal investment strategy problem for the port authority is solved, subject to the previous problem. Model results are used to answer questions regarding the optimal investment strategy of a port authority in order to maximize the net social benefit; the impact of this strategy to the terminal operators and users; the effect of competition or cooperation between carriers; and the shippers' behavior in terms of quantity and price of goods shipped over the intermodal network.

The paper concludes with a practical interpretation of the results of the theoretical models. Further improvements that would capture real world issues that are not adequately treated by the current models are discussed.

INTERACTIONS AMONG PLAYERS IN THE PORT INDUSTRY

Port facilities consist of channels, berths, docks, and land, managed by a Port Authority, which is typically a public or quasi-public agency operating in the public interest. A Port Authority may have several terminals within its port complex and may operate them as is the case in operating ports, or lease the land and facilities to private operators as is the case in landlord ports. The terminal operators together with the other public and private transportation carriers that own and operate transportation facilities serving the port constitute a multimodal freight transportation system. Through this system move vehicles and containers carrying commodities from the shippers to the receivers located in spatially separated markets. The various players in the port industry have been classified in the literature (Harker et al. 1986-a, Harker et al. 1986-b) in three broad categories, namely shippers, carriers and regulatory agencies. Specific characteristics and behavior of the players that will be examined in this paper are presented in this section.

Port authorities are the regulatory and development agencies involved in the supply side of the network operation by being responsible for major port infrastructure investments. Increased competition and sustained congestion problems has resulted in ports rethinking how to bolster capacity and improve service quality to maintain current and attract new business. The increased competition and service deterioration due to lack of capacity adds to the pressure for infrastructure investment. In response to these opportunities and challenges, most ports have started to or plan to redesign and reorganize their operations and have come up with a long-term investment plan. Port authority investments are carried out during a long time period. Infrastructure and capacity improvements and selection of the best investment strategies are the port authority decisions considered in this paper.

Carriers, also representing the supply side of the players, are the economic agents who operate transportation facilities and/or equipment and provide the transportation service. Carriers may compete or collude in setting prices for the services they provide. They can observe each other's pricing behavior and react accordingly, which assumes oligopolistic market conditions.

Shippers represent the demand side of the players involved in freight transportation. A shipper is the economic agent who engages in moving commodities over the spatial network to explore the potential economic benefit arising from the difference in commodity price between different regions (Friesz et al. 1986). Shippers make decisions on the production and consumption location and select a carrier or sequence of carriers to ship the commodities between different markets.

The decisions made by each player are graphically shown in Figure 1 below.

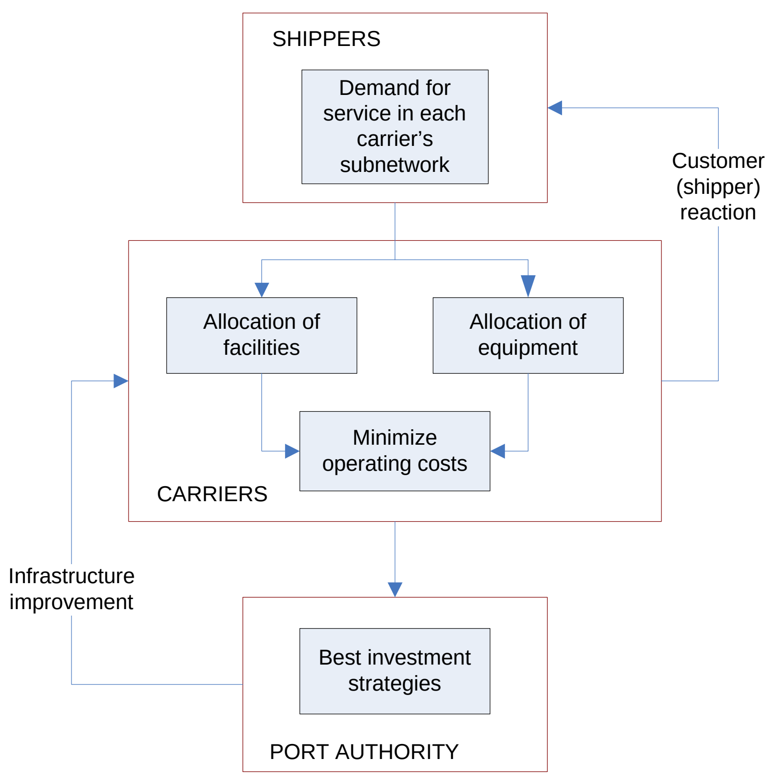


Figure 1: Major player decisions

In making decisions the players interact with each other within an intermodal transportation network environment. Each shipper makes commodity production and consumption decisions based on knowledge of the pattern of the market prices in the spatially separated markets, the pattern of the service charges set by the carriers, and the travel time and service functions on the carriers' network. These decisions determine the level of demand for service in the carriers' networks. Carriers make pricing and routing decisions based on knowledge of the Port Authority's investment decision and forecast of the shippers' and competing carriers' reaction. The sequential nature of the interaction between the Port Authority and the carriers is determined by the fact that the Port Authority's investment decision is a long-term decision while the carriers' pricing and routing decision is a short-term decision. The shippers' and carriers' behaviors in turn influence the Port Authority's decision. How the shippers and the carriers react to the Port Authority's investment strategy determines the effectiveness of this strategy. The Port Authority makes its investment decision based on its forecast of the effects on carriers' pricing and routing decision and the shift of the production, consumption and flow pattern under different investment strategies.

BEHAVIORAL MODELING

The interactions discussed above can be formulated using a three-level model. The first level describes the behavior of the Port Authority in choosing the best investment strategy to maximize net social benefit. The second level describes the behavior of the carriers in choosing optimal service charge and routing pattern to maximize their profits. The third level formulates the behavior of the shippers, which is to determine the supply and demand of each commodity at each market and the distribution pattern of each commodity on the network.

This three level model aims to answer behavioral questions related to each major player. In terms of port authority behavior, the model answers questions on which investment strategy out of a finite set of alternatives should the port authority implement to maximize the net social benefit; and what will be the impact of this strategy on the terminal operators and consequently the shippers. Related to the carriers' behavior, the model aims to answer questions on the level of the equilibrium service charge and routing pattern on each carrier's sub-network given the competitive pricing game among the carriers; the optimal set of service charge and routing pattern on each terminal sub-network and the resulting profit if the carriers choose to price collusively; and the impact of the competitive or collusive pricing on the shippers' decisions. In terms of the shippers' behavior, the model aims to answer questions on the optimal locations at which goods are produced and consumed and their optimal quantity and price; and the equilibrium flow on the shipper network and the resulting cost.

In modeling shippers' behavior, the assumed market condition is perfect competition. Each individual shipper acts as a price taker in the market. By purchasing in the market with the lowest cost, selling in the market with the highest price, and shipping the commodity via the path with the lowest cost, each shipper aims to maximize profit. The equilibrium resulting from this rent seeking behavior can be described as a *spatial price equilibrium* (SPE) (Sheffi, 1985), at which the demand price at any destination market with positive consumption of certain commodity equals the sum of the supply price at any origin market and the transaction cost on any path with positive flow between these two markets. The shippers' behavior may be modeled as a variational inequality problem.

Competition among carriers may be modeled using a non-cooperative game-theoretic model based on the *Nash oligopolistic market equilibrium principle*. This principle is founded on the economic theory of imperfect competition. Terminal operators can observe each other's pricing behavior and react accordingly, which assumes oligopolistic market conditions. The terminal operators can either compete or collude in making pricing decisions. When pricing competitively, the equilibrium is reached at the price that is optimal to each individual operator. This equilibrium is referred to as *Bertrand Equilibrium* (Tirole, 1988), which is a type of Nash equilibrium for the pricing game among finite number of players. It states that no player can be better of by unilaterally adjusting the price. If the operators choose to price collusively, the equilibrium is reached at the price that is optima from the viewpoint of all operators together. The *compensation principle* of Hicks and Kalder can be used as a criterion to evaluate whether the collusion can be established and sustained. The principle states that the collusion should achieve better collective outcome than the sum of the outcomes achieved individually without any cooperation among players.

The interaction between shippers and carriers may be formulated as a *Stakelberg game*, which is also called the leader and follower game in game theory. In this type of game, the leader makes a decision based on forecast of the follower's reaction, and the follower makes a decision based on knowledge of the leader's decision. In this case the carrier is assumed to be the leader who sets the service fare based on forecast of the shipper's behavior concerning production, consumption and shipping. The shipper is the follower in the game. The service fare set by the leader affects the shipper's decision. On the other hand, the decision of the follower, the shipper, determines the demand for service and consequently influences the profit of the terminal operator.

The mathematical formulations of the above-described problems are presented in detail in Boile and Wang (2000). A graphical representation of the modeling procedure is shown in Figure 2 below.

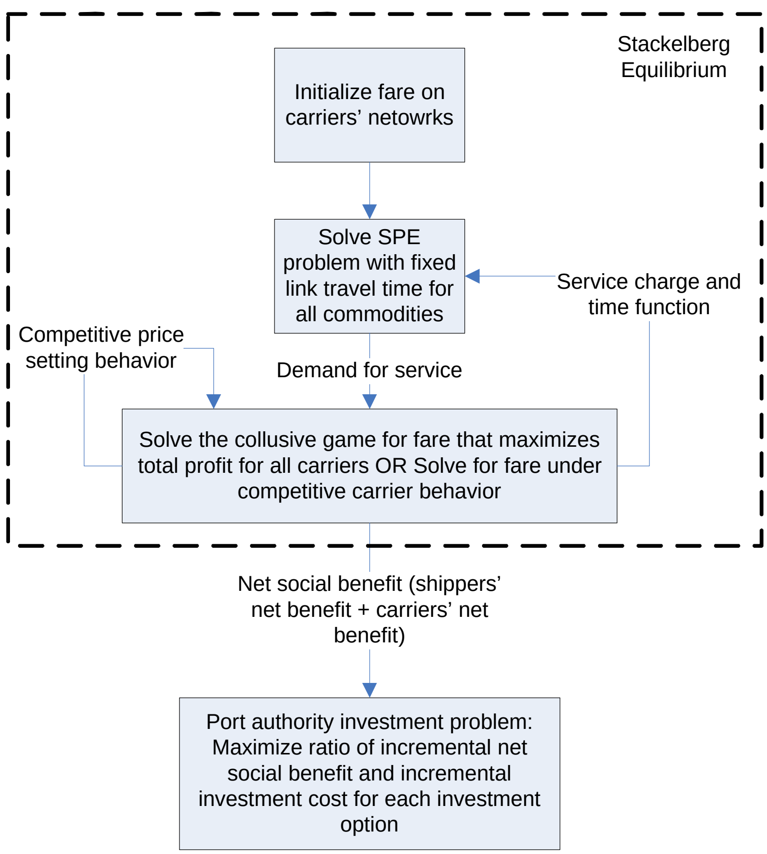


Figure 2: Modeling framework of player interactions

The shippers' and carriers' reaction to the Port Authority's investment strategy determines the effectiveness of the strategy. The Port Authority makes its investment decision based on its forecast of the effects on carriers' pricing and routing decision and the shift of the production, consumption and flow pattern under different investment strategies.

THE PORT AUTHORITY'S INVESTMENT PROBLEM

Boile and Wang (2002) provided a bi-level programming approach to solve the Stackelberg equilibrium between carriers and shippers. This approach is used to facilitate the Port Authority's investment decisions as it is demonstrated in the following sections (mathematical notation is explained at the end of the paper). The criterion used in comparing alternative investment strategies determines whether the incremental net social benefit brought by the investment is greater than the incremental investment cost. The net social benefit is defined as the sum of the net benefits of all players in the port vicinity affected by the port authority's investment in the port infrastructure. The investment cost is the capital expense associated with an investment strategy. For an investment strategy to be feasible, the incremental net social benefit should exceed the incremental investment cost.

<u>Terminal Operators' Net Benefit</u>

Terminal operators are the producers of the port service. For the terminal operators, the monetary value of their net benefits is indicated by the total profits earned from their services. Let (R^u, e^u) denote the terminal operators' decision at the Stackelberg equilibrium under investment strategy u. Then, the terminal operators' net benefit under investment strategy u (TNB^u) is given as Eq. (1).

$$\begin{aligned} TNB^u &= \sum_{t \in TM} Z_t(g_t(R_t^u, R_{-t}^u), R_t^u, e_t^u) \\ &= \sum_{t \in TM} \Big(\sum_{v \in V^t} \sum_{c \in C} g_{v,c}(R_t^u, R_{-t}^u) * R_{v,c}^u - \sum_{a \in A^t} \sum_{c \in C} AC_{a,c}(e_{a,c}^u) * e_{a,c}^u \Big) \end{aligned} \qquad \forall u \in U \qquad (1)$$

The service demand $g_{v,c}(R_t^u, R_{-t}^u)$ and the link flow $e_{a,c}^u$ are in units of flow per hour; TNB^u is in dollars per hour. Assuming that all terminals' profits occur in the port vicinity, 100% of TNB^u are included in the calculation of the net social benefit. As to the carriers other than the terminal operators (who, in this paper, are considered to be a special form of carriers), their profits may or may not occur in the port vicinity. Here, for the sake of simplification, the profits of the carriers other than the port terminal operators are not included in the analysis.

<u>Shippers' Net Benefit</u>

Shippers are the users of the terminal service. According to the economic theory (Wohl, 1984), their net benefit is the monetary value of their total willingness to pay minus the amount they do pay. The shippers can be either the consumers or the producers of the transported commodities. The shippers' net benefit is broken down into two sources. Consumer surplus (*CS*), which is the consumer's total willingness to pay minus what the consumer actually pays for the transported commodities; and producer surplus (*PS*), which is the total sales revenue minus the total production cost.

To estimate the shippers' net benefit, the inverse supply and inverse demand functions may be formulated as shown in Eqs. (2) and (3).

$$\pi^u_{b,c}(S_{b,c}) = (\gamma_{b,c} + \sum_{c' \in C, c' \neq c} \lambda_{b,c',c} * S^u_{b,c'}) + \lambda_{b,c,c} * S_{b,c}$$
$$= \gamma^u_{b,c} + \lambda_{b,c,c} * S_{b,c} \qquad \forall\, b \in CN, c \in C, u \in U \qquad (2)$$
$$\text{where } \gamma^u_{b,c} = (\gamma_{b,c} + \sum_{c' \in C, c' \neq c} \lambda_{b,c',c} * S^u_{b,c'})$$

$$\rho^u_{b,c}(D_{b,c}) = (\alpha_{b,c} - \sum_{c' \in C, c' \neq c} \beta_{b,c',c} * D^u_{b,c'}) - \beta_{b,c,c} * D_{b,c}$$
$$= \alpha^u_{b,c} - \beta_{b,c,c} * D_{b,c} \qquad \forall\, b \in CN, c \in C, u \in U \qquad (3)$$
$$\text{where } \alpha^u_{b,c} = (\alpha_{b,c} - \sum_{c' \in C, c' \neq c} \beta_{b,c',c} * D^u_{b,c'})$$

In Eqs. (2) and (3), $\pi^u_{b,c}(S_{b,c})$ is the inverse supply function of commodity *c* at centroid *b* given that the supply vector for the other commodities is $S^u_{b,-c}$. $\rho^u_{b,c}(D_{b,c})$ is the inverse demand function of commodity *c* at centroid *b* given that the demand vector for the other commodities is $D^u_{b,-c}$. Using these functions in Eqs. (2) and (3), the consumer surplus and the producer surplus at centroid *b* for commodity *c* can be calculated.

The consumer surplus at centroid *b* from the consumption of commodity *c* ($CS^u_{b,c}$) is calculated using the formula in Eq. (4).

$$CS^u_{b,c} = \int_0^{D^u_{b,c}} \rho^u_{b,c}(D^u_{b,c})dD^u_{b,c} - \rho^u_{b,c}(D^u_{b,c}) * D^u_{b,c} \quad \forall\, b \in CN, c \in C, u \in U \qquad (4)$$

The producer surplus at centroid *b* from the production of commodity *c* ($PS^u_{b,c}$) is calculated using the formula in Eq. (5).

$$PS^u_{b,c} = \pi^u_{b,c}(S^u_{b,c}) * S^u_{b,c} - \int_0^{S^u_{b,c}} \pi^u_{b,c}(S^u_{b,c})dS^u_{b,c} \quad \forall\, b \in CN, c \in C, u \in U \qquad (5)$$

Combining Eqs. (4) and (5), the shippers' net benefit at centroid *b* from the consumption and the production of commodity *c* ($SNB^u_{b,c}$) is obtained as follows:

$$SNB^u_{b,c} = CS^u_{b,c} + PS^u_{b,c}$$
$$= \int_0^{D^u_{b,c}} \rho^u_{b,c}(D^u_{b,c})dD^u_{b,c} - \rho^u_{b,c}(D^u_{b,c}) * D^u_{b,c} + \pi^u_{b,c}(S^u_{b,c}) * S^u_{b,c} - \int_0^{S^u_{b,c}} \pi^u_{b,c}(S^u_{b,c})dS^u_{b,c}$$
$$\forall\, b \in CN, c \in C, u \in U \qquad (6)$$

The shippers' net benefit (SNB^u) is calculated as the sum of $SNB^u_{b,c}$ for each centroid and each commodity type as follows:

$$
\begin{aligned}
SNB^{u} &= \sum_{b\in CN,c\in C} SNB^{u}_{b,c} = \sum_{b,c}\left(CS^{u}_{b,c} + PS^{u}_{b,c}\right) \\
&= \sum_{b,c}\left(\int_{0}^{D^{u}_{b,c}} \rho^{u}_{b,c}(D^{u}_{b,c})dD^{u}_{b,c} - \rho^{u}_{b,c}(D^{u}_{b,c}) * D^{u}_{b,c} + \pi^{u}_{b,c}(S^{u}_{b,c}) * S^{u}_{b,c} - \int_{0}^{S^{u}_{b,c}} \pi^{u}_{b,c}(S^{u}_{b,c})dS^{u}_{b,c} \right) \\
&= \sum_{b,c}\left(\int_{0}^{D^{u}_{b,c}} \rho^{u}_{b,c}(D^{u}_{b,c})dD^{u}_{b,c} - \int_{0}^{S^{u}_{b,c}} \pi^{u}_{b,c}(S^{u}_{b,c})dS^{u}_{b,c} \right) - \sum_{b,c}\left(\rho^{u}_{b,c}(D^{u}_{b,c}) * D^{u}_{b,c} - \pi^{u}_{b,c}(S^{u}_{b,c}) * S^{u}_{b,c} \right) \\
&= \sum_{b,c}\left(\int_{0}^{D^{u}_{b,c}} \rho^{u}_{b,c}(D^{u}_{b,c})dD^{u}_{b,c} - \int_{0}^{S^{u}_{b,c}} \pi^{u}_{b,c}(S^{u}_{b,c})dS^{u}_{b,c} \right) - \sum_{b1,b2\in CN,c}\left(GC^{u}_{b1,b2,c} * Q^{u}_{b1,b2,c} \right) \\
&= \sum_{b,c}\left(\int_{0}^{D^{u}_{b,c}} \rho^{u}_{b,c}(D^{u}_{b,c})dD^{u}_{b,c} \right) - \sum_{b,c}\left(\int_{0}^{S^{u}_{b,c}} \pi^{u}_{b,c}(S^{u}_{b,c})dS^{u}_{b,c} \right) - \sum_{l\in L,c}\left(GC^{u}_{l,c} * f^{u}_{f,c} \right) \\
& \qquad \forall\, u \in U \qquad (7)
\end{aligned}
$$

In Eq. (7), the first element is the sum of consumers' willingness to pay for each commodity at each market. The second element is the sum of production cost for each commodity at each market. The third element is the total generalized transportation cost. The supply $S^{u}_{b,c}$, the demand $D^{u}_{b,c}$, and the link flow $f^{u}_{l,c}$ are all in units of flow per hour; SNB^{u} is in dollars per hour.

<u>Adjustments to the Shippers' Net Benefit</u>

It is important to note that two adjustments need to be made before including the shippers' net benefit in the calculation of the net social benefit. First, only part of the commodity transported via the port terminals is produced or consumed in the local region. The rest is just a passing through traffic to the designations outside the region and as such it doesn't contribute to the region's net social benefit. The portion of the shippers' net benefit, which directly contributes to the net social benefit of the local region, is called the local shippers' net benefit. A ratio (υ_c) is used to denote the passing through traffic as a percentage of the total freight of commodity c. Then, the local shippers' net benefit as a percentage of the total shippers' net benefit of commodity c is given as 1-υ_c.

Second, besides the shippers' net benefit directly related to the local production and consumption of these traded commodities, other economic sections in the port vicinity are involved and benefited in a meaningful way from the trade in these commodities and all associated manufacturing and services. To account for this external net benefit, a multiplier (ζ) is used to denote the ratio of external benefit to the localized shippers' net benefit.

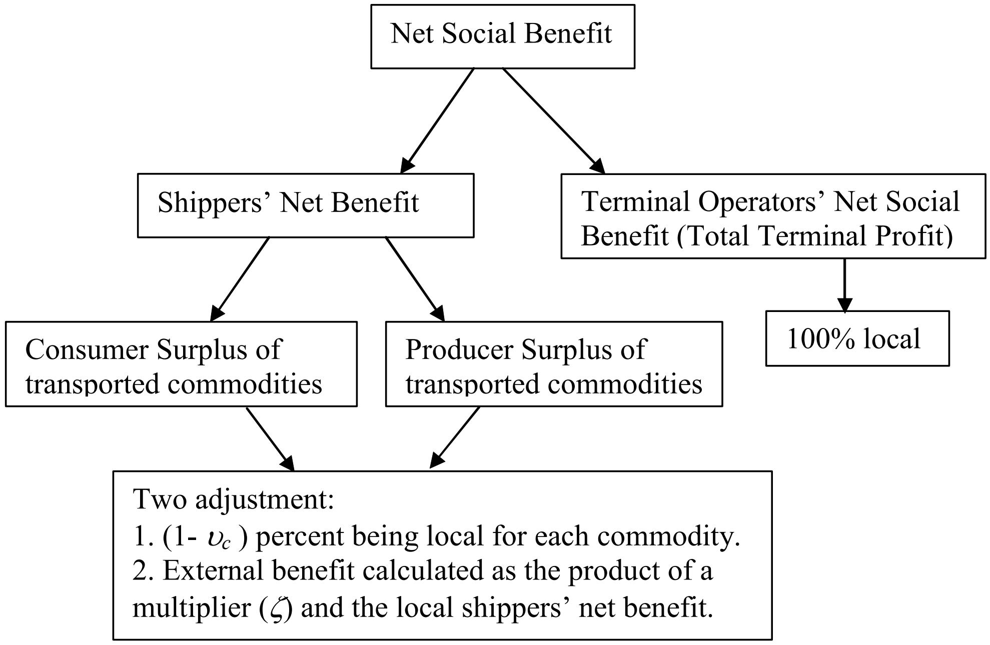


Figure 3: Net Social Benefit

Taking into account the passing through traffic and the external economy, the adjusted shippers' net benefit under investment strategy u ($ASNB^u$) is calculated in Eq. (8).

$$\begin{aligned} & ASNB^u = \\ & (1+\zeta)*\sum_{c\in C}\left((1-\upsilon_c)*\sum_{b\in CN} SNB^u_{b,c}\right) \qquad \forall\, u \in U \qquad (8) \\ & = (1+\zeta)*\sum_{c\in C}\left((1-\upsilon_c)*\sum_{b\in CN}(CS^u_{b,c}+PS^u_{b,c})\right) \end{aligned}$$

Given the Stackelberg equilibrium (S^u, f^u, D^u, R^u, e^u) (Boile and Wang, 2002), the net social benefit under investment strategy u (NSB^u) is calculated as:

$$\begin{aligned} & NSB^u(S^u, f^u, D^u, R^u, e^u) = \\ & TNB^u(R^u, e^u) + ASNB^u(S^u, f^u, D^u) \end{aligned} \qquad \forall\, u \in U \qquad (9)$$

The above discussion of the various sources of net social benefit is illustrated in Figure 3.

<u>Investment Cost</u>

There is a finite number of alternative investment strategies. Associated with each investment strategy ($u\in U$) is a specific vector of capacity improvement pattern $\Delta\overline{E}^u = (\cdots,\Delta\overline{E}^u_a,\cdots)_{|A|}$. Under the do-nothing-strategy, $\Delta\overline{E}^u = 0$. The investment cost associated with an investment strategy must be defined and expressed in units that allow for comparison between different investment strategies, which vary in their service lives. For this purpose, investment costs are expressed as hourly costs using a method presented in Boile and Wang (2000).

The hourly investment cost on link *a* under investment strategy *u* is a function of the capacity improvement $\Delta \overline{E}_a^u$, the analysis period designated, the service life of the facility improved, and the discount rate. The total investment cost of the Port Authority is the sum of the investment costs on all improved links.

For the investment cost, a linear function similar to that shown in Yang and Meng (2000) is implemented. The flow dependent investment cost such as the maintenance cost is not considered. The investment cost on link *a* under investment strategy *u* (IC_a^u) is defined as follows:

$$IC_a^u = p2_a^u * (p1_a^u * \Delta \overline{E}_a^u) \qquad \forall a \in A, u \in U \qquad (10)$$

In Eq. (10), $p1_a^u$ is a parameter that represents cost of one additional unit of capacity. The value of $p1_a^u$ is determined by the type of facility represented by link *a* and the resources such as technology used for investment strategy *u*. In Eq. (10), $p1_a^u * \Delta \overline{E}_a^u$ represents the capital expense for the capacity improvement of $\Delta \overline{E}_a^u$ on link *a*. $p2_a^u$ is a factor that converts the capital expense into hourly investment cost. The value of $p2_a^u$ depends on the analysis period, the service life of this capital expense, and the discount rate. The method to calculate $p2_a^u$ is illustrated in Boile and Wang (2000).

Let $p_a^u = p1_a^u * p2_a^u$. Then, Eq. (10) can be restated as: $IC_a^u = p_a^u * \Delta \overline{E}_a^u$. The total hourly investment cost under investment strategy *u* (IC^u) can be calculated as the sum of IC_a^u over all links. Thereby:

$$IC^u(\Delta \overline{E}^u) = \sum_a IC_a^u(\Delta \overline{E}_a^u) = \sum_a p_a^u * \Delta \overline{E}_a^u \qquad \forall a \in A, u \in U \qquad (11)$$

<u>Mathematical Formulation of the Port Authority's Investment Problem</u>

The Port Authority aims to maximize the ratio between the incremental net social benefit brought about to the region through an investment, and the incremental investment cost. The incremental net social benefit through investment strategy *u* (ΔNSB^u) is calculated as follows:

$$\begin{aligned} &\Delta NSB^u(S^u, f^u, D^u, R^u, e^u) \\ &= NSB^u(S^u, f^u, D^u, R^u, e^u) - NSB^0(S^0, f^0, D^0, R^0, e^0) \end{aligned} \qquad \forall u \in U \qquad (12)$$

Where, $NSB^u(S^u, f^u, D^u, R^u, e^u)$ is the net social benefit under investment strategy *u*. $NSB^0(S^0, f^0, D^0, R^0, e^0)$ is the net social benefit under the do-nothing strategy.

The incremental investment cost is calculated as follows:

$$\Delta IC^u(\Delta \overline{E}^u) = IC^u(\Delta \overline{E}^u) - IC^0(\Delta \overline{E}^0) = IC^u(\Delta \overline{E}^u) \qquad \forall u \in U \qquad (13)$$

Where, $IC^u(\Delta \overline{E}^u)$ is the hourly investment cost under investment strategy *u*. $IC^0(\Delta \overline{E}^0)$ is the hourly investment cost under the do-nothing strategy, which equals to zero since $\Delta \overline{E}^0$=0.

Combining Eq. (12) and Eq. (13), the investment problem for the Port Authority is defined and stated in Table 1.

Table 1 Port Authority's Investment Problem

$$\frac{\Delta NSB^{u^*}(S^{u^*}, f^{u^*}, D^{u^*}, R^{u^*}, e^{u^*})}{IC^{u^*}(\Delta \overline{E}^{u^*})} = \underset{u\in U}{Max}\left[\frac{\Delta NSB^{u}(S^{u}, f^{u}, D^{u}, R^{u}, e^{u})}{IC^{u}(\Delta \overline{E}^{u})}\right] \qquad (14)$$

s.t.

P1 for competitive game or **P2** for collusive game

Where u^* is the most desirable investment strategy.

P1 competitive game

$$\sum_{t\in T}\left(-\nabla_{R_t} Z_t(g_t(R_V^*), R_t^*, e_t^*)(R_t - R_t^*) - \nabla_{e_t} Z_t(g_t(R_V^*), R_t^*, e_t^*)(e-e^*)\right) \geq 0$$
$$\forall (R_V, e) \in KT$$

where $g_{v,c}(R_V) = \sum_l \xi_{l,v} * f_{l,c}(R_L)$

s.t. $\pi(S^*)\bullet(S-S^*) + GC(f^*, R_L^0)\bullet(f-f^*) - \rho(D^*)\bullet(D-D^*) \geq 0$
$\forall (S, f, D) \in KS$

$$R_L^0 = \left[\xi_{l,v}\right]_{|L|*|V|} * R_V^*.$$

or

P2 collusive game

$$\underset{R_{vl,c}, e_{a,c}}{Max} : \sum_t Z_t(g_t(R_V), R_t, e_t)$$

s.t. (R_V, e)□KT,

where $g_{v,c}(R_V) = \sum_l \xi_{l,v} * f_{l,c}(R_L)$

s.t. $\pi(S^*)\bullet(S-S^*) + GC(f^*, R_L^0)\bullet(f-f^*) - \rho(D^*)\bullet(D-D^*) \geq 0$
$\forall (S, f, D) \in KS$

$$R_L^0 = \left[\xi_{l,v}\right]_{|L|*|V|} * R_V^*.$$

Problem 1 (P1) above is the formulation of the bi-level program formulating the Stackelberg game between carriers and shippers for the competitive game. Problem 2 (P2) is the bi-level program for the collusive game. In these formulations the shippers' Spatial Price Equilibrium model (SPE) is combined with the carriers' pricing and routing problem. The port authority investment problem is solved subject to the bi-level shipper / carrier model. This behavioral problem determines capacity limitations within the marine terminal's sub-network.

NUMERICAL EXAMPLE

This section presents results from the application of the model to the test network shown in figure 4. The Stackerberg equilibrium between two oligopolistic carriers, in this case private port terminal operators, and various shippers is solved. Strategies that the port authority can use to invest in terminals' infrastructure are evaluated. Numerical results are presented and discussed to demonstrate the applicability of the model.

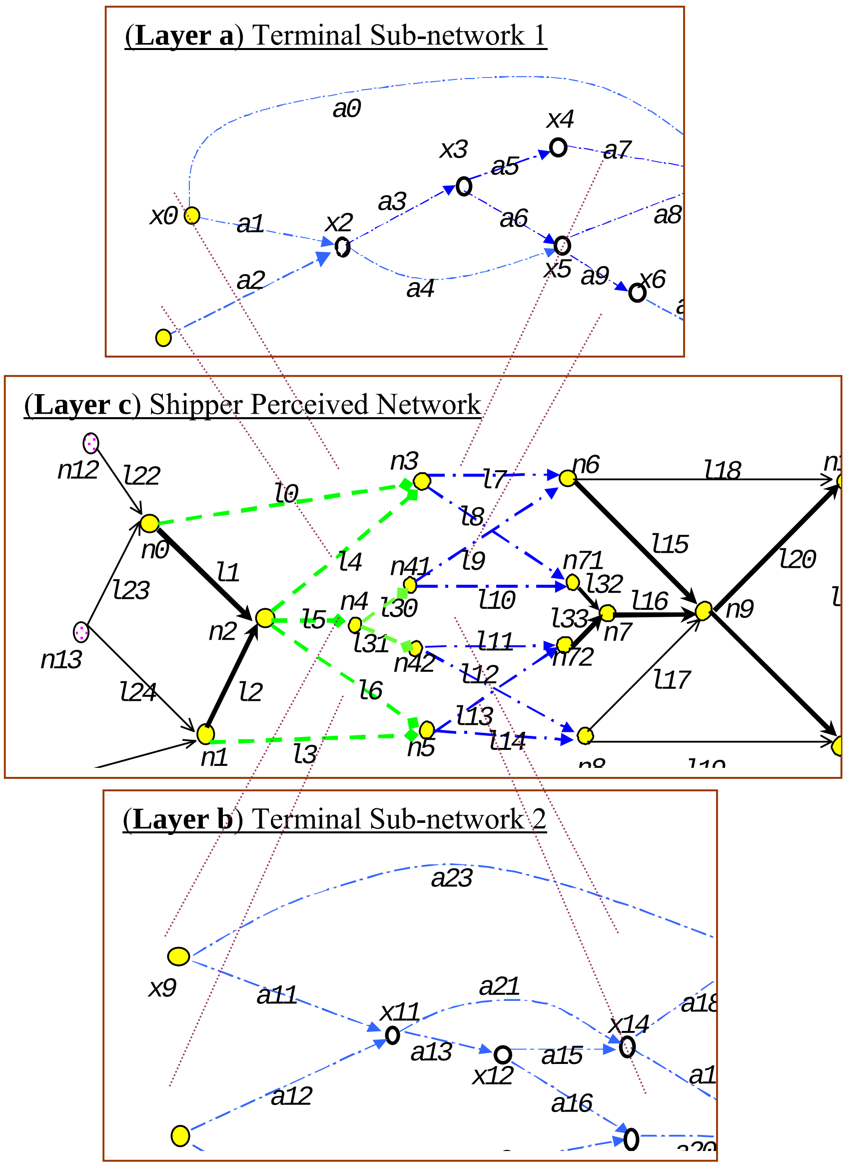
The test network consists of two terminal operators with their two respective sub-networks shown in figure 3 as layers (a) and (b). Terminal 1 sub-network shown in layer (a), consists of nine nodes (*x0*-*x8*), twelve links (*a0*-*a10*, *a22*) and four O-D pairs [*v0*=(*x0*, *x7*), *v1*=(*x0*, *x8*), *v2*=(*x1*, *x7*), *v3*=(*x1*, *x8*)]. Terminal 2 sub-network, shown in layer (b), also consists of nine nodes (*x9*-*x17*), twelve links (*a11*-*a21*, *a23*) and four O-D pairs [*v4*=(*x9*, *x16*), *v5*=(*x9*, *x17*), *v6*=(*x10*, *x16*), *v7*=(*x10*, *x17*)].

The shipper network is shown in layer (c). The network consists of 22 nodes (*n0*-*n17, n41, n42, n71, n72*) and 34 links (*l0*-*l33*). Among the 22 nodes, *n*12-*n*17 are centroid nodes, which comprise nine O-D pairs ((*n12*, *n15*), (*n12*, *n16*), (*n12*, *n17*), (*n13*, *n15*), (*n13*, *n16*), (*n13*, *n17*), (*n14*, *n15*), (*n14*, *n16*), (*n14*, *n17*)).

The shipper network and the terminal sub-networks are related through the incidence matrix between the port links *l7-l14* in the shipper network and the O-D pairs *v0-v7* in the terminal sub-networks. This relationship indicates that a shipper sees only the starting and ending points in a carriers subnetwork, meaning that the shipper is interested in the cost and service associated with the commodity movement between two points and not necessarily on the detailed routing of the commodity. This relationship is also manifested in figure 4. For example, (*x0*, *x7*) that are the origin and destination nodes of O-D pair *v0* corresponds to (*n3*, *n6*) that are the starting and ending nodes of link *l7*.

In figure 4, the shipper links other than *l7* to *l14* correspond to certain O-D pairs on the sub-networks of the carriers other than the terminal operators. The pricing and routing behavior of these other carriers is not the focus of this application and their service charges are assumed to be constant, which indicates that the shippers will not change their perception of the cost between the O-D pairs on the sub-networks of these carriers. Hence, the presentation of the sub-networks of these carriers as a set of links on the shipper network is sufficient for the demonstration purpose of this application.

A heuristic algorithm that iteratively solves the upper level terminal operators' pricing and routing problem and the lower level shippers' SPE problem has been implemented. Details on the algorithm's implementation, solution and convergence are given in Boile and Wang (2000). The solution to this bi-level problem is then used to facilitate the Port Authority's investment decisions. The net social benefit (*NSB*) is used as a measure of the worthiness of an investment strategy. The net social benefit for both shippers and carriers, the players impacted by the port authority's investment decision, is estimated. The investment improves the terminal operators' operating cost and the shippers' generalized cost. In response to the improvement in these costs, the terminal operators as well as the other carriers and the shippers will adjust their behavior until a new Stackelberg equilibrium is attained. The bi-level program in Boile and Wang (2002) is used to predict this new equilibrium, based on which the terminal operators' net benefit (*TNB*) and the shippers' net benefit (*SNB*) associated with the port authority investment decision are estimated. In formulating the port authority's investment problem the ratio between the incremental net social benefit brought about to the region through an investment and the incremental investment cost is estimated for each investment strategy.



Legends:

	Highway links		Railway links
	Waterway links		Port links
	Links in terminal sub-networks		Intermediate node on the shipper network and origin/destination nodes on the terminal networks
	Centroid nodes		Intermediate node at a terminal network

Figure 4: Transportation Networks for the Example

Numerical results are used to identify the candidate terminal links for improvement, and evaluate the economic impact of various investment strategies. In this example it is assumed that both terminal operators are under the same Port Authority.

Denote the do-nothing strategy as *S0*. Using the bi-level model the candidate terminal links for improvement can be identified by comparing the current flow on a link (e_a^{S0}) with its capacity ($\overline{E}_a^{S0}$). The links on which the flow exceeds capacity ($e_a^{S0} > \overline{E}_a^{S0}$) are the candidates for improvement. Model results indicate that these links are *a0*, *a1*, *a4*, *a12*, *a13*, *a14*, *a17*, *a19*, *a20*, *a21*, *a22*, *a23*.

The Port Authority can choose to invest in expanding the capacity of those candidate links to the current flow level, that is $\Delta\overline{E}_a = (e_a^{S0} - \overline{E}_a^{S0})$. Depending on the availability of funds, all candidate links or only a partial set of those candidate links may be improved. In this example, three investment strategies besides the do-nothing strategy *S0* are envisioned and their impact on the shippers and terminal operators are predicted. The first investment strategy *S1* adds capacity on those links belonging to the first terminal: *a0 a1*, *a4*, and *a22*. The second investment strategy *S1* adds the capacity on those links belonging to the second terminal: *a12*, *a13*, *a14*, *a17*, *a19*, *a20*, *a21*, and *a23*. The third investment strategy *S3* adds capacity on all the candidate links. Table 2 presents the intended capacity improvement associated with investment strategies *S1*, *S2* and *S3*.

Table 2 Capacity Improvement under Three Investment Strategies

	Collusive game			Competitive game		
Link	$\Delta\overline{E}_a^{S1}$ (units/hr)	$\Delta\overline{E}_a^{S2}$ (units/hr)	$\Delta\overline{E}_a^{S3}$ (units/hr)	$\Delta\overline{E}_a^{S1}$ (units/hr)	$\Delta\overline{E}_a^{S2}$ (units/hr)	$\Delta\overline{E}_a^{S3}$ (units/hr)
a0	10.8813	0	10.8813	15.7695	0	15.7695
a1	5.04124	0	5.04124	9.08145	0	9.08145
a4	0.27189	0	0.27189	4.61153	0	4.61153
a12	0	8.93173	8.93173	0	12.6596	12.6596
a13	0	7.02437	7.02437	0	11.0276	11.0276
a14	0	5.67637	5.67637	0	8.87876	8.87876
a17	0	10.1764	10.1764	0	13.3788	13.3788
a19	0	7.63156	7.63156	0	10.3923	10.3923
a20	0	9.929	9.929	0	14.3758	14.3758
a21	0	13.2731	13.2731	0	17.7631	17.7631
a22	15.1843	0	15.1843	23.4173	0	23.4173
a23	0	24.587	24.587	0	31.1064	31.1064

Table 2 shows that the competitive game has higher investment requirements than the collusive game. This is due to the fact that under the competitive game the terminal operator tends to charge lower service fee and thus more service demand is induced resulting in higher capacity requirements.

For the investment strategies *S1*, *S2* and *S3* proposed above the bi-level problem is solved to determine the equilibrium supply, demand and routing decision of the shippers and the equilibrium service charge and routing pattern in each terminal sub-network Based on the equilibrium solution, various indexes regarding the port operation are calculated and compared for different investment strategies as shown in Tables 3 and 4 below.

Table 3 Demand, Travel Time, Revenue and Profit under Four Investment Strategies

		Terminal 1				Terminal 2			
	Strategy	Total demand (units/hr)	Average Travel Time (hrs/unit)	Total Revenue ($/hr)	Total Profit ($/hr)	Total Demand (units/hr)	Average Travel Time (hrs/unit)	Total Revenue ($/hr)	Total Profit ($/hr)
Collusive Game	*S0*	102	3.87	3634	3279	123	2.81	4908	4181
	S1	108	4.05	3714	3429	120	2.77	4806	4104
	S2	96	3.68	3390	3075	139	3.14	5190	4623
	S3	102	3.85	3473	3219	137	3.09	5091	4542
Competitive Game	*S0*	123	4.41	3520	3012	141	3.14	4762	3798
	S1	132	4.69	3534	3171	139	3.10	4612	3682
	S2	118	4.23	3236	2771	163	3.59	4883	4199
	S3	127	4.51	3250	2918	160	3.53	4739	4078

Table 3 shows that the investment of the port authority will generate additional service demand at the port terminal with capacity improvement and may decrease the service demand at the port terminal without capacity improvement. For example, under the collusive game with the investment strategy *S1*, the service demand at port terminal 1 will increase from 102 units per hour to 108 units per hour. However, it will decrease at port terminal 2 from 123 to 120 units per hour. This is explained by the fact that with the cost advantage resulted from the capacity improvement, terminal 1 is able to adjust the service charge and hence attract some business from terminal 2.

The increase on average travel time from the increase in demand at terminal 1 will more than offset the savings of average travel time resulted from the increase in capacity, thus causing an overall increase in the average travel time. For example, for the competitive game, the investment strategy *S1* causes the average travel time at port terminal 1 to increase from 4.41 hours per unit to 4.69 hours per unit.

It is also shown in Table 3 that the investment on one terminal has a reverse effect on the other terminal competing for the service demand but without any investment. For example, the investment strategy *S2* increases the revenue and profit at port terminal 2 while it decreases the revenue and profit at port terminal 1 under both the collusive and competitive games. This is attributed to the decrease of the service charge at terminal 1 in order to maintain business and to the resulting decrease in demand for terminal 1 service.

Table 3 also shows that the investment strategy *S3* may have different effects on the revenue and profit at the two terminals even though both terminals receive investment. For example, the investment strategy *S3* under the competitive game decreases the profit at port terminal 1 while it increases the profit at port terminal 2. This phenomenon can be explained by the fact that the investment intensifies the price competition between the two terminals, hence reducing the equilibrium service charges at both terminals as shown in Table 4. Depending on the elasticity of the service demand with respect to the service charge, the revenue may increase or decrease with the reduction of the service charge. If the elasticity of the service demand is less than 1, the negative effect on the revenue from the decrease in service charge dominates the positive effect from the increase in demand. Hence, the revenue will decrease. Otherwise, the revenue will increase. In this example, investment strategy *S3* decreases the revenue at port terminal 1 in spite of the increase in the service demand from 123 to 127 units per hour at this terminal. This indicates that at the current equilibrium point, the demand elasticity at port terminal 1 is less than 1.

Table 4 Equilibrium Service Charge of Commodity *c3*

O-D Pair		*R* ($/unit) for Collusive Game				*R* ($/unit) for Competitive Game			
		S0	*S1*	*S2*	*S3*	*S0*	*S1*	*S2*	*S3*
Terminal 1	*v0*	47	45	46	45	37	34	35	33
	V1	46	44	45	44	35	32	33	31
	v2	45	44	45	44	34	32	32	31
	v3	46	45	46	45	34	32	33	31
Terminal 2	*v4*	49	49	46	46	39	38	34	34
	v5	48	48	46	46	39	38	35	34
	v6	50	49	47	47	40	40	36	35
	v7	51	51	49	49	43	42	39	38

The Port Authority is interested in the net social benefit (*NSB*) and the investment cost (*IC*) under different investment strategies. In calculating *NSB*, the economic multiplier ς is set to 4 to account for the external economy. To demonstrate the effect on *NSB* from the percentage of the passing through traffic, a set of different percentages of passing through traffic ($\upsilon_c=\upsilon$ $\forall c\in C$) is used. In calculating *IC*, p_a^u is set to \$5 per unit for any terminal link *a* and any investment strategy *u*. Similar as the values selected for the value-of-time, the values for these three parameters are also selected arbitrary. Same as before, the accuracy of these arbitrary selected values doesn't affect the demonstration purpose of the numerical example. Based on these values of the parameters (ς, p_a^u and υ_c) and the Stackelberg equilibrium results, *NSB* and *IC* for each investment strategy are calculated as shown in Table 5.

Table 5 Net Social Benefit (*NSB*) and Investment Cost (*IC*)

Strategy	Collusive Game					Competitive Game				
	NSB ($/hr)				*IC* ($/hr)	*NSB* ($/hr)				*IC* ($/hr)
	$\upsilon=1$	$\upsilon=0.7$	$\upsilon=0.4$	$\upsilon=0$		$\upsilon=1$	$\upsilon=0.7$	$\upsilon=0.4$	$\upsilon=0$	
S0	7460	9855	12249	15442	0	6811	9926	13042	17196	0
S1	7534	9992	12451	15729	157	6853	10116	13379	17729	264
S2	7698	10281	12863	16307	436	6970	10447	13923	18559	598
S3	7761	10401	13041	16560	593	6995	10607	14219	19034	862

Two observations can be made based on Table 5. First, if no freight is produced or consumed locally ($\upsilon=1$), the net social benefit is higher for the collusive game under all investment strategies. This can be explained by the fact that the collusion brings more profit to the terminal operation as a whole, thus increasing the operators' contribution in the net social benefit formula. Second, with the decrease in the percentage of the passing through traffic, the net social benefit becomes higher for the competitive game. This can be explained by the fact that the competitive game generates more commodity production, consumption and shipment. With the percentage of passing through traffic low enough, the positive effect on the net social benefit from the gain in local shippers' net benefit will more than offset the negative effect on the net social benefit from the loss of total profit at port operation for the competitive game.

Based on the net social benefit and the investment cost shown in Table 5, the ratios between the incremental net social benefit and the incremental investment cost under the various investment strategies are calculated and are shown in Table 6

Table 6 Ratio of Incremental Net Social Benefit and Incremental Investment Cost

	Collusive Game				Competitive Game			
	ΔNSB/ΔIC				*ΔNSB/ΔIC*			
Strategy	$\upsilon=1$	$\upsilon=0.7$	$\upsilon=0.4$	$\upsilon=0$	$\upsilon=1$	$\upsilon=0.7$	$\upsilon=0.4$	$\upsilon=0$
S0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
S1	0.47	0.88	1.29	1.83	0.16	0.72	1.27	2.02
S2	0.55	0.98	1.41	1.98	0.27	0.87	1.47	2.28
S3	0.51	0.92	1.33	1.89	0.21	0.79	1.36	2.13

Table 6 shows that when the passing through traffic is high, the do-nothing strategy may become the best choice since the gain in net social benefit may not be enough to cover the increase in investment cost. For example, for both the competitive and collusive games, the ratio between the incremental net social benefit and the incremental investment cost is less than 1 under investment strategies *S1*, *S2* and *S3* when the percentage of passing through traffic is 100% or 70%. Table 7.6 also shows that with the decrease of the passing through traffic, the investment strategy for the competitive game results in higher ratio between the incremental net social benefit and the incremental investment cost, while it is the opposite case when the percentage of the passing through traffic is high.

Observations from Tables 5 and 6 together with the result in Table 3 indicate that the collusion of the terminal operators under the control of one port authority is favored by the terminal operators and the port authority alike if a very high percentage of the freight is not produced or consumed locally. In this case, the social benefit to the local region brought by the terminal operations is mainly contributed by the terminal profits. The terminal profits are higher under the collusive game. With the decrease of this percentage and the investment, the weight of the shippers' contribution to the net social benefit becomes higher. Port authority starts to prefer the competitive game since it will bring about more shippers' net benefit and accordingly more net social benefit to the local region.

DISCUSSION

In the previous section, various strategies that the port authority can use to invest in terminals have been evaluated. The numerical example demonstrates the capability of the bi-level programming method in solving the Stackelberg equilibrium and the applicability of the model in facilitating the port authority's investment decision. Four proposed investment strategies are compared, by evaluating criteria from the perspective of various players. The applicability of the results is limited to the initial assumptions related to the behaviour of the major players. Although the dynamics of the interaction among these key players are captured, there are several real world questions that cannot be addressed by the current formulations. The shipping industry is very dynamic with players integrating their businesses vertically and horizontally and forming various types of collaborations. To study and analyse these industry models new mathematical techniques are required.

In this paper, for example, the case of a landlord port and private terminal operators has been analysed. Future formulations could consider an operating type port authority. In this case the port authority, in addition to landlord, is operator of the port facilities. Future models should also examine the “spillover effect” for the net benefit of global (or regional) port operators’, operating several terminals in different parts of the world. In addition, the case in which a port authority is involved in joint investment and operations with a terminal operator should be examined. Another industry model is that of a terminal operator being at the same time a shipping line or having a corporate affiliation with a shipping line. This formulation needs to examine the difference in services to the sister carrier and to third carriers. Finally, competitive and collusive situations prevailing at the same time between carriers (e.g. two carriers operating within an alliance and a third carrier competing to the alliance) and situations of two competing port authorities under various circumstances for the other players need to be modelled.

CONCLUSIONS

A mathematical approach has been developed to assist port authorities in decision making for infrastructure investment. The problem formulation examines the port investment decisions within the context of a multimodal transportation system. The problem is solved for both competitive and for collusive behavior of carriers. Carriers’ behavior (including marine terminal operators) and the interaction between shippers and terminal operators are modeled through a bi-level problem. The complexity of the relationships between port authorities, marine terminal operators, ocean carriers and shippers can be captured through further elaboration of the modeling approach. Since Port Authority investment decision making has a long-term character and it is associated with a high level of risk, it is imperative that the complex relationships between industry players and stakeholders are taken seriously and consistently into account. The modeling approach presented provides a systematic methodology in considering these relationships when deciding infrastructure investments in the port sector.

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APPENDIX - MATHEMATICAL NOTATION

Parameters and Variables

S:	Vector of supplies, $S = \left(\cdots, S_{b\in CN}, \cdots\right)^T_{\lvert CN\rvert}$.
$D_{b,c}$:	Demand of commodity c at centroid b.
D:	Vector of demands, $D = \left(\cdots, D_{b\in CN}, \cdots\right)^T_{\lvert CN\rvert}$.
f:	Vector of flows on all shipper links, $f = \left(\cdots, f_{l\in L}, \cdots\right)^T_{\lvert L\rvert}$.
R_l:	Vector of commodity specific service charges on link l, $R_l = (\cdots, R_{l,c\in C}, \cdots)^T_{\lvert C\rvert}$.
$e_{a,c}$:	Flow of commodity c on the link a.
e_t:	Vector of link flows on the carrier t's sub-network, $e_t = \left(\cdots, e_{a\in A^t}, \cdots\right)^T_{\lvert A^t\rvert}$.
e:	Vector of link flows on the carriers' network, $e = \left(\cdots, e_{a\in A}, \cdots\right)^T_{\lvert A\rvert}$.
R_v:	Vector of service charges between O-D pair v, $R_v = \left(\cdots, R_{v,c\in C}, \cdots\right)^T_{\lvert C\rvert}$.
R_t:	Vector of service charges between O-D pairs on the carrier t's sub-network, $R_t = \left(\cdots, R_{v\in V^t}, \cdots\right)^T_{\lvert V^t\rvert}$.
R_{-t}:	Vector of service charges between O-D pairs on all other carriers' sub-networks except the carrier t's, $R_{-t} = (\cdots, R_{t'\in T, t'\neq t}, \cdots)^T_{\lvert T\rvert-1}$.
$\overline{E}_a$:	Capacity on link a.

Functions

$S_{b,c}(\pi_b)$:	Supply function of commodity c at centroid b.
$D_{b,c}(\pi_b)$:	Demand function of commodity c at centroid b.
$\pi_{b,c}(S_b)$:	Inverse supply function of commodity c at centroid b.
$\gamma_{b,c}, \lambda_{b,c',c}$:	Constants in the inverse supply function.
$\rho_{b,c}(D_b)$:	Inverse demand function of commodity c at centroid b.
$\alpha_{b,c}, \beta_{b,c'c}$:	Constants in the inverse demand function.
$Z_t(g_t(R_t, R_{-t}), R_t, e_t)$:	Profit of carrier t as a function of the vector of service charges at this carrier's sub-network (R_t) and the vector of service charges at the other carriers' sub-networks (R_{-t}) and the vector of link flows at this carrier's sub-network (e_t)
$g_{v,c}(R_V)$:	Demand function of commodity c between O-D pair v as a function of the vector of service charges, R_V.
$AC_{a,c}(e_{a,c})$:	Average operating cost function for commodity c on link a.

Other Notation

U:	Set of investment strategies available to the Port Authority, $u\in U$.
$\overline{E}^u_a$:	Capacity on terminal link a under investment strategy u.
$\Delta\overline{E}^u_a$:	Capacity improvement on terminal link a under investment strategy u.
(S^u, f^u, D^u):	Spatial price equilibrium solution under investment strategy u.
(R^u, e^u):	Equilibrium service charge and link flow under investment strategy u.
υ_c:	Percentage of passing through freight of commodity c for the local region the port authority is located.
ζ:	Economic multiplier.
TNB^u:	The terminal operators' net benefit under investment strategy u.

$CS_{b,c}^{u}$: The consumer surplus at centroid b from the consumption of commodity c under investment strategy u.

$PS_{b,c}^{u}$: The producer surplus at centroid b from the production of commodity c under investment strategy u.

$SNB_{b,c}^{u}$: The shippers' net benefit at centroid b from the consumption and the production of commodity c under investment strategy u.

SNB^{u}: The shippers' net benefit under investment strategy u.

$ASNB^{u}$: The shippers' net benefit adjusted to account for the passing through traffic and the external economy under investment strategy u.

NSB^{u}: Net social benefit under investment strategy u, $NSB^{u} = TNB^{u} + ASNB^{u} \quad \forall u \in U$.

I_{a}^{u}: Capital expense for the capacity improvement on link a under investment strategy u.

PIC_{a}^{u}: Present value of all capital expenses on link a under investment strategy u in the analysis period.

AIC_{a}^{u}: Annual investment cost of all capital expenses on link a under investment strategy u in the analysis period

IC_{a}^{u}: Hourly investment cost of all capital expenses on link a under investment strategy u in the analysis period,

$$IC_{a}^{u} = p2_{a}^{u} * (p1_{a}^{u} * \Delta\overline{E}_{a}^{u}) \qquad \forall a \in A, u \in U \ .$$

$p1_{a}^{u}$, $p2_{a}^{u}$: Constants.

IC^{u}: Total hourly investment cost under investment strategy u,

$$IC^{u} = \sum_{a \in A} IC_{a}^{u} \ .$$