Title: **MEASURING AIRCRAFT-TAXIING DELAY IN THE ASPM**

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Abstract.

Minimizing the time needed for aircraft to taxi from gates to runways is an important part of efforts to manage the national airways system. For that reason, the Federal Aviation Administration (FAA) currently collects detailed information on all ground traffic in order to help estimate delays in taxiing out to runways. The agency bases its definition of taxi-out delay on the difference between actual taxi-out times and a constructed value referred to as "unimpeded taxi-out time." Currently, unimpeded times are estimated applying a statistical relationship between historical taxi-out times and the levels of outgoing and incoming traffic under the further assumption of "ideal" ground conditions: that is, (1) the weather is optimal, and (2) just one flight is leaving and no flights are arriving.

In a completely different way, using a novel approach that groups flights by airport, airline, season, day, and hour in order to control for variations in taxiing conditions, the paper would use FAA information on the observed minimum of actual taxi-out times to identify a new "time reference" estimate of the unimpeded taxi-out times. The paper describes criteria for identifying and discarding data errors that hamper the current FAA data. It presents an example of the estimation for one time reference grouping. Importantly, the paper also describes how to provide a measure of the uncertainty in that deterministic estimate of the unimpeded time by hypothesizing and testing a probability distribution for those times and presenting examples of how those probability distributions can be used to make inferences.

Introduction.

As a part of its mission to maintain a safe and efficient airspace system, the Federal Aviation Administration (FAA) closely monitors delays in aircraft movements on the ground. The reasons are many. Such delays can disrupt airline schedules locally and nationwide, add to costs both for airlines (for fuel, aircraft maintenance, and crews) and for the flying public, and add to environmental emissions. The two basic parts of on-ground delay, from arrival to departure, are the taxi-in delay and the taxi-out delay. The FAA collects data related to each in a data base known as the Aviation System Performance Metrics System or for short ASPM. With permission from the FAA, online access to the ASPM is available at aspm.faa.gov/aspm/entryASPM.asp .

In addition to basic information related to delays, the ASPM retains details of current and historical information on air traffic operations, airline schedules, runway conditions, and weather. (A third source of delay for arriving aircraft, time waiting for a gate assignment, is not distinguished in the ASPM records from total taxi-in time[1]). Despite the efforts of the Aviation Industry to minimize ground delays, such delays are becoming more problematic. FAA research over a decade ago pointed to a strong trend of increasing delays[2]. And recently, incidents involving passengers held on the ground for many hours in part as a result of congestion at airport gates or of pilots unwilling to return to the gate and lose their position in the take-off queue, have received national attention. FAA data on taxi-out delays are critical to understanding the scope and causes of that congestion. But over time, questions have arisen as to whether the FAA's methods for estimating delays should be revised, especially in light of much-improved methods of analysis and much higher-powered computing and simulation capabilities.

To anticipate such a move, the paper proposes an alternative approach. The next section will introduce the FAA data specifically related to delays and very succinctly the current method for estimating delays. The following section illustrates one way to improve the current point estimates of delay, short of gathering improved data on ideal ground conditions that underlie any calculation of delays. In its simplest form that approach presents a "deterministic" estimate of the unimpeded taxi-out times based on taxi-out times that controls for important time-referenced determinants of delay. The section following the deterministic approach introduces the stochastic approach by constructing a probability distribution of those time reference estimates that can be used to describe the level of uncertainty surrounding the deterministic point estimates. It is shown that the probability distributions for the minimum taxi-out times (the time reference values) can be described as uniform distributions. That is a powerful assumption, since it makes it possible to infer the likelihood of the actual unimpeded time (and by extension the actual taxi-out delay) matching or exceeding the deterministic estimate. A concluding section discusses further research needs related to the issue of reporting taxi-out delay in the ASPM. An ideal measure of delay should be based on actual data, the expected amount of delay reflected in the actual take-off time for every flight (given the uncertainty surrounding estimates of unimpeded times) and some measure of the level of uncertainty around that deterministic estimate.

ASPM data and the current approach to delays.

The FAA records the details of air traffic operations and delays, airline schedules, runway information, and weather in a database known as ASPM. The ASPM includes flight data for all carriers (including international flights) at 77 domestic airports. The data are organized by individual flights. Altogether, the ASPM data for 2007 (the most recent full year available for this study) includes 7,435,031 observations. The data for each flight include the airport, the airline, and the season, as well as other data to estimating delays. Those times and durations are:

- the time the airline pushes back from the gate and the time of take off (wheels off),
- the actual taxi-out time (calculated as the difference between gate-out and wheels off),
- the unimpeded taxi time (estimated from historical taxi-out times under defined assumptions), and
- the taxi-out delay (calculated as the difference between the actual and the unimpeded taxi-out times).

In the ASPM, the reported values for taxi-out delay are currently calculated directly as:

$$\textbf{delay} = \textbf{taxi\_out\_time} - \textbf{unimpeded\_taxi\_out\_time}$$

The same relation will be used in this paper. But to avoid any confusion, the unimpeded taxi-out-time will now be called **time reference** and denoted by time_ref as defined below.

A new concept of unimpeded taxi-out times and delays based on "time reference".

In the new concept, delays are considered in a framework broader than the one used in the current FAA approach. In particular, identifying unimpeded taxi-out times by the actual time of day that flights depart can yield proxies that more closely control for the more predictable delays related to daily weather patterns (like fog), flight crew changes, full flights, etc. Given a set of taxi-out times (grouped within 24 hourly slots throughout the day), selecting the "best" taxiing times for each flight departing within those time increments yields a new measure of unimpeded times that will be called here the "**time reference**." (Or with more mathematical rigor, given a set of actual taxi-out times, the time reference that produces the best, i.e., the smallest, delay is the minimum of that set. It will be referred to as **the chosen time reference** and denoted as **time_ref** (underscores are used in variable names)**.** Combined with real-time data on actual taxiing times, delays now would be calculated as

$$\textbf{delay} = \textbf{taxi\_out\_time} - \textbf{time\_ref}$$

But, in the absence of direct empirical measurements of unimpeded times, what type of measurable information of ground movements can best serve as a proxy for that new concept? The new measure should also conform to the common-sense condition that delays should not be negative–that is, the time reference should never be greater than any actual taxi-out times. This section describes the ideal properties of such a proxy using taxi-out data and some simple rules for omitting observations that are likely in error or that otherwise would undermine those properties.

Desirable Properties of "Time Reference" Data and Generated Delays. The time reference for a particular flight should be defined by several properties considered desirable for generating an estimate of respective flight delay.

- First, the unique time reference has to be one of the taxi times in the identifiable set of data. That is, it should be readily "measurable" to capture as much of the variation in optimal conditions as possible.

- Second, the chosen time reference for a flight should never be greater than the actual taxi-out times observed. That is, the generated taxi-out delays should never be negative.
- Third, because the data chosen to serve as a proxy for unimpeded times will have a minimum, choosing that minimum value as the time reference will guarantee that the time reference is indeed unique and that the generated taxi-out delays so defined will be the smallest among all possible non negative delays. In an absolutely hypothetical case of an airport where all taxi-out times are equal to one another, we will say that taxi-out delay is equal to zero (a closed airport is a case in point). That is, there is no delay.

If from a theoretical point of view choosing that minimum does not raise any special problems, from a practical point of view it does. One obvious set of observations to discard are those with zero taxi-out times. More than two percent of the total 7,435,031 observations in the ASPM 2007 include such zero-time reports. Of course, a zero-time taxi-out is not possible, and thus, not acceptable. Therefore, such observations can easily be discarded as recording mistakes. The question, then, is how to identify a correct minimum time reference, from a set of non-zero taxi-out times. In what follows, we are going to present first a quick and dirty way to discard non-acceptable minima, and then to introduce the definition of the **theoretical absolute minimum** for taxi-out times above which they are deemed correct and reliable and below which they should to be dropped from the database because they most likely represent reporting errors.

A quick way to identify minimum taxi-out times as non-acceptable. Comparing taxi-in and taxi-out times in similar time slots provides a quick way to screen for non-acceptable taxi-out times. The logic is straightforward: Under the same conditions, the time taken to cover the “distance from gate to wheels-off” should approximate the time needed to cover the “distance from wheels-on to gate.” That is, it should be acceptable to use any lower taxi in numbers as good estimates of the minima taxi-out times under similar conditions for individual flights in similar time slots. The key word here is “estimate.”

The first step in illustrating the merits of that screening criteria focuses on the 35 largest commercial airports, which account for more than 75 percent of passengers moving through the nation’s airports. The ASPM refers to those airports as the OEP35, or Operational Evolution Partnership Plan. The average taxi-in and taxi-out times are presented for the OEP35 and, within that group, the three airports in the New York City area for all flights through those airports in 2007 are presented in Table 1.

(The path to Table 1 is as follows:

ASPM/MANAGEMENT REPORTS/TAXI TIMES/YEARLY/AVERAGES/SUMMARY BY PERIOD/FROM2007/TO 2007/CALENDAR YEAR/CARRIER:OEP35,EWR,JFK,LGA/

where the "/" indicates the elements of the path and a "," indicates an option. In the field "CARRIER," use "OEP35", and subsequently replace OEP35 by EWR, or JFK, or LGA.)

Table 1. Taxi Out/In
2007 Average in min

Airport or Group of Airports	Average Taxi Out	Average Taxi In
OEP35	18.15	7.59
EWR	28.62	9.34
JFK	36.18	10.56
LGA	27.03	7.53

The data indicate that average taxi-in times are always smaller than taxi-out times regardless of the airports, airlines, or time periods. The average taxi-out times are more than twice the average of the taxi-in times. Those differences underscore the value of this quick approach for identifying non-acceptable taxi-out times: discard any times that are below the lowest taxi-in time in a particular slot. Such times should be considered as recording mistake and dropped from the database.

An "Acceptable" Way to Identify Non-Acceptable Taxi-Out Times. Of course, the best way to screen non-acceptable times would be to know the actual distances from gates to runways and the allowable speeds. No flights should be able to travel those distances faster than the airports would allow. Usually for airports under consideration, there are two defined ground speed limits, one for aircraft and one for all other vehicles. Under those circumstances, **we take as a rule there are no taxi-out times smaller than the (physical) time to go from gate to the take-off point on the runway at the aircraft ground speed limit for that airport.** That (physical) time is the **theoretical absolute minimum.** That time also will be the **theoretical reference time**. Although the allowable ground speeds and the ground distances for individual flights are not known for this study, the correct approach for identifying non-acceptable taxi out times would be to discard any reported actual taxi-out times smaller than the corresponding theoretical reference time. Such times should be considered as recording mistake and dropped from the database.

For any flights for which the taxi-out time is discarded as non-acceptable, whether by the quick approach or the correct approach, it will not be possible to calculate a taxi-out delay.

A Deterministic Approach to Time Reference. To get the current ASPM estimates of unimpeded taxi-out times, the FAA relies on a statistical regression of taxi-out times against outgoing and incoming traffic, with data grouped by airport, airline and season, and with extreme values tossed out of relationship. The new approach described here is structured to select measurable data on taxi-out times from the ASPM to estimate the so-called **time_ref** by airport, airline, and season, as well as by the time of the day, day of week, and month. The two approaches are not directly comparable. However, to just guess how the two systems might differ, a limited example is presented here based on 2007 data as shown in Table 2. The path to Table 2 is as follows:

ASPM/MANAGEMENT REPORTS/TAXI TIMES/UNIMPEDED/CARRIER: AAL/2007.

The comparison suggests that the new times will be much lower than the current times, with the result that the estimates of taxi-out delays generated with the new times will be greater than is now the case. In particular, the example focuses on just one airport (JFK) and one airline (American, or AA), with new time-reference numbers calculated for 3 times of the day (7:00, 12:00, 17:00) and seven days of the week. Monthly estimates are not included. In this example, flights with zero taxi-out times are discarded. That is, no time reference numbers or estimates of delay are reported. To keep things simple no further criteria for discarding low times are included. The following conventions apply to notation:

- Airports: 3-character-notation (such as JFK for John F. Kennedy),
- Airlines: 3- or 2-character names (such as AAL or AA for American Airlines),
- Season: 1 for Winter (December to February), 2 for Spring (March to May), 3 for Summer (June to August), and 4 for Fall (September to November),
- Day: **day_ind**: day of the week, starting with 1 for Sunday,
- Hour: **cat** is the hourly increment in local time, numbered from 0 to 23 (for midnight to 1am).

Given the airport, airline, and season, the current ASPM unimpeded taxi-out is located in the ASPM column in Table 2. With the proposed methodology, the same number should now be computed as the average of numbers coming from every hour, every day within the season. It is located in the last row of the table in the time_ref column. Two results of that comparison are particularly noteworthy.

- First, the time reference estimates for "unimpeded" taxi-out time are consistently lower than the current ASPM estimates of the unimpeded taxi-out times. For 54 out of the 63 time slots considered here, the time reference value is below the reported ASPM estimate of unimpeded taxi-out time. **In general, the time reference estimates are consistently 4 to 5 minutes less than the reported ASPM unimpeded taxi-out times**. Delays based on those new values would be accordingly higher than those delays now reported.

- And second, the current FAA approach of effectively assigning a single value over an entire season misses important daily and hourly variations in unimpeded times that can be seen with the new approach. With the current approach, for the same airport, airline, and season, the ASPM now records just a single value for all months, weeks, and hours of the day. By design, the time reference approach captures those variations.

Stochastic variations in taxiing times.

The time reference approach to approximating the unimpeded taxi out times has the advantage of relying on data that most closely matching those flight times to many of the key factors that reflect their individual "best" conditions for taxiing. But other, unobserved variables certainly exist as well, so some uncertainty will always surround any deterministic estimate of the unimpeded time and, hence, the delay. For that reason, it may be important that future enhancements to ASPM data include not only the time referenced estimates of unimpeded times, but also some information that sheds light on the uncertainty surrounding those deterministic values. It is especially important to know how much uncertainty surrounds the delay estimates for specific flights and what factors may contribute to it.

Table 2. The ASPM Unimpeded taxi out compared with the time reference (time_ref) of the new system for JFK/AAL/Four seasons/Three pre-chosen times:07:00, 12:00 and 17:00.

dep	oag carr	ASPM	sea son	local hour	day of week	time _ref	ASPM	sea son	local hour	day of week	time _ref	ASPM	sea son	local hour	day of week	time _ref	ASPM	sea son	local hour	day of week	time _ref
JFK	AA	**20.1**	1	7	1	17	**20.1**	2	7	1	13	**20.7**	3	7	1	16	**19.9**	4	7	1	14
JFK	AA				2	14				2	13				2	15				2	14
JFK	AA				3	17				3	13				3	12				3	15
JFK	AA				4	14				4	16				4	15				4	16
JFK	AA				5	14				5	14				5	16				5	13
JFK	AA				6	18				6	15				6	12				6	14
JFK	AA				7	14				7	15				7	13				7	13
JFK	AA			12	1	11			12	1	19			12	1	13			12	1	13
JFK	AA				2	15				2	15				2	15				2	12
JFK	AA				3	19				3	13				3	21				3	14
JFK	AA				4	15				4	11				4	17				4	12
JFK	AA				5	15				5	13				5	19				5	17
JFK	AA				6	14				6	16				6	22				6	15
JFK	AA				7	17				7	15				7	13				7	14
JFK	AA			17	1	14			17	1	16			17	1	23			17	1	16
JFK	AA				2	13				2	19				2	14				2	16
JFK	AA				3	15				3	21				3	13				3	21
JFK	AA				4	15				4	15				4	18				4	18
JFK	AA				5	21				5	16				5	15				5	26
JFK	AA				6	17				6	14				6	19				6	21
JFK	AA				7	18				7	19				7	18				7	24
						15.57					**15.29**					**16.14**					**16.10**

It is important to insist that, to our knowledge, so far no distribution has been found for the time-reference. **It is not based on assumptions. It is based on a novel, but simple idea** that scheduled flights –to be defined in the example below- present some kind of regularity that confers to the minimum of their taxi-out times a uniform distribution.

This section has three parts. In the first, data and assumptions are presented to show the derivation of a distribution of time references for a specific example using taxi-out times for JFK airport, American Airlines, winter 2007, Mondays, 7:00am. In the second sub-section, statistical tests from the resulting distribution of times are presented to support the hypothesis of a uniform probability distribution for those time reference values. Third, the paper demonstrates, using a simple example, how a uniform probability distribution function for the time reference values can be used to make inferences.

Data and Assumptions to Derive Distributions of Taxiing Times. The interest here is in the distribution for the minimum taxi-out times (or time references). Data are available to define that distribution for each grouping of airports, airlines, seasons, days, and hours of departure. But for simplicity, the paper narrowly focuses on just one airport (JFK), one airline (American Airlines), one season (January to March), one day of the week (Monday) and one time slot (7am). As a result, each data point represents the minimum taxi-out time from all the 7am American Airline flights from JFK on each Monday in that three-month period. Two important points should be noted:

- That data selection ignores further variations attributable to the month by grouping all data for the three months in that season.
- Additional examples for other time slots and other airports are presented in Appendix B.

Altogether, there are 13 Mondays in that period. Accordingly, the actual time-out values for all the JKF/AA flights for each of those thirteen Monday morning slots can be organized into thirteen groups of data. If there is just one flight in that time slot, that time will be defined to be the time reference value (**time_ref**). If there are more than one flight within a given hour, then the minimum of all the associated taxi out times within that hour will be extracted to serve as the time_ref for that hour. That time_ref for each day is then ranked along with the number of occurrences for each of its values. Data in Table 3 feed the two graphs shown below and support the investigation of the nature of the probability distribution of time_ref in the following section.

Table A in the Appendix A is complete to allow the reader to follow tables and graphs presented in this section. It shows all the taxi out times for all the 7:00 (cat=7) Sunday flights (day_ind=1) during the first three months of 2007 (either mm in yyyymmdd from 1 to 3 or moindex also from 1 to3) from John F. Kennedy (JFK) airport. When grouped by day (that is, yyyymmdd), there are 13 groups. Each of those 13 groups of data are screened to extract the minimum of the taxi out times. Thirteen minima altogether define the two empirical probability distributions—the probability distribution function or pdf and the cumulative distribution function or cdf—of time_ref. Time_ref is now ranked along with the number of observations for each of its values.

Table 3. **The Time_Ref: PDF & CDF**

JFK/Day=1/Cat=7/M=1 to 3

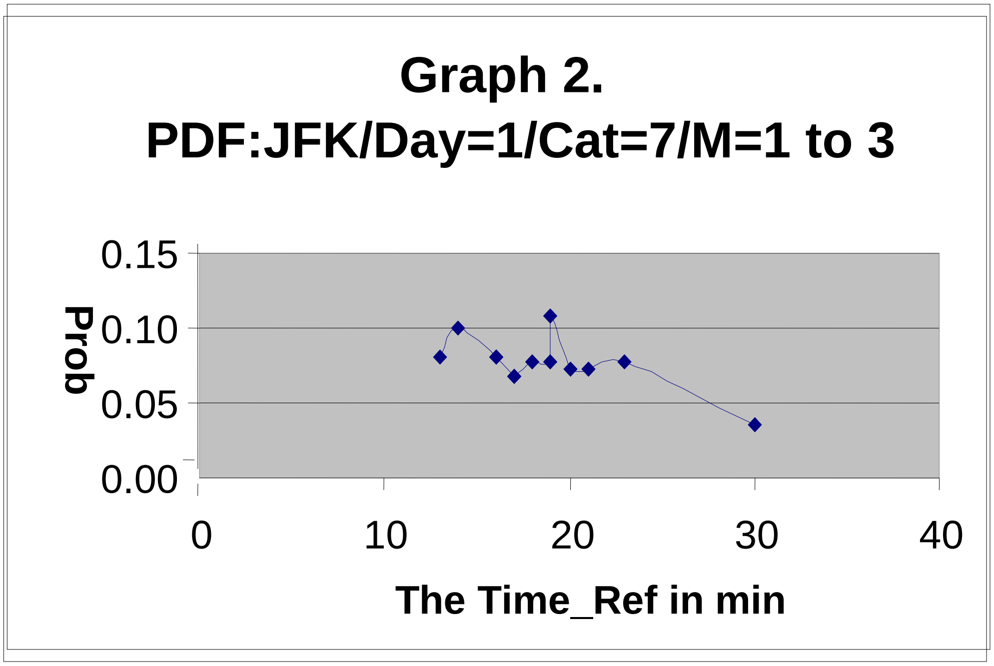
day_ind	Cat	yyyymmdd	#obs	time_ref	Cum	cum_pct	Predicted	Residuals
1	7	20070305	18	13	0.08145	0.08145	0.21374	-0.13230
1	7	20070326	22	14	0.09955	0.18100	0.19417	-0.09462
1	7	20070122	18	16	0.08145	0.26244	0.21838	-0.13693
1	7	20070129	18	16	0.08145	0.34389	0.13693	-0.05548
1	7	20070101	15	17	0.06787	0.41176	0.11735	-0.04948
1	7	20070319	15	17	0.06787	0.47964	0.04948	0.01839
1	7	20070205	17	18	0.07692	0.55656	0.04348	0.03344
1	7	20070108	17	19	0.07692	0.63348	0.02844	0.04849
1	7	20070219	24	19	0.10860	0.74208	-0.04849	0.15708
1	7	20070312	16	20	0.07240	0.81448	-0.09521	0.16761
1	7	20070212	16	21	0.07240	0.88688	-0.10573	0.17813

1	7	20070115	17	23	0.07692	0.96380	-0.05438	0.13130
1	7	20070226	8	30	0.03620	1.00000	0.30183	-0.26563

With more mathematical rigor, the process is this: From the database, define sets of taxi out times under the constraints that the airport (JFK), the airline (AA), the day index, **day_ind**, and at the local time, **cat,** remain unchanged. Identify the set of the minima of the preceding sets. The thirteen minimum values for taxi out times constitute a probability distribution for the time reference value for that one departure slot, Sunday 7am, for that one airport and airline.

A Uniform Distribution of Time Reference Values?—Hypothesis Testing Using Kolmogorov-Smirnov Distance. Except for periodic changes in the published schedules, the same flights generally arrive and depart on the same days of the week at the same times of the day. Of course, schedules do change from time to time. However, data always records most recent changes. In other words, schedules may imply that for a given airport, on a given day at a given local time, there is a possibility that some flight characteristics would be the same, up to some probabilistic and generally small variations. We are only interested in the variation for one of these characteristics: time references (time_ref). Practically, how would we go about showing whether this characteristic would or would not obey that rule reflecting generally uniform taxiing times? If the time reference estimates of unimpeded times are generally the same for the same times and days, the observations of minimum taxi-out times that underlie those estimates should correspond to a uniform distribution.

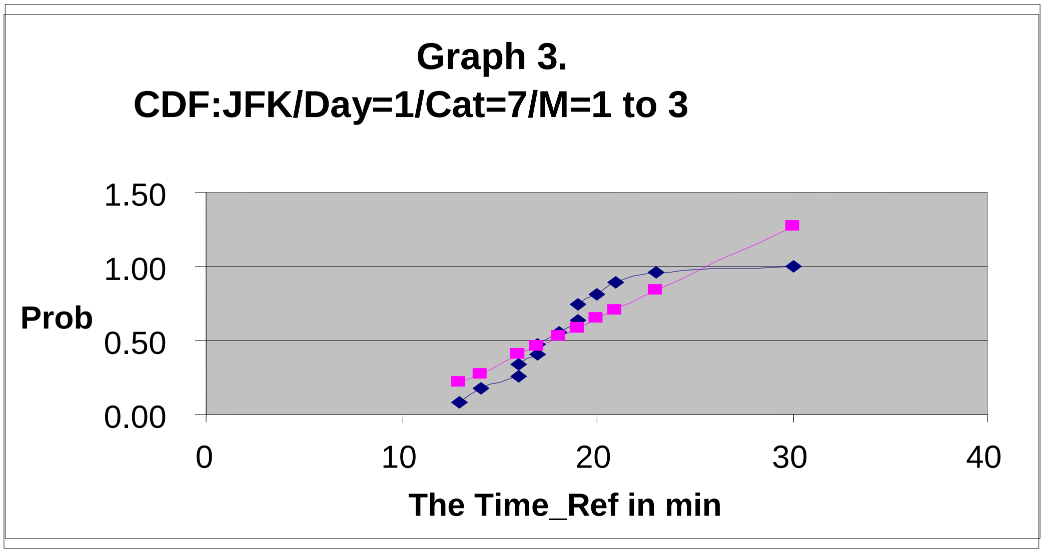
The thirteen observations of minimum times first are presented here as a probability density function, which makes the shape of distribution easier to visualize (see Graph 2). Consistent with the idea of a uniform probability distribution, those points should appear as a horizontal straight line. In this case the value all align somewhat horizontally, with an ordinate somewhere between 0.06 and 0.08. If the time reference values are indeed stochastic, one should look at points on Graph 2 as all lying on a horizontal line, within some probabilistic variation. The equation of the horizontal line (y=0.06188) will be easily identified with the next graph.



A second presentation of the time reference values in the form of a cumulative probability distribution (cdf) facilitates a statistical test of the shape of the distribution (see Graph 3). In that form, a uniform distribution of the underlying data would be indicated if the associated cumulative probability distribution approximates a straight line. Visually, that again appears to be the case. That approximation is supported by the comparison with the straight line fitted to the data. Specifically, we regress cdf on time_ref. All the usual regression statistics are good. In particular, the R^2 is close to 0.8. The equation of the regression line is

cdf = -.59064 + .06188***time_ref**

In Graph 3, points on the blue line are 13 empirical observations of time reference (only 11 distinct points show up, because of duplications of a couple values). The red line is the regression line that represents the theoretical cdf for a uniform distribution. Only the last point of the cdf (the upper tail) is clearly off that line, so only the segment of the red line below y = 1 counts.



This section develops a more rigorous statistical test to reject or accept the null hypothesis H_0 that the empirical distribution (time_ref values on the blue line) and the theoretical distributions (values on the straight red line, consistent with a uniform distribution) are identical. The null hypothesis to be tested is:

> **The probability of occurrence of time_ref is the same for all of its values. Or equivalently, the population distribution from which the data sample is drawn is a uniform distribution.**

In other words, it is expected that data that respect the dual conditions day_ind=1 (same day in the week) and cat=7 (same hour of the day) are drawn from a uniform distribution. It is, therefore, expected that the distribution derived from the actual data—what should be called **the empirical distribution**—should be close enough to the theoretical distribution. And to measure this closeness, we propose to use the Kolmogorov-Smirnov (KS) distance. In a formal way,

> Let $X_1, X_2, \ldots X_n$ be a sample from a (cumulative) distribution function (cdf) F and let F^*_n be the corresponding empirical distribution, the statistic
>
> $$D_n = \sup_x | F^*_n (x) - F(x)|$$
>
> is called the (two-sided) KS statistic (or distance).[3]

To test the hypothesis that H_0: $F^*_n(x) = F(x)$ for all x at a given confidence level α, the KS rejects H_0 if $D_n > D^\alpha_n$ where D^α_n is the critical value of the test for a given confidence level α and the number of observations n.

The theoretical distribution function F(x) is the appropriate segment of the regression line going through points from the sample. Very conveniently, the KS distance D_n turns out to be the absolute value of "Residual" from the regression. The 0.05 is the most often used level of significance. And the critical values of the test are given in the above reference Table 7, p.661. For the number of observations in our example, critical values are in the (.361, .454) range.

Since the largest absolute value in the column "Residuals" in Table 3 is 0.26 we conclude that at the 5% confidence level, one should accept the null hypothesis that the two distributions, empirical and theoretical, are the same.

Making Inferences with Uniform Distributions—Three Examples. Several numerical examples may help to explain the nature of the stochastic variation in unimpeded taxi-out times and demonstrate the value of the uniform distribution for conveying that information to policy makers and to the airline industry. Specifically, for a specific estimate of the unimpeded time, the examples demonstrate the calculation of the probabilities that the actual unimpeded time could be: (1)lower than that estimate, (2) greater than that estimate, (3) within a specific range of values related to the standard deviation for that distribution.

First some background on the properties of continuous uniform distributions. If **x** is a random variable that is uniformly distributed over the range [a, b], then its probability density function (**f(x)**), cumulative distribution function (**F(x)**), mean (**E(x)**), and variance (**V(x)**) and the standard deviation (**STD(x)**) are defined as:

$$f(x) = \begin{cases} 1/(b-a) & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} 0 & \text{if } x \le a \\ (x-a)/(b-a) & \text{if } a \le x \le b \\ 1 & \text{if } x \ge b \end{cases}$$

$E[x] = (a + b) / 2$ and

$V[x] = (b - a)^2 / 12$ and $STD = (b - a) / (2\sqrt{3})$

The numerical examples below will consider taxi-out times over a range where a = 20 and b = 80. That implies that the mean of the distribution is E = 50 and the standard deviation is STD = 17.32. The example begin with consideration of an estimate of the unimpeded time c = 30 minutes. What can be said about the actual unimpeded time (for a given airport, airline, month, day, and hour) given that point estimate and the information on the underlying distribution? In Question/Answer format:

Q1. What is the probability for the unimpeded taxi-out time to be equal to or smaller than c minutes?

Time_ref is $c \le 30$ minutes

A. Prob($X \le c$) = F(c) = (c – a)/(b – a)

Prob($X \le 30$) = (30 – 20)/(80 – 20) = 10/60 = **0.17**

Q2. What is the probability for the unimpeded time to be equal to or larger than c minutes?

A. Use answer to Q1 to get

Prob($X \ge c$) = 1- F(c) = 1 - (c – a)/(b – a)

Prob($X \ge 30$) = 1 - .17 = **0.83**

Note. The probability 1 – F(c) is often called **the survival probability**.

Q3. What is the probability for the unimpeded time to be in the within an interval of one-half standard deviation of the distribution

Time_ref is c = 30 minutes ± -.5*STD[x]

A. Note that 0.5*STD [x] = 0.5*17.32

Thus, one is looking for a probability that X is in the interval [30 – 8.66, 30 + 8.66] or [32.68, 67.32]. Thus,

$$\text{Prob}(X \in [21.32, 38.66]) = \frac{38.66-20}{80-20} - \frac{21.32-20}{80-20} = \frac{18.66-1.32}{60} = \frac{17.34}{60} = 0.29$$

CONCLUSIONS AND FUTURE RESEARCH.

This paper provides a new way of defining delays associated with airplanes taxiing out to runways and a method for estimating them. It starts from the current ASPM definition of delay. Then it proceeds in two steps. Its first step is a deterministic point of view. If the "unimpeded" taxi-out times means the "best" (or lowest) taxi-out time, then it should reflect the smallest physical time to travel from the gate to wheels-off under given weather and traffic conditions, subject to the allowed ground speed at each airport. Because direct information on that travel time is generally not available from the airports, this paper replaces it with the minimum of a chosen set of observed taxi-out times. The second step is to add some uncertainty to this deterministic view. This is to allow, as little as possible, some variation around the preceding minimum. And one way to achieve it is to turn to scheduled flights. The direct implication of this simple idea is to consider only flights from the same airports, for a given airline, departing the same day of the week at the same local time. The length of the period under consideration is not addressed in the paper. Intuitively, it should be correct as long as the scheduled flights remain unchanged. It turns out the idea applied to JFK airport, AA Airlines shows all the taxi out times for all the 7:00 o'clock Sunday flights during the first three months in 2007 can be described as following a uniform distribution.

More work in this area remains to be done. None of the traditional impacts of delays—such as transmission of delays, cost of delays, etc.—were considered in the study. The extreme values of the taxi-out times that are usually termed as tarmac times have never been considered. And with the focus here on unimpeded times, additional sources of stochastic variations in total taxi-out times are not considered. Left for future papers are also efforts to extend the new approach to the identification of distribution functions for total taxi-out times and the demonstration of how to infer likelihoods of total times being reached or exceeded for particular airports, airlines, or time slots.

APPENDIX A.

Table A: Dataset used to derive some PDFs and CDFs in the "Stochastic Approach" section.

dep	yyyymmdd	day _ind	taxi _out	mo index	cat	dep	yyyymmdd	day _ind	taxi _out	mo index	cat	dep	yyyymmdd	day _ind	taxi _ou t	mo index	cat	dep	yyyymmdd	day _ind	taxi _out	mo index	cat
JFK	20070101	1	18	1	7	JFK	20070108	1	48	1	7	JFK	20070212	1	21	2	7	JFK	20070326	1	30	3	7
JFK	20070108	1	29	1	7	JFK	20070129	1	43	1	7	JFK	20070212	1	24	2	7	JFK	20070305	1	26	3	7
JFK	20070115	1	38	1	7	JFK	20070101	1	38	1	7	JFK	20070212	1	31	2	7	JFK	20070312	1	25	3	7
JFK	20070122	1	22	1	7	JFK	20070108	1	29	1	7	JFK	20070219	1	25	2	7	JFK	20070326	1	39	3	7
JFK	20070129	1	35	1	7	JFK	20070108	1	29	1	7	JFK	20070219	1	28	2	7	JFK	20070305	1	22	3	7
JFK	20070108	1	19	1	7	JFK	20070115	1	43	1	7	JFK	20070219	1	28	2	7	JFK	20070305	1	15	3	7
JFK	20070115	1	47	1	7	JFK	20070122	1	21	1	7	JFK	20070219	1	22	2	7	JFK	20070305	1	31	3	7
JFK	20070122	1	18	1	7	JFK	20070129	1	45	1	7	JFK	20070219	1	36	2	7	JFK	20070305	1	28	3	7
JFK	20070108	1	41	1	7	JFK	20070101	1	24	1	7	JFK	20070219	1	24	2	7	JFK	20070305	1	25	3	7
JFK	20070101	1	17	1	7	JFK	20070108	1	46	1	7	JFK	20070219	1	26	2	7	JFK	20070305	1	17	3	7
JFK	20070108	1	33	1	7	JFK	20070122	1	16	1	7	JFK	20070226	1	35	2	7	JFK	20070305	1	31	3	7
JFK	20070115	1	36	1	7	JFK	20070129	1	20	1	7	JFK	20070205	1	30	2	7	JFK	20070312	1	46	3	7
JFK	20070122	1	45	1	7	JFK	20070122	1	19	1	7	JFK	20070212	1	28	2	7	JFK	20070312	1	43	3	7
JFK	20070129	1	33	1	7	JFK	20070129	1	38	1	7	JFK	20070226	1	74	2	7	JFK	20070312	1	20	3	7
JFK	20070101	1	30	1	7	JFK	20070108	1	28	1	7	JFK	20070205	1	20	2	7	JFK	20070312	1	24	3	7
JFK	20070122	1	24	1	7	JFK	20070108	1	37	1	7	JFK	20070205	1	29	2	7	JFK	20070312	1	37	3	7
JFK	20070129	1	41	1	7	JFK	20070108	1	23	1	7	JFK	20070212	1	27	2	7	JFK	20070319	1	45	3	7
JFK	20070101	1	20	1	7	JFK	20070115	1	31	1	7	JFK	20070219	1	27	2	7	JFK	20070319	1	39	3	7
JFK	20070101	1	20	1	7	JFK	20070115	1	36	1	7	JFK	20070219	1	38	2	7	JFK	20070319	1	17	3	7
JFK	20070108	1	39	1	7	JFK	20070129	1	16	1	7	JFK	20070226	1	75	2	7	JFK	20070319	1	51	3	7
JFK	20070115	1	38	1	7	JFK	20070129	1	26	1	7	JFK	20070205	1	20	2	7	JFK	20070319	1	21	3	7
JFK	20070122	1	18	1	7	JFK	20070129	1	26	1	7	JFK	20070212	1	31	2	7	JFK	20070319	1	17	3	7
JFK	20070129	1	46	1	7	JFK	20070101	1	18	1	7	JFK	20070219	1	21	2	7	JFK	20070319	1	24	3	7
JFK	20070101	1	25	1	7	JFK	20070108	1	37	1	7	JFK	20070212	1	21	2	7	JFK	20070326	1	24	3	7
JFK	20070101	1	23	1	7	JFK	20070115	1	27	1	7	JFK	20070219	1	29	2	7	JFK	20070326	1	35	3	7
JFK	20070101	1	24	1	7	JFK	20070122	1	25	1	7	JFK	20070205	1	36	2	7	JFK	20070326	1	14	3	7
JFK	20070101	1	30	1	7	JFK	20070129	1	30	1	7	JFK	20070212	1	30	2	7	JFK	20070326	1	22	3	7
JFK	20070101	1	25	1	7	JFK	20070115	1	39	1	7	JFK	20070219	1	30	2	7	JFK	20070326	1	19	3	7
JFK	20070101	1	21	1	7	JFK	20070122	1	18	1	7	JFK	20070219	1	47	2	7	JFK	20070326	1	22	3	7
JFK	20070108	1	25	1	7	JFK	20070129	1	22	1	7	JFK	20070219	1	28	2	7	JFK	20070326	1	25	3	7
JFK	20070108	1	38	1	7	JFK	20070212	1	31	2	7	JFK	20070219	1	29	2	7	JFK	20070326	1	21	3	7
JFK	20070108	1	38	1	7	JFK	20070219	1	21	2	7	JFK	20070205	1	18	2	7	JFK	20070326	1	30	3	7
JFK	20070108	1	24	1	7	JFK	20070226	1	55	2	7	JFK	20070219	1	27	2	7	JFK	20070326	1	42	3	7
JFK	20070115	1	29	1	7	JFK	20070205	1	32	2	7	JFK	20070226	1	38	2	7	JFK	20070326	1	34	3	7
JFK	20070115	1	34	1	7	JFK	20070212	1	42	2	7	JFK	20070205	1	46	2	7	JFK	20070305	1	30	3	7
JFK	20070115	1	36	1	7	JFK	20070219	1	37	2	7	JFK	20070212	1	28	2	7	JFK	20070312	1	33	3	7
JFK	20070115	1	23	1	7	JFK	20070205	1	58	2	7	JFK	20070219	1	26	2	7	JFK	20070305	1	24	3	7
JFK	20070115	1	26	1	7	JFK	20070212	1	32	2	7	JFK	20070219	1	19	2	7	JFK	20070312	1	33	3	7
JFK	20070115	1	39	1	7	JFK	20070219	1	26	2	7	JFK	20070226	1	64	2	7	JFK	20070319	1	37	3	7
JFK	20070115	1	35	1	7	JFK	20070226	1	30	2	7	JFK	20070212	1	29	2	7	JFK	20070326	1	31	3	7
JFK	20070122	1	22	1	7	JFK	20070212	1	31	2	7	JFK	20070305	1	20	3	7	JFK	20070305	1	13	3	7
JFK	20070122	1	25	1	7	JFK	20070219	1	19	2	7	JFK	20070312	1	27	3	7	JFK	20070312	1	32	3	7
JFK	20070122	1	18	1	7	JFK	20070212	1	27	2	7	JFK	20070319	1	23	3	7	JFK	20070326	1	26	3	7
JFK	20070122	1	20	1	7	JFK	20070219	1	28	2	7	JFK	20070326	1	15	3	7	JFK	20070305	1	33	3	7
JFK	20070122	1	28	1	7	JFK	20070226	1	48	2	7	JFK	20070305	1	30	3	7	JFK	20070312	1	21	3	7
JFK	20070122	1	27	1	7	JFK	20070219	1	43	2	7	JFK	20070312	1	24	3	7	JFK	20070319	1	38	3	7
JFK	20070122	1	19	1	7	JFK	20070205	1	24	2	7	JFK	20070326	1	31	3	7	JFK	20070326	1	38	3	7
JFK	20070129	1	43	1	7	JFK	20070205	1	32	2	7	JFK	20070305	1	24	3	7	JFK	20070305	1	20	3	7
JFK	20070129	1	17	1	7	JFK	20070205	1	36	2	7	JFK	20070312	1	30	3	7	JFK	20070312	1	23	3	7
JFK	20070129	1	45	1	7	JFK	20070205	1	28	2	7	JFK	20070319	1	22	3	7	JFK	20070326	1	24	3	7
JFK	20070129	1	25	1	7	JFK	20070205	1	22	2	7	JFK	20070326	1	27	3	7	JFK	20070305	1	22	3	7
JFK	20070129	1	26	1	7	JFK	20070205	1	23	2	7	JFK	20070319	1	34	3	7	JFK	20070312	1	22	3	7
JFK	20070101	1	26	1	7	JFK	20070205	1	33	2	7	JFK	20070305	1	19	3	7	JFK	20070319	1	26	3	7
JFK	20070115	1	42	1	7	JFK	20070205	1	26	2	7	JFK	20070312	1	30	3	7	JFK	20070326	1	29	3	7
JFK	20070122	1	20	1	7	JFK	20070212	1	26	2	7	JFK	20070319	1	21	3	7	JFK	20070326	1	30	3	7

One observation is left out: JFK 20070326 1 30 3 7

APPENDIX B.

This appendix presents summary tables to extend the picture of the JFK airport to other times, days at that airport and at two other airports in the New York Metropolitan Area, EWR and LGA. The extended results for will be considered conclusive as long as

- the usual regression statistics are "good" and
- the requirements of the K_S distance are satisfied.

The summary entries in the next table are either the R^2 of the regression line and the maximum of the absolute values of the K-S distance. The bolded values are those larger than the critical value 0.361, which requires that the null hypothesis that the empirical and theoretical distribution are the same should be rejected.

Table 6. Time_Ref on Selective Days and Local Hours

		Sundays			**Tuesdays**			**Fridays**		
		7:00	12:00	17:00	7:00	12:00	17:00	7:00	12:00	17:00
JFK	Rsquare	0.7943	0.8224	0.9271	0.1750	0.3094	0.3390	0.9571	0.6584	0.9101
	|Max(K-S)|	0.2656	0.2119	0.1458	**0.4671**	**0.4133**	**0.3849**	0.1259	0.3126	0.2042
EWR	Rsquare	0.9254	0.8192	0.9208	0.5147	0.2970	0.8325	0.3685	0.8382	0.8978
	|Max(K-S)|	0.1627	0.2267	0.1567	0.3512	**0.4129**	0.1978	**0.3664**	0.2296	0.1831
LGA	Rsquare	0.8213	0.5272	0.7568	0.5953	0.7497	0.9192	0.3801	0.3557	0.8766
	|(Max(K-S)|	0.2457	0.3041	0.2678	0.2948	0.2998	0.1435	**0.3860**	**0.3942**	0.2019

The clear conclusion is that the present methodology does not allow any generalization from the results. For some time slots the hypothesis of a uniform distribution can be accepted, at others it is rejected. That suggests that specific inferences about uncertainty for some airports, airlines, or time slots will not be as valid as for others. The results appear to argue for a case-by-case approach and, at the minimum, for more analysis of the differences across cases. For example, for certain time references, hypothesizing other forms of distribution functions may yield more useful results.

NOTES.

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[1] John F. Shortle et al., "Analysis of Gate-Hold Delays at the OEP-35 Airports," Center for Air Transportation Systems Research, 2009 paper submitted to the 8th USA/Europe ATM R&D Seminar, Napa Valley, CA, available at catsr.ite.gmu.edu/pubs/ATM2009_GateAnalysis.pdf .

[2] Federal Aviation Agency, AO-130, "Total Cost for Air Carrier Delay For the Years 1987-1994," December 1995, available at www.faa.gov/library/reports/**delay**_analysis/media/DCOS1995.doc .

[3] Rohatgi, V.K., *An Introduction to Probability Theory and Mathematical Statistics*, John Wiley & Sons, p.539, 1976.