A System-Wide Approach for Analyzing Japanese Wheat Import Allocation Decisions

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Introduction

Japan is one of the largest and most diverse importers of wheat in the world. Japan imported over 6.2 million metric tonnes (MMT) of wheat in 1994/95, representing approximately 50 kilograms of wheat imports per capita. Of that amount, 56% originated in the United States, 24% originated in Canada, and 20% originated in Australia. In a typical year, Japan imports at least 85% of its total wheat requirements. The allocation of Japanese wheat imports among different source countries depends on relative market conditions within exporting countries, world market conditions, wheat class, grade, and other quality characteristics (Stiegert and Blanc). It also depends on the policies implemented by the Japanese Food Agency, which has held a monopoly on wheat imports from all sources since 1952. Japan imports significant quantities of durum, hard red winter, hard red spring, and white wheat from the United States, durum and hard red spring wheat from Canada, and white and prime hard wheat from Australia. Hence, an analysis of Japanese import allocation decisions would not be complete without differentiating wheat imports by source and by type.

This paper uses a system-wide import allocation model to determine the degree of substitutability among different types of wheat imports in Japan. Wheat imports are differentiated by class and by source country. The procedure used to estimate demand is based on the system-wide approach to demand analysis (Barten, 1964 and Theil, 1965). The estimation procedure is based on the maximum likelihood estimation of a complete system of demand equations developed by Barten (1969). It is expected that durum wheat from Canada and the United States, hard red spring wheat from Canada and the
United States, and Australian and U.S. white wheat will exhibit a high degree of substitutability.

**The Differential Approach**

The differential approach to estimating a system of demand equations was formulated by Barten (1964) and Theil (1965). It results from the maximization of a general utility function with respect to a vector of quantities, subject to a linear budget constraint under the assumption of Walras' Law. Total differentiation of the budget constraint allows for a series of substitutions into the first order conditions resulting from utility maximization. The resulting system of equations is known as Barten's fundamental matrix (Barten, 1964). The solution to Barten's fundamental matrix generates a demand system for \( n \) commodities. If we let \( p_i \) and \( q_i \) be the price and quantity of commodity \( i \) and \( E \) be the total expenditure on goods \( i = 1, \ldots, n \), then the \( i^{th} \) equation in the differential demand system is typically written as

\[
(1) \quad \Delta (\ln q_i) = b_i \sum_{k=1}^{n} \Delta \ln p_k + s_{ij} \Delta (\ln p_j) + v_{it}
\]

Where \( \Delta \) denotes the change in the variable, \( w_{it} = \frac{w_{it} + w_{i,t-1}}{2} \) and \( w_{it} = \frac{p_i q_i}{E} \)

\[
\Delta \ln(p_i) = \ln p_i - \ln p_{i,t-1} \quad \text{and} \quad \Delta \ln(p_j) = \ln p_j - \ln p_{j,t-1}
\]

\[
b_i = \frac{\partial (p_i q_i)}{E} \quad \text{and} \quad s_{ij} = \frac{p_i p_j}{E} \left[ \frac{\partial q_i}{\partial p_j} + \frac{\partial q_j}{\partial E} q_j \right]
\]

In the above formulation, \( t \) denotes the period of observation, \( w_{it} \) is the budget share of good \( i \), \( b_i \) is the marginal budget share of good \( i \), and \( s_{ij} \) are the Slutsky price parameters. If one estimates system (1) under the assumption that the \( b_i \)'s and \( s_{ij} \)'s are constant over
time, then the demand system generated using these estimates is known as the absolute price version of the Rotterdam model. However, there are alternative parameterizations where the $b_i$ or $s_{ij}$ are allowed to vary over time (for examples, see Lee, Brown, and Seale and Seale et al.).

Another less popular representation of the differential demand system can be attributed to Barten (1969). In this representation, the system is rewritten in matrix form as a system of $t = 1,\ldots,T$ equations where the $t^{th}$ equation is specified as

$$(2) \quad y_t = b_t' y_t + S z_t + v_t$$

where $y_t$ is the $n$-element vector of observations on the left-hand variables in period $t$, such that $\bar{w}_i \Delta (\ln q_{it})$ is the $i^{th}$ element of $y_t$. $z_t$ and $v_t$ are $n$-element vectors of log-changes in prices and of disturbances, respectively, in period $t$. Moreover, the $b_i$ ($i = 1,\ldots,n$) are represented by the column vector $b$, while $S$ denotes the $n \times n$ matrix of Slutsky coefficients $s_{ik}$ ($i,k = 1,\ldots,n$). Finally, $\iota$ is the summation vector which is a $n$-element column vector comprised of ones.

Demand systems (1) and (2) are equivalent. By construction of the unrestricted system, the following relationships are mathematical identities:

$$t'b = 1 \quad , \quad t'S = 0 \quad , \quad t'v_t = 0$$

The latter relationship implies that the variance-covariance matrix associated with the disturbances is singular. The standard way to circumvent this problem is to eliminate one of the $n$ equations from system (1), estimate the system using ordinary least squares or seemingly unrelated regressions, and then recover the remaining equation. However, the advantage to system (2) is that maximum likelihood estimates can be obtained for the
complete system of demand equations without the need to eliminate one equation. This facilitates a smoother programming implementation because estimation of the full system
does not require dropping one equation, a process that can be cumbersome. Specifically, if we define

where ii’ is a nxn matrix of ones divided by n, then Barten (1969) shows that the
concentrated version of the unrestricted likelihood function can be expressed as

For the unrestricted estimation of the parameters, ordinary least squares can be used on each equation separately. Hence, in the unrestricted case, the parameters for system (1) or system (2) can be easily estimated without the need to drop one of the equations. However, imposing homogeneity and symmetry on the differential demand system requires inverting the true variance/covariance matrix, which in system (1) is singular. The advantage to using system (2) is that the matrix A is invertible and can be used in place of the true variance/covariance matrix.

Using system (2), homogeneity and symmetry can be imposed through the restrictions St = 0 (homogeneity) and S = S’ (symmetry). Define the following:

Let  \( D = [b, S] \) and  \( x'_i = [t', y_i', z_i'] \)

\( V' = [v_1, \Lambda, v_n] \quad X' = [x_1, \Lambda, x_n] \quad Y' = [y_1, \Lambda, y_n] \)
Making use of these definitions, the entire system represented by (2) can be rewritten as

\[ Y = XD' + V. \]

The restricted maximum likelihood estimates of this system result from the solution to the following problem:

where \( \kappa \) is a \( nx1 \) vector of Lagrange Multipliers, \( \tau \) is a \((n+1)x1\) vector with a zero in the first row and ones everywhere else, \( d \) is VEC(D), \( \mu \) is a \(.5(n)(n-1)x1\) vector of Lagrange Multipliers, and \( R \) is the symmetry restriction matrix constructed so that \( r'd = s_{ik} - s_{ki} \).

Barten (1969) showed that the solution to this system is \( d_3 = Hd_2 \) where

\[ H = I - (A \otimes G)R[R(A \otimes G)R']^T R \quad \text{and} \]

\[ G = (XX')^{-1} - \frac{(XX')^{-1} \tau'(XX')^{-1}}{\tau'(XX')^{-1} \tau} \]

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and \( d_2 = \text{VEC}(D_2) \), where \( D_2 = GX'Y \) is the solution to the system with homogeneity imposed only. In this case, the variance/covariance matrix for \( d_3 \) is an approximation because \( H \) is a function of \( A \), which contains only an estimate of the variance/covariance matrix of errors. The asymptotic variance/covariance matrix for \( d_3 \) is

The solution to \( d_3 \) requires non-linear maximum likelihood estimation techniques. One can proceed by computing \( d_2 \) and using this vector as the starting values for \( d_3 \). Initial
estimates of A are then computed and d₃ is then re-estimated using the new values for A.

This procedure is performed iteratively until d₃ converges.

There is an additional advantage to using system (2) rather than system (1). The Slutzky matrix can be rewritten as \( S = C - \phi b b' \) using the solution to Barten's Fundamental Matrix, where

Here, \( u_{ij} \) is the \( ij \)th element of the inverse of the Hessian matrix of the original utility function in expenditure terms. Also, \( \phi \) is the reciprocal of the derivative of the marginal utility with respect to income, which is referred to as the income flexibility coefficient.

By construction, \( \iota' C = \phi b \) and \( \iota' C \iota = \phi \). Hence, the entire system can be reformulated yet again as \( y_t = C f_t + v_t \) for \( t = (1, \ldots, T) \) where:

This can be used to impose and test different separability conditions by restricting certain off-diagonal elements of the Hessian matrix (through the C matrix) to be zero. In this paper we test the hypothesis of preference independence to determine whether there is any substitutability among different wheat classes from different origins. A portion of the results in the next section are derived using the procedure developed by Barten (1969) for imposing blockwise independence. A detailed description of the procedure for imposing different separability restrictions are beyond the scope of this paper (see Theil,
1981 for restrictions of uniform substitutability under preference independence, and Seale et al. for restrictions of uniform substitutability under blockwise dependence).

**Data**

Data on Japanese imports of wheat distinguished by country of origin and class for the period from 1970-1994 were compiled from different sources. Canadian exports were compiled from the publication entitled "Canadian Grain Exports" which reports annual data for the August/July crop year. Most of the years contained exports by grade, class, and destination. Data on individual grades within each class were aggregated. Over the duration of this study Canada exported significant quantities of durum (DURCN) and Canadian western red spring (HRSCN) to Japan. Feed and white wheat exports from Canada to Japan represent an insignificant share of Canadian wheat exports and were added to a general category (OTHER).

U.S. wheat exports by destination and class for 1970-1994 were obtained from the Federal Grain Inspection Service, reported in various issues of the "Wheat Yearbook" based on a June/July crop year. The United States exports significant quantities of durum (DURUS), hard red spring (HRSUS), hard red winter (HRWUS), and white wheat (WHTUS) to Japan. U.S. feed and soft red winter wheat exports represent an insignificant share of Japanese wheat imports and were added to the OTHER category.

Data on wheat exports from Argentina, the European Union, and Australia were obtained from various issues of "World Grain [Wheat] Statistics" published by the International Wheat Council. Japan has not imported any wheat from Argentina since 1974 and any Argentinian wheat prior to that was placed in the OTHER category.
Similarly, Japanese imports of wheat from the European Union are negligible and are also placed in the OTHER category. Australia exports two major classes of wheat to Japan — Australian white and Australian prime hard. Unfortunately, these two classes of wheat could not be separated due to data limitations. Hence, all wheat exported from Australia is placed in one category (AUWHT) for the purposes of this study. All Japanese wheat import data were converted to a per-capita consumption basis using yearly population figures provided by the Economic Research Service.

Japanese budget shares for imports of different classes of wheat from different countries are given in Table 1. Mean budget shares are calculated for 4 different periods within the 1970-1994 time frame (the first five years, the middle five years, the last five years, and all years). Canadian hard red spring wheat has the largest budget share, which remained consistently around 27% throughout the entire period. The budget share for U.S. hard red spring wheat increased over time along with the budget share for Canadian durum wheat. The Australian budget share decreased over time. Budget shares for all other wheat types fluctuated from 1970-1994. Durum wheat imports comprise a small portion of Japanese wheat imports (less than a 2% budget share) with relative U.S. and Canadian durum shares fluctuating over time.

Monthly export prices published by the International Wheat Council were used to obtain (unweighted) average yearly prices for different classes of wheat on a July/June crop year basis. All prices were converted to U.S. dollars per metric tonne. The "best" representative price series for each wheat class exported by each country was chosen. Following is a list of the price series used: (1) HRWUS - U.S. hard red winter #2
ordinary protein, Gulf port; (2) HRSUS - U.S. hard red spring #2, 14% protein, Pacific Northwest (PNW) port; (3) WHTUS - U.S. winter wheat #2, PNW port; (4) DURUS - U.S. hard amber durum #3, Great Lakes; (5) HRSCN - Canadian hard red spring #1, 13.5% protein, St. Lawrence; (6) DURCN - Canadian western amber durum #1, Thunder Bay/St. Lawrence; (7) WHTAU - Australian standard white, Sydney; and (8) OTHER - U.S. soft red winter #2, Gulf port. While there are other data sets available for Japanese import prices, the above data set was used because the ultimate objective of these modeling efforts is to use a consistent data set to formulate a similar import demand system for each of the major wheat importing countries in the world.

**Analysis and Results**

The procedure outlined in the previous section was used to analyze Japanese wheat import allocation decisions from 1970 through 1994. In the following analysis, it is implicitly assumed that the total amount spent on wheat imports by Japanese consumers is independent of the amount spent on all other goods. This assumption is made in order to simplify the analysis by focusing on Japanese wheat import allocation decisions only.

Four different models were estimated using the maximum likelihood estimation of the complete system of demand equations. Each of these models distinguishes among eight different types of wheat imports. These different types are durum, hard red spring, hard red winter, and white wheat from the United States, durum and hard red spring wheat from Canada, Australian (white) wheat, and other wheat. The first model estimates the marginal budget shares and Slutsky parameters under no additional
restrictions. The second model imposes homogeneity. The third model imposes both symmetry and homogeneity. The fourth model imposes preference independence on all eight wheat import types.³

Due to space considerations, parameter estimates are provided for only the full Rotterdam model (i.e., the system-wide formulation under the assumption of constant parameters with both homogeneity and symmetry imposed). Similarly, income elasticities and price elasticities are provided for the latter model only.⁴ Finally, the likelihood ratio test $LRT = -2[\log L(\theta^*) - \log L(\theta)]$ is used to test the three restricted models against the unrestricted model. In this case, $\theta$ is the vector of unrestricted parameter estimates and $\theta^*$ is the vector associated with different restricted estimates. Under the null hypothesis that the unrestricted Rotterdam model best describes the data, $LRT$ has an asymptotic Chi-square distribution where the degrees of freedom equal the difference between the number of parameters in the general vs. the restricted model.

Estimates of the marginal budget shares and Slutsky price coefficients generated by the full maximum likelihood procedure constrained by homogeneity and symmetry for Japanese wheat imports are provided in Table 2. The parameters are estimated at the sample mean budget share for the entire period. The standard error associated with each parameter is below the estimated value provided in Table 2. The analysis makes use of 24 observations for each of the 8 wheat types. The first column shows the marginal budget share associated with each type of wheat. For all wheat types, except Australian wheat imports, the change in the amount spent on each type of wheat increases as the total expenditure on wheat imports increases. The marginal budget share of -.046
associated with Australian wheat imports is not significant. The diagonals (the bottom value in each column in Table 2) provide the own Slutsky price parameters associated with each type of wheat. Five of the eight wheat classes have the expected negative sign. These are U.S. durum, Canadian Hard Red Spring, U.S. Hard Red Spring, Australian wheat, and U.S. white wheat. The other three wheat classes, Canadian durum, U.S. hard red winter, and other wheat, each have positive signs, but are also insignificant.

The off-diagonal elements of Table 2 show the cross-price Slutsky coefficients for the different types of wheat. The positive coefficients associated with Canadian and U.S. durum (0.03) and Australian and U.S. white wheat (0.5) confirm that these types of wheat are considered specific substitutes for each other by the Japanese. However, the parameter associated with Canadian and U.S. hard red winter wheat is -.03, indicating that these two types of wheat are viewed as specific complements. However, of these three cross-price parameters, only the parameter associated with Australian and U.S. white wheat is significant.

Income elasticities for each wheat type at different periods in time are reported in Table 3. The income elasticity for the \(i\)th wheat type is computed from the formula \(\eta_i = \frac{\partial \ln q_i}{\partial \ln E} = \frac{b_i(E/p_i q_i)}{w_i} = b_i/w_i\). The first two columns indicate that Canadian and U.S. durum wheat are highly income elastic and that the income elasticity varies significantly across time periods. One of the reasons for this high level of variability is that the income elasticity is inversely proportional to the budget share, which is very small in the case of Japanese durum imports. All other wheat types are income inelastic with positive values except for Australian wheat. The income elasticity for Australian wheat is -0.27.
indicating that Japan allocates less of its expenditure to Australian wheat imports as its wheat import expenditure increases. One reason for this unexpected result may be that data that separates Japanese imports of Australian white wheat vs. Australian prime hard are currently unavailable to the authors. If Australian wheat were to be divided into these two separate classes, the resulting estimates would most likely improve.

Cournot (uncompensated) price elasticities for each of the different wheat types are provided in Table 3 as well. U.S. Durum wheat imports, Australian wheat imports, and U.S. white wheat imports are all price elastic. Both Canadian and U.S. hard red spring imports are price inelastic. The elasticity for U.S. hard red winter wheat imports is slightly positive. Finally, Canadian durum and the “other” category both have large positive own-price demand elasticities. One reason for the latter unrealistic results is that the budget share allocated to each of these wheat types is quite small, so that small changes in volume have exaggerated effects on differential demand systems.

Comparisons of the alternative models are provided in Table 4. The results indicate that, assuming that the unrestricted Rotterdam model is the "correct" model, homogeneity and homogeneity with symmetry, can not be rejected at the 95% confidence level. The last column of Table 4 shows the log-likelihood values under the assumption of preference independence and/or uniform substitutes. This assumption is readily rejected when compared to the unrestricted model. In fact, even if the model with homogeneity and symmetry imposed were considered "correct", the hypothesis of preference independence or uniform substitutes would still have to be rejected. Hence,
when considering Japanese wheat import allocation decisions, one cannot assume that Japanese consumers treat different types of wheat as independent.

**Concluding Remarks**

An estimation procedure based on the maximum likelihood estimation of a complete system of demand equations (Barten, 1969) was used to estimate an import allocation model for Japanese wheat imports. The results indicate that durum wheat imports from Canada and the United States, and Australian and U.S. white wheat imports are specific substitutes. However, hard red spring wheat from Canada and the United States were found to be specific complements. These results should be viewed as preliminary. Future research will extend the analysis to incorporate multi-level allocation schemes for Japanese imports of similar types of wheat from different countries. In addition, procedures will be developed to estimate and test various blockwise separability conditions. It is expected that these additional structural modifications will improve the parameter estimates associated with Japanese wheat import allocation decisions.
Endnotes

1 In earlier years, Canadian durum export prices were quoted out of Thunder Bay. This series was spliced with the data from St. Lawrence by adding $15/MT U.S. to the Thunder Bay price.

2 The U.S. price for soft red winter wheat was used as a proxy for the OTHER category because the largest share of Japanese wheat imports in this category are U.S. soft red winter wheat imports.

3 Three other sets of results were obtained by adding an intercept to each of the models described above. However, the inclusion of an intercept did not alter the results significantly. Moreover, two additional sets of results were obtained which imposed symmetry restrictions only. These additional results are available upon request from the authors.

4 Detailed parameter results for the other three cases are available upon request from the authors.
REFERENCES


