INDIRECT PRODUCTION FUNCTION AND THE OUTPUT EFFECT OF PUBLIC TRANSIT SUBSIDIES

By

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(February 2009**)**

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This paper uses an indirect production function to decompose the effects of subsidies on output into the lump-sum, cost and inefficiency effects. Using 2006 data for U.S. transit systems it estimates an indirect production function and uses the results to calculate these effects. It finds that the lump-sum effects exceed the other effects and that the average total effect of the subsidies is a 73.23% increase in output. The range of the output change shows that in many transit systems the subsidies more than double the outputs they produce. The paper suggests that reductions in allocative inefficiencies from the subsidies would result in very large increases in output.

Previous research has shown that when operating and capital subsidies are offered they create allocative inefficiencies by distorting the optimal rate of input substitution (Obeng et al. 1997). These allocative inefficiencies assume output remains unchanged. It can be argued that this assumption is unjustified because Mohring (1972) and Pederson (2003) show that based on user cost economies of scale the subsidies increase service frequencies and output. According to van Reeven (2008), this increase could make service frequency higher than its socially optimum level. If so, then the subsidies could lead to oversupply of services, and the increase in output from the higher frequencies would be due to inefficient use of resources. On the other hand Small and Gomez-Ibanez (1999) argue that the subsidies could lead to inefficiencies and reduced productivity, which if true could lead to lower levels of output. Thus, there appears to be two counteracting effects of operating and capital subsidies worthy of further investigation. One hand, the user cost economies of scale argument shows that output increases with these subsidies through increased service frequency. On the other, the allocative inefficiency and reduced productivity arguments, show possible reductions in output from the subsidies. If these effects exactly offset each other then the subsidies are used to maintain existing services and do not increase output. There are no studies that attempt to bring these two effects together in the public transit economics literature. However, there are many studies that examine the impacts of the subsidies on cost and inefficiency. For example, Kim (1987) studies the effects of lump-sum subsidies within a constrained optimization framework, while Kerstens (1996), Nolan (1996), Karlaftis and McCarthy (2002) address the effects of operating and capital subsidies on cost and efficiency.

While providing useful information this focus on cost can be critiqued on several fronts. First, if subsidies increase output then they must increase total cost, since it requires more inputs to produce the additional output unless there is a gain in productivity or technological improvement from the subsidies that makes it cheaper to produce each level of output. Second, since both cost and output increase we should be examining the relationship between subsidies and average cost not total cost. Alternatively, the focus should be on comparing the increase in cost from the subsidies to the increase in output. Third, focusing only on cost, past research completely ignores the possible effects of these subsidies on output by changing input prices. As recent developments in the public transit economics literature show, operating and capital subsidies make transit systems misperceive input prices as lower (Obeng et al. 1997) thereby making them employ more inputs than they would do otherwise. In turn this change in inputs increases the amount of output produced, a result consistent with what Cervero (1984) and Bly and Oldfield (1986) found. Therefore, the effects of the subsidies on output cannot be completely ignored, unless it is assumed they only support existing but not expanded services. Since this assumption raises empirical questions and cannot be supported in practice, ignoring the output effects of the subsidies leaves a void in the transit economics literature that requires examination.

To fill this void this paper determines the impact of operating and capital subsidies on output and argues that while this impact could be seen as a benefit, it is a result of input overuse and, therefore, inefficiency. It surveys the literature on public transit objectives and follows it with an indirect production function to decompose the effects of operating and capital subsidies on output into lump-sum, cost and inefficiency effects. To the best knowledge of this author, this decomposition is unique to this paper and this is the first estimation of indirect production function using public transit data. Finally, to illustrate the usefulness of the decomposition the paper specifies an empirical model and estimates it with 2006 data for U.S. bus transit systems. Using the results it calculates the proportions of output due to the lump-sum, allocative inefficiency and other effects and sums the results to obtain the total effects of operating and capital subsidies on output. It finds that the cost and inefficiency effects consistently reduce output while the lump-sum effects increase output. The combined effect of these two sources of output change is a 73.23% increase in output on the average and the range of the output change shows that in many transit systems the subsidies more than doudle the outputs produced. Thus, the positive lump-sum output effects of the subsidies exceed their negative output effects.

I. Literature Review

Some possible reasons for the absence of focus on the effects of operating and capital subsidies on output are the conceptual difficulties in applying production analysis to transit systems. These include ambiguities about whether what is produced is an intermediate or a final output (Small 1990) and output heterogeneity with its resultant aggregation problems (Kerstens and Eeckaut 1999). Hanushek (1979), writing in a different context but relevant to transportation, includes in these difficulties output heterogeneity in terms of quality, the existence of multiple outputs that may be related or simultaneously produced, and the relatively fixed ratios between some inputs implying they would explain little variation when used in production functions. Another reason is the presence of many objectives in public transit and the lack of consensus about which to use to model decision making in transit systems (De Borger et al. 2002). Berechman (1993) groups these objectives into political, managerial, bureaucratic and cost minimization and evaluates how each applies to public transit. He argues against the political and managerial objectives and notes that cost minimization is also not the main objective. Next, he examines the public sector models of Niskanen (1968) and De Alessi (1969), particularly bureaucratic objectives, in terms of budget surplus maximization and argues against them too. He postulates that it is likely managers allow their costs to increase to meet their budgets (i.e., catch-up effect). Thus, with a large budget possibly from increases in subsidies, managers are likely to overspend by showing expense preference for some inputs (such as visible inputs) and exercising less control over cost.

Over time, these actions increase costs, exhaust the subsidies, and make transit systems want more subsidies.

The recent spate of research addressing technical efficiency in public transit systems shows a new trend toward using it as an empirical objective (e.g., Nolan et al. 2002) implying output maximization or input minimization. Other studies suggest that transit systems pursue a modified cost minimization objective such as after-subsidy cost minimization, which could lead to after-subsidy profit (Obeng 2000), or that directed by legislation they pursue social objectives. Savage (2004) writing on management objectives and the causes of urban transit deficits surmises that "managers may be motivated to maximize social welfare, number of passengers, or the amount of service provided" and notes that maximizing the amount of service is a favored objective in some Australian and U.K. transit systems. Novaes (2001) notes the work of Talley (1988) which argues that transit systems maximize their output in terms of passenger miles. Besides these single objectives, managers may face multiple objectives (e.g., universal availability of service, minimize cost per passenger, provide service to diversified population, provide high quality service, provide environmentally friendly service) making it difficult to develop a single empirical model to analyze their behavior. Or as Fabbri (1996) suggests they may have "non-conventional objectives and therefore non-standard behavioral programs."

Despite these various objectives and their limitations pointed out by Berechman (1993), most recent research show that cost minimization is an empirical objective commonly used in the transit economics literature, where cost is a function of competitive input prices, output, firm and environmental characteristics. Using the cost function minimizes some of the conceptual problems enumerated, for example, those regarding the proportions in which the physical inputs are used, and leads to the estimation of frontier and non-frontier cost functions that in some cases include subsidies as variables. The signs of subsidies in these functions allow inferences to be made about their impacts on cost but not on output. Despite their advantages, cost functions have been critiqued on the grounds that they do not allow the effects of input price changes on output to be studied. Since production decisions depend on input prices, the critics argue that input demand elasticities from cost functions are imprecise and are output constant input elasticities of demand instead of Marshallian demand elasticities needed for policies. According to Gajanan and Ramaiah (1996) the problem with these elasticities is that they preclude the estimation of the output effect of input price changes.

Echoing these problems too, Garofalo and Malhotra (1990) write that the main drawback of demand elasticities from cost functions is their assumption that a change in input price occurs only through substitution effects, ignoring the output effect. To overcome this drawback these authors assume output maximization and derive a set of Marshallian input demand equations which they estimated and used to calculate the substitution and output effects of input price changes via the Slutsky decomposition. Kako (1978) used a translog cost function to show that changes in factor prices can be decomposed into output effect, substitution effect and technical change effect. Chambers (1982) used comparative statics and an indirect production function to derive the output effect of a change in factor prices from a cost function. He showed that for a homothetic production function the output effect of a change in the price of an input is the negative of its corresponding optimal share in cost.

Output maximization as an empirical objective asks how much output would be obtained from a given level of expenditure. According to Shephard (1973) this objective is most appropriate in public and service sector studies where decision makers are concerned with how much benefit in terms of output would be obtained for given levels of expenditures. In such instances, market prices are unobserved perhaps because of subsidization and regulation, or are not exogenous and profit maximization or cost minimization is inappropriate (Fare et al., 1988). As these latter authors further note, output maximization is most “appropriate to producer performance evaluation when resource usage can be reliably compared on the basis of cost but benefits (outputs produced or services provided) cannot be priced reliably enough to allow revenue comparisons” (p. 73). Additionally, it allows the impacts of changes in input prices on output to be determined. However, it can be critiqued on the grounds that it could lead to excessive output and wasteful services. As Nash (1978) shows, such an objective could lead to output levels that are not Pareto optimal.

These critiques notwithstanding, this paper assumes output maximization. Unlike previous works analyzing the impacts of input price changes on output cited above, however, the paper does not use the Slutsky decomposition. Instead, it shows through derivation that the indirect production function is flexible enough to permit a decomposition of the effects of operating and capital subsidies on output into the lump-sum effect, cost and allocative inefficiencies effects. In addition, our use of the indirect production function also is based upon the premise that transit systems are often given subsidies to increase and improve their services and make them generally available to the population served. Decision makers, therefore, are interested in knowing how much the subsidies increase output beyond the existing output levels they help maintain. Another reason is that efficiency studies using data envelopment analysis and Malmquist indices or their variations assume output maximization so the choice of this objective conforms to current practices in efficiency studies.

II. Output Decomposition

Assume a transit system produces vehicle miles of service (Q) with labor (L), capital (K) in terms of vehicles, and fuel (F). In producing the output, the transit system incurs total actual resource cost $C = w_L L + w_K K + w_F F$ where w_L, w_K, w_F are the respective market prices of the inputs, and receives capital subsidies, $A_K(L, K, F)$, and operating subsidies $A_o(L, K, F)$. The firm maximizes output subject to the constraint that its after-subsidy cost must not exceed B. The Lagrangian of this constrained optimization is,

$$Max \;\; Q(L, K, F) + \lambda\left(w_L L + w_K K + w_F F - A_K(L, K, F) - A_o(L, K, F) - B\right) \qquad (1)$$

From the first order conditions of Eq. (1), the ratio of the marginal products f_i, f_j of any input pair $(i,\; j)$ is,

$$\frac{f_i}{f_j} = \frac{w_i\left(1 - \mu_{oi} H_{oi} - \mu_{Ki} H_{Ki}\right)}{w_j\left(1 - \mu_{oj} H_{oj} - \mu_{Kj} H_{Kj}\right)} = \frac{w_i^*}{w_K^*} \quad for \; i = L, K, F \;\; and \; i \neq j \qquad (2)$$

Where, for any input x_i, the ratios of the subsidies to its costs are $H_{oi} = A_o / w_i x_i$, $H_{Ki} = A_K / w_i x_i$. μ_{oi} and μ_{Ki} respectively are the coefficients of H_{oi} and H_{Ki}, and $\sum_i \mu_{oi} = \sum_i \mu_{Ki} = 1$ ensures full allocation of the subsidies to all inputs. The term $(1 - \mu_{oi} H_{oi} - \mu_{Ki} H_{ki}) / (1 - \mu_{oj} H_{oj} - \mu_{Kj} H_{kj})$ measures allocative distortion or the inefficiency between input pairs because it is how much the optimal rate of input substitution deviates from what it should be without the subsidies. If its value is less (more) than one, the subsidy makes the price of input i very low relative to the price of input j; the reverse being true also. This makes transit systems overuse some inputs relative to others when subsidies are given to them.

From the dual of this constrained optimization problem, the after-subsidy minimum cost function can be written as $C^* = C^*(w_L^*, w_K^*, w_F^*, Q)$ and the after-subsidy total cost as $C^* = w_L^* L + w_K^* K + w_F^* F = C \sum_i S_i (1 - \mu_{oi} H_{oi} - \mu_{Ki} H_{Ki})$, where S_i is the share of an input in actual total cost. This after-subsidy cost is also the implied cost to the transit system, and it is what influences a transit system's production plans.[1] Thus, though total resource cost is C, a firm receiving capital and operating subsidies misperceives it as C^* and uses the latter as the basis for making production decisions. Because $\sum_i S_i (1 - \mu_{oi} H_{oi} - \mu_{Ki} H_{Ki})$ is less than one, it is the proportion of total resource cost transit systems misperceive to be their own and must pay with their own internally generated funds from passenger revenues, rentals, advertisements and investments. Further, because transit systems misperceive their costs as low they overuse some of their inputs resulting in overproduction and an increase in overall resource cost. This increase in output also results from the subsidies increasing service frequency (van Reeven 2008, Tistato 2007, Mohring 1972).

Because a higher service frequency results from subsidies, the question is how much output will be obtained by offering subsidies to public transit systems? That question can be answered using an indirect production function. Using the duality between cost and production functions, if a transit system's minimum implied cost function is $C^*(w_L^*, w_K^*, w_F^*, Q)$ then under output maximization there exists an indirect production function $Q(w_L^*, w_K^*, w_F^*, C^*)$ which is the solution to solving for output at the cost minimization point.[2] This function is non-decreasing in implied cost C^*, non-increasing in implied input prices w_L^*, w_K^*, w_F^*, homogeneous of degree zero in C^* and w_i^* and quasi-convex in input prices, i.e. $\partial^2 C^* / \partial w_i^{*2} > 0$.

Assume a flexible technology of the translog type.[3] Then, expanding $Q\left(w_L^*, w_K^*, w_F^*, C^*\right)$ using Taylor's series gives the translog indirect production function,

$$\ln Q = \beta_0 + \sum_i \beta_i \ln\left(\frac{C^*}{w_i^*}\right) + 0.5 \sum_i \sum_j \beta_{ij} \ln\left(\frac{C^*}{w_i^*}\right) \ln\left(\frac{C^*}{w_j^*}\right) \qquad (3).$$

Substituting the implied cost $C^* = C \sum_i S_i \left(1 - \mu_{oi} H_{oi} - \mu_{Ki} H_{Ki}\right)$ and the implied input price $w_i^* = w_i \left(1 - \mu_{oi} H_{oi} - \mu_{Ki} H_{Ki}\right)$ provided earlier into Eq. (3), taking the logarithms of both sides and adding an error term ε, gives,

$$\begin{aligned} \ln Q &= \beta_0 + \sum_i \beta_i \ln\left(\frac{C}{w_i} U_i\right) + 0.5 \sum_i \sum_j \beta_{ij} \ln\left(\frac{C}{w_i} U_i\right) \ln\left(\frac{C}{w_j} U_j\right) + \varepsilon. \\ &= \beta_0 + \sum_i \beta_i \ln\left(\frac{C}{w_i}\right) + 0.5 \sum_i \sum_j \beta_{ij} \ln\left(\frac{C}{w_i}\right) \ln\left(\frac{C}{w_j}\right) + \sum_i \beta_i \ln U_i + \\ &\qquad 0.5 \sum_i \sum_j \beta_{ij} \left(\ln\left[\frac{C}{w_i}\right] \ln\left[U_j\right] + \ln\left[\frac{C}{w_j}\right] \ln\left[U_i\right] + \ln\left[U_j\right] \ln\left[U_i\right] \right) + \varepsilon. \qquad (4) \end{aligned}$$

$$\text{where} \quad U_i = \left(\sum_i S_i \left(1 - \mu_{oi} H_{oi} - \mu_{Ki} H_{Ki}\right)\right) / \left(1 - \mu_{oi} H_{oi} - \mu_{Ki} H_{Ki}\right)$$

Simplifying this equation through substitution, we have,

$$\begin{aligned} \ln(Q) = \ln\hat{Q} &- \sum_i \beta_i \ln\left(1 - \mu_{oi} H_{oi} - \mu_{Ki} H_{Ki}\right) + \left(\sum_i \beta_i \ln\left[\sum_i S_i \left(1 - \mu_{oi} H_{oi} - \mu_{Ki} H_{Ki}\right)\right] \right) + \\ &0.5 \sum_i \sum_j \beta_{ij} \left(\ln\left[\frac{C}{w_i}\right] \ln\left[U_j\right] + \ln\left[\frac{C}{w_j}\right] \ln\left[U_i\right] + \ln\left[U_j\right] \ln\left[U_i\right] \right) + \varepsilon. \qquad (5) \end{aligned}$$

Where, $\ln(\hat{Q})$ is the sum of the first three terms on the second line of Eq. (4), and it is the output that would have been produced if there were no subsidies. The sum of the second, third and fourth terms on this line show how the changes in input use from the subsidies affect output. More specifically, because the second term of Eq. (5) does not affect input shares or input proportions, it is the lump-sum effect of the subsidies on output by changing implied input prices. Further, because $(1-\mu_{oi}H_{oi}-\mu_{Ki}H_{Ki})$ is positive and less than one, its logarithm is negative and this makes the whole second term positive. Therefore, operating and capital subsidies increase output through their lump-sum effects.

On the other hand, the third term $\sum_i \beta_i \ln\left(\sum_i S_i(1-\mu_{oi}H_{oi}-\mu_{Ki}H_{Ki})\right)$ is the weighted effect of the cost impacts of the subsidies on output as a result of increased input use. Specifically, $\sum_i \beta_i$ is the weight and $\ln\left(\sum_i S_i(1-\mu_{oi}H_{oi}-\mu_{Ki}H_{Ki})\right)$ is how much the subsidies make implied total cost less than actual total cost. Thus, $\sum_i \beta_i$ is a factor that converts cost into output. The value of this third term is always negative and shows the opportunity cost of the subsidies in terms of forgone output.

The fourth term adds to, or reduces output. This term measures the effects of allocative inefficiency because U_i is the sum of the share weighted relative input price distortions from the subsidy as can be seen in Eq. (6) by expanding U_i for labor, capital and fuel and using labor as the denominator.

$$
\begin{aligned}
&\left(\sum_i S_i[1=\mu_{oi}H_{oi}-\mu_{Ki}H_{Ki}]\right)/(1-\mu_{oi}H_{oi}-\mu_{Ki}H_{Ki})=S_L+S_K\gamma_{KL}+S_F\gamma_{FL} \\
&\quad \textit{where } \gamma_{KL}=(1-\mu_{oL}H_{oK}-\mu_{KK}H_{KK})/(1-\mu_{oL}H_{oL}-\mu_{KL}H_{Kl}), \\
&\gamma_{FL}=(1-\mu_{oF}H_{oF}-\mu_{KF}H_{KF})/(1-\mu_{oL}H_{oL}-\mu_{KL}H_{Kl}).
\end{aligned}
\tag{6}
$$

In this equation, γ_{KL} and γ_{FL} are capital-labor and fuel-labor allocative distortions from the subsidies. Both $\ln(C/w_i)$ and $\ln(U_j)$ are positive, so it is the sign of β_{ij} that determines the direction of the contribution of the fourth term of Eq. (4) to output. If this sign is positive then the whole term is positive and the result is a further increase in output. If β_{ij} is negative an additional decrease in output would result from this term.

Adding the second, third, and fourth terms of Eq. (5) together gives the total effect of the subsidies on output. If the result of this addition is positive then the lump-sum output effect of the subsidies is greater than their output reduction effects and the subsidies increase output more than they decrease it. If negative it shows that the output reduction effects of the subsidies are larger than their output increasing effects. If zero, it shows that both the negative and positive output effects of the subsidies exactly offset each other thus leaving output unchanged. Here, the subsidies are used to maintain existing services without increasing them and this makes it appropriate to examine the impacts of the subsidies on costs only. Thus, it is in the latter case where the subsidies maintain existing services, but not increase them, that there is support for studies that use cost functions to examine the impact of the subsidies.

Another interpretation of the terms in Eq. (5) is in terms of inefficiency. The second, third and fourth terms of this equation are the changes in output from the subsidies making transit systems overuse their inputs. In other words, their sum is the overproduction of transit services because the subsidies distort the ratios of input prices and result in overuse of some inputs relative to others. To facilitate this interpretation, rewrite this equation as below.

$$\begin{aligned}\ln(Q) = {} & \ln\hat{Q} + \sum_i \beta_i \left\{ \ln\left(\frac{\sum S_i (1-\mu_{oi}H_{oi} - \mu_{Ki}H_{Ki})}{(1-\mu_{oi}H_{oi} - \mu_{Ki}H_{Ki})} \right) \right\} + 0.5 \sum_i \sum_j \beta_{ij} \left(\ln\left[\frac{C}{w_i}\right] \ln[U_j] + \ln\left[\frac{C}{w_j}\right] \ln[U_i] \right) \\ & + 0.5 \sum_i \sum_j \beta_{ij} \ln[U_j] \ln[U_i] + \varepsilon \\ = {} & \ln\hat{Q} + \sum_i \beta_i \ln(U_i) + 0.5 \sum_i \sum_j \beta_{ij} \left(\ln\left[\frac{C}{w_i}\right] \ln[U_j] + \ln\left[\frac{C}{w_j}\right] \ln[U_i] \right) + 0.5 \sum_i \sum_j \beta_{ij} \ln[U_j] \ln[U_i] + \varepsilon \qquad (7).\end{aligned}$$

From this equation, since U shows allocative distortion, it follows that the second, third and fourth terms are the changes in output from overusing some inputs relative to others and, therefore, inefficiency from overproduction. Alternatively, the sum of these terms is the percentage change in output due to allocative inefficiency $(\ln(\xi))$. Therefore, though output increases from the subsidies, and decision-makers may be glad about it, that increase is a result of inefficient input use. Eq. (7) may also be written as $\ln(Q) = \ln(\hat{Q}) + \ln(\xi) + \varepsilon$, making its form similar to what Kumbhakar (1997) derived for translog cost functions, except in this paper we are dealing with production instead of cost. Similar statements can be made about Eq. (5).

The decomposition, therefore, fills the void in the transit economics literature mentioned earlier in terms of lack of focus on the effect of subsidies on output and links the results to the well-known Kumbahkar equation. Notice that though output increases or decreases from the subsidies cost does not behave likewise and the change in cost is $-\ln\left(\sum_i S_i\left(1-\mu_{oi}H_{oi}-\mu_{Ki}H_{Ki}\right)\right)$. Since the impacts of subsidies on cost and output can be both determined from Eq. (5), it is advantageous to estimate indirect production functions instead of cost functions when studying the impacts of operating and capital subsidies.

III. The Empirical Model

To apply the decomposition cross-sectional data are used. This requires that Eq. (4) is modified to account for heterogeneity. One approach to do so is to estimate latent class models. However, because Eq. (2) is inherently nonlinear this approach is not suitable. A second is to cluster transit systems by some measures of their characteristics and estimate a different equation for each cluster as Karlaftis and McCarthy (2002) did in their work. A third is to add variables reflecting the characteristics of the observations (i.e., transit systems) to capture heterogeneity. Using the third approach, many previous studies examine the relationships between organizational and environmental characteristics and public transit performance. Among them, Pina and Torres (2001) consider a city's industrial characteristics, geographical extent, population density, income per capita and the age of the population as exogenous variables in their study of transit performance. They find that the population of an urban area, environmental variables and type of management do not have significant impacts on efficiency. Obeng (1987) provides a framework for classifying bus transit policy variables based upon their effects on input productivity and total performance defined as cost. The variables he finds affect cost, partial measures of productivity and the measure of economies of scale are average vehicle speed, the ratio of employer to employee paid benefits, subsidies, capacity utilization, route miles, the peak-base ratio, average fleet age, number of modes operated, and the ratio of supervisors, professionals and executives to total employment.

Guiliano (1980) studied the effects of environmental variables on public transit efficiency and identified market conditions (e.g., hours of service availability and the peak-base ratio), system size (e.g., service area), age of the firm and unionization as affecting efficiency. Kerstens (1996) surveyed the literature on the variables affecting public transit performance and classified the variables into competition (e.g., the extent of privatization or contracting), organizational differences (e.g., ownership) and operating environment (e.g., network length, number of lines, peak-base ratio, number of stops, vehicle speed), vehicle age, and method of financing (e.g., subsidies).

Similar variables as those listed above are used in this paper to account for heterogeneity. In particular the specified indirect production function includes population density (D), average vehicle speed (V), average vehicle age (z), and network size in terms of route miles (N). Thus, the empirical cost function to be estimated is,

$$\ln Q = \beta_0 + \sum_i \beta_i \ln\left(\frac{C.}{w_i} U_i\right) + 0.5 \sum_i \sum_j \beta_{ij} \ln\left(\frac{C}{w_i} U_i\right) \ln\left(\frac{C}{w_j} U_j\right) + \sum_m \eta_m \ln(N)$$
$$+ \eta_Z \ln(z) + \eta_V \ln(V) + \eta_D \ln(D) + \varepsilon \qquad (8)$$

Imposing the symmetry constraints $\beta_{ij} = \beta_{ji}$ and employing Roy's (1943) identity, the observed share (S_i) of an input in cost is,

$$S_i = \frac{(-\partial \ln Q / \partial \ln w_i)}{(\partial \ln Q / \partial \ln C)} = \frac{\left(\beta_i + \sum_j \beta_{ij} \left(\ln(C/w_j) + \ln(U_i)\right)\right)}{\left(\sum_i \beta_i + \sum_i \sum_j \beta_{ij} \left\{\ln\left(C/w_j\right) + \ln\left(U_j\right)\right\}\right)} \qquad (9)$$

Both Eq. (8) and $i-1$ of the share equations from (9) form a system to be estimated jointly. For unique identification of their parameters, we follow Gajanan and Ramaiah (1996), Hilmer and Holt (2005) and the discussion in the decomposition section of this paper and impose the following restrictions on the coefficients.

$$\sum_i \beta_i = 1, \; \sum_i \mu_{oi} = 1, \quad \sum_i \mu_{Ki} = 1, \quad \sum_i \sum_j \beta_{ij} = 0, \; \beta_{ij} = \beta_{ji} \; \textit{for all i and j}. \qquad (10)$$

Additionally, to improve convergence these equations are estimated jointly with the subsidy functions below.

$$\left.\begin{aligned}
&A_o = \phi_0 + \sum_i \mu_{oi} x_i + \phi_n \ln(N) + \phi_D \ln(D) + \phi_{UAF}(Y_{UAF}) \\
&A_K = \upsilon_0 + \sum_i \mu_{ki} x_i + \upsilon_Z \ln(z) + \upsilon_{CAP}(Y_{CAP}) + \upsilon_{GEN}(Y_{GEN}) + \upsilon_{PM} \ln(Y_{PM}) \\
&where\ \ x = L, K, F,
\end{aligned}\right\} \quad (11)$$

Where ϕ and υ are parameters to be estimated, Y_{UAF} and Y_{CAP} are respectively binary variables showing receipt of funds from the federal urban area formula grant and capital subsidy programs. Y_{GEN} is a binary variable showing receipt of funds from state and local general revenues, Y_{PM} is passenger miles and all other variables are defined already. The signs of the coefficients of all the variables are expected to be positive.

IV. Data

The data used pertain to the bus transit systems included in the 2006 U.S. National Transportation Statistics (NTS) database. Initially all such transit systems reporting their data were included in the sample providing 100% enumeration.[4] Later, observations missing relevant data on operating subsidies, labor hours, and gallons of fuel, vehicle miles and route miles were deleted. Similarly, transit systems whose data on key variables (e.g., ratio of operating subsidies to capital cost, ratio of capital subsidy to labor cost) were judged unreasonable or whose data were listed as questionable in the NTS database were deleted.[5] These deletions left 227 observations to be used in this study.

The data for these observations include operating cost, total annual vehicle miles of service, total annual hours worked by labor, gallons of fuel which are used as a proxy for all non-labor and non-capital inputs, fare revenue, total capital subsidies, total operating subsidies, fleet age, fleet size, transit background data, and the shares of labor and fuel in total operating cost. Other variables are labor cost calculated as the sum of wages, salaries and fringe benefits, fuel cost which is total operating cost less labor cost, population density, service area and capital user costs. Capital cost is calculated as $rK = Kw_K(R+d)e^{-d(z)}$ where K is fleet size, w_K is the weighted average price of a new public transit bus in 2006. This price was calculated from awarded bus purchase contracts reported in various issues of METRO magazine by dividing the contract amount by the number of vehicles bought.[6] R is the average prime rate for 2006, d is a straight line rate of depreciation assuming a bus useful life of 20 years and r is bus user cost. Total capital cost calculated as above was added to total operating cost to obtain total cost and the shares of labor, fuel and capital in total cost calculated as each input's cost divided by total cost. After that, the cost of purchased transportation was allocated to the inputs according to their shares in cost and this cost was also added to total cost. Finally, input shares were recalculated and input prices calculated as input cost divided by input quantity. Table 1 shows descriptive statistics for the transit systems used.

V. Results

A. *Estimation*

Equations (8), (9) and (11) are estimated jointly by iterative nonlinear seemingly unrelated equations methods using the Marquardt optimization technique after imposing the restrictions and the non-negativity constraint, $\beta_i > 0$ on the coefficients. The choice of this method is because upon convergence it gives similar estimates as do maximum likelihood methods. Table 2 shows the results of the estimation. Convergence was achieved in 35 iterations and at that point 149 observations were used and 78 rejected.[7] For those rejected their implied input prices were negative and the model did not fit their data well. From the adjusted coefficients of determination the indirect production function explains 78.32% of the variation in output while 58.68% and 66.07% of the variation in capital subsidy and operating subsidy respectively are explained by their equations. Additionally, the equations explain 67.73% and 47.25% of the variation in the labor and fuel shares respectively. Most of the estimated coefficients are highly significant statistically and the test statistics for all the restrictions, except one, are statistically significant showing they are valid.

Examining the signs of the coefficients, those of the subsidy equations are consistent with prior expectation. Surprisingly, route mile does not have a statistically significant coefficient in the operating subsidy equation. In comparison, receipt of funds from the federal urban area formula grant has a positive and statistically weak coefficient. The coefficient of passenger miles in the capital subsidy equation is statistically significant as are the coefficients of receipt of funds from capital subsidy programs and local and state general revenues. And, contrary to our expectation, the coefficient of fleet age is negative and statistically significant in the capital subsidy equation showing that transit systems that keep their buses longer generally receive less capital subsidies.

Regarding the estimated coefficients of the indirect production function, the signs of average bus speed and population density in it are positive and statistically significant while the coefficient of average fleet age and route miles are non-significant. These results show that transit systems that maintain relatively high average speed and operate in high population density areas produce large outputs. Also, all the coefficients of the ratios of subsidies to input costs are positive and statistically significant. Using these results, on the average, a transit system's share in total cost that it must pay with its non-subsidy funds is 35.32% leaving 64.68% to be accounted for by subsidies.

Similarly, using the results and the equation below, there are diseconomies of scale in the transit systems studied. These diseconomies show that a percentage increase

$$\partial \ln C / \partial \ln Q = 1 / \left(\sum_i \beta_i + \sum_i \sum_j \beta_{ij} \left\{ \ln(C / w_i) + \ln(U_i) \right\} \right) \quad (12)$$

in output increases cost by 0.9720 with a range of 0.8002 to 1.045. The average value of the diseconomies of scale, however, is not statistically different from the value of one for constant returns to scale $(t = -0.6829)$. Therefore, we surmise that there are near constant returns to scale in the transit systems studied.

B. *Effects of both Operating and Capital Subsidies on Output*

Based upon the second, third and fourth terms of Eq. (5) we calculate the effects of the subsidies on output first by considering both subsidies together and then each subsidy separately. A particularly noteworthy point in this calculation is that the third term of this equation which is $\left(\sum \beta_i\right)\left(\ln\left[\sum_i S_i\left(1-\mu_{oi}H_{oi}-\mu_{Ki}H_{Ki}\right)\right]\right)$ reduces into $\ln\sum_i S_i\left(1-\mu_{oi}H_{oi}-\mu_{Ki}H_{Ki}\right)$ after imposing the constraint $\sum_i \beta_i = 1$. In Table 3 when we consider a transit system that receives both subsidies, the lump-sum effects of these subsidies are positive and increase output by 132.51% per transit system. This shows that the subsidies by reducing implied input prices increase the quantities of inputs demanded and make transit systems more than double their outputs. The size of this increase is affected by the effects of the cost impacts of the subsidies which reduce output by 55.08% per transit system when both subsidies are received. The fourth row of Table 3 shows that allocative inefficiencies from the subsidies add 4.20% on the average to the output reduction. Combining these results, the subsidies increase output by 73.23% per transit system with a range of 0.36% to 220.31%. This range shows that while in some transit systems the effects of the subsidies on output are quite small in others they are quite large.

Overall, the results suggest that the subsidies increase output and that it is inappropriate to assume that the outputs in these transit systems remain constant when capital and operating subsidies are offered as cost studies assume. For the transit systems studied, a proper accounting of the effects of operating and capital subsidies would be obtained by estimating indirect production functions as in this study. Though there are potentials for output to change by a large proportion in all the transit systems studied, inefficiencies in input overuse reduce that change.

C. *Effects of Either Subsidy on Output*

Surprisingly enough, when the individual impacts of both subsidies are considered and compared, the table shows that the total effects of operating subsidies on output are far larger than the total effects of capital subsidies on output. When only operating subsidies are considered the results show that on the average the lump-sum effects of operating subsidies result in 101.96% increase in output while the cost impacts and allocative inefficiencies reduce output by 43.41% and 2.64% respectively resulting in a 55.91% increase in output overall. Comparatively, when only capital subsidy is considered the lump-sum effects of capital subsidies increase output by 17.74% while the cost impacts and allocative inefficiencies reduce output by 7.78% and 0.43% respectively. The net result is a 9.53% output gain from capital subsidies. Thus, while both types of subsidies are important, operating subsidies have larger impacts on output than capital subsidies do, at least in the transit systems studied. In fact, in this study capital subsidies have little effect on output increasing, suggesting that they support current services.

This finding may be because capital subsidies are mainly for equipment replacement and do not add to output but maintains it. To operate bus services more intensively to increase output requires the same capital but more labor and fuel whose costs are partially supported by operating subsidies. It could also be that operating subsidies cover those costs that heavily influence short run production decisions. For example, decisions to purchase or replace capital are made quite infrequently and involve large expenditures which increase the scale of transit operations. Once such decisions are made what influences how much service to produce is a transit system's ability to cover its short run costs, and this makes it important to have operating subsidies.

D. *Sources of Inefficiencies*

Eq. (7) shows that all the increases in output can be considered as due to allocative inefficiency because the subsidies affect the optimal rate of input substitution. That interpretation shows that operating subsidies cause more allocative inefficiencies than do capital subsidies. Given this result it is important to examine the sources of the allocative inefficiencies. To do so labor-capital, capital-fuel and labor-fuel allocative distortions from the subsidies are calculated using the equation $\left(1-\mu_{oi}H_{oi}-\mu_{Ki}H_{Ki}\right)/\left(1-\mu_{oj}H_{oj}-\mu_{Ki}H_{Kj}\right)$ and the results are also shown in Table 3. Focusing only on the results when both subsidies are received, the value of the labor-capital allocative distortion is 1.7333 and it shows distortions in the optimal rate of substitution between these inputs. Since this distortion is more than one it shows that the subsidies have made capital relatively cheap leading to its overuse relative to labor. For capital-fuel allocative distortion its value of 2.6354 shows that because of subsidies fuel is more than 2.64 times overused relative to capital. That is, they have reduced the cost of fuel so much that transit management misperceives it as relatively cheap compared to other inputs therefore, leading to its substantial overuse relative to capital. This overuse could take the form of buying and running less fuel efficient vehicles, routing services through congested routes, excessive idling of vehicles, extended service, improper vehicle maintenance, and possibly wrong engine choices during the bus purchase decision-making process. Finally, the labor-fuel allocative distortion is 1.5821 showing that fuel is also overused relative to labor. This could take the form of operating larger and longer buses that increase fuel use and reduce the number of drivers per shift. From these results, input overuse from the subsidies has led to overproduction of transit services, particularly overuse of capital and fuel relative to labor.

VI. Conclusion

This paper’s purpose is to fill a gap in the public transit economics literature by estimating the effects of subsidies on output, recognizing that previous studies fail to do so. Its main contributions are as follows. Using an indirect production function, it shows that the effects of capital and operating subsidies on public transit output can be decomposed into the lump-sum, cost and allocative inefficiency effects. This decomposition is unique to this paper and, as has been shown, it is the production counterpart of what Kumbhakar (1997) derives in his work on translog cost functions. Further, the paper estimates an indirect production function and finds that the positive lump-sum effects of the subsidies on output are larger than the negative effects of the cost impacts and allocative inefficiencies from the subsidies. This implies that the subsidies together increase output beyond maintaining existing output. Overall, this increase in output is 73.23% on the average mainly due to the effects of operating subsidies. Given these findings, future work must incorporate them to provide a better understanding of the impacts of subsidies on cost. This is because previous studies, particularly those based upon cost functions, assume output remains unchanged and that changes in cost occur only through allocative inefficiencies or the substitution effects of the subsidies. As the results show, that assumption is questionable and may apply to some but not those transit systems studied in this paper. Focusing only on the substitution effect would be clearly misleading. This is because as shown in this paper, the lump-sum effects of the subsidies on output are very large and exceed the combined impacts of the cost and allocative inefficiencies on output and could lead to lower average cost.

END NOTES

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[1] These authors showed that this relationship is exact if the production function underlying the cost function is Cobb-Douglas. In another context, Kumbhakar (1997) generalized the relationship between implied and actual cost to situations where the cost function is translog.

[2] These implied prices can be obtained by maximizing output subject to a net cost constraint where net cost is total cost less the amounts of operating subsidies and capital subsidies expended. These subsidies are functions of all inputs. They can also be obtained by minimizing net cost subject to a production function constraint.

[3] If the implied cost function is Cobb-Douglas of the form $C^* = (1/\eta_0)^\theta w_L^{*\beta_L\theta} w_K^{*\beta_K\theta} w_F^{*\beta_F\theta} Q^\theta$ which is homogeneous of degree one in input prices implying that $\theta(\beta_L + \beta_K + \beta_F) = 1$ then the indirect production function is, $Q = \eta_0 \left(C^* / w_L^*\right)^{\beta_L} \left(C^* / w_K^*\right)^{\beta_K} \left(C^* / w_F^*\right)^{\beta_F}$.

[4] Notice that these are the transit systems submitting their annual data to the Federal Transit Administration and that not all transit systems do so. Therefore, they do not represent all the transit systems in the U.S.

[5] Some of the ratios of operating subsidies to capital cost were 100 or higher, and the ratios of capital subsidies to labor cost were in some case greater than 50.

[6] This is comparable to the average 2007 and 2008 new bus price of $424,880 reported by APTA (2008).

[7] This result was obtained after many trials using different starting values. In all cases the values of the coefficients at convergence were very close suggesting a global convergence point had been reached.

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Table 1: Descriptive Statistics

Variable	N	Mean	Std. Dev.	Minimum	Maximum
Total Cost ($)	227	18,946,934.26	29,553,144.59	983,648.73	276,868,066.00
Passenger miles		16,111,996.22	30,280,491.94	8,025.00	289,297,904
Vehicle miles	227	2,959,289	4,348,777	25,950	39,504,428
Labor wage ($)	227	18.21	58.77	6.12	894.96
Capital user cost per vehicle ($)	227	44,422.73	6,075.01	3,636.42	56,646.74
Fuel price per gallon ($)	227	8.45	9.64	3.38	99.87
Labor hours	227	6,20,217.04	843,732.79	9,152.00	6,810,714.00
Fleet size	227	93.9736	119.0029	7.0000	905.0000
Gallons of fuel	227	583,620.75	1,004,167.93	21,609.00	10,091,084.00
Capital subsidy ($)	227	2,608,230.23	4,156,500.35	1,239.00	30,114,012.00
Operating subsidy ($)	227	11,211,794.83	17,813,901.89	6,017.00	154,588,939.00
Received funds from capital program (yes =1, No = 0)	227	0.4846	0.5009	0.0000	1.0000
Received funds from local dedicated subsidy sources (Yes = 1, No = 0)	227	0.3700	0.4839	0.0000	1.0000
Received subsidy from state dedicated subsidy sources (Yes = 1, No = 0)	227	0.4405	0.4975	0.0000	1.0000
Received funds from federal urban area formula funds (Yes =1, No = 0)	227	0.8899	0.3137	0.0000	1.0000
Received funds from local and state general revenues (Yes = 1, No = 0)	227	0.3524	0.4788	0.0000	1.0000
Route miles	227	328.61	349.16	7.00	2,674.76
Average fleet age	227	5.50	4.23	0.33	55.25
Service area (square miles)	227	293.83	510.00	14.00	3,353.00
Population density	227	2,327	1,182	1,055	7,068
Average vehicle speed (mph)	227	14.37	3.55	9.24	47.73

1. Fuel is a proxy for all non-labor and non-capital inputs. Therefore, its costs include the costs of materials, tires and all types of liquid fuels, and a portion of the cost of purchased service.

Table 2: Nonlinear Iterative Seemingly Unrelated Regression Estimation Results

Parameter	Estimate	Std. Error	t-value	Probability
Shared Variables				
Share of operating subsidy in labor cost (H_{oL})	0.5439	0.0036	152.6200	<.0001
Share of operating subsidy in fuel cost (H_{oF})	0.2406	0.0026	91.1700	<.0001
Share of capital subsidy in capital cost (H_{KK})	0.4326	0.0133	32.5800	<.0001
Share of operating subsidy in capital cost (H_{oK})	0.2155	0.0032	67.6600	<.0001
Share of capital subsidy in labor cost (H_{KL})	0.4598	0.0116	39.5100	<.0001
Share of capital subsidy in fuel cost (H_{KF})	0.1076	0.0057	18.9200	<.0001
Operating Subsidy Equation				
Constant term	-1.9667	0.1519	-12.9400	<.0001
Population density (logarithm)	0.3370	0.2012	1.6700	0.0961
Route miles (logarithm)	0.0803	0.0598	1.3400	0.1816
Allocation from urban area formula grant	0.2862	0.1589	1.8000	0.0738
Capital Subsidy Equation				
Constant term	-1.5141	0.1307	-11.5800	<.0001
Passenger miles (logarithm)	0.1469	0.0661	2.2200	0.0279
Average fleet age (logarithm)	-0.3090	0.1518	-2.0400	0.0436
Allocation from federal capital program (Yes =1, No =0)	0.4908	0.1698	2.8900	0.0044
Funds allocated out of general revenue (Yes = 1, No = 0)	0.0012	0.0004	3.4200	0.0008
Indirect Production Function Equation				
Constant term	-0.1674	0.0399	-4.2000	<.0001
$\log(w_L(U_L)/C)$	0.5041	0.0047	108.0900	<.0001
$\log(w_F(U_F)/C)$	0.2274	0.0038	59.3000	<.0001
$\log(w_K(U_K)/C)$	0.2686	0.0063	42.9300	<.0001
$0.5\log(w_L(U_L)/C)\ \log(w_L(U_L)/C)$ h11	-0.0473	0.0027	-17.7800	<.0001
$\log(w_L(U_L)/C)\ \log(w_F(U_F)/C)$ h12	0.0219	0.0026	8.4000	<.0001
$\log(w_L(U_L)/C)\ \log(w_K(U_K)/C)$ h13	0.0254	0.0031	8.3400	<.0001
$0.5\log(w_F(U_F)/C)\ \log(w_F(U_F)/C)$ h22	-0.0389	0.0031	-12.5000	<.0001
$\log(w_F(U_F)/C)\ \log(w_K(U_K)/C)$ h23	0.0170	0.0033	5.2100	<.0001
$0.5\log(w_K(U_K)/C)\ \log(w_K(U_K)/C)$ h33	-0.0424	0.0050	-8.4300	<.0001
Population density (logarithm)	0.2587	0.1427	1.8100	0.0719
Average fleet age (logarithm)	-0.0183	0.0657	-0.2800	0.7810
Average speed (logarithm)	0.7125	0.1867	3.8200	0.0002
Route miles (logarithm)	0.0354	0.0428	0.8300	0.4096
Tests of Restrictions				
Operating subsidy: $\sum_i \mu_{oi} = 1$	1501.3110	156.4000	9.6000	<.0001
Capital subsidy: $\sum_i \mu_{Ki} = 1$	359.6371	49.3003	7.2900	<.0001
Tests of Linear Homogeneity Restrictions				
$\sum_i \beta_i = 1$	30.2528	18.0017	1.6800	0.0929
$\beta_{11} + \beta_{12} + \beta_{13} = 0$	-335.4450	141.4000	-2.3700	0.0172
$\beta_{12} + \beta_{22} + \beta_{23} = 0$	486.5259	236.9000	2.0500	0.0395
$\beta_{13} + \beta_{23} + \beta_{33} = 0$	116.5248	113.0000	1.0300	0.3041

Table 3: Output Effects of Operating and Capital Subsidies

Effects of both Subsidies	**Total systems**	**Mean**	**Standard Deviation**	**Min.**	**Max.**
Implied system share in cost	149	0.3610	0.1420	0.0893	0.9924
Total effect of subsidies on output	149	0.7323	0.3029	0.0036	2.2031
Effect of cost impact of the subsidies	149	-0.5508	0.1987	-1.2178	-0.0039
Allocation inefficiency	149	-0.0420	0.0667	-0.3423	0.0546
Lump-sum effects of subsidies on output	149	1.3251	0.5018	0.0075	3.5202
Labor-capital allocative distortion	149	1.7333	5.4501	0.0396	58.7631
Labor-fuel allocative distortion	149	1.5821	2.6823	0.0530	23.1287
Capital-fuel allocative distortion	149	2.6354	5.8192	0.0071	53.0131
Effect of Operating Subsidies					
Total effect of subsidies on output	211	0.5591	0.1887	0.0003	1.2931
Effect of cost impacts of the subsidies	211	-0.4341	0.1259	-0.7327	-0.0003
Allocative inefficiency	211	-0.0264	0.0582	-0.5500	0.1119
Lump-sum effects of subsidies on output	211	1.0196	0.3186	0.0005	2.4011
Labor-capital allocative distortion	211	2.4640	15.9231	0.1037	223.7523
Labor-fuel allocative distortion	211	1.4107	2.1134	0.0751	27.2745
Capital-fuel allocative distortion	211	1.7638	2.0328	0.0062	22.4986
Effect of only Capital Subsidies					
Total effect of subsidies on output	221	0.0953	0.1059	0.0002	0.6488
Effect of cost impacts of he subsidies	221	-0.0778	0.0816	-0.4911	-0.0002
Allocative inefficiency	221	-0.0043	0.0124	-0.1311	0.0174
Lump-sum effects of subsidies on output	221	0.1774	0.1931	0.0004	1.0818
Labor-capital allocative distortion	221	1.2694	0.8600	0.6049	11.9672
Labor-fuel allocative distortion	221	0.9316	0.0897	0.4314	1.0157
Capital-fuel allocative distortion	221	0.8209	0.1921	0.0651	1.0317