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An anchoring-adjusted option pricing model is developed in which the expected return of the underlying stock is used as a starting point that gets adjusted upwards to form expectations about corresponding call option returns. Anchoring bias implies that such adjustments are insufficient. In continuous time, the anchoring price always lies within the bounds implied by expected utility maximization when there are proportional transaction costs. Hence, an expected utility maximizer may not gain utility by trading against the anchoring prone investors. The anchoring model is consistent with key features in option prices such as implied volatility skew, superior historical performance of covered call writing, inferior performance of zero-beta straddles, smaller than expected call option returns, and large magnitude negative put returns. The model is also consistent with the puzzling patterns in leverage adjusted option returns, and extends easily to jump-diffusion and stochastic-volatility approaches.

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Anchoring Heuristic in Option Pricing

One of the major achievements in financial economics is the no-arbitrage approach of pricing options pioneered in Black and Scholes (1973), and Merton (1973). The approach is appealingly simple as it does not have any role for investor demand. Due to its simplicity, the Black-Scholes model is still the most widely used model among practitioners despite its several well-known shortcomings. The key shortcomings are:

1) The existence of the implied volatility skew in index options, and average implied volatility of at-the-money options being larger than realized volatility. (Rubinstein (1994))

2) Superior historical performance of covered-call writing. (Whaley (2002))

3) Worse-than-expected performance of zero-beta straddles. (Coval and Shumway (2002))

4) Average call returns appear low given their systematic risk. (Coval and Shumway (2002))

5) Average put returns are far more negative than expected. (Bondarenko (2014))

6) Leverage adjusted call and put index returns exhibit patterns inconsistent with the model. (Constantinides, Jackwerth, and Savov (2013))

These shortcomings imply that at least one (possibly more) assumption in the Black-Scholes model is wrong. Relaxing the Black-Scholes assumptions has led to several fruitful directions of research; however, no existing reduced form option pricing model convincingly explains the above findings (see the discussion in Bates (2008)). The two most frequently questioned assumptions in the Black-Scholes model are: 1) The use of geometric Brownian motion as a description for the underlying stock price dynamics. 2) Assuming that there are no transaction costs.

Dropping the assumption of geometric Brownian motion to processes incorporating stochastic jumps in stock prices, stochastic volatility, and jumps in volatility has been the most active area of research. Initially, it was assumed that the diffusive risk premium is the only priced risk factor (Merton (1976), and Hull and White (1987)), however, state of the art models now include risk factors due to stochastic volatility, stochastic jumps in stock prices, and in some cases also stochastic jumps in volatility. Based on the assumption that all risks are correctly priced, this approach empirically searches for various risk factors that could potentially matter (see Constantinides, Jackwerth, and Savov (2013), and Broadie, Chernov, & Johannes (2009) among others). These models typically attribute the divergences between objective and risk neutral probability measures to the free "risk premium" parameters within an affine model.[1] Bates (2008) reviews the empirical evidence on stock index option pricing and concludes that options do not price risks in a way which is consistent with existing option pricing models. Many of these models are also critically discussed in Hull (2011), Jackwerth (2004), McDonald (2006), and Singleton (2006).

Another area of research deviates from the Black-Scholes no-arbitrage approach by allowing for transaction costs. Leland (1985) considers a class of imperfectly replicating strategies in the presence of proportional transaction costs and derives bounds in which an option price must lie when there are proportional transaction costs. Hodges and Neuberger (1989) and Davis et al (1993) explicitly derive and numerically compute the bounds under the assumption that utility is exponential with a given risk aversion coefficient. Their bounds are comparable to Leland bounds. Constantinides and Perrakis (2002) show that expected utility maximization in the presence of proportional transaction costs implies that European option prices must lie within certain bounds. Constantinides and Perrakis bounds are generally tighter than the Leland bounds, and are the tightest bounds derived in the literature for European option prices.

Yet another line of research provides evidence that options might be mispriced. Jackwerth (2000) shows that the pricing kernel recovered from option prices is not everywhere decreasing as predicted by theory, and concludes that option mispricing seems the most likely explanation. Others researchers (see Rosenberg and Engle (2002) among others) also find that empirical pricing kernel is oddly shaped. Constantinides, Jackwerth, and Perrakis (2009) find that S&P 500 index options are possibly overpriced relative to the underlying index quite frequently.

Earlier, Shefrin and Statman (1994) put forward a structured behavioral framework for capital asset pricing theory that allows for systematic treatment of various biases. A particular bias that stands out in recent empirical literature on financial markets is anchoring which implies that *adjustments in assessments away from some initial value are often insufficient.* Starting from the early experiments in Kahneman and Tversky (1974), over 40 years of research has demonstrated the

[1] Bates (2003) writes, "To blithely attribute divergences between objective and risk-neutral probability measures to the free "risk premium" parameters in an affine model is to abdicate one's responsibilities as a financial economist" (page 400).

relevance of anchoring in a variety of decision contexts (see Furnham and Boo (2011) for a literature review). Starting from a self-generated initial value, adjustments are insufficient because people tend to stop adjusting once a plausible value is reached (see Epley and Gilovich (2006)). Hence, assessments remain biased towards the starting value known as the anchor.

Intriguingly, there is considerable evidence that anchoring matters for call option prices: 1) A series of laboratory experiments (see Rockenbach (2004), Siddiqi (2012), and Siddiqi (2011)) show that the hypothesis that a call option is priced by equating its expected return to the expected return from the underlying stock outperforms other pricing hypotheses. The results are consistent with the idea that return of the underlying stock is used as a starting point with the anchoring heuristic ensuring that adjustments to the stock return to arrive at call return are insufficient. Hence, expected call returns do not deviate from expected underlying stock returns as much as they should. 2) Furthermore, market professionals with decades of experience often argue that a call option is a surrogate for the underlying stock.[2] Such opinions are surely indicative of the importance of the underlying stock as a starting point for thinking about a call option, and point to insufficient adjustment, which creates room for the surrogacy argument.

This article puts forward an anchoring adjusted option pricing model in which the expected return on the underlying stock is used as a starting point which gets adjusted upwards to form expectations about call returns. In principle, there is nothing wrong in doing that, and one can potentially make the correct adjustment to arrive at the correct expected return. However, the anchoring heuristic may cause insufficient adjustment. This article explores the implications of the anchoring bias for option pricing. I show that the anchoring price (in continuous time) always lies within the bounds derived in Constantinides and Perrakis (2002). Hence, an expected utility maximizer may not gain utility by trading against the anchoring prone investors if there are proportional transaction costs. That is, proportional transaction costs not only rule out the possibility of riskless arbitrage, but also may not allow risky expected utility gain at the expense of anchoring prone investors.

[2] As illustrative examples, see the following:
http://finance.yahoo.com/news/stock-replacement-strategy-reduce-risk-142949569.html
http://ezinearticles.com/?Call-Options-As-an-Alternative-to-Buying-the-Underlying-Security&id=4274772,
http://www.investingblog.org/archives/194/deep-in-the-money-options/
http://www.triplescreenmethod.com/TradersCorner/TC052705.asp,
http://daytrading.about.com/od/stocks/a/OptionsInvest.htm

The anchoring model has a closed-form solution, and the pricing formula is almost as simple as the Black-Scholes formula if the underlying stock price follows geometric Brownian motion. The anchoring model converges to the Black-Scholes model in the absence of the anchoring bias. The anchoring model is consistent with all of the key features of option prices mentioned earlier. Hence, it provides a unified framework for understanding the key puzzles.

The anchoring approach relates to all of the three directions of research described earlier, namely, 1) incomplete markets due to jumps and stochastic volatility, 2) bounds implied by transaction costs, and 3) option mispricing. Specifically, it shows that an option can be mispriced in such a way that it lies within the bounds implied by expected utility maximization when there are proportional transaction costs (Constantinides and Perrakis (2002) bounds), and remains so even when markets are incomplete due to jumps and stochastic volatility.

Hirshleifer (2001) considers anchoring to be an *"important part of psychology based dynamic asset pricing theory in its infancy"* (p. 1535). Shiller (1999) argues that anchoring appears to be an important concept for financial markets. This argument has been supported quite strongly by recent empirical research on financial markets: 1) Anchoring has been found to matter in the bank loan market as the current spread paid by a firm seems to be anchored to the credit spread the firm had paid earlier (see Douglas, Engelberg, Parsons, and Van Wesep (2015)). 2) Baker, Pan, and Wurgler (2012) provide evidence that peak prices of target firms become anchors in mergers and acquisitions. 3) The role of anchoring bias has been found to be important in equity markets in how analysts forecast firms' earnings (see Cen, Hillary, and Wei (2013)). 4) Campbell and Sharpe (2009) find that expert consensus forecasts of monthly economic releases are anchored towards the value of previous months' releases. 5) Johnson and Schnytzer (2009) show that investors in a particular financial market (horse-race betting) are prone to the anchoring bias.

Given the key role that anchoring appears to play in financial decision making, it seems only natural that anchoring matters for option pricing too, especially given the fact that an option derives its existence from the underlying stock. After all, a call option is equivalent to a leveraged position in the underlying stock. Hence, a clear starting point exists. In fact, one can argue that it would be rather odd if the return on the underlying stock is ignored while forming return expectations about a call option. The Black-Scholes model by-passed the stock return only by assuming perfect replication, which is impossible, if transaction costs are allowed or markets are incomplete. This article shows that if anchoring matters then the resulting option pricing formula is almost as simple as the Black-Scholes formula. It seems that anchoring provides the minimum deviation from the Black-Scholes framework that is needed to capture the key empirical properties mentioned earlier. To my knowledge, the anchoring approach developed here is the simplest reduced-form framework that captures the key empirical features of option returns and prices.

If the marginal investor in a given call option is anchoring prone, then it follows that:

$$E[R_C] = E[R_S] + A \tag{0.1}$$

where $E[R_C]$ is the expected return from the call option, $E[R_S]$ is the expected return from the underlying stock, and A is the adjustment term. The anchoring prone investor realizes that a call option is riskier than the underlying stock, as it is a leveraged position in the underlying stock. However, the adjustment that he makes to get to the call return from the underlying stock return is insufficient due to the anchoring bias.

To contrast (0.1) with a particular case of correct adjustment, consider the expected return implied by the Black-Scholes model:

$$E[R_C] = R_F + \Omega \cdot (E[R_S] - R_F) \tag{0.2}$$

$\Omega > 1$ (it is the call price elasticity w.r.t the underlying stock price) and typically takes very large values (typically varies from 3 to 35 for index options), and R_F is the risk-free rate. Note that (0.1) and (0.2) are equal if $A = (\Omega - 1)(E[R_S] - R_F)$. So, the presence of anchoring bias implies that $A < (\Omega - 1)(E[R_S] - R_F)$.

This article is organized as follows. Section 2 builds intuition by providing a numerical illustration of option pricing with anchoring in a complete market context. Section 3 provides a numerical example of an incomplete market. Section 4 puts forward the anchoring adjusted option pricing formulas in continuous time, and shows that the anchoring price always lies within the Constantinides and Perrakis (2002) bounds. Section 5 shows that if anchoring determines option prices, and the Black-Scholes model is used to back-out implied volatility, the skew arises, which flattens as time to expiry increases. Section 6 shows that the anchoring model is consistent with key empirical findings regarding returns from covered call writing and zero-beta straddles. Section 7 shows that the anchoring model is consistent with empirical findings regarding leverage adjusted option returns. Section 8 puts forward an anchoring adjusted pricing model when the underlying stock returns exhibit stochastic volatility. It integrates anchoring with the stochastic volatility model developed in Hull and White (1987). Section 9 integrates anchoring with the jump diffusion approach of Merton (1976). Section 10 concludes.

2. Anchoring Adjusted Option Pricing: A Numerical Illustration

To fix ideas, I provide an example in this section, which assumes a complete market. Consider an investor in a two-state, two-asset complete market world with one time period marked by two points in time: 0 and 1. The two assets are a stock (S) and a risk-free zero coupon bond (B). The stock has a price of $140 today (time 0). Tomorrow (time 1), the stock price could either go up to $200 or go down to $94. Each state has a 50% chance of occurring. There is a riskless bond (zero coupon) that has a price of $1 today. Its price stays at $1 at time 1 implying that the risk free rate is zero. Suppose a new asset "C" is introduced to him. The asset "C" pays $100 in cash in the red state and nothing in the blue state. How much should the investor be willing to pay for this new asset?

Finance theory provides an answer by appealing to the principle of no-arbitrage: *portfolios with identical state-wise payoffs must have the same price*. Hence, to price asset "C" correctly, one solves the following system of equations:

$$200x + B = 100$$

$$94x + B = 0$$

It follows that $x = 0.9434$, and $B = -88.6796$.

The payoffs from asset "C" can be perfectly replicated by creating a portfolio in which one buys 0.9434 of the underlying stock, and borrow 88.6796. The cost of setting up this portfolio is $140 \times 0.9434 - 88.6796 = 43.3964$. Hence, the correct price of asset "C" is 43.3964. Note, that this price implies a net expected return of 15.22% from asset "C", whereas, the expected return from the underlying stock is 5%.

The payoffs from asset "C" are strongly correlated with the payoffs from "S". In fact, "C" is equivalent to a call option on "S" with a strike price of 100. Buying "C" creates a leveraged position in the underlying stock. As leveraged positions amplify risks and returns, one expects higher returns from them. But, how much higher? In other words, what should be the value of A in the following?

$$E[R_C] = E[R_S] + A$$

where $E[R_C]$ is the expected return from the call option, and $E[R_S]$ is the expected return from the underlying stock. Continuing with the example, $E[R_S]$ is 1.05, and correct $E[R_C]$ is 1.1522, so correct adjustment to the stock return to arrive at the call return is 0.1022.

An anchoring prone investor starts from the return of the underlying stock; however, he makes insufficient adjustment. To take one example, suppose he only adjusts about halfway, that is, $A = 0.05$. It follows that, for the anchoring prone investor, the expected return from the given call option is 1.1. Consequently, he prices "C" at 45.4545 $\left(\frac{0.5\times100+0.5\times0}{1.1} = 45.4545\right)$.

If there are no transaction costs, then the anchoring prone investor who values "C" at 45.4545 creates an arbitrage opportunity for the rational investor who values "C" at 43.3964. The rational investor should write "C" and buy the replicating portfolio to make a riskless profit of $45.4545 - 43.3964 = 2.0581$. However, if the transaction costs are introduced, then this arbitrage opportunity may disappear.

Suppose buying and selling the underlying stock incurs a proportional transaction cost of 1%. That is, the buyer of "S' pays 141.4 $(1.01 \times 140 = 141.4)$, and the seller receives 138.6 $(0.99 \times 140 = 138.6)$. As is standard practice in option pricing literature with transaction costs, continue to assume that there are no transaction costs involved in trading the option or the bond. Replicating a long position in "C" now requires solving the following system of equations:

$$0.99 \times 200x + B = 100$$

$$0.99 \times 94x + B = 0$$

It follows that $x = 0.9529$, and $B = 88.67$. So, the cost of replication goes up to 46.07 $(1.01 \times 140 \times 0.9529 - 88.67 = 46.07)$. Hence, the rational investor can no longer make arbitrage profits by trading against the anchoring prone investor unless the anchoring price is larger than 46.07.

The transaction cost upper bound derived in this simple one period example is tight, however, it is well known that as more trading periods are added, the total transaction cost keeps on rising, and grows without bound in continuous time. Hence, it is useful to focus on the upper bound implied by expected utility maximization when there are proportional transaction costs. So, instead of asking, "can a rational investor make arbitrage profits by trading against the anchoring prone investor?" (the answer to which is always no in continuous time), one asks, "can a risk averse expected utility maximizer gain utility by trading against the anchoring prone investor when there are proportional transaction costs?"

Surprisingly, it turns out that the answer to the second question is also always no in continuous time even though the expected utility maximization upper bound is tight (with stock price allowed to be in the full range $[0, \infty)$). That is, the anchoring price is less than the expected utility maximization upper bound when there are proportional transaction costs. Hence, the argument that an expected utility maximizer gains utility by trading against the anchoring prone investor cannot be made. The upper bound is derived in Constantinides and Perrakis (2002), (henceforth called CP upper bound), and in this example, is equal to 46.07. The CP upper bound and the transaction cost upper bound are equal in this example; however, they diverge substantially as more trading period are added. In particular, the CP upper bound remains tight even when the limit of continuous time is reached. Constantinides and Perrakis (2002) show that in continuous time, the CP upper bound is the price at which the expected return from the call option is equal to the expected return from the underlying stock net of round-trip transaction costs. The CP upper bound is the tightest bound in the literature, and is considerably tighter than the Leland (1985) upper bound, in general. Section 3 discusses the continuous time case, and illustrates the CP upper bound and the Leland (1985) upper bound in relation with the anchoring price.

Clearly, anchoring lowers the expected return from the call option. What about the expected return from a put option? Continuing with the same example, consider a put option with the strike of 100. It pays 6 when the stock price is 100, and pays 0 when the stock price is 200. With anchoring, the price of the call option is 45.4545, so the price of the corresponding put option is 5.4545 (from put-call parity). Hence, the (net) expected return from the put option is -0.45 or -45%. With rational pricing, the price of the put option is 3.3964 (from put-call parity), and the expected return from the option is -0.1167 or -11.67%. One can see that anchoring implies much larger magnitude put returns when compared with rational pricing.

3. Anchoring Adjusted Option Pricing: An Incomplete Market Example

To illustrate anchoring in an incomplete market, the example in the previous section is modified slightly. Specifically, at time 1, instead of two states, now assume that there are three states. Namely, the stock price can go up to 200 or go down to 94 with a probability of 0.495 each. That is, the up factor is $\frac{200}{140}$ and the down factor is $\frac{94}{140}$. In addition, there is a 0.01 probability that the stock price jumps to 0 at time 1. Everything else is kept the same. As before, the question is how much to pay for a call option on the stock with a strike of 100.

As the market is no longer complete, the call option's payoffs cannot be perfectly replicated by a combination of the stock and the risk free bond. Hence, pricing by replication is not possible. A pertinent question is: What is the range of prices consistent with risk-averse expected utility maximization? A body of literature has developed in response to this question (see Perrakis and Ryan (1984), Ritchken (1985), Levy (1985), Perrakis (1986)(1988), and Ritchken and Kuo (1988) among others). Oancea and Perrakis (2007) provide a detailed review of this literature. Following Oancea and Perrakis (2007), the call price upper bound and lower bound consistent with risk-averse expected utility maximization in discrete time are:

$$\bar{C} = \frac{1}{R}\left[\frac{R-\hat{z}_1}{\hat{z}_n-\hat{z}_1}\hat{c}_n + \frac{\hat{z}_n-R}{\hat{z}_n-\hat{z}_1}c_1\right]$$

$$\underline{C} = \frac{1}{R}\left[\frac{R-\hat{z}_h}{\hat{z}_{h+1}-\hat{z}_h}\hat{c}_{h+1} + \frac{\hat{z}_{h+1}-R}{\hat{z}_{h+1}-\hat{z}_h}\hat{c}_h\right]$$

where $\hat{z}_j = \frac{\sum_{i=1}^{j}\frac{s_i}{S_0}\pi_i}{\sum_{i=1}^{j}\pi_i}$, state-wise stock payoffs are $s_1 \leq s_2 \leq \cdots \leq s_n$ with physical probabilities π_j, S_0 is the current stock price, R is the (gross) riskless rate, state-wise call payoffs are $c_1 \leq c_2 \leq \cdots \leq c_n$, and $\hat{c}_j = \frac{\sum_{i=1}^{j}c_i\pi_i}{\sum_{i=1}^{j}\pi_i}$. In the expressions, h is a state index such that $\hat{z}_h \leq R \leq \hat{z}_{h+1}$.

Using the above formulas, the call price upper bound is 47.619, and the call price lower bound is 44.373. It follows that call prices in the interval $[44.373, 47.619]$ are consistent with risk-averse expected utility maximization. It is possible to show that various specific models pick out different points from this interval. For example, Merton's jump diffusion model (Merton (1976)) assumes that the jump risk is not priced. Merton's model is discretized in Amin (1993). Following Amin's approach, it is easy to see that the Merton's model picks out a price very close to the lower bound of 44.373. So, the expected return from the call option under Merton's approach is approximately 1.116 (gross) or 11.6% (net). The corresponding put option (one with the strike of 100) has a price of 4.373, yielding an expected return of 0.9078 (gross) or -9.22% (net).

In contrast with Merton's approach, the anchoring prone investor picks out a price close to the upper bound. To see this, recall that the anchoring prone investor starts from the return of the underlying stock and adds to it to form return expectations regarding the call option. The expected return from the underlying stock, in this example, is 3.95%. The anchoring prone investor would add to it to arrive at the return he demands from the call option with the anchoring heuristic ensuring that the addition is not too large. To take one example, suppose he adds 2% to demand a return of 5.95% from the call option. It follows that the anchoring price is 46.72. Note, if the adjustment term is zero, that is, if the anchoring prone investor demands the same return from the call option as is available from the underlying stock, then the anchoring price coincides with the call price upper bound.

From put-call parity, one can deduce that the corresponding put option is priced at 6.72, yielding an expected return of 0.59 (gross) or -41% (net). So, just like in the complete market case, the expected put return has a larger magnitude with anchoring when compared with the Black-Scholes counterpart in incomplete markets, which is Merton's jump diffusion model.

Next, we formalize the intuition developed in these examples. The complete market case and its implications for key puzzles are discussed in the next few sections, followed by the jump diffusion and stochastic volatility cases in sections 8 and 9.

4. Anchoring Adjusted Option Pricing: The Continuous Case

I maintain all the assumptions of the Black-Scholes model except two. The first point of departure is the assumption that the marginal investor in a given call option is anchoring prone. That is, the marginal investor starts from the return of the underlying stock, and adds to it to arrive at the expected call return:

$$\frac{1}{dt}\frac{E[dC]}{C} = \frac{1}{dt}\frac{E[dS]}{S} + A_K \tag{4.1}$$

Where $C, and\ S,$ denote the call price, and the stock price respectively. $A_K \geq 0$. The anchoring prone investor understands that leverage increases expected returns. As leverage rises with strike, A_K rises with strike.

If the risk free rate is r and the risk premium on the underlying stock is δ, then, $\frac{1}{dt}\frac{E[dS]}{S} = \mu = r + \delta$. So, (4.1) may be written as:

$$\frac{1}{dt}\frac{E[dC]}{C} = (r + \delta + A_K) \tag{4.2}$$

The underlying stock price follows geometric Brownian motion:

$$dS = \mu S dt + \sigma S dZ$$

Where dZ is the standard Guass-Weiner process.

From Ito's lemma:

$$E[dC] = \left(\mu S \frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 C}{\partial S^2}\right) dt \tag{4.3}$$

Substituting (4.3) in (4.2) leads to:

$$(r + \delta + A_K)C = \frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}(r + \delta)S + \frac{\partial^2 C}{\partial S^2}\frac{\sigma^2 S^2}{2} \tag{4.4}$$

(4.4) describes the partial differential equation (PDE) that must be satisfied if anchoring determines call option prices.

To appreciate the difference between the anchoring PDE and the Black-Scholes PDE, consider the expected return under the Black-Scholes approach, which is given in (0.2). Over a small time interval, dt, one may re-write (0.2) as:

$$\frac{1}{dt}\frac{E[dC]}{C} = r + \Omega \cdot (\mu - r) \tag{4.5}$$

Substituting (4.3) in (4.5) and realizing that $\Omega = \frac{S}{C}\frac{\partial C}{\partial S}$ leads to the following:

$$rC = \frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{\partial^2 C}{\partial S^2}\frac{\sigma^2 S^2}{2} \tag{4.6}$$

(4.6) is the Black-Scholes PDE.

In the Black-Scholes world, the correct adjustment to stock return to arrive at call return is $A = (\Omega - 1)\delta$. By substituting $A = (\Omega - 1)\delta$ in (4.4), it is easy to verify that the Black-Scholes PDE in (4.6) follows. That is, with correct adjustment (4.4) and (4.6) are equal to each other. Clearly, with insufficient adjustment, that is, with the anchoring bias $(A < (\Omega - 1)\delta)$, (4.4) and (4.6) are different from each other.

The second, and final, departure from the Black-Scholes assumptions is that proportional transaction costs are allowed. Such transaction costs capture brokerage fee, bid-ask spread, transaction taxes, market impact costs etc. Specifically, I allow for proportional transaction costs in the underlying stock. I assume that the buyer of the stock pays$(1 + \theta)S$, and the seller receives $(1 - \theta)S$, where S is the stock price, and θ is a constant less than one. There are no transaction costs in the bond or the option market. In the presence of transaction costs, the Black-Scholes portfolio replication argument breaks down.

Constantinides and Perrakis (2002) derive a tight upper bound (CP upper bound) on a call option's price in the presence of proportional transaction costs. In particular, under very general conditions ($\frac{(1-\theta)E[S]}{(1+\theta)S} > 1 + r$, and stock price is allowed to be in the full range $[0, \infty)$), risk-averse expected utility maximization implies the following upper bound. It is the call price at which the expected return from the call option is equal to the expected return from the underlying stock net of round-trip transaction cost:

$$\bar{C} = \frac{(1 + \theta)S \cdot E[C]}{(1 - \theta)E[S]}$$

It is easy to see that the anchoring price is always less than the CP upper bound. The anchoring prone investor expects a return from a call option which is at least as large as the expected return from the underlying stock. That is, with anchoring, $\frac{E[C]}{C} \geq \frac{E[S]}{S} > \frac{(1-\theta)E[S]}{(1+\theta)S}$. It follows that the maximum price under anchoring is: $\bar{C}_A < \bar{C} = \frac{(1+\theta)S \cdot E[C]}{(1-\theta)E[S]}$. Hence, if there are proportional transaction costs, not only the riskless arbitrage is eliminated, but also the risky expected utility gain, as an expected utility maximizer may not gain utility by trading against the anchoring prone investor.

Re-writing the anchoring PDE with the boundary condition, we get:

$$(r + \delta + A_K)C = \frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}(r + \delta)S + \frac{\partial^2 C}{\partial S^2}\frac{\sigma^2 S^2}{2} \tag{4.7}$$

where $0 \leq A_K < (\Omega - 1)\delta$, and $C_T = max\{S - K, 0\}$

Note, that the presence of the anchoring bias, $A_K < (\Omega - 1)\delta$, guarantees that the CP lower bound is also satisfied. The CP lower bound is weak and lies substantially below the Black-Scholes price. As the anchoring price is larger than the Black-Scholes price, it follows that it must be larger than the CP lower bound.

One way to interpret the anchoring approach is to think of it as a mechanism that substantially tightens the Constantinides and Perrakis (2002) option pricing bounds. The anchoring price always lies in the narrow region between the Black-Scholes price and the CP upper bound.

There is a closed form solution to the anchoring PDE. Proposition 1 puts forward the resulting European option pricing formulas.

Proposition 1 *The formula for the price of a European call is obtained by solving the anchoring PDE. The formula is* $\boldsymbol{C = e^{-A_K(T-t)}\{SN(d_1^A) - Ke^{-(r+\delta)(T-t)}N(d_2^A)\}}$ *where* $\boldsymbol{d_1^A = \frac{ln(S/K)+(r+\delta+\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}}$ *and* $\boldsymbol{d_2^A = \frac{ln\left(\frac{S}{K}\right)+\left(r+\delta-\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}}$

Proof.

See Appendix A. ■

Corollary 1.1 *There is a threshold value of* $\boldsymbol{A_K}$ *below which the anchoring price stays larger than the Black-Scholes price.*

Proof.

See Appendix B.

■

Corollary 1.2 *The formula for the anchoring adjusted price of a European put option is*

$$Ke^{-r(T-t)}\{1 - e^{-\delta(T-t)}N(d_2)e^{-A_K(T-t)}\} - S\left(1 - e^{-A_K(T-t)}N(d_1)\right)$$

Proof.

Follows from put-call parity.

■

As proposition 1 shows, the anchoring formula differs from the corresponding Black-Scholes formulas due to the appearance of δ, and A_K. If the marginal investor is risk neutral, then the two sets of formulas are identical.

It is interesting to analyze put option returns under anchoring. Proposition 2 shows that put option returns are more negative under anchoring when compared with put option returns in the Black Scholes model.

Proposition 2 *Expected put option returns (for options held to expiry) under anchoring are more negative than expected put option returns in the Black Scholes model as long as the underlying stock has a positive risk premium and there is anchoring bias.*

Proof.

See Appendix F.

■

Proposition 2 is quite intriguing given the puzzling nature of empirical put option returns when compared with the predictions of popular option pricing models. Chambers et al (2014) analyze nearly 25 years of put option data and conclude that average put returns are, in general, significantly more negative than the predictions of Black Scholes, Heston stochastic volatility, and Bates SVJ model. See also Bondarenko (2014). Hence, anchoring offers a potential explanation.

Clearly, expected call return under anchoring is a lot less than what is expected under the Black Scholes model due to the anchoring bias. Empirical call returns are found to be a lot smaller given the predictions of the Black Scholes model (see Coval and Shumway (2001)). Hence, anchoring seems consistent with the empirical findings regarding both call and put option returns.

5. The Implied Volatility Skew with Anchoring

If anchoring determines option prices (formulas in proposition 1), and the Black Scholes model is used to infer implied volatility, the skew is observed. For illustrative purposes, the following parameter values are used: $S = 100, T - t = 0.25\ year, \sigma = 15\%, r = 0\%, and\ \delta = 4\%.$

An anchoring prone investor uses the expected return of the underlying stock as a starting point that gets adjusted upwards to arrive at the expected return of a call option. Anchoring bias implies that the adjustment is not sufficient to reach the Black-Scholes price.

As long as the adjustment made is smaller than the adjustment required to reach the Black-Scholes price, the implied volatility skew is observed. To reach the Black-Scholes price, it must be: $ln\left(\frac{SN(d_1^A)-Ke^{-(r+\delta)(T-t)}N(d_2^A)}{SN(d_1)-Ke^{-r(T-t)}N(d_2)}\right) \cdot \frac{1}{(T-t)} = \bar{A}_K$. For the purpose of this illustration, assume that the actual adjustment is only a quarter of that.

Table 1 shows the Black-Scholes price, the anchoring price, and the resulting implied volatility. The skew is seen. Table 1 also shows the CP upper bound and Leland prices for various trading intervals by assuming that $\theta = 0.01$. The anchoring price lies within a tight region between the Black-Scholes price and the CP upper bound. Furthermore, implied volatility is always larger than actual volatility.

The observed implied volatility skew also has a term-structure. Specifically, the skew tends to be steeper at shorter maturities. Figure 1 plots the implied volatility skews both at a longer time to maturity of 1 year and at a considerably shorter maturity of only one month. As can be seen, the skew is steeper at shorter maturity (the other parameters are $S = 100,\ \sigma = 20\%, r = 2\%, \delta = 3\%, and\ A_K = 0.25\bar{A}_K$).

Table 1						
K/S	Black-Scholes	Anchoring	Implied Volatility	CP Upper Bound	Leland Price (Trading Interval 1/250 years)	Leland Price (Trading Interval 1/52 years)
0.95	6.07	6.61	18.32%	6.93	8.36	7.59
1.0	2.99	3.37	16.88%	3.57	5.42	4.50
1.05	1.19	1.40	16.21%	1.50	3.28	2.38

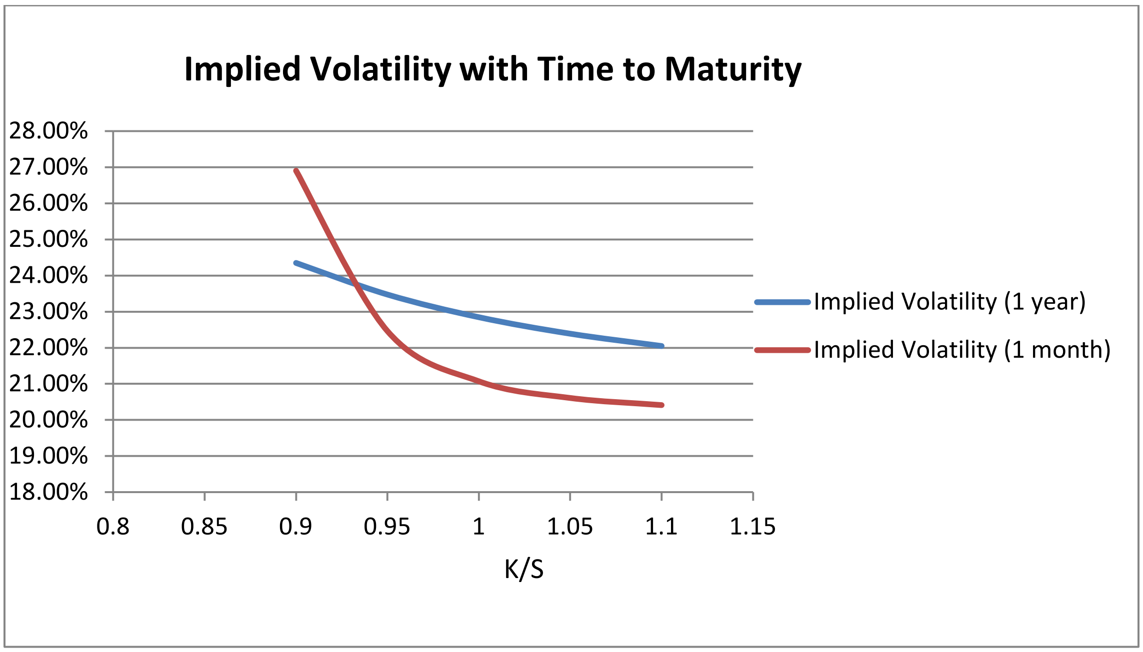

Figure 1

6. The Profitability of Covered Call Writing with Anchoring

The profitability of covered call writing is quite puzzling in the Black Scholes framework. Whaley (2002) shows that BXM (a Buy Write Monthly Index tracking a Covered Call on S&P 500) has significantly lower volatility when compared with the index, however, it offers nearly the same return as the index. In the Black Scholes framework, the covered call strategy is expected to have lower risk as well as lower return when compared with buying the index only. See Black (1975). In fact, in an efficient market, the risk adjusted return from covered call writing should be no different than the risk adjusted return from just holding the index.

The covered call strategy (S denotes stock, C denotes call) is given by:

$$V = S - C$$

With anchoring, this is equal to:

$$V = S - e^{-A_K(T-t)}\left\{SN(d_1^A) - Ke^{-(r+\delta)(T-t)}N(d_2^A)\right\}$$

$$=> V = \left(1 - e^{-A_K(T-t)}N(d_1^A)\right)S + e^{-A_K(T-t)}N(d_2^A)Ke^{-(r+\delta)(T-t)} \tag{6.1}$$

The corresponding value under the Black Scholes assumptions is:

$$V = \left(1 - N(d_1)\right)S + N(d_2)Ke^{-r(T-t)} \tag{6.2}$$

A comparison of 6.1 and 6.2 shows that covered call strategy is expected to perform much better with anchoring when compared with its expected performance in the Black Scholes world. With anchoring, covered call strategy creates a portfolio which is equivalent to having a portfolio with a weight of $1 - e^{-A_K(T-t)}N(d_1^A)$ on the stock and a weight of $e^{-A_K(T-t)}N(d_2^A)$ on a hypothetical risk free asset with a return of $r + \delta + A_K$. The stock has a return of $r + \delta$ plus dividend yield. This implies that, with anchoring, the return from covered call strategy is expected to be comparable to the return from holding the underlying stock only. The presence of a hypothetical risk free asset in 6.1 implies that the standard deviation of covered call returns is lower than the standard deviation from just holding the underlying stock. Hence, the superior historical performance of covered call strategy is consistent with anchoring.

6.1 The Zero-Beta Straddle Performance with Anchoring

Another empirical puzzle in the Black-Scholes/CAPM framework is that zero beta straddles lose money. Goltz and Lai (2009), Coval and Shumway (2001) and others find that zero beta straddles earn negative returns on average. This is in sharp contrast with the Black-Scholes/CAPM prediction which says that the zero-beta straddles should earn the risk free rate. A zero-beta straddle is constructed by taking a long position in corresponding call and put options with weights chosen so as to make the portfolio beta equal to zero:

$$\theta \cdot \beta_{Call} + (1-\theta) \cdot \beta_{Put} = 0$$

$$=> \theta = \frac{-\beta_{Put}}{\beta_{Call} - \beta_{Put}}$$

Where $\beta_{Call} = N(d_1) \cdot \frac{Stock}{Call} \cdot \beta_{Stock}$ and $\beta_{Put} = (N(d_1) - 1) \cdot \frac{Stock}{Put} \cdot \beta_{stock}$

It is straightforward to show that with anchoring, where call and put prices are determined in accordance with proposition 1, the zero-beta straddle earns a significantly smaller return than the risk free rate (with returns being negative for a wide range of realistic parameter values). See Appendix E for proof. Intuitively, with anchoring, both call and put options are more expensive when compared with Black-Scholes prices. Hence, the returns are smaller, and are typically negative.

Anchoring adjusted option pricing not only generates the implied volatility skew, it is also consistent with key empirical findings regarding option portfolio returns such as covered call writing and zero-beta straddles.

7. Leverage Adjusted Option Returns with Anchoring

Leverage adjustment dilutes beta risk of an option by combining it with a risk free asset. Leverage adjustment combines each option with a risk-free asset in such a manner that the overall beta risk becomes equal to the beta risk of the underlying stock. The weight of the option in the portfolio is equal to its inverse price elasticity w.r.t the underlying stock's price:

$$\beta_{portfolio} = \Omega^{-1} \times \beta_{call} + (1 - \Omega^{-1}) \times \beta_{riskfree}$$

where $\Omega = \frac{\partial Call}{\partial Stock} \times \frac{Stock}{Call}$ (i.e price elasticity of call w.r.t the underlying stock)

$$\beta_{call} = \Omega \times \beta_{stock}$$

$$\beta_{riskfree} = 0$$

$=> \beta_{portfolio} = \beta_{stock}$

Constantinides, Jackwerth and Savov (2013) uncover a number of interesting empirical facts regarding leverage adjusted index option returns. They find that over a period ranging from April 1986 to January 2012, the average percentage monthly returns of *leverage-adjusted index call and put options are decreasing in the ratio of strike to spot.* They also find that *leverage adjusted put returns are larger than the corresponding leverage adjusted call returns*. The empirical findings in Contantinides et al (2013) are inconsistent with the Black-Scholes/CAPM framework, which predicts that the leverage adjusted returns should be equal to the return from the underlying index. That is, they should not fall with strike, and the leverage adjusted put option returns should not be any different than the leverage adjusted call returns.

If anchoring determines call prices, then the behavior of leverage adjusted call and put returns should be a lot different than their predicted behavior under the Black-Scholes assumptions. For call options (suppressing subscripts for simplicity):

$$\Omega^{-1} \cdot \frac{1}{dt}\left[\frac{E[dC]}{C}\right] + (1 - \Omega^{-1})r \tag{7.1}$$

where $$E[dC] = \left((r+\delta)S\frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 C}{\partial S^2}\right)dt \tag{7.2}$$

According to the anchoring PDE:

$$(r + \delta + A)C = \frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}(r+\delta)S + \frac{\partial^2 C}{\partial S^2}\frac{\sigma^2 S^2}{2} \tag{7.3}$$

Substituting (7.3) and (7.2) in (7.1) and simplifying leads to:

$$Leverage\ Adjusted\ Call\ Return = \Omega^{-1}(\delta + A) + r \tag{7.4}$$

Note that in (7.4), if $A = (\Omega - 1)\delta$, that is if the adjustment is correct (no anchoring bias), then the leverage adjusted call return is equal to the return from the underlying stock. With the anchoring bias, that is, when $< (\Omega - 1)\delta$, the leverage adjusted call return falls with strike-to-spot. To take an example, suppose the adjustment is only a quarter of the correct adjustment. That is, assume that $A = 0.25(\Omega - 1)\delta$. It follows:

$$Leverage\ Adjusted\ Call\ Return = 0.25 + \frac{0.75\delta}{\Omega} + r \qquad (7.4b)$$

As Ω rises rapidly with strike-to-spot ratio, leverage adjusted call return falls as strike-to-spot ratio rises. Hence, anchoring adjusted option pricing is consistent with empirical evidence regarding leverage adjusted call returns.

The corresponding leverage adjusted put option return with anchoring adjusted option pricing is:

$$Leverage\ Adjusted\ Put\ Return = r + \left\{\frac{\delta S - (\delta + A)C}{S(1 - \Delta_{call})}\right\} \qquad (7.5)$$

Where $\Delta_{call} = \frac{\partial C}{\partial S}$

If there is no anchoring bias, that is, if the adjustment is correct, then $A = (\Omega - 1)\delta$. Substituting this value in (7.5) leads to leverage adjusted put return being equal to the return from the underlying stock. What happens if $A < (\Omega - 1)\delta$? To fix ideas, as before, let's take the following example: $A = 0.25(\Omega - 1)\delta$. (7.5) then becomes:

$$Leverage\ Adjusted\ Put\ Return = r + \delta\left\{\frac{S - (0.75 + 0.25\Omega)C}{S(1 - \Delta_{call})}\right\} \qquad (7.5b)$$

It is straightforward to check that for realistic parameter values, (7.5b) falls with strike-to-spot, and (7.5b) is larger than (7.4b). Hence, empirical findings in Constantinides et al (2013) regarding leverage adjusted option returns are consistent with anchoring adjusted option pricing.

8. Anchoring Adjusted Option Pricing with Stochastic Volatility

In this section, I put forward an anchoring adjusted option pricing model for the case when the underlying stock price and its instantaneous variance are assumed to obey the uncorrelated stochastic processes described in Hull and White (1987):

$$dS = \mu S dt + \sqrt{V} S dw$$

$$dV = \varphi V dt + \varepsilon V dz$$

$$E[dwdz] = 0$$

Where $V = \sigma^2$ (Instantaneous variance of stock's returns), and φ and ε are non-negative constants. dw and dz are standard Guass-Weiner processes that are uncorrelated. Time subscripts in S and V are suppressed for notational simplicity. If $\varepsilon = 0$, then the instantaneous variance is a constant, and we are back in the Black-Scholes world. Bigger the value of ε, which can be interpreted as the volatility of volatility parameter, larger is the departure from the constant volatility assumption of the Black-Scholes model.

Hull and White (1987) is among the first option pricing models that allowed for stochastic volatility. A variety of stochastic volatility models have been proposed including Stein and Stein (1991), and Heston (1993) among others. Here, we use Hull and White (1987) assumptions to show that the idea of anchoring is easily combined with stochastic volatility. Clearly, with stochastic volatility it does not seem possible to form a hedge portfolio that eliminates risk completely because there is no asset which is perfectly correlated with $V = \sigma^2$.

If anchoring determines call prices and the underlying stock and its instantaneous volatility follow the stochastic processes described above, then the European call option price (no dividends on the underlying stock for simplicity) must satisfy the partial differentiation equation given below (see Appendix C for the derivation):

$$\frac{\partial C}{\partial t} + (r+\delta)S\frac{\partial C}{\partial S} + \varphi V\frac{\partial C}{\partial V} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2} + \frac{1}{2}\varepsilon^2 V^2\frac{\partial^2 C}{\partial V^2} = (r+\delta+A)C \qquad (8.1)$$

Where δ is the risk premium that a marginal investor in the call option expects to get from the underlying stock.

By definition, under anchoring, the price of the call option is the expected terminal value of the option discounted at the rate which the marginal investor in the option expects to get from investing in the option. The price of the option is then:

$$C(S_t, \sigma_t^2, t) = e^{-(r+\delta+A)(T-t)} \int C(S_T, \sigma_T^2, T) p(S_T | S_t, \sigma_t^2) dS_T \tag{8.2}$$

Where the conditional distribution of S_T as perceived by the marginal investor is such that $E[S_T | S_t, \sigma_t^2] = S_t e^{(r+\delta)(T-t)}$ and $C(S_T, \sigma_T^2, T)$ is $max(S_T - K, 0)$.

By defining $\bar{V} = \frac{1}{T-t} \int_t^T \sigma_\tau^2 d\tau$ as the means variance over the life of the option, the distribution of S_T can be expressed as:

$$p(S_T | S_t, \sigma_t^2) = \int f(S_T | S_t, \bar{V})\, g(\bar{V} | S_t, \sigma_t^2) d\bar{V} \tag{8.3}$$

Substituting (8.3) in (8.2) and re-arranging leads to:

$$C(S_t, \sigma_t^2, t) = \int \left[e^{-(r+\delta+A)(T-t)} \int C(S_T) f(S_T | S_t, \bar{V}) dS_T \right] g(\bar{V} | S_t, \sigma_t^2) d\bar{V} \tag{8.4}$$

By using an argument that runs in parallel with the corresponding argument in Hull and White (1987), it is straightforward to show that the term inside the square brackets is the anchoring price of the call option with a constant variance $\bar{V}$. Denoting this price by $Call_{AM}(\bar{V})$, the price of the call option under anchoring when volatility is stochastic (as in Hull and White (1987)) is given by (proof available from author):

$$C(S_t, \sigma_t^2, t) = \int Call_{AM}(\bar{V}) g(\bar{V} | S_t, \sigma_t^2)\, d\bar{V} \tag{8.5}$$

Where $Call_{AM}(\bar{V}) = e^{-A(T-t)} \{ SN(d_1^M) - Ke^{-(r+\delta)(T-t)} N(d_2^M) \}$

$$d_1^M = \frac{ln\left(\frac{S}{K}\right) + \left(r + \delta + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} ;\ d_2^M = \frac{ln\left(\frac{S}{K}\right) + \left(r + \delta - \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}$$

Equation (8.5) shows that the anchoring adjusted call option price with stochastic volatility is the anchoring price with constant variance integrated with respect to the distribution of mean volatility.

8.1 Option Pricing Implications

Stochastic volatility models require a strong correlation between the volatility process and the stock price process in order to generate the implied volatility skew. They can only generate a more symmetric U-shaped smile with zero correlation as assumed here. In contrast, the anchoring stochastic volatility model (equation 8.5) can generate a variety of skews and smiles even with zero correlation. What type of implied volatility structure is ultimately seen depends on the parameters δ and ε. It is easy to see that if $\varepsilon = 0$ and $\delta > 0$, only the implied volatility skew is generated, and if $\delta = 0$ and $\varepsilon > 0$, only a more symmetric smile arises. For positive δ, there is a threshold value of ε below which skew arises and above which smile takes shape. Typically, for options on individual stocks, the smile is seen, and for index options, the skew arises. The approach developed here provides a potential explanation for this as ε is likely to be lower for indices due to inbuilt diversification (giving rise to skew) when compared with individual stocks.

9. Anchoring Adjusted Option Pricing with Jump Diffusion

In this section, I integrate the idea of analogy making with the jump diffusion model of Merton (1976). As before, the point is that the idea of anchoring is independent of the distributional assumptions that are made regarding the behavior of the underlying stock. In the previous section, anchoring is combined with the Hull and White stochastic volatility model to illustrate the same point.

Merton (1976) assumes that the stock returns are a mixture of geometric Brownian motion and Poisson-driven jumps:

$$dS = (\mu - \gamma\beta)Sdt + \sigma Sdz + dq$$

Where dz is a standard Guass-Weiner process, and $q(t)$ is a Poisson process. dz and dq are assumed to be independent. γ is the mean number of jump arrivals per unit time, $\beta = E[Y - 1]$

where $Y-1$ is the random percentage change in the stock price if the Poisson event occurs, and E is the expectations operator over the random variable Y. If $\gamma = 0$ (hence, $dq = 0$) then the stock price dynamics are identical to those assumed in the Black Scholes model. For simplicity, assume that $E[Y] = 1$.

The stock price dynamics then become:

$$dS = \mu S dt + \sigma S dz + dq$$

Clearly, with jump diffusion, the Black-Scholes no-arbitrage technique cannot be employed as there is no portfolio of stock and options which is risk-free. However, with anchoring, the price of the option can be determined as the return on the call option demanded by the marginal investor is equal to the return he expects from the underlying stock plus an adjustment term.

If anchoring determines the price of the call option when the underlying stock price dynamics are a mixture of a geometric Brownian motion and a Poisson process as described earlier, then the following partial differential equation must be satisfied (see Appendix D for the derivation):

$$\frac{\partial C}{\partial t} + (r+\delta)S\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + \gamma E[C(SY,t) - C(S,t)] = (r+\delta+A)C \qquad (9.1)$$

If the distribution of Y is assumed to log-normal with a mean of 1 (assumed for simplicity) and a variance of v^2 then by using an argument analogous to Merton (1976), the following anchoring adjusted option pricing formula for the case of jump diffusion is easily derived (proof available from author):

$$Call = \sum_{j=0}^{\infty} \frac{e^{-\gamma(T-t)}\left(\gamma(T-t)\right)^j}{j!} Call_{AM}\left(S,(T-t),K,r,\delta,\sigma_j\right) \qquad (9.2)$$

$$Call_{AM}\left(S,(T-t),K,r,\delta,\sigma_j\right) = e^{-A(T-t)}\left\{SN(d_1^M) - Ke^{-(r+\delta)(T-t)}N(d_2^M)\right\}$$

$$d_1^M = \frac{ln\left(\frac{S}{K}\right) + \left(r+\delta+\frac{\sigma_j^2}{2}\right)(T-t)}{\sigma_j\sqrt{T-t}} \qquad d_2^M = \frac{ln\left(\frac{S}{K}\right) + \left(r+\delta-\frac{\sigma_j^2}{2}\right)(T-t)}{\sigma_j\sqrt{T-t}}$$

$\sigma_j = \sqrt{\sigma^2 + v^2\left(\frac{j}{T-t}\right)}$ and $v^2 = \frac{f\sigma^2}{\gamma}$

Where f is the fraction of volatility explained by jumps.

As expected, the formula in (9.2) is identical to the Merton jump diffusion formula except for two parameter, δ andA.

9.1 Option Pricing Implications

Merton's jump diffusion model with symmetric jumps around the current stock price can only produce a symmetric smile. Generating the implied volatility skew requires asymmetric jumps (jump mean becomes negative) in the model. However, with anchoring, both the skew and the smile can be generated even when jumps are symmetric. In particular, for low values of δ, a more symmetric smile is generated, and for larger values of δ, skew arises.

Even if we one assumes an asymmetric jump distribution around the current stock price, Merton formula, when calibrated with historical data, generates a skew which is a lot less pronounced (steep) than what is empirically observed. See Andersen and Andreasen (2002). The skew generated by the anchoring formula (with asymmetric jumps) is typically more pronounced (steep) when compared with the skew without anchoring. Hence, anchoring potentially adds value to a jump diffusion model as well.

10. Conclusions

Intriguing option pricing puzzles include: 1) the implied volatility skew, 2) superior historical performance of covered call writing, 3) worse-than-expected performance of zero beta straddles, and 4) the puzzling findings regarding leverage adjusted index option returns. Furthermore, it is well known that average put returns are far more negative than what theory predicts, and average call returns are smaller than what one would expect given their systematic risk.

If the return of the underlying stock is used as a starting point which gets adjusted upwards to arrive at call option return, then the anchoring bias implies that such adjustments are insufficient.

There is considerable field and experimental evidence of the role of anchoring in option pricing. In this article, an anchoring-adjusted option pricing model is put forward. The model provides a unified explanation for the puzzles mentioned above.

The challenge for financial economics is to enrich the elegant option pricing framework sufficiently so that it captures key empirical regularities. This article shows that incorporating the anchoring bias in the option pricing framework provides the needed enrichment, while preserving the elegance of the framework. Furthermore, anchoring works regardless of the distributional assumptions that are made about the underlying stock behavior. As shown in this article, it is easy to combine anchoring with jump diffusion and stochastic volatility approaches.

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Appendix A

The anchoring adjusted PDE can be solved by converting to heat equation and exploiting its solution.

Start by making the following transformations in (4.4):

$$\tau = \frac{\sigma^2}{2}(T-t)$$

$$x = ln\frac{S}{K} => S = Ke^x$$

$$C(S,t) = K \cdot\ c(x,\tau) = K \cdot c\left(ln\left(\frac{S}{K}\right), \frac{\sigma^2}{2}(T-t)\right)$$

It follows,

$$\frac{\partial C}{\partial t} = K \cdot \frac{\partial c}{\partial \tau} \cdot \frac{\partial \tau}{\partial t} = K \cdot \frac{\partial c}{\partial \tau} \cdot \left(-\frac{\sigma^2}{2}\right)$$

$$\frac{\partial C}{\partial S} = K \cdot \frac{\partial c}{\partial x} \cdot \frac{\partial x}{\partial S} = K \cdot \frac{\partial c}{\partial x} \cdot \frac{1}{S}$$

$$\frac{\partial^2 C}{\partial S^2} = K \cdot \frac{1}{S^2} \cdot \frac{\partial^2 C}{\partial x^2} - \ K \cdot \frac{1}{S^2}\frac{\partial C}{\partial x}$$

Plugging the above transformations into (4.4) and writing $\tilde{r} = \frac{2(r+\delta)}{\sigma^2}$, and $\tilde{\epsilon} = \frac{2A}{\sigma^2}$ we get:

$$\frac{\partial c}{\partial \tau} = \frac{\partial^2 c}{\partial x^2} + (\tilde{r} - 1)\frac{\partial c}{\partial x} - (\tilde{r} + \tilde{\epsilon})c \qquad (D1)$$

With the boundary condition/initial condition:

$$C(S,T) = max\{S-K,0\}\ becomes\ c(x,0) = max\{e^x - 1,0\}$$

To eliminate the last two terms in (D1), an additional transformation is made:

$$c(x,\tau) = e^{\alpha x + \beta \tau}u(x,\tau)$$

It follows,

$$\frac{\partial c}{\partial x} = \alpha e^{\alpha x+\beta\tau}u + e^{\alpha x+\beta\tau}\frac{\partial u}{\partial x}$$

$$\frac{\partial^2 c}{\partial x^2} = \alpha^2 e^{\alpha x+\beta\tau}u + 2\alpha e^{\alpha x+\beta\tau}\frac{\partial u}{\partial x} + e^{\alpha x+\beta\tau}\frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial c}{\partial \tau} = \beta e^{\alpha x+\beta\tau}u + e^{\alpha x+\beta\tau}\frac{\partial u}{\partial \tau}$$

Substituting the above transformations in (D1), we get:

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2} + (\alpha^2 + \alpha(\tilde{r}-1) - (\tilde{r}+\tilde{\epsilon}) - \beta)u + \big(2\alpha + (\tilde{r}-1)\big)\frac{\partial u}{\partial x} \qquad \text{(D2)}$$

Choose $\alpha = -\frac{(\tilde{r}-1)}{2}$ and $\beta = -\frac{(\tilde{r}+1)^2}{4} - (\tilde{\epsilon})$. (D2) simplifies to the Heat equation:

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2} \qquad (D3)$$

With the initial condition:

$$u(x_0,0) = max\{(e^{(1-\alpha)x_0} - e^{-\alpha x_0}),0\} = max\left\{\left(e^{\left(\frac{\tilde{r}+1}{2}\right)x_0} - e^{\left(\frac{\tilde{r}-1}{2}\right)x_0}\right),0\right\}$$

The solution to the Heat equation in our case is:

$$u(x,\tau) = \frac{1}{2\sqrt{\pi\tau}}\int_{-\infty}^{\infty} e^{-\frac{(x-x_0)^2}{4\tau}}u(x_0,0)dx_0$$

Change variables: $= \frac{x_0-x}{\sqrt{2\tau}}$, which means: $dz = \frac{dx_0}{\sqrt{2\tau}}$. Also, from the boundary condition, we know that $u > 0 \; iff \; x_0 > 0$. Hence, we can restrict the integration range to $z > \frac{-x}{\sqrt{2\tau}}$

$$u(x,\tau) = \frac{1}{\sqrt{2\pi}}\int_{-\frac{x}{\sqrt{2\pi}}}^{\infty} e^{-\frac{z^2}{2}}\cdot e^{\left(\frac{\tilde{r}+1}{2}\right)(x+z\sqrt{2\tau})}dz - \frac{1}{\sqrt{2\pi}}\int_{-\frac{x}{\sqrt{2\tau}}}^{\infty} e^{-\frac{z^2}{2}}\cdot e^{\left(\frac{\tilde{r}-1}{2}\right)(x+z\sqrt{2\tau})}dz$$

$$=: H_1 - H_2$$

Complete the squares for the exponent in H_1:

$$\frac{\tilde{r}+1}{2}\left(x+z\sqrt{2\tau}\right)-\frac{z^2}{2}=-\frac{1}{2}\left(z-\frac{\sqrt{2\tau}(\tilde{r}+1)}{2}\right)^2+\frac{\tilde{r}+1}{2}x+\tau\frac{(\tilde{r}+1)^2}{4}$$

$$=:-\frac{1}{2}y^2+c$$

We can see that $dy = dz$ and c does not depend on z. Hence, we can write:

$$H_1=\frac{e^c}{\sqrt{2\pi}}\int_{-x/\sqrt{2\pi}-\sqrt{\tau/2}(\tilde{r}+1)}^{\infty} e^{-\frac{y^2}{2}}dy$$

A normally distributed random variable has the following cumulative distribution function:

$$N(d)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{d} e^{-\frac{y^2}{2}}dy$$

Hence, $H_1 = e^c N(d_1)$ where $d_1 = x/\sqrt{2\pi} + \sqrt{\tau/2}\,(\tilde{r}+1)$

Similarly, $H_2 = e^f N(d_2)$ where $d_2 = x/\sqrt{2\pi} + \sqrt{\tau/2}\,(\tilde{r}-1)$ and $f = \frac{\tilde{r}-1}{2}x + \tau\frac{(\tilde{r}-1)^2}{4}$

The analogy based European call pricing formula is obtained by recovering original variables:

$$C = e^{-A(T-t)}\left\{SN(d_1^A) - Ke^{-(r+\delta)(T-t)}N(d_2^A)\right\}$$

Where $\boldsymbol{d_1^A} = \frac{\boldsymbol{ln(S/K)+(r+\delta+\frac{\sigma^2}{2})(T-t)}}{\boldsymbol{\sigma\sqrt{T-t}}}$ and $\boldsymbol{d_2^A} = \frac{\boldsymbol{ln\left(\frac{S}{K}\right)+\left(r+\delta-\frac{\sigma^2}{2}\right)(T-t)}}{\boldsymbol{\sigma\sqrt{T-t}}}$

Appendix B

By definition, at the threshold:

$$e^{-A_K(T-t)}\left\{SN\left(d_1{}^A\right) - Ke^{-(r+\delta)(T-t)}N\left(d_2{}^A\right)\right\} = SN(d_1) - Ke^{-r(T-t)}N(d_2) \qquad \text{(E1)}$$

Solving (E1) for A_K gives the threshold value:

$$ln\left(\frac{SN(d_1^A)-Ke^{-(r+\delta)(T-t)}N(d_2^A)}{SN(d_1)-Ke^{-r(T-t)}N(d_2)}\right)\cdot\frac{1}{(T-t)} = |\overline{A_K}| \qquad \text{(E2)}$$

Appendix C

By Ito's Lemma (time subscript is suppressed for simplicity):

$$dC = \frac{\partial C}{\partial t}dt + \frac{\partial C}{\partial S}dS + \frac{\partial C}{\partial V}dV + \frac{1}{2}VS^2\frac{\partial^2 C}{\partial S^2}dt + \frac{1}{2}V^2\varepsilon^2\frac{\partial^2 C}{\partial V^2}dt \tag{F1}$$

Substituting:

$dS = \mu S dt + \sigma S dw$ and,

$dV = \varphi V dt + \varepsilon V dz$

in (F1) and taking expectations leads to:

$$\frac{\partial C}{\partial t} + (r+\delta)S\frac{\partial C}{\partial S} + \varphi V\frac{\partial C}{\partial V} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2} + \frac{1}{2}\varepsilon^2 V^2\frac{\partial^2 C}{\partial V^2} = (r+\delta+A)C \tag{F2}$$

Appendix D

By following a very similar argument as in appendix C, and using Ito's lemma for the continuous part and an analogous Lemma for the discontinuous part, the following is obtained:

$$\frac{\partial C}{\partial t} + (r+\delta)S\frac{\partial C}{\partial S} + \frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2} + \gamma E[C(SY,t) - C(S,t)] = (r+\delta+A)C$$

Appendix E

Following Coval and Shumway (2001) and some algebraic manipulations, the return from a zero-beta-straddle can be written as:

$$r_{straddle} = \frac{-\Omega_c C + S}{\Omega_c P - \Omega_c C + S}\cdot r_{call} + \frac{\Omega_c P + S}{\Omega_c P - \Omega_c C + S}\cdot r_{put}$$

Where C and P denote call and put prices respectively, r_{call} is expected call return, r_{put} is expected put return, and Ω_c is call price elasticity w.r.t the underlying stock price.

Under anchoring:

$$r_{call} = \mu + A$$

$$r_{put} = \frac{(\mu + A)C - \mu S + rPV(K)}{P}$$

Substituting r_{call} and r_{put} in the expression for $r_{straddle}$, and simplifying implies that as long as the risk premium on the underlying is positive, it follows that:

$$r_{straddle} < r$$

Appendix F

Note that for a put option, if the underlying stock has a positive risk premium, then the expected put payoff must be less than its price. That is, expected put return is negative. The proof follows directly from realizing that if the risk premium on the underlying stock is positive, the price of a put option under anchoring is larger than the price of a put option in the Black Scholes model.

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